Mathematical Model

Bungee-Jump

Newton's Second Law : $F = ma$

$$F = F_D + F_U$$

$$ma = mg - C_d \cdot v^2 \qquad \left(\begin{array}{l} g \fallingdotseq 9.81 \ m/s^2 \quad \text{gravity} \\ C_d : \text{drag coefficient } (kg/m) \end{array} \right.$$

$$a = g - \frac{C_d}{m} \cdot v^2$$

$$\boxed{\frac{dv}{dt} = g - \underbrace{\frac{C_d}{m} \cdot v^2}_{\text{Air resistance}}} \quad (A) \quad \text{differential equation}$$

(1) Algebraic Solution:

$$v(t) = \sqrt{\frac{g \cdot m}{C_d}} \tanh\left(\sqrt{\frac{g \cdot C_d}{m}} \, t\right) \quad \text{where } \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Ex.1.1 $C_d = 0.25 \ kg/m$

$m = 68.1 \ kg$

$$v(t) = \sqrt{\frac{(9.81)(68.1)}{0.25}} \cdot \tanh\left(\sqrt{\frac{(9.81)(0.25)}{68.1}} \, t\right)$$

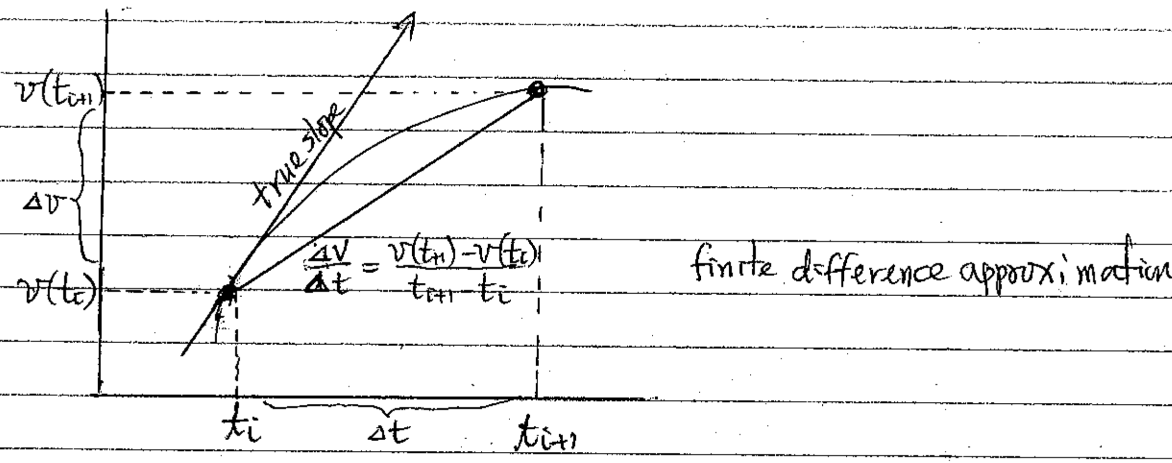
$$= 51.693752 \cdot \tanh(0.1897714 \, t)$$

t (sec)	v (m/s)
0	0
2	18.7292
4	33.11
⋮	⋮

(2) Numerical Solution

$$\frac{dv}{dt} \simeq \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} \qquad (B)$$

Note $\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$



$$\frac{dv}{dt} = g - \frac{C_d}{m} v^2 \qquad (A)$$

$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{C_d}{m} v(t_i)^2$$

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{C_d}{m} v(t_i)^2\right](t_{i+1} - t_i)$$

$t_0 = 0, \; v = 0.$

$t_1 = 2, \quad v = 0 + \left[9.81 - \frac{0.25}{68.1} \cdot (0)^2\right] \times 2 = 19.62$ m/s

$t_2 = 4 \quad v = 19.62 + \left[9.81 - \frac{0.25}{68.1}(19.62)^2\right] \times 2 = 36.4137$ m/s

$\vdots$

t, s	Theorical v, m/s	Numerical v, m/s
0	0	0
2	18.7292	19.6200
4	33.1118	36.4137
6	42.0762	46.2983
8	46.9575	50.1802
10	49.4214	51.3123
12	50.6175	51.6008
∞	51.6938	51.6938

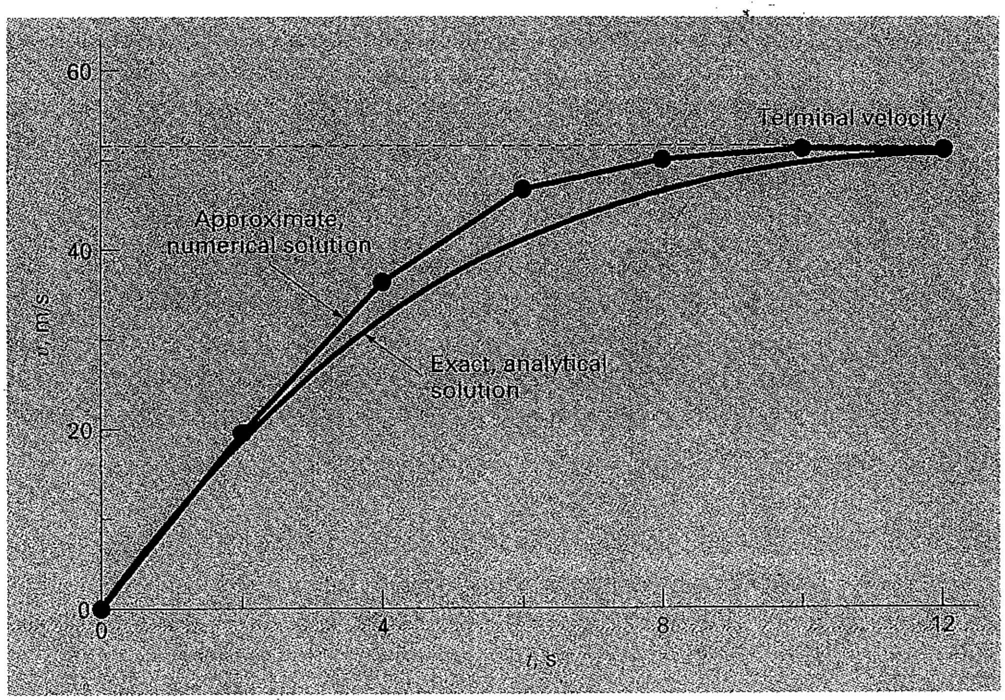


FIGURE 1.4
Comparison of the numerical and analytical solutions for the bungee jumper problem.

How to reduce the gap ?