To prove a language L is not context free:

Show that for any p there is a string s ∈ L with $|s| \geq p$ such that for any

↳ step 1

uvxyz = s with $|vxy| \leq p$ and $|vy| > 0$, there exists an i such that

$uv^i xy^i z \notin L$

Step 1: provide a candidate string s within L that is longer than p

- Make sure $s \in L$ and $|s| \geq p$ for any p

Step 2: show that if uvxyz = s, $|vxy| \leq p$, and $|vy| > 0$, then uvxyz must have certain characteristics or falls into certain cases.

- If you use cases, make sure they are exhaustive and broad
- In general, focus on vxy and $|vxy| \leq p$

Step3: show that there is some i such that $uv^i xy^i z \notin L$

- in many cases you can use i = 0 or i = 2 will work
- if you used cases in step two, show that for any case you can make $uv^i xy^i z \notin L$

Example 1: $\{a^{n^2} : n \geq 0\}$

Step 1: finding a candidate string s

$S = a^{p^2}$ $S \in L$ ✓ $|S| = p^2 > p$ ✓

Step2: characteristics of uvxyz using $|vy| > 0$

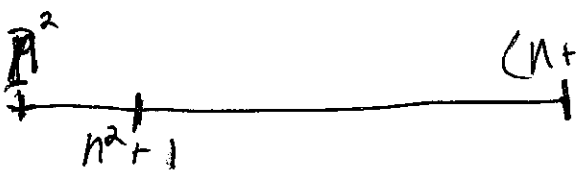
$vy = a^{m}$ for some $m > 0$

Step 3: finding i so that $uv^ixy^iz \notin L$

$uv^0xy^0z = a^{p^2-m}$

$uv^2xy^2z = a^{p^2-m+2m}$

$uv^ixy^iz = a^{p^2-m+im} = a^{p^2+(i-1)m}$



$(n+1)^2 = n^2+2n+1 > n^2+1$

$i = p^2 s + 1$

$uv^ixy^iz = a^{p^2+(p^2 m)m} = a^{p^2(1+m^2)}$

$uv^ixy^iz \notin L$

Example 2: $\{a^n b^m a^n : n \geq m\}$

Step 1: finding a candidate string s

$s = a^p b^p a^p$ $|s| = 3p \geq p$ ✓ $s \in L$ ✓

Step2: cases of v and y using $|vxy| \leq p$

Case 1: v, y contain an a from that 1st run of a's

Case 2: doesn't contain a from 1st group of a's
→ v or y contain at least one b
→ all a's

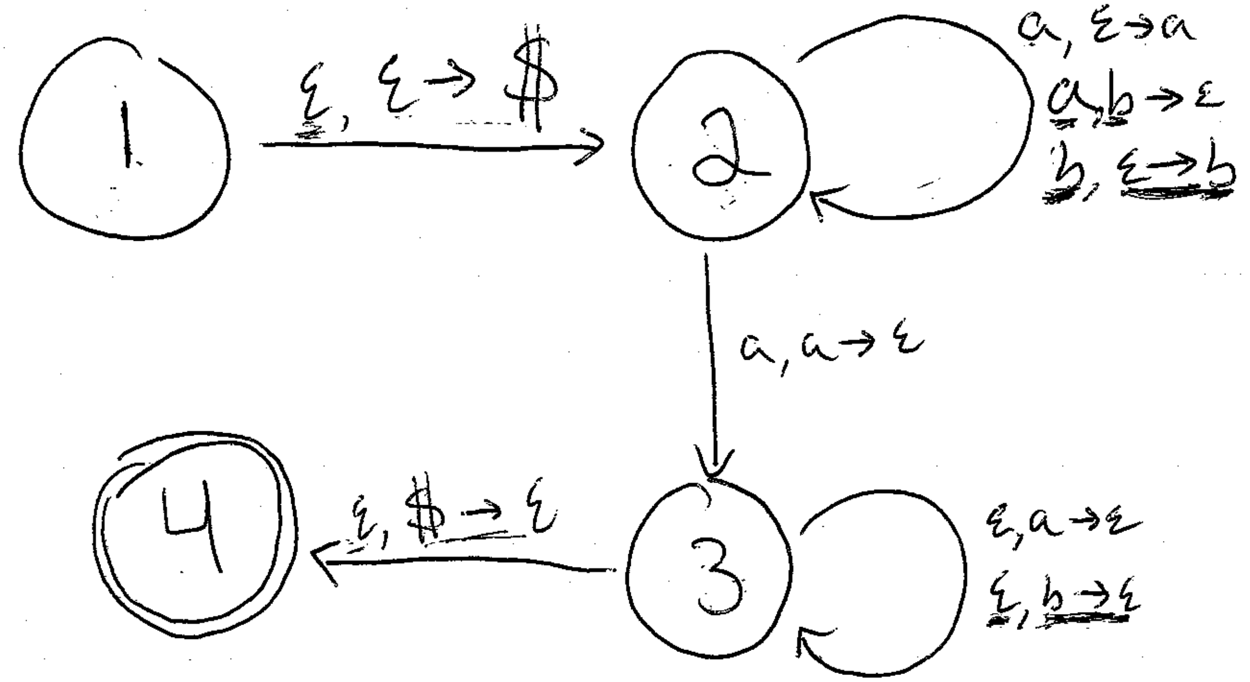
Step 3: finding i so that $uv^ixy^iz \notin L$

Case 1: uv^2xy^2z will have more a's at the start so $uv^2xy^2z \notin L$

Case 2:

→ uv^2xy^2z p+m # of b's so
$uv^2xy^2z \notin L$

→ $uv^2xy^2z \notin L$



$A_{pp} \to \varepsilon$ $A_{pq} \to A_{pr} A_{rq}$ $A_{pq} \to a A_{rs} b$

$A_{11} \to \varepsilon$ $A_{12} \to A_{13} A_{32}$ $A_{22} \to b A_{22} b$

$A_{22} \to \varepsilon$ $A_{14} \to \varepsilon A_{23} \varepsilon$

$A_{44} \to \varepsilon$ $A_{23} \to b A_{23} \varepsilon$