CSCI 338, Second Midterm – March 9th, 2016

Name: Sample Solution

Question One. 25 points. Consider language $A, A = \{a^{2n}b^{3n} | n \geq 0\}$.

(a) 4 points. Write a context-free grammar (CFG) for $A$ using as few rules as possible.

$$S \rightarrow anSbbb \mid \varepsilon$$

(b) 4 points. What is $V$ in your CFG?

$$\{S\}$$

(c) 4 points. What is $\Sigma$ in your CFG?

$$\{a, b\}$$

(d) 4 points. What is $R$ in your CFG?

$$\{S \rightarrow anSbbb, S \rightarrow \varepsilon\}$$

(e) 4 points. What is $S$ in your CFG?

$$S$$

(f) 5 points. Rewrite your CFG in Chomsky-Normal Form.

$$S_{0} \rightarrow S + \varepsilon$$

$$S \rightarrow \alpha a S b b b$$

$$S \rightarrow A C L \varepsilon$$

$$C \rightarrow A D$$

$$A \rightarrow a$$

$$B \rightarrow b$$

$$S_{0} \rightarrow S E$$

$$E \rightarrow B F$$

$$F \rightarrow B B$$
Question Two. 25 points. Consider language A from Question One.

(a) 10 points. Draw a 7-state PDA that recognizes A.

\[ \text{State Transitions} \]

\[ \begin{array}{c}
\text{Start State} \\
\rightarrow (q_0, a, \varepsilon) \\
\text{a,} \varepsilon \rightarrow \text{a,} \varepsilon \\
\varepsilon, \varepsilon \rightarrow \varepsilon \\
\end{array} \]

\[ \begin{array}{c}
\text{ACCEPT State} \\
\rightarrow (q_2, b, \varepsilon) \\
b, \varepsilon \rightarrow \varepsilon \\
\varepsilon, \varepsilon \rightarrow \varepsilon \\
\end{array} \]

(b) 3 points. What is Q in your PDA?

\[ \{ q_0, q_1, q_2, q_3, q_4, q_5, q_{\text{ACCEPT}} \} \]

(c) 3 points. What is \( \Sigma \) in your PDA?

\[ \{ a, b \} \]

(d) 3 points. What is \( \Gamma \) in your PDA?

\[ \{ a, b \} \]

(e) 3 points. What is \( q_0 \) in your PDA?

\[ q_0 \]

(f) 3 points. What is \( F \) in your PDA?

\[ \{ q_{\text{ACCEPT}} \} \]
Question Three. 25 points. Prove that language $B = \{a^{2^n} \mid n \geq 0\}$ is not a context-free language.

Consider the string $a^{2^p}$

$uvxyz = a^{2^p}$  \hspace{1cm} |uvxyz| = 2^p$

Since $|vxy| \leq p$ and $|vy| \geq 1$,

\[ |vy| = k \]

Pump twice

$|uv^nxyz| = |uvxyz| + |vy|$

\[ = 2^p + k < 2^p + 2^p \]

because $p < 2^p$

$\therefore uv^2xyz \notin B$
Question Four. 25 points.

(a) 5 points. True or False. Adding a second stack to a PDA increases the set of languages that can be recognized. If True, provide a language that can now be recognized. If False, briefly explain why the two are equivalent.

True. Consider $\exists a^n b^n c^n | n \geq 0$.

(b) 5 points. What is the key difference between a finite state automaton and a pushdown automaton?

PDA has a stack

(c) 5 points. Add one or more rules to the following CFG to generate the Kleene star of the original language. Do not change what either S or A generates.

$S_0 \rightarrow SS_0 | \epsilon$

$S \rightarrow aSa | bSb | A$

$A \rightarrow a | b$

(d) 5 points. True or False. The above CFG can be captured by a DPDA. Briefly explain.

False. Palindromes require guessing when to start the matching process.

(e) Consider a PDA that has 10 states and accepts strings that contain x's, y's and z's. What is the maximum number of rules of the form $A_{pq} \rightarrow a A_{rs} b$ that can be produced? Briefly explain.

$A \rightarrow q a r s b$

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 4 = 160,000$