CSCI 132:
Basic Data Structures and Algorithms

Recursion (Part 1)

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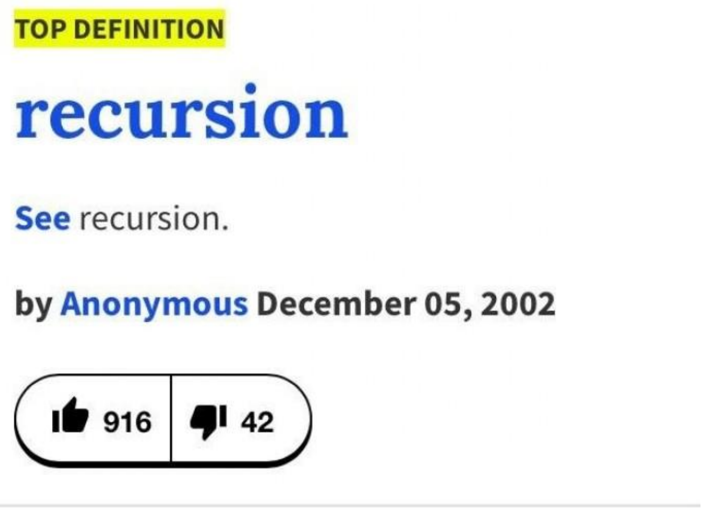
Fall 2023

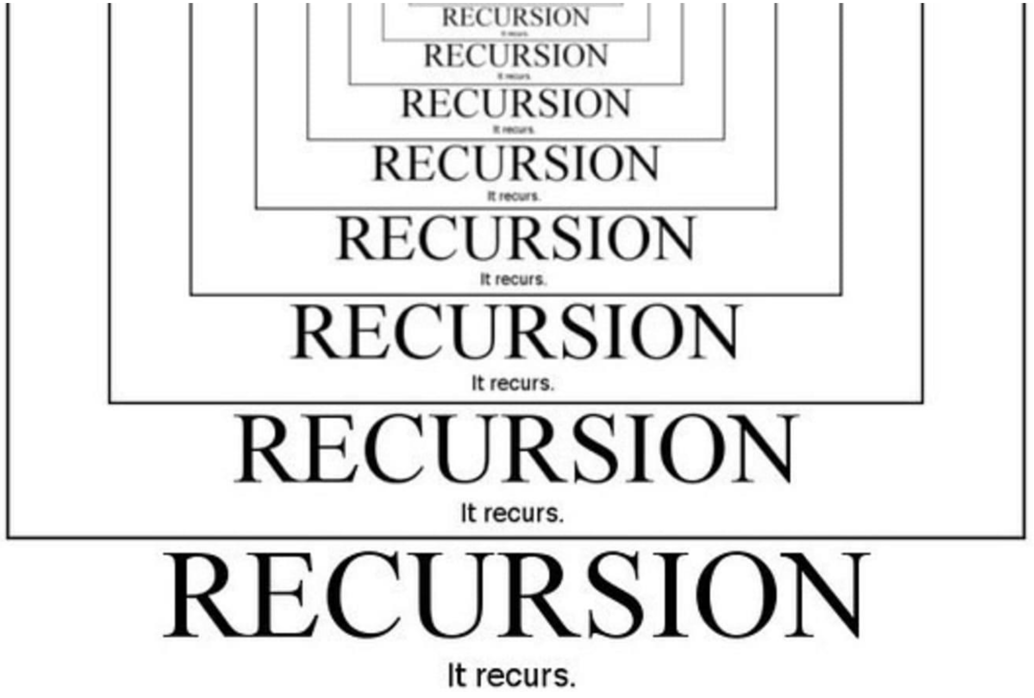
*All images are stolen from the internet

Recursion is a problem-solving technique that involves a *method calling itself* to solve some smaller problem

```java
static int factorial(int n)
	{
		if (n == 0)
			return 1;

		return n * factorial(n - 1);
	}
```

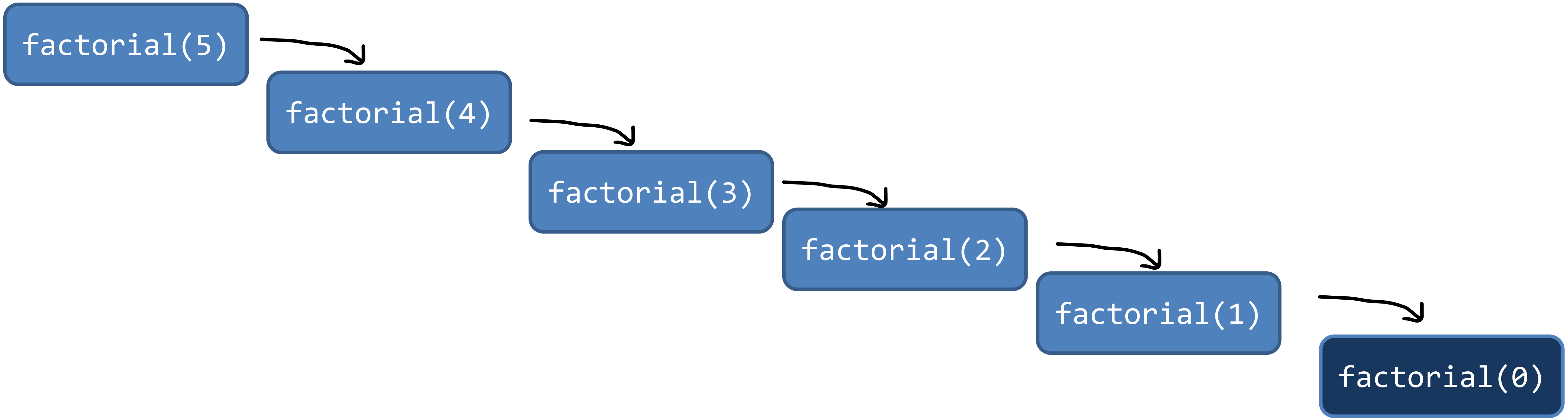




```java
static int factorial(int n)
    {
        if (n == 0)
            return 1;

        return n * factorial(n - 1);
    }
```

We can solve the factorial for n by solving smaller problems (factorial of n-1) !



```
static int factorial(int n)
    {
        if (n == 0)
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        return n * factorial(n - 1);
    }
```

(base case)

(recursive case)

We can solve the factorial for n by solving smaller problems (factorial of n-1) !

