Pumping Lemma

Consider $\{s \mid s = ww^R w\}$ $\Sigma = \{0, 1\}$

$0^p 1 1 0^p 0^p 1$ Can be pumped

$u = 0^{p-3}$
$v = 0^3$
$x = 11$
$y = 0^6$
$z = 0^{2p-6} 1$

$uv^i xy^i z = 0^{p-3}\, 0^{3i}\, 11\, 0^{6i}\, 0^{2p-6} 1$

$= 0^{p-3+3i}\, 11 0^{p-3+3i}\, 0^{p-3+3i} 1$

Consider $10^p 110^p 110^p 1$ which cannot be pumped!

Suppose xyz vxy $\exists$ v or y contain 1st 1.
$vxy = 10^n$ $0 \le n < p$ since $|vxy| < p$

$uv^0 xy^0 z = 0^{p-n} 110^p 110^p 1$ so w_i ends in 01
$w = 0^{p-n} 1$ not acceptable
$w = 0^{p-n} 11 0^p 1$ not acceptable
thus v and y doesn't include 1st 1
For similar reasons, v and y doesn't include last 1.
$uv^i xy^i z$ begins and ends in 1 so w_i begins in 1.
$w_i w_i^R w_i$ has at least 6 1s. So v and y cannot contain 1 otherwise $uv^0 xy^0 z$ doesn't have enough 1s. v and y must be just zeroes. Suppose v or y include zeroes from the middle 0^p. Then vxy can include zeroes from the left 0^p or the right 0^p not both since $|vxy| < p$. In which case $uv^0 xy^0 z$ will take the form $10^n 110^n 110^s 1$ where only one of the numbers $n, n,$ and s as $10^m 110^n 110^s 1$ cannot be devided into $ww^R w$ and so is not an element of the language. For similar reasons v and y cannot include zeroes from the left 0^p or the Right 0^p

1

$\{S \mid S = ww^R w,\ w \in \Sigma^*\}$

$0^p 1 1 0^p 0^p 1$

$U V X Y Z$

$U = 0^{p-3}$

$V = 0^3$

$X = 11$

$Y = 0^6$

$Z = 0^{2p-6} 1$

$U V^i X Y^i Z = 0^{p-3} 0^{3i} 11 0^{6i} 0^{2p-6} 1$

$0^p 1^{2p} 0^{2p} 1^p \qquad 0^{p-3+3i} 1$

$0^p 1^p 1^p 0^p \quad 0^p 1^p$

$U = 0^p$

$V = 1^{p_L}$

$W = 1^{p_R}$

$X = 0^{p_L}$

$Y = w$

$10^p 110^p 110^p 1$

$V = \varepsilon$

$V = 1$

$X = \varepsilon$

$Y = \varepsilon$

$Z = 0^p 110^p 110^p 1$

$\cup V^0 X Y^0 Z = 0^p 110^p 110^p 1$

$10^p 110^p 110^p 1$

$U = \varepsilon$

$U V^0 X Y^0 Z = 0^{p-n} 110^p 110^p 1$

$|X Y| \leq p \qquad V X Y = 10^n$

$\cup V^i X Y^i Z = 1$ ——— 1

$\cup V^0 X Y^0 Z$ does not have enough 1s if V or Y have a 1

$A \to aAa \mid Bb$

$B \to b \mid AB \mid \varepsilon$

↓

$S \to A$

$A \to aAa \mid Bb$

$B \to b \mid AB \mid \varepsilon$

Create new Start State

↓

$S \to A$

$A \to aAa \mid Bb \mid b$

$B \to b \mid AB \mid A$

remove ε-rule

↓

$S \to aAa \mid Bb \mid b$

$A \to aAa \mid Bb \mid b$

$B \to b \mid AB \mid aAa \mid Bb \mid b$

$A \to a$

$A \to BC$

remove unit rules

(4)

$S \to A_1 A_2 \mid BB_1 \mid b$

$A_1 \to a$

$A_2 \to A_3 A_3$

$A_3 \to a$

final step

$B_1 \to b$

$A \to A_1 A_2 \mid BB_1 \mid b$

$B \to b \mid AB \mid A_1 A_2 \mid BB_1$

↓

$S \to A_1 A_2 \mid BB_1 \mid b$

$A_1 \to a$

$A_2 \to AA_1$

$B_1 \to b$

$A \to A_1 A_2 \mid BB_1 \mid b$

$B \to b \mid AB \mid A_1 A_2 \mid BB_1$