1. A. Sweep the tape input from left to right if there are no more Os or Is, REJECT. B. Sweep the tape input from right to left mark two Os with an X and one 1 if this was possible, go to A.
   Otherwise ACCEPT.

2. Problem 3.15 (d)
   For any decidable language L, let M be the TM that decides it.
   Construct TM M' to decide the complement of L as follows.
   "On input w:
   1. Run M on w. If M accepts, then REJECT.
      If M rejects, then ACCEPT."
3. Problem 4.2

\[ EQ_{\text{DFA,REX}} = \{ \langle A, R \rangle \mid A \text{ is a DFA, and } R \text{ is a regular expression and } L(A) = L(R) \} \]

\( EQ_{\text{DFA,REX}} \) is decidable.

\( \exists = \) "On input \( \langle A, R \rangle \) where \( A \) is a DFA and \( R \) is a regular expression:

1. Convert regular expression \( R \) to an equivalent NFA \( N \) using the procedure in Theorem 1.54.

2. Convert NFA \( N \) to an equivalent DFA \( B \) using the procedure in Theorem 1.39.

3. Run \( \text{TM} \) from Theorem 4.5 on \( \langle A, B, \rangle \).

4. If \( \text{TM} \) accepts, ACCEPT.
   If \( \text{TM} \) rejects, REJECT."
4. Problem 4.3

\[ \text{All}_{\text{DFA}} = \{ A \mid A \text{ is a DFA and } L(A) = \Sigma^* \} \text{ is decidable} \]

Construction:

"On input \(<A>\) where \(A\) is a DFA

1. Construct \(\text{DFA } B\) as follows

\[ Q = \{ q_1, q_2 \} \]

\[ \Sigma = \{ \Sigma \} \]

\[ S(q_1, a) = q_2 \quad \forall a \in \Sigma \]

\[ q_{\text{start}} = q_1 \]

\[ q_{\text{accept}} = q_2 \]

Always accepts

2. Run TM \(F\) from Theorem 4.5 on \(<A, B>\)

3. If \(F\) accepts, ACCEPT.

   If \(F\) rejects, REJECT."

4.
5. **Problem 4.7**

Suppose \( B \) is countable. We could then create a table such as

<table>
<thead>
<tr>
<th>( N )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 1 1 0 1 1 1 0 ...</td>
</tr>
<tr>
<td>2</td>
<td>1 1 0 0 0 0 1 1 ...</td>
</tr>
<tr>
<td>3</td>
<td>1 1 1 0 1 0 1 1 ...</td>
</tr>
<tr>
<td>4</td>
<td>0 0 0 0 0 1 0 1 ...</td>
</tr>
</tbody>
</table>

However, we can always find a number that is not in the table using diagonalization, e.g., 1 0 0 0 ...

Contradiction! Therefore \( B \) is uncountable.
6. Problem 4.8 (Many answers are possible.)

<table>
<thead>
<tr>
<th>N</th>
<th>i</th>
<th>j</th>
<th>k</th>
</tr>
</thead>
</table>
| 1  | 1   | 1   | 1   | \( i + j + k = 3 \)  
| 2  | 1   | 1   | 2   | \( i + j + k = 4 \)  
| 3  | 1   | 2   | 1   | \( i + j + k = 4 \)  
| 4  | 2   | 1   | 1   | \( i + j + k = 4 \)  
| 5  | 1   | 1   | 3   | \( i + j + k = 5 \)  
| 6  | 1   | 2   | 2   | \( i + j + k = 5 \)  
| 7  | 1   | 3   | 1   | \( i + j + k = 5 \)  
| 8  | 2   | 1   | 2   | \( i + j + k = 5 \)  
| 9  | 2   | 2   | 1   | \( i + j + k = 5 \)  
| 10 | 3   | 1   | 1   | \( i + j + k = 5 \)  

Notes: (go in ascending order over i, j, k)