`factorial(5)`

```java
static int factorial(int n)
    {
        if (n == 0)
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```

(base case)

(recursive case)

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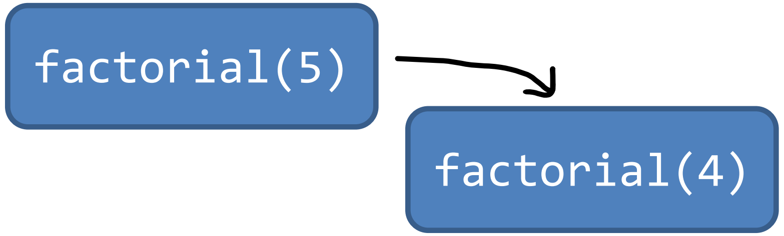

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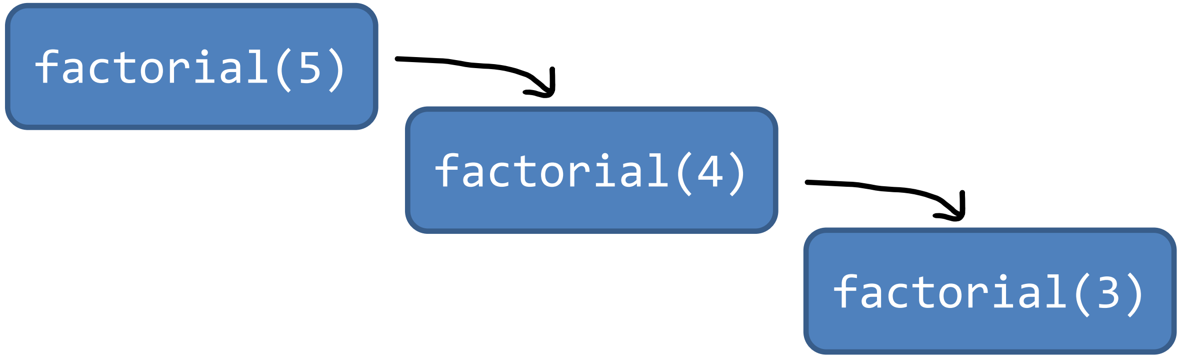

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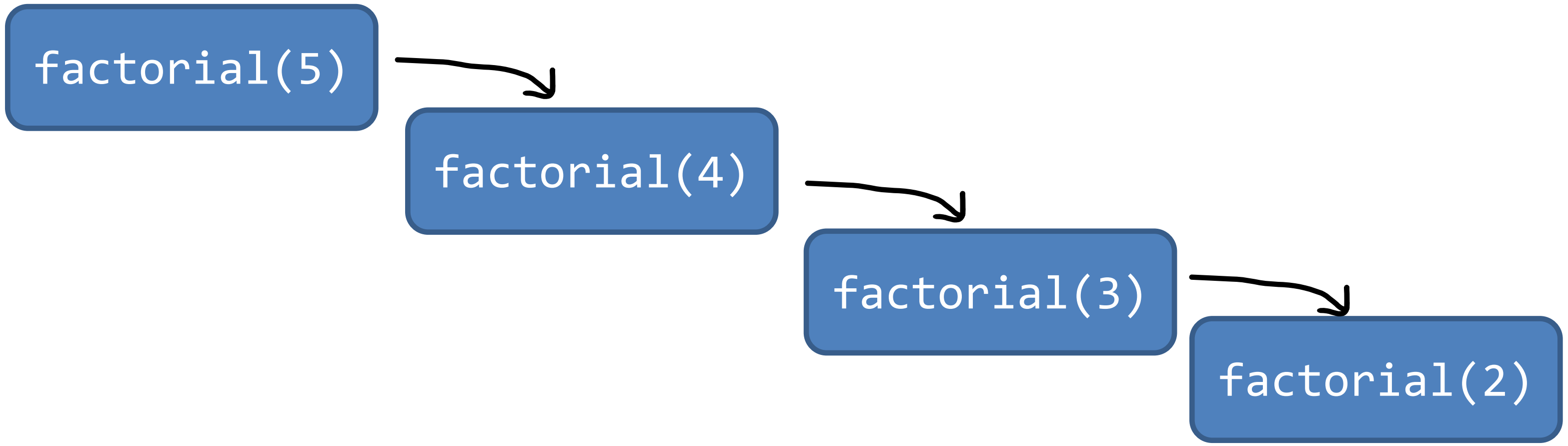

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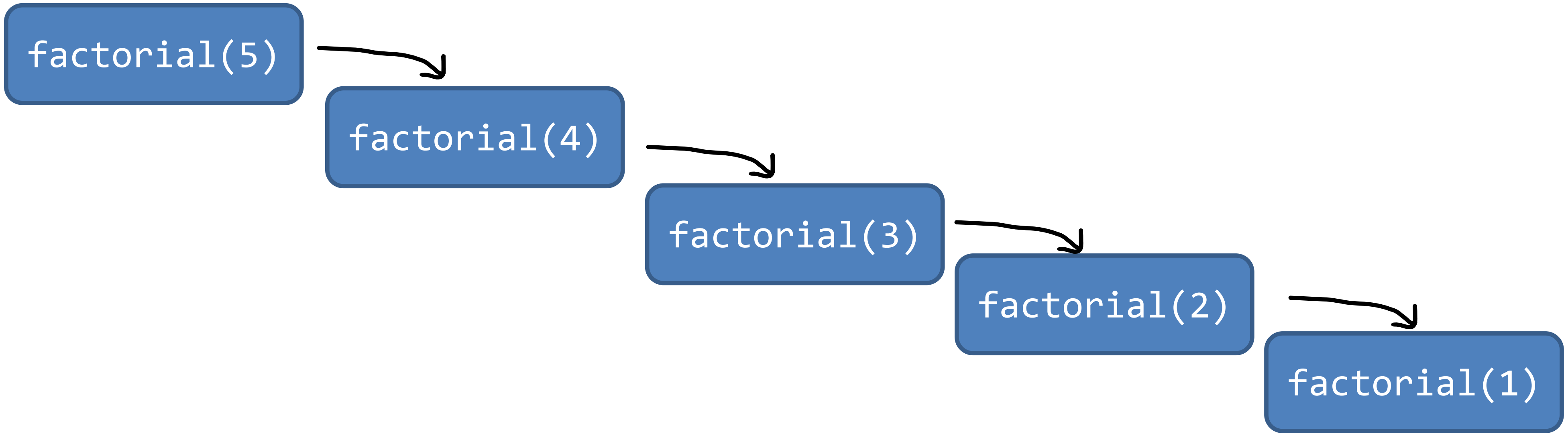

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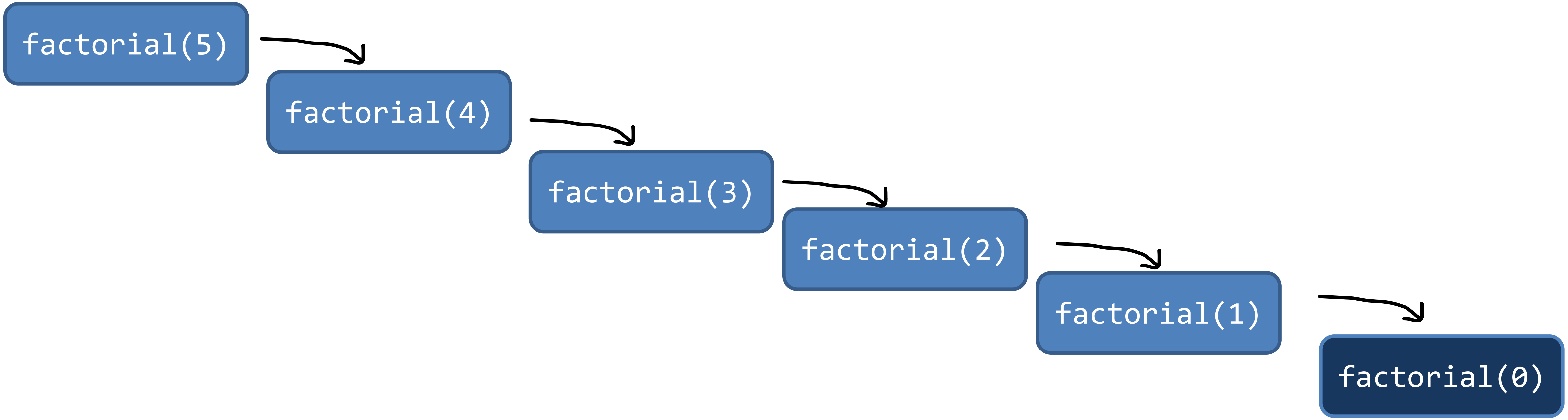

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    }
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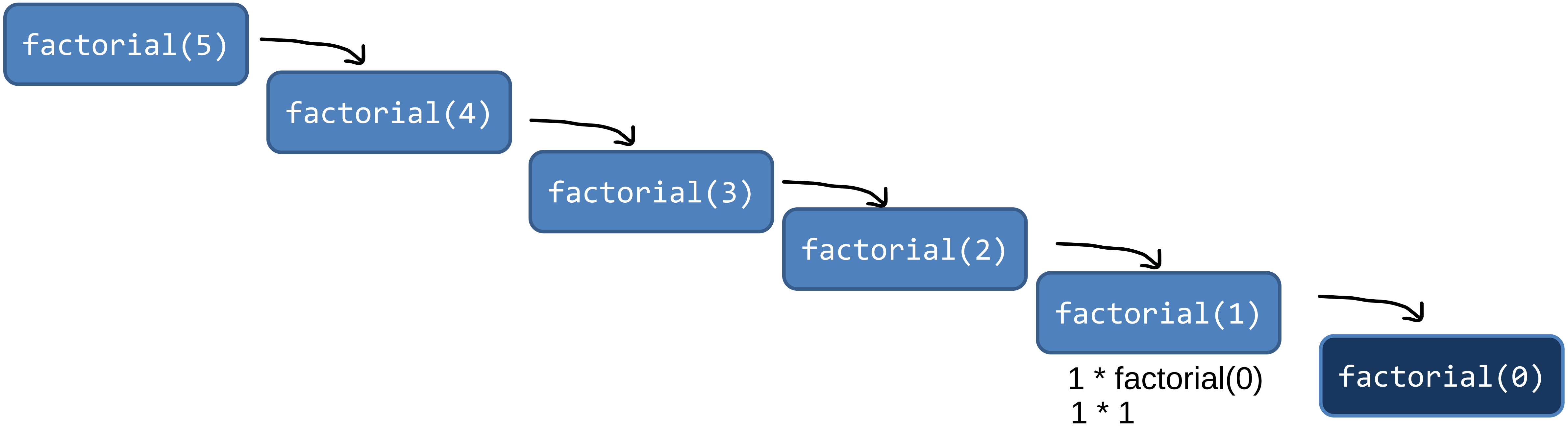
(base case)

(recursive case)

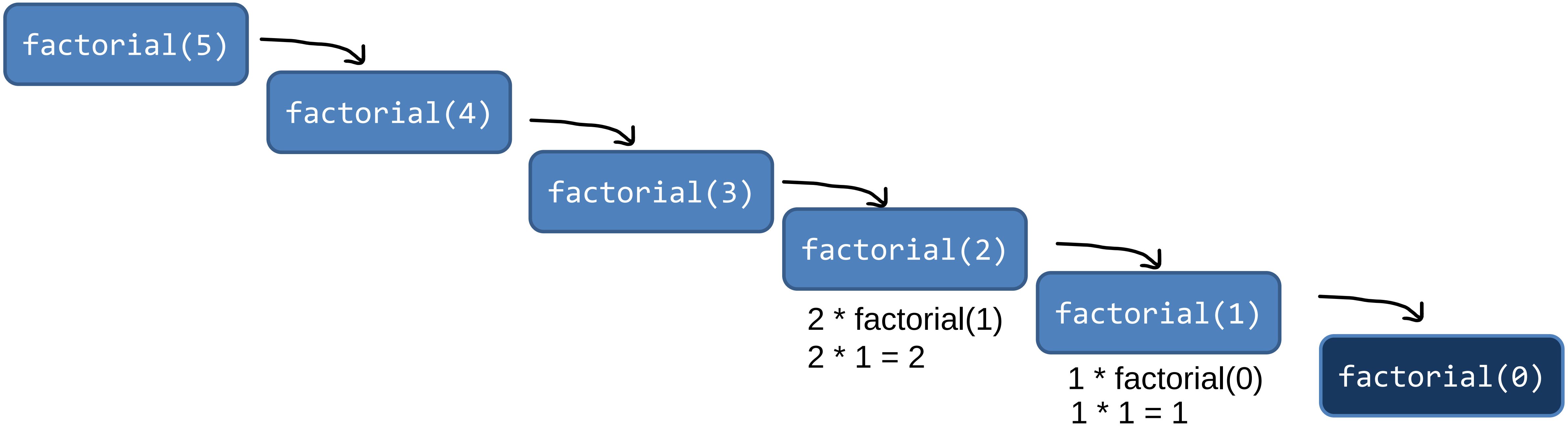
We can solve the factorial for n by solving smaller problems (factorial of n-1) !



