#1 2.30(a)

Prove $\{0^n 1^n 0^n 1^n \mid n \geq 0\} = A$ is not context-free

Consider the string $s = 0^p 1^p 0^p 1^p = uvxyz$

$|s| \geq p$

Because $|vy| \geq 0$ and $|vxy| \leq p$, the following cases must be considered

1) ~~z~~ vy contains only 0s or only 1s

~~xxx~~ if $|vy| = n \geq 1$

then uv^2xy^2z is either

(a) $0^{p+n} 1^p 0^p 1^p \notin A$

(b) $0^p 1^{p+n} 0^p 1^p \notin A$

(c) $0^p 1^p 0^{p+n} 1^p \notin A$

(d) $0^p 1^p 0^p 1^{p+n} \notin A$

2) vy contains both 0s and 1s (in either order)

let the # of 0s be m

and the # of 1s be n, $m \geq 1, n \geq 1$

#1 uv^0xy^0z is either

(a) $0^{p-m}1^{p-n}0^p1^p \notin A$

(b) $0^p1^{p-n}0^{p-m}1^p \notin A$

(c) $0^p1^p0^{p-m}1^{p-n} \notin A$

A contradiction is reached in all cases.

Therefore, A is not context-free.

#2 Prove $Q = \{a^k \mid k \text{ is prime}\}$ is not Context-free.

Consider string $s = a^p$ where p is prime

Note: $uvxyz = a^p$, $|uvxyz| = p$, $|vy| = n \geq 1$

Pump string s $p+1$ times:

$$|uv^{p+1}xy^{p+1}z| = |uvxyz| + |v^p y^p|$$

$$= p + p \cdot n = p(1+n)$$

However, $p(1+n)$ is not prime

Since a contradiction is reached,
language Q is not context-free.

#3 Original grammar $G = (V, \Sigma, R, S)$

Construct G_0 as follows, assuming $S_0 \notin V$,

$$G_0 = (V_0, \Sigma, R_0, S_0)$$

$$V_0 = V \cup S_0$$

$$R_0 = R \cup S_0 \to S_0 S \mid \epsilon$$

#4 Many answers are possible, e.g.

DCFLs yield efficient parser implementations for a compiler.

#5

laaabbb $\longrightarrow$

laaSbb $\longrightarrow$

laSb $\longrightarrow$

lS $\longrightarrow$

R