Consider Quicksort

	best case	average case	worst case
ω			$\omega(1)$ $\omega(\log n)$ $\omega(n)$ ...
Ω			$\Omega(1)$ $\Omega(\log n)$ ... $\Omega(n^2)$
Θ	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n^2)$
O			$O(n^3)$ $O(n^2 \log n)$ $O(n^2)$
o			$o(n^3)$ $o(n^2 \log n)$ ...

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

Quicksort is . . . :

$\omega(n)$ . . . .

$\Omega(n \log n)$ $\quad$ $\cancel{\Omega(n)}$

$O(n^2)$ $\quad$ $O(\cancel{n^3})$

$o(n^2 \log n)$ . . .

O example

$$f(n) = 2n^3 + 7n + 3$$

Show $f(n)$ is $O(n^3)$

$$2n^3 + 7n + 3 \le \underline{12} \cdot n^3 \;\; \forall\, n \ge \underline{1}$$

Show $f(n)$ is $O(n^3 \log n)$

$$2n^3 + 7n + 3 \le \underline{12}\, n^3 \log n \;\; \forall\, n \ge 1$$

Prove $2n^3 + 7n + 3$ is $O(n^4)$

$$\lim_{n\to\infty} \frac{2n^3 + 7n + 3}{n^4}$$

$$= \underbrace{\lim_{n\to\infty} \frac{2n^3}{n^4}}_{0} + \underbrace{\lim_{n\to\infty} \frac{7n}{n^4}}_{0} + \underbrace{\lim_{n\to\infty} \frac{3}{n^4}}_{0} = 0$$

Prove $2n^3 + 7n + 3$ is $O(n^3)$

$$\lim_{n\to\infty} \frac{2n^3 + 7n + 3}{n^3} \to 2$$

Prove $\log n$ is $O(n^{1/2})$

$$\lim_{n\to\infty} \frac{\log n}{n^{1/2}} = \lim_{n\to\infty} \frac{1/n}{\frac{1}{2} n^{-1/2}}$$

$$\left(\log_2 n = \underbrace{\log_2 e}_{c} \log_e n \right)$$

$$= \lim_{n \to \infty} \frac{2n^{1/2}}{n} = \lim_{n \to \infty} \frac{2}{n^{1/2}} \to 0$$

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p. 279 #1 $O(n)$

#2/3 $O(n^2)$

#4 $O(n)$

reject $\Omega(1)$

$O(n^2)$

Accept case $\Theta(n^2)$

Reject case $\Omega(1)$ $O(n^2)$