```java
static int factorial(int n)
    {
        if (n == 0)
            return 1;        (base case)

        return n * factorial(n - 1);   (recursive case)
    }
```

We can solve the factorial for n by solving smaller problems (factorial of n-1) !



```
static int factorial(int n)
    {
        if (n == 0)
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        return n * factorial(n - 1);
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```
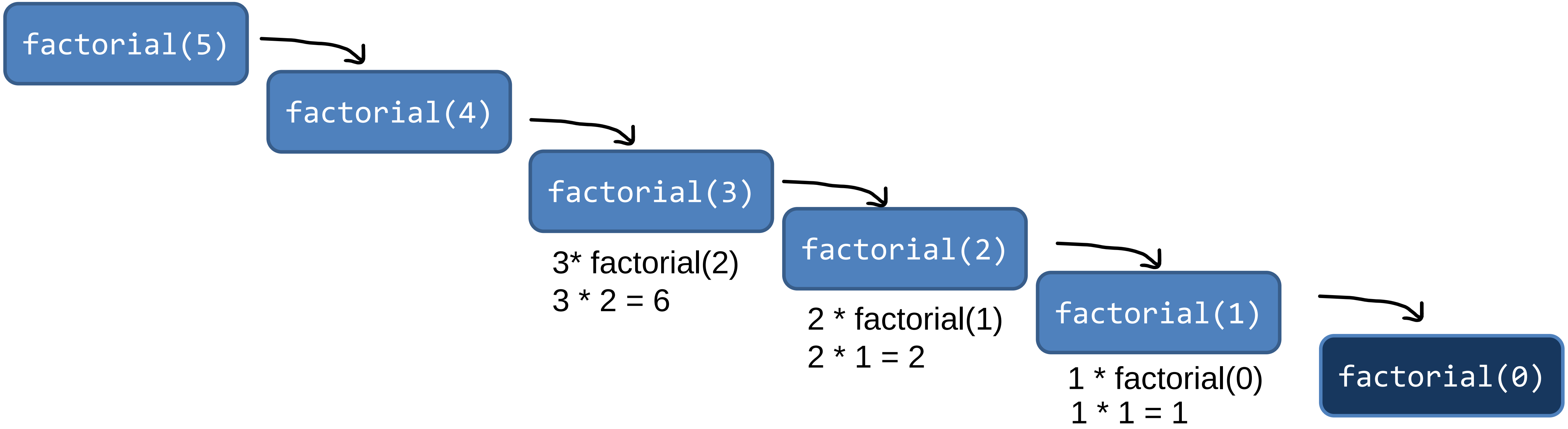

(base case)

(recursive case)

