CSCI 338, First Midterm – February 10th, 2016

Name

Sample Solutions Covered in Class

Question One. 15 points. Draw a state diagram for a 3-state NFA that recognizes $1^*(0011^*)^*$.

Question Two. 15 points. Consider the DFA below.

(a) What is $Q$?

$Q = \{ s_0, s_1, s_2 \}$

(b) What is $\Sigma$?

$\Sigma = \{ 0, 1 \}$

(c) What is $\delta$?

$\begin{array}{c|cc} \delta & 0 & 1 \\ \hline s_0 & s_1 & s_0 \\ s_1 & s_1 & s_2 \\ s_2 & s_1 & s_2 \end{array}$

(d) What is $q_0$?

$\delta(0, s_0) = s_1$

(e) What is $F$?

$F = \{ s_0, s_1, s_2 \}$
Question Three. 20 points. Consider the GNFA below. Show the resulting GNFA when state $q_1$ is ripped out.

\[
\quad \rightarrow S \rightarrow q_2 \rightarrow a \\
\quad \quad \varepsilon \cup \ \ (auv)^*a \\
\quad \quad b^* \varepsilon \quad b \ (auv)^*a \\
\quad \quad \text{answer} \quad ab \cup b \ (auv)^*a \\
\]

Question Four. 10 points. Consider converting an NFA that has 5 states to a DFA.

(a) What is the maximum number of states that the DFA could have?

2

(b) What is the minimum number of states that the DFA could have?

→ 0 1
Question Five. 15 points. Use the pumping lemma to show that language $A = \{w \mid \text{1}^m\text{0}^n\text{1}^m\}^n$ where $m, n \geq 1$. 

$S = 1^p01^p0^p \in A \quad |S| \geq p$

$S = xyz \quad |xy| \leq p \quad |y| > 0 \quad y = 1^i$

$xyyz = 1^{p+i}0^p0^i1^{2p}$

$x = \varepsilon$

$y = 1^p$

$z = 0^p0^i1^{2p}$

$xyz \notin A$

Question Six. 10 points. Write a regular expression that captures $A = \{w \mid w \text{ contains exactly three } a\text{'s} \}$ over $\Sigma = \{a, b\}$.

$\ast b a b a b a b \ast$
Question Seven. 15 points. Consider NFAs NFA₁ and NFA₂. NFA₁ has $n_1$ states (including $f_1$ accept states) and $t_1$ transitions. NFA₂ has $n_2$ states (including $f_2$ accept states) and $t_2$ transitions.

(a) Consider NFA₃, that is constructed to recognize NFA₁ $\cup$ NFA₂ (union) using the construction technique in the book. How many total states will NFA₃ have?

$$n_1 + n_2 + 1$$

(b) How many transitions will NFA₃ have?

$$t_1 + t_2 + 2$$

(c) Consider NFA₄, that is constructed to recognize NFA₁ $\cdot$ NFA₂ (concatenation) using the construction technique in the book. How many accept states will NFA₄ have?

$$f_2$$

(d) How many transitions will NFA₄ have?

$$t_1 + t_2 + f_1$$