1. A. Sweep the tape input from left to right

   if there are no more 0s or 1s, REJECT

   B. Sweep the tape input from right to left

   mark two 0s with an X and one 1 with an X

   if this was possible, go to A

   otherwise ACCEPT

2. Problem 3.15 (d)

   For any ~~Turing-recognizable~~ decidable language L,

   let M be the TM that ~~recognizes~~ decides it.

   Construct TM M' to ~~recognize~~ decide the complement of L as follows:

   "On input w:

   1. Run M on w. If M accepts, then REJECT.

      If M rejects, then ACCEPT."

3. Problem 4.2

$EQ_{DFA,REX} = \{ \langle A, R \rangle \mid A$ is a DFA, ~~and~~ R is a regular expression and $L(A) = L(R) \}$

$EQ_{DFA,REX}$ is decidable

Construction

Z = "On input $\langle A, R \rangle$ where A is a DFA and R is a regular expression

1. Convert regular expression R to an equivalent NFA N using the procedure in Theorem 1.54
2. Convert NFA N to an equivalent DFA B using the procedure in Theorem 1.39
3. Run TM F from Theorem 4.5 on $\langle A, B \rangle$
4. If F accepts, ACCEPT.
   If F rejects, REJECT."

4. Problem 4.3

$ALL_{DFA} = \{A \mid A$ is a DFA and $L(A) = \Sigma^*\}$ is decidable

Construction:

"On input $\langle A \rangle$ where A is a DFA

1. Construct DFA B as follows

$$\left.\begin{aligned} &Q = \{q_1\} \\ &\Sigma \\ &\delta(q_1, a) = q_1 \quad \forall a \in \Sigma \\ &q_{start} = q_1 \\ &q_{accept} = q_1 \end{aligned}\right\} \text{always accepts}$$

2. Run TM F from Theorem 4.5 on $\langle A, B \rangle$

3. If F accepts, ACCEPT.

If F rejects, REJECT."

5. Problem 4.7

Suppose B is countable. We could then create a table such as

$\mathbb{N}$	B							
1	0	1	1	0	1	1	0	...
2	1	1	0	0	0	1	1	...
3	1	1	1	0	1	0	1	...
4	0	0	0	1	1	0	1	...
⋮								

However, we can always find a number that is not in the table using diagonalization,

e.g. 1 0 0 0 ...

Contradiction! Therefore B is uncountable.

6. Problem 4.8 (Many answers are possible.)

N	i	j	k	
1	1	1	1	$i+j+k=3$
2	1	1	2	
3	1	2	1	$i+j+k=4$
4	2	1	1	
5	1	1	3	
6	1	2	2	$i+j+k=5$
7	1	3	1	(go in ascending order over i, j, k)
8	2	1	2	
9	2	2	1	
10	3	1	1	

⋮ ⋮