We can solve the factorial for n by solving smaller problems (factorial of n-1) !



```java
static int factorial(int n)
    {
        if (n == 0)
            return 1;

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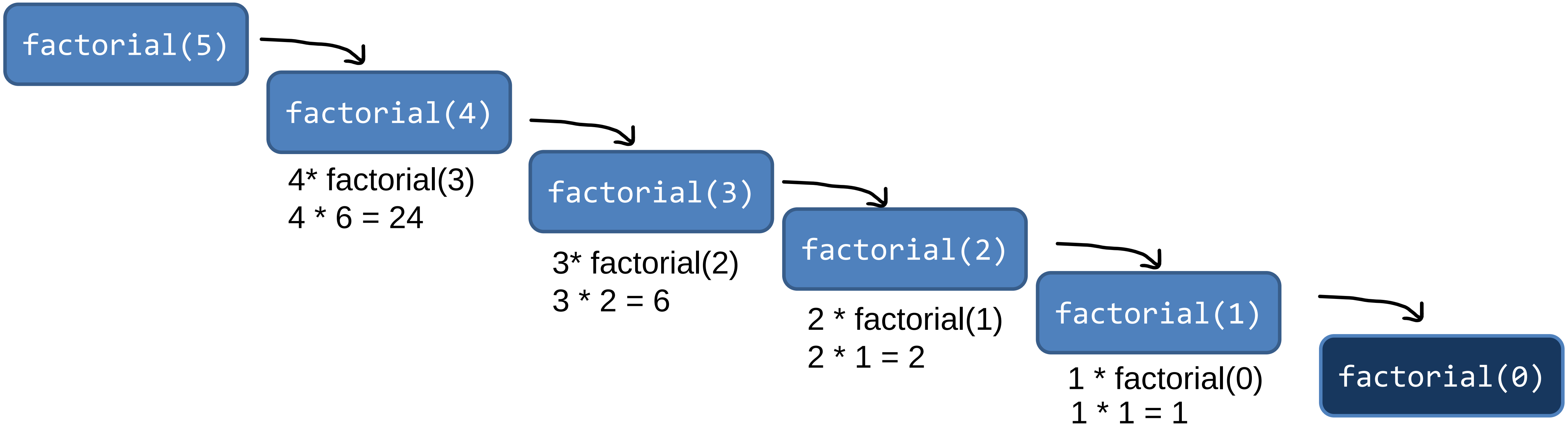
(base case)

(recursive case)

We can solve the factorial for n by solving smaller problems (factorial of n-1) !



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static int factorial(int n)
    {
        if (n == 0)
            return 1;

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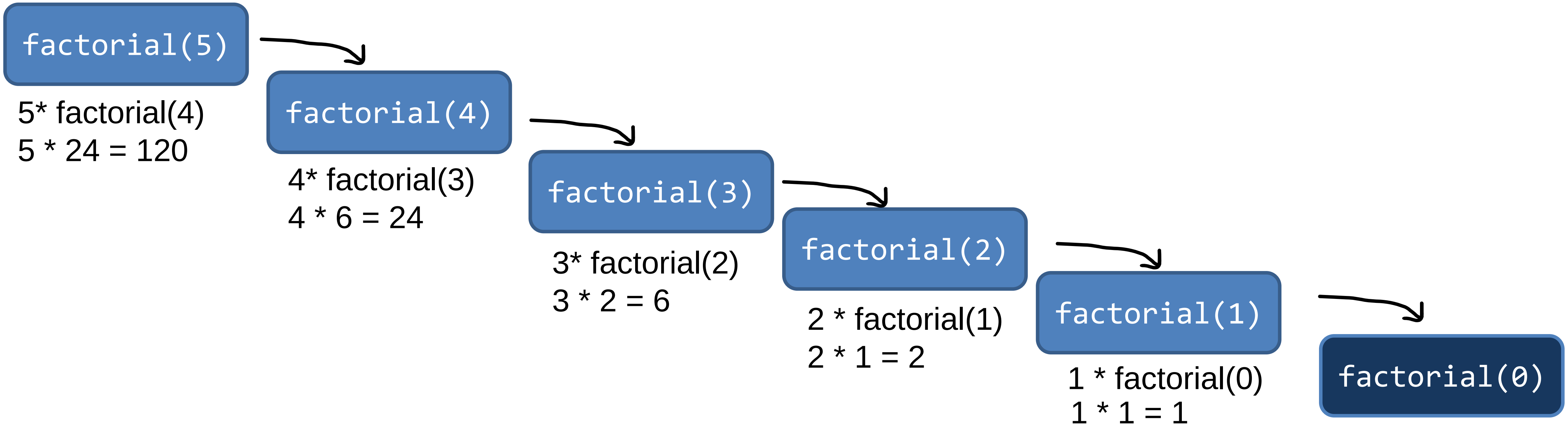
(base case)

(recursive case)

We can solve the factorial for n by solving smaller problems (factorial of n-1) !



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(base case)

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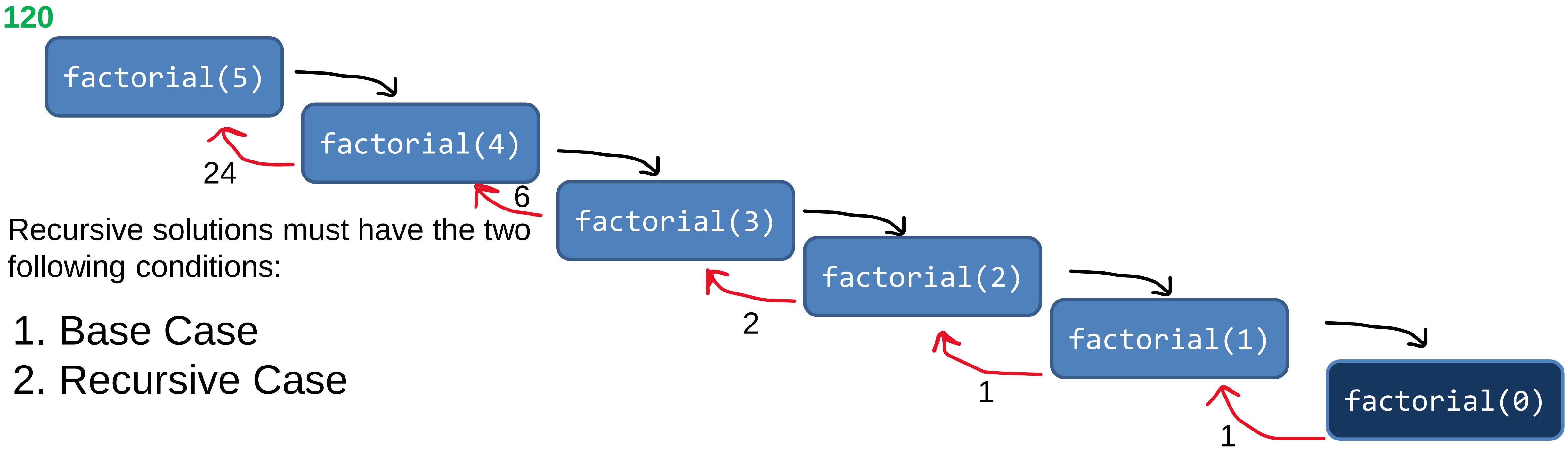
We can solve the factorial for n by solving smaller problems (factorial of n-1) !



Recursive solutions must have the two following conditions:

1. Base Case
2. Recursive Case

The Fibonacci sequence is a sequence in which each number is the sum of the two preceding ones

So, the N^th^ digit of the Fibonacci Sequence = f(N-1) + f(N-2)

The Fibonacci Sequence

1,1,2,3,5,8,13,21,34,55,89,144,233,377…

1+1=2	**13+21=34**
1+2=3	**21+34=55**
2+3=5	**34+55=89**
3+5=8	**55+89=144**
5+8=13	**89+144=233**
8+13=21	**144+233=377**

Because the solution to some problem can be expressed in terms of some smaller problem(s), recursion may be a good fit here

The Fibonacci sequence is a sequence in which each number is the sum of the two preceding ones

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Base Case?

Recursive Case?

Calculate

The Fibonacci sequence is a sequence in which each number is the sum of the two preceding ones

So, the N^{th} digit of the Fibonacci Sequence = f(N-1) + f(N-2)

The Fibonacci Sequence

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3+5=8	**55+89=144**
5+8=13	**89+144=233**
8+13=21	**144+233=377**

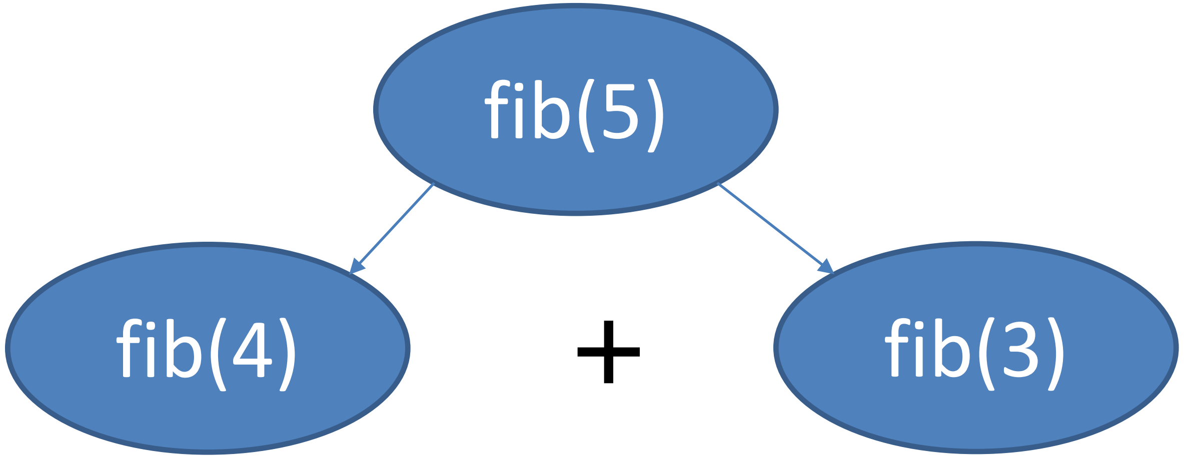
Base Case?

If finding the 1^{st} or 2^{nd} digit, return 1

Recursive Case?

Calculate the previous two digits, f(n-1), f(n-2) fib(5)

```java
private static int fib(int n) {
  if(n == 1 || n == 2) {
            return 1;
  }
  else {
            return fib(n-1) + fib(n-2);
  }
}
```




