$n \backslash m$	1	2	3	4	
1	1	2	3	4	$\cdots$
2	$1/2$	~~1~~	$3/2$	2	$\cdots$
3	$1/3$	$2/3$	1	$4/3$	$\cdots$
4	$1/4$	$1/2$	$3/4$	1	$\cdots$
$\cdots$					

$\mathbb{N}$	m/n
1	1
2	$1/2$
3	2
4	$1/3$
5	3
$\vdots$	

✓

$\mathbb{N}$	$\rightarrow$	m/n
1		1
2		2
3		3
4		4
		$\vdots$
		XXX

this correspondence won't work

$\mathbb{R}$

$\mathbb{N}$	$\mathbb{R}$
1	7.00 . . . .
2	1.00 . . . .
3	2.70 . . . .

$\mathbb{N}$	$\mathbb{R}$
1	.000 . . . .
2	.000 . . . .
3	.700 . . . .

Consider the following # $.126$

$\neq \mathbb{R}_1 \quad \neq \#\mathbb{R}_2 \quad \neq \#\mathbb{R}_3$

the decimal
value of Real
#1, position 1

$\boxed{\mathbb{R} \text{ is uncountable}}$

"Proof by Diagonalization"

$\Sigma_1 = \{0, 1\}$

$\Sigma_2 = \{a, b, c\}$

$\mathbb{N}$	Σ_2^*
1	ϵ
2	a
3	b
4	c
5	aa

strings are Countable

languages are uncountable

$\Sigma^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 100, \dots\}$

$A = \{ \quad 1 \quad 1 \quad 1 \quad 1 \quad \}$

$XA = \{0 \; 0 \; 1 \; 0 \; 1 \; 1 \; 0 \; 1 \dots\}$

Characteristic function

$L(A) = \{$binary strings with odd # of 1s$\}$

$\mathbb{N}$	all languages								
1	0	0	1	0	1	1	0	1	...
2	0	1	0	1	1	0	0	0	...

Using diagonalization, it can be shown that ~~the~~ languages are uncountable

Since TMs are countable, there must be some undecidable languages!

For example, A_{TM} is undecidable