2.30 (a)

Prove $\exists 0^1 0^n 1^p \mid n \geq 0 \Rightarrow A$ is not context-free

Consider the string $s^* = 0^p 1^p 0^p 1^p \Rightarrow uvxvyz$

$|s| \geq p$

Because $|vy| \geq 0$ and $|vxy| \leq p$, the following cases must be considered

1) $vy$ contains only 0s or only 1s

if $|vy| = n \geq 1$

then $uv^2xy^2z$ is either

(a) $0^{p+n} 1^p 0^p 1^p \notin A$

(b) $0^p 1^{p+n} 0^p 1^p \notin A$

(c) $0^p 1^p 0^{p+n} 1^p \notin A$

(d) $0^p 1^p 0^p 1^n \notin A$

2) $vy$ contains both 0s and 1s (in either order)

let the # of 0s be $m$

and the # of 1s be $n$, $m \geq 1$, $n \geq 1$
\#1  \( uv^0xy^0z \) is either

(a)  \( O^p \) \( \rho^p \) \( O^p \) \( \rho \) \( O^p \) \( \rho \) \& A

(b)  \( O^p \) \( \rho^p \) \( O^p \) \( \rho^m \) \( \rho \) \& A

(c)  \( O^p \) \( \rho \) \( O^p \) \( \rho^m \) \( \rho \) \& A

A contradiction is reached in all cases.

Therefore, \( A \) is not context-free.
#2  Prove \( Q = \{a^k \mid k \text{ is prime} \} \) is not context-free.

Consider string \( s = a^p \) where \( p \) is prime.

Note: \( uvxyz = a^p \), \( |uvxyz| = p \), \( |uy| = n \geq 1 \)

Damp string \( s \) \( p+1 \) times:

\[
|uv^{p+1}xy^{p+1}z| = |uvxyz| + \underbrace{\cdots}_{p+1} + |uv^{p+1}y^{p+1}| = p + pn = p(1+n)
\]

However, \( p(1+n) \) is not prime.

Since a contradiction is reached,
\( Q \) is not context-free.
#3
Original grammar \( G = (V, \Sigma, R, S) \)

Construct \( G_0 \) as follows, assuming \( S_0 \notin V \),

\[
G_0 = (V_0, \Sigma, R_0, S_0)
\]

\[
V_0 = V \cup S_0
\]

\[
R_0 = R \cup S_0 \rightarrow S_0 S \mid \varepsilon
\]

#4
Many answers are possible, e.g.,

NCFLs yield efficient parser implementations for a compiler.
#5

\[
\frac{aaabbb}{aabb} \to \\
\frac{aaSbb}{aSb} \to \\
\frac{S}{\mathcal{R}} \to
\]