```
private static int fib(int n) {
  if(n == 1 || n == 2) {
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  }
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  }
}
```
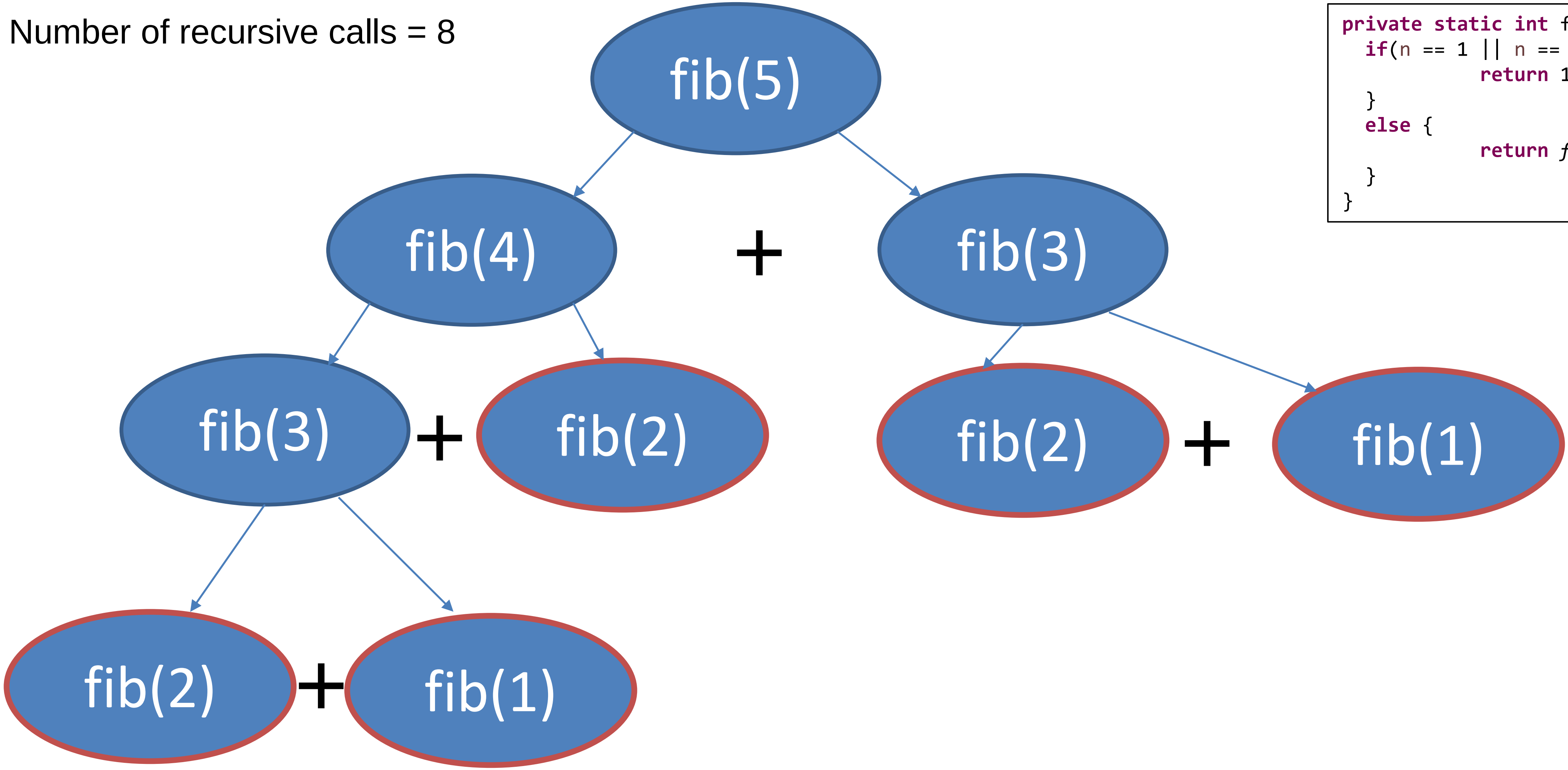

Number of recursive calls = 8

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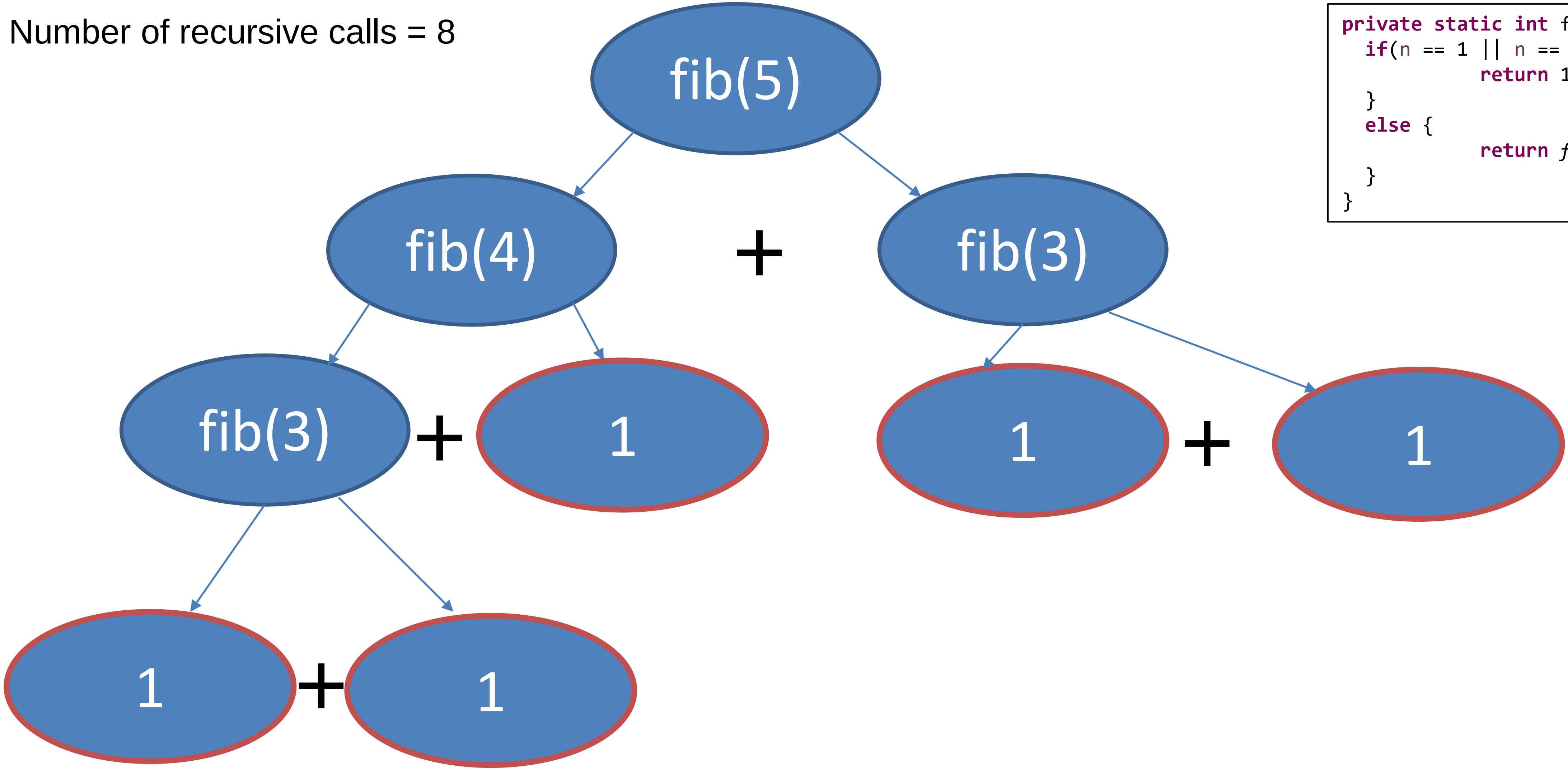


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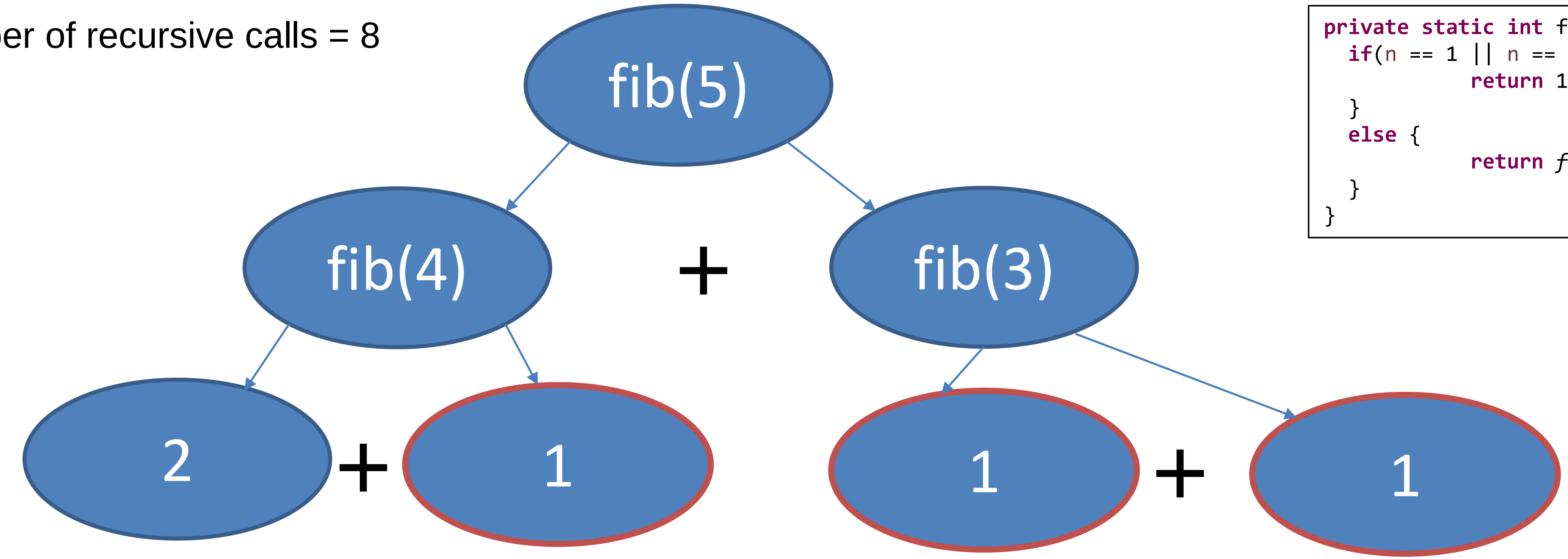


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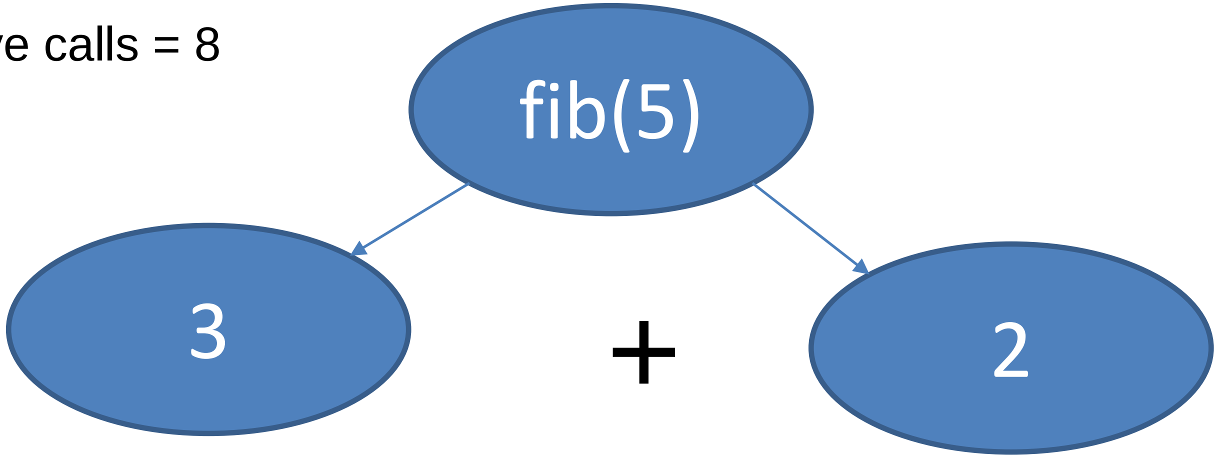



```java
private static int fib(int n) {
  if(n == 1 || n == 2) {
            return 1;
  }
  else {
            return fib(n-1) + fib(n-2);
  }
}
```

Number of recursive calls = 8

5

Final answer!

```
private static int fib(int n) {
  if(n == 1 || n == 2) {
            return 1;
  }
  else {
            return fib(n-1) + fib(n-2);
  }
}
```

```java
private static int fib(int n) {
  if(n == 1 || n == 2) {
       return 1;
  }
  else {
       return fib(n-1) + fib(n-2);
  }
}
```

Running Time?

```java
private static int fib(int n) {
  if(n == 1 || n == 2) { O(1)
      return 1; O(1)
  }
  else {        O(1)          O(1)
      return fib(n-1) + fib(n-2);
  }
}
```

Running Time?

O(1) ?

```java
private static int fib(int n) {
  if(n == 1 || n == 2) { O(1)
       return 1; O(1)
  }
  else {           O(1)          O(1)
       return fib(n-1) + fib(n-2);
  }
}
```

Running Time?

~~**O(1) ?**~~

No!

When we are analyzing recursive algorithms, we have to calculate running time slightly different

```java
private static int fib(int n) {
  if(n == 1 || n == 2) {
        return 1;
  }
  else {
        return fib(n-1) + fib(n-2);
  }
}
```

Generally speaking, we can compute the running time of a recursive algorithm by using the following formula:

Running time = **# of recursive calls made** * **amount of work done in each call**

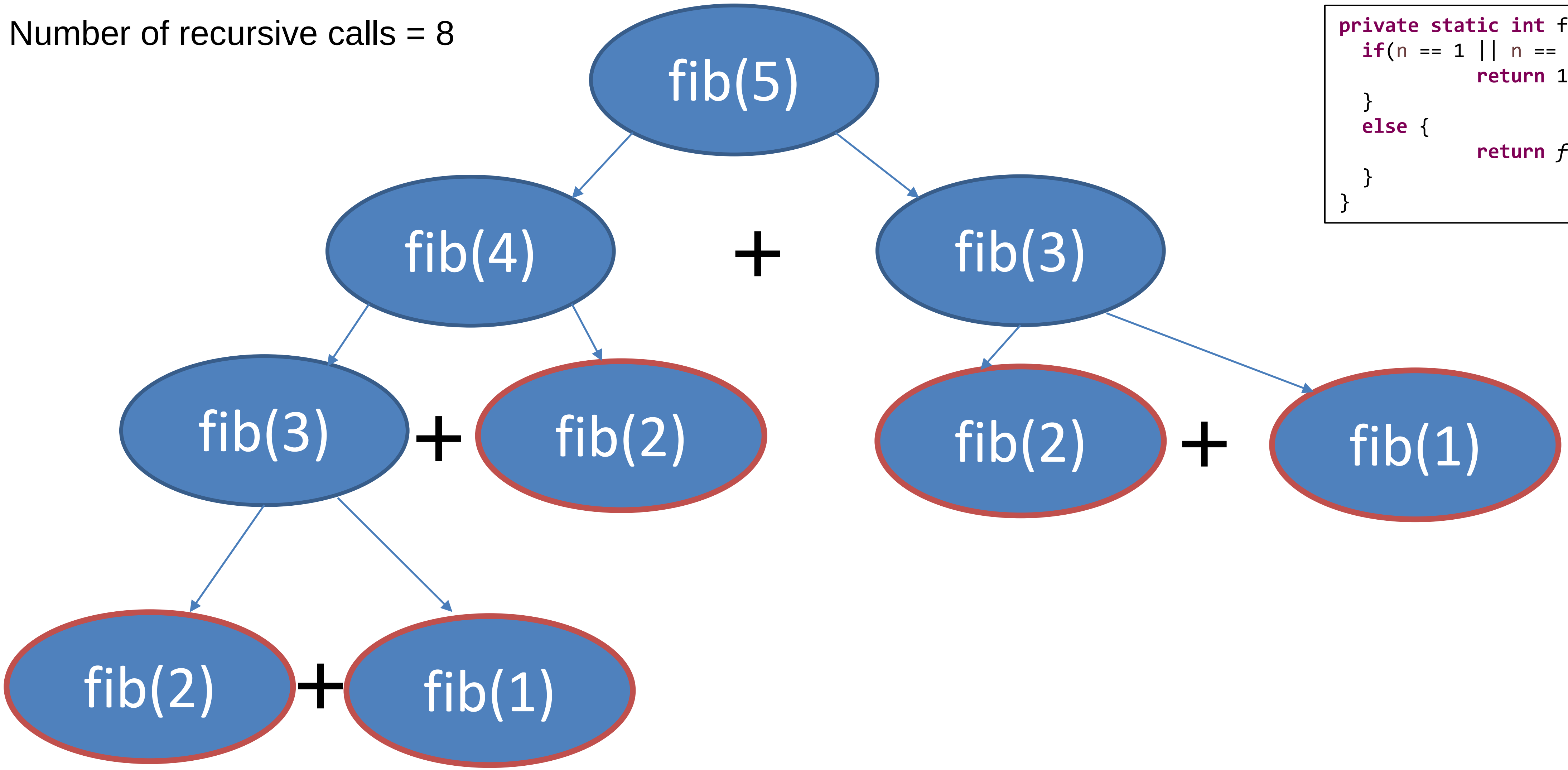
```java
private static int fib(int n) {
  if(n == 1 || n == 2) { O(1)
      return 1;  O(1)
  }
  else {              O(1)          O(1)
      return fib(n-1) + fib(n-2);
  }
}
```

Generally speaking, we can compute the running time of a recursive algorithm by using the following formula:

Running time = **# of recursive calls made** * **amount of work done in each call**

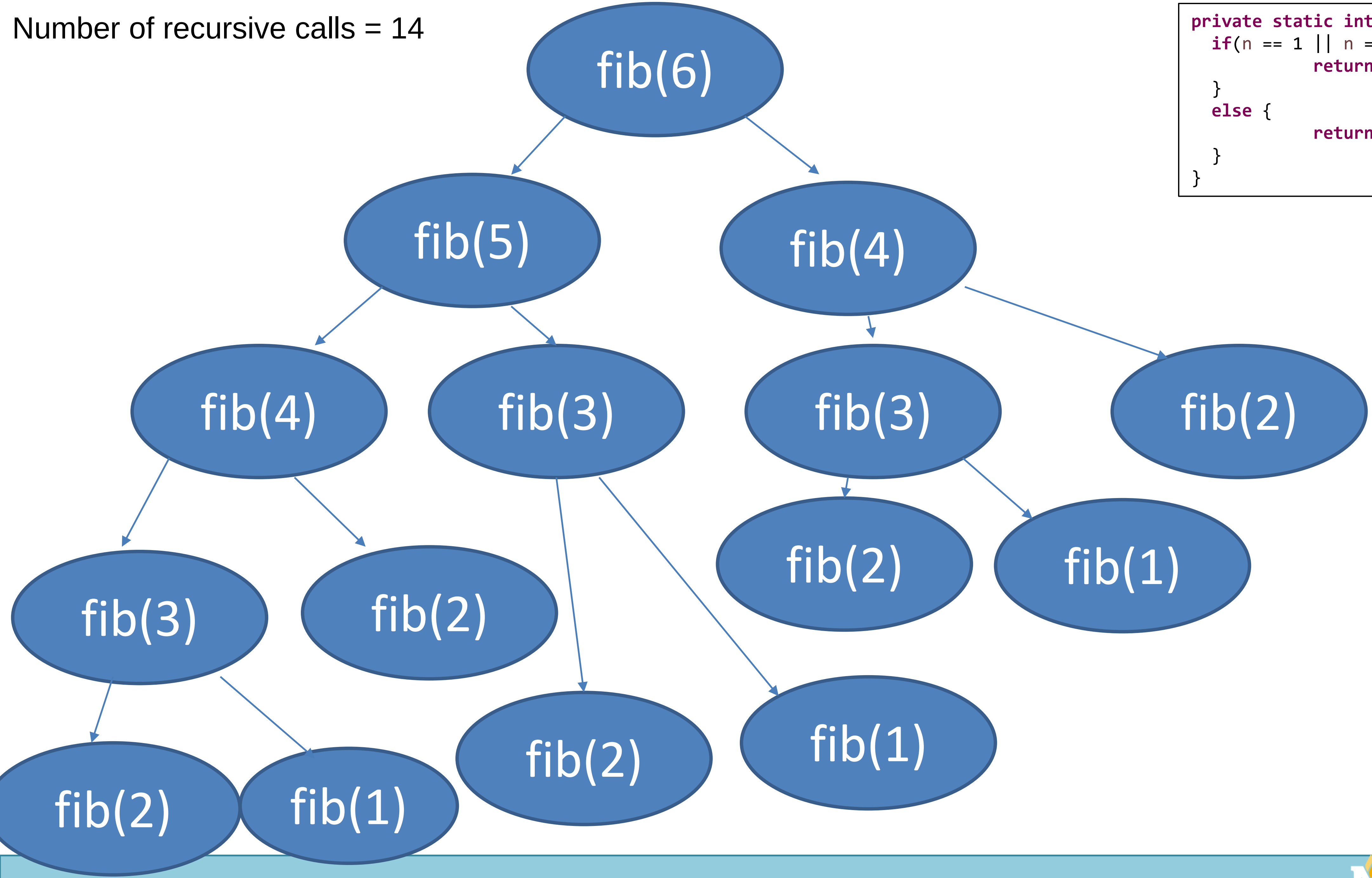
Running time = ??? * **O(1)**

Number of recursive calls = 8




```java
private static int fib(int n) {
  if(n == 1 || n == 2) {
            return 1;
  }
  else {
            return fib(n-1) + fib(n-2);
  }
}
```

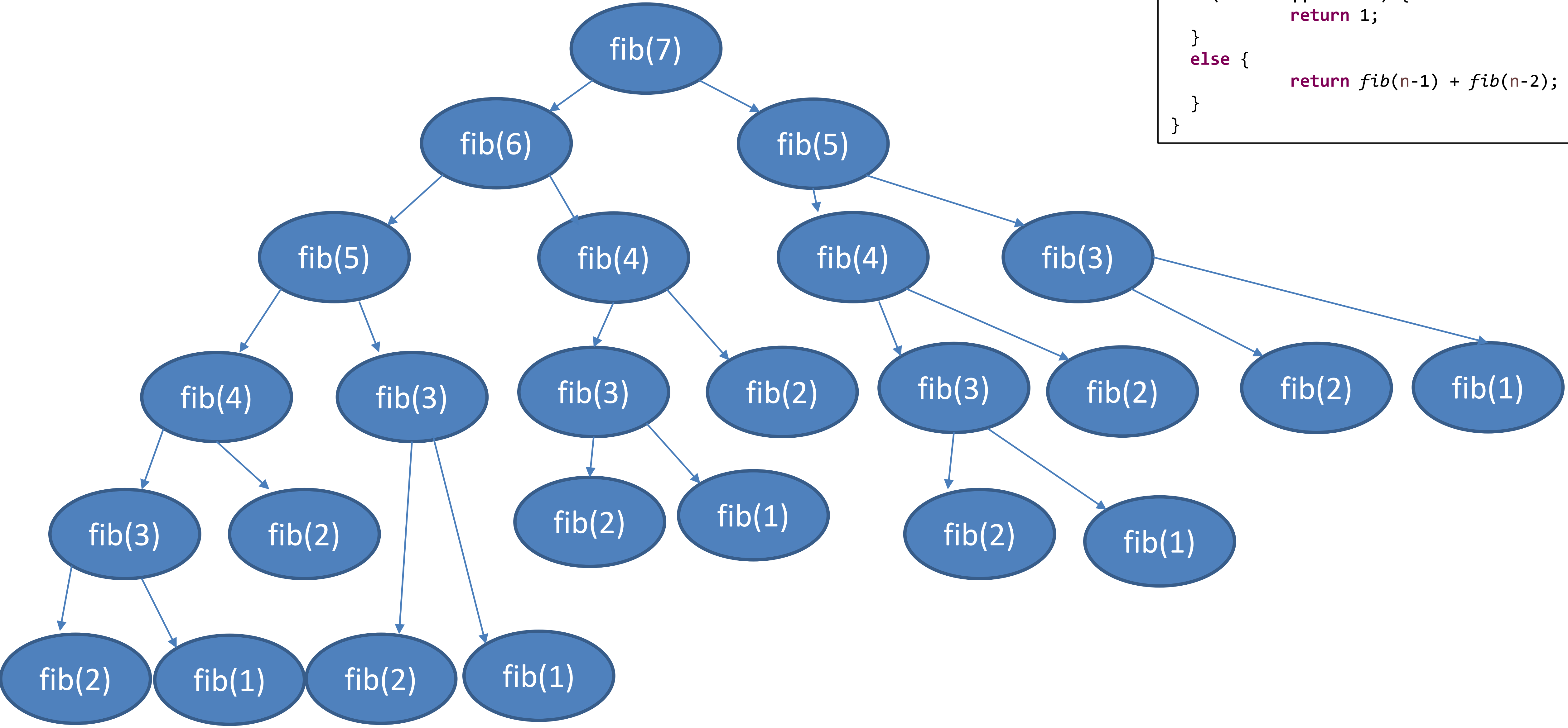
Number of recursive calls = 14




```java
private static int fib(int n) {
  if(n == 1 || n == 2) {
            return 1;
  }
  else {
            return fib(n-1) + fib(n-2);
  }
}
```

Number of recursive calls = 24

```java
private static int fib(int n) {
  if(n == 1 || n == 2) {
            return 1;
  }
  else {
            return fib(n-1) + fib(n-2);
  }
}
```



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  if(n == 1 || n == 2) {
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  else {
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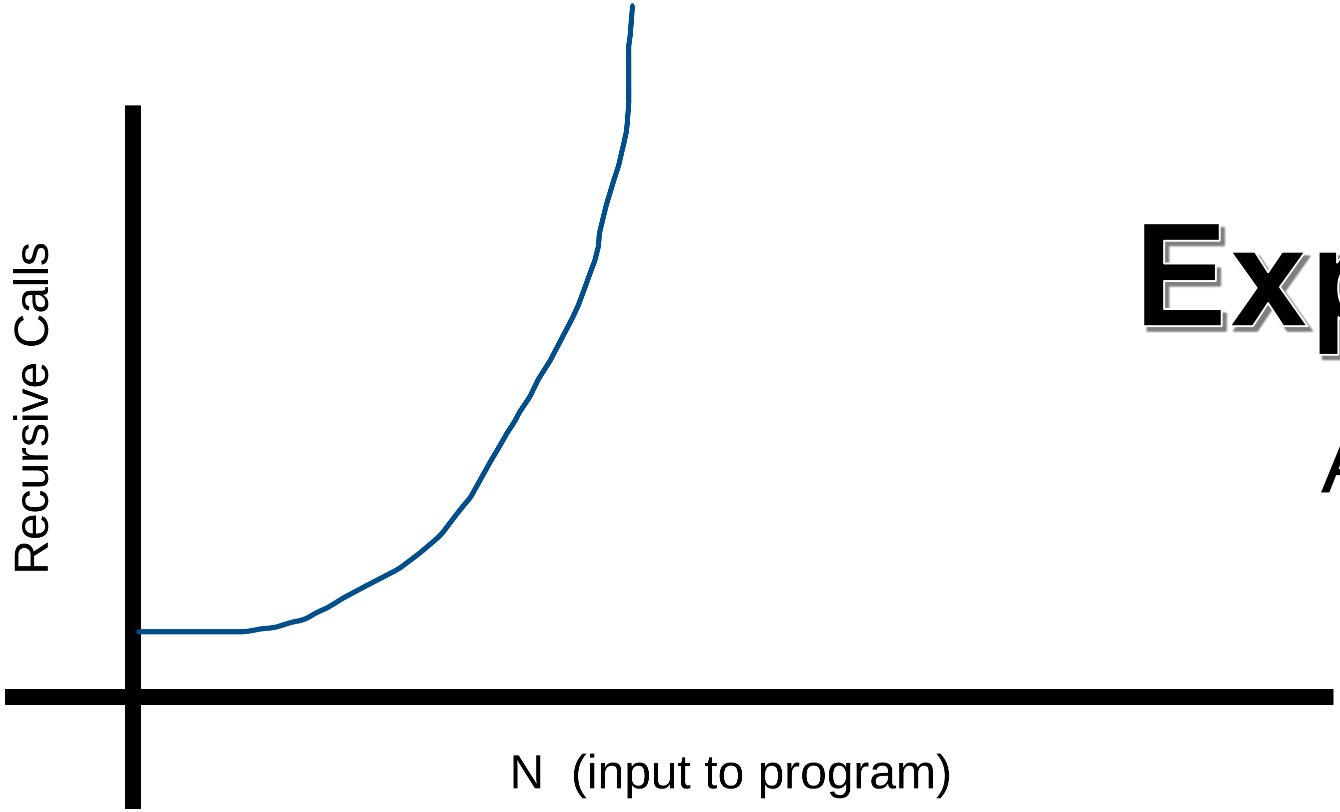
  }
}
```

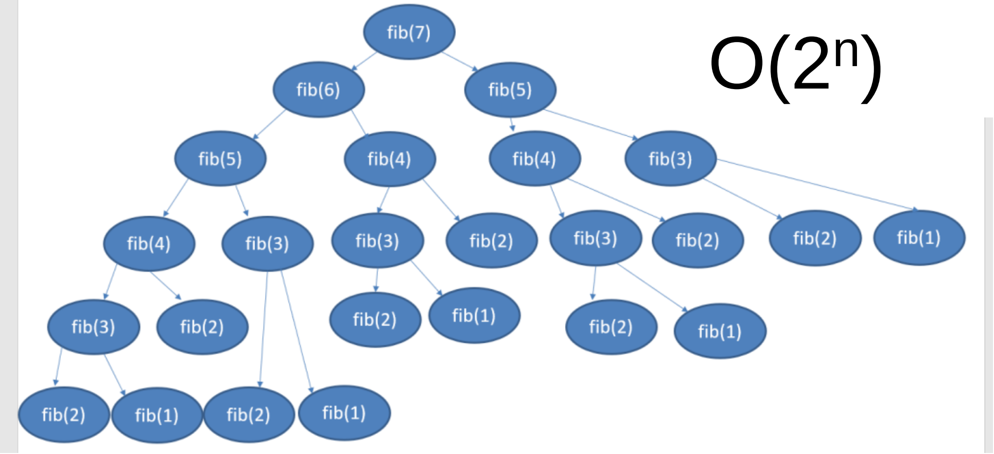
If we were to plot the number of recursive calls made as n increases, it would look something like his:



Exponential

Aka. $O(2^n)$

```
private static int fib(int n) {
  if(n == 1 || n == 2) { O(1)
      return 1;  O(1)
  }
  else {          O(1)           O(1)
      return fib(n-1) + fib(n-2);
  }
}
```



$O(2^n)$

Generally speaking, we can compute the running time of a recursive algorithm by using the following formula:

Running time = **# of recursive calls made** * **amount of work done in each call**

Running time = $O(2^n)$ * $O(1)$

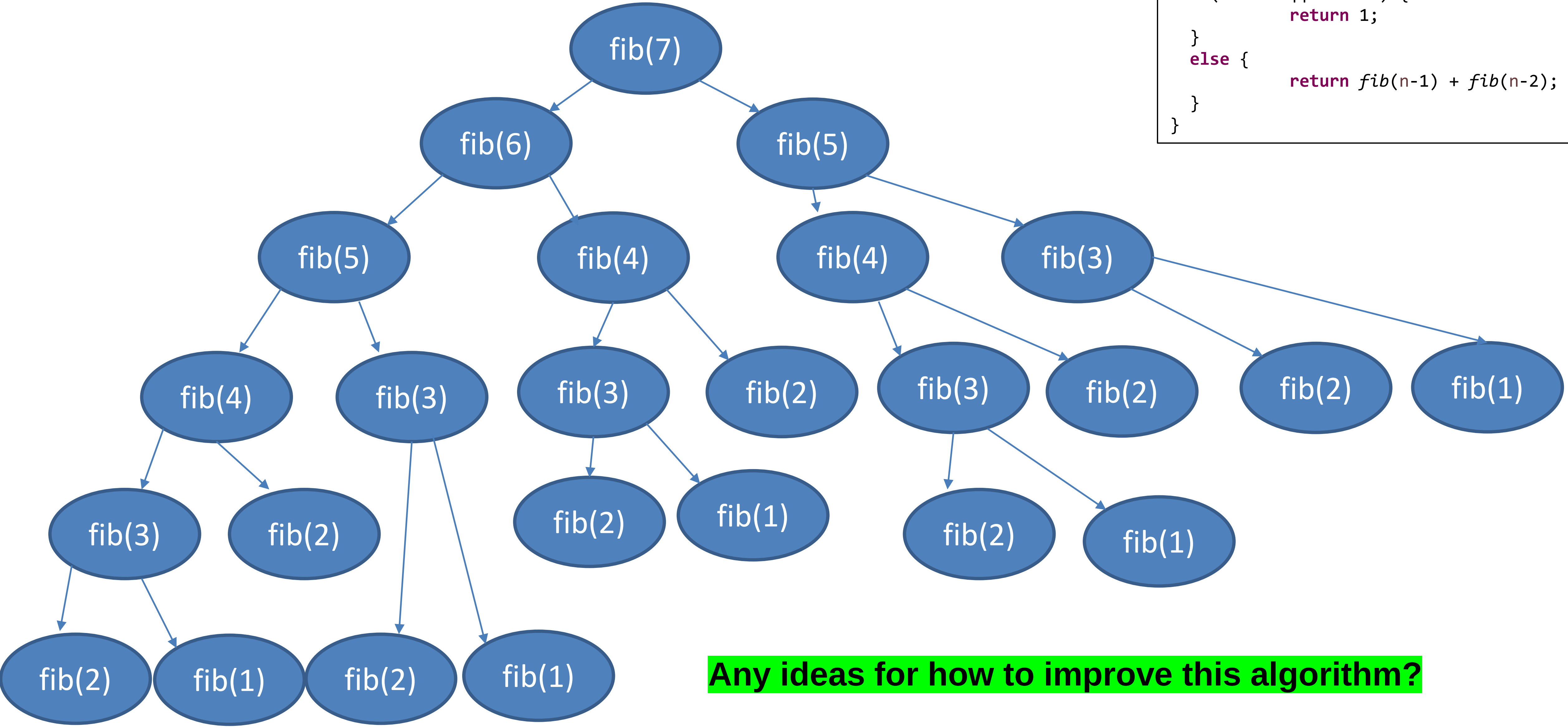
Total running time = $O(2^n)$

n = requested Fibonacci digit

$O(2^n)$ is very bad…

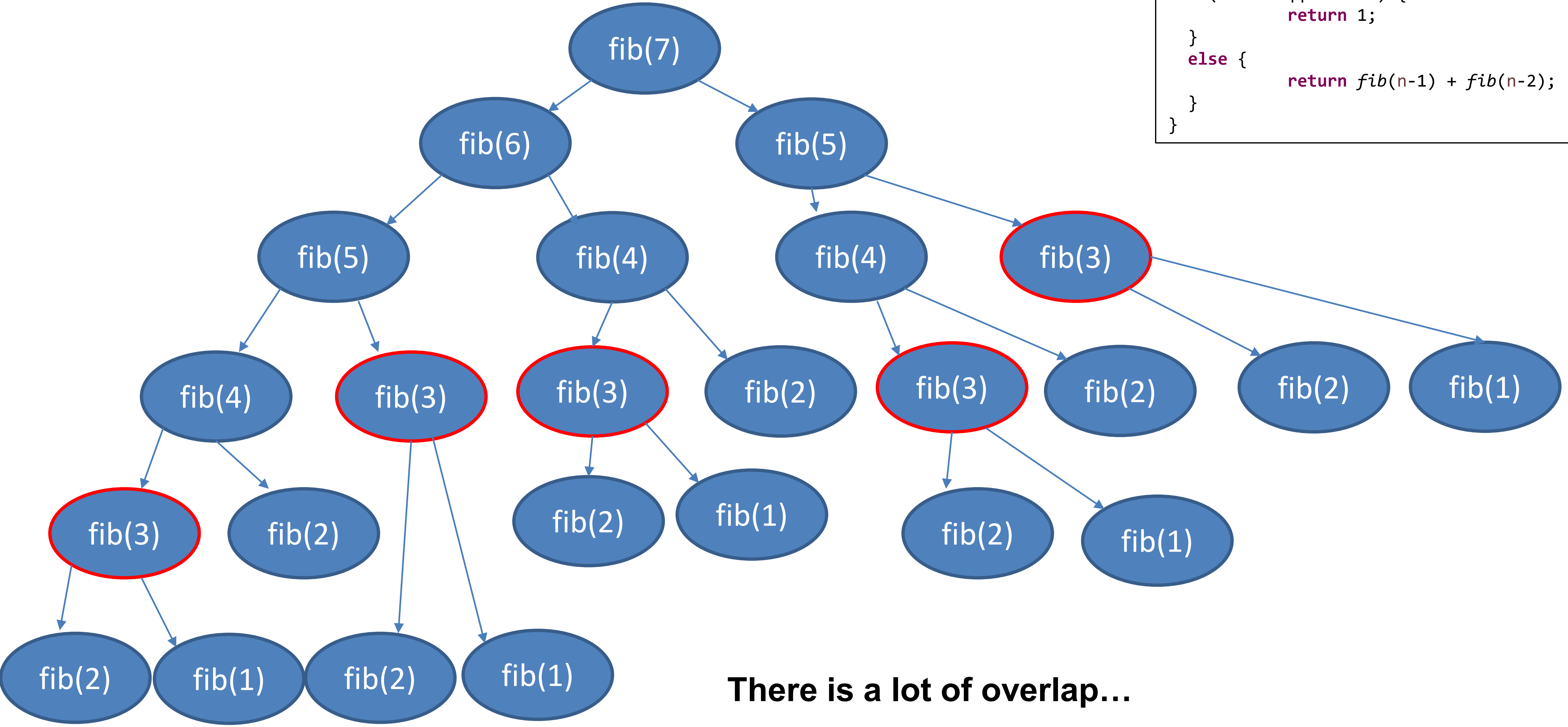
Number of recursive calls = 24

```java
private static int fib(int n) {
  if(n == 1 || n == 2) {
            return 1;
  }
  else {
            return fib(n-1) + fib(n-2);
  }
}
```



Any ideas for how to improve this algorithm?

Number of recursive calls = 24

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private static int fib(int n) {
  if(n == 1 || n == 2) {
            return 1;
  }
  else {
            return fib(n-1) + fib(n-2);
  }
}
```

There is a lot of overlap…

Number of recursive calls = 24

fib(1)	1
fib(2)	1
fib(3)	3
fib(4)	…
fib(5)	…

```
private static int fib(int n) {
  if(n == 1 || n == 2) {
            return 1;
  }
  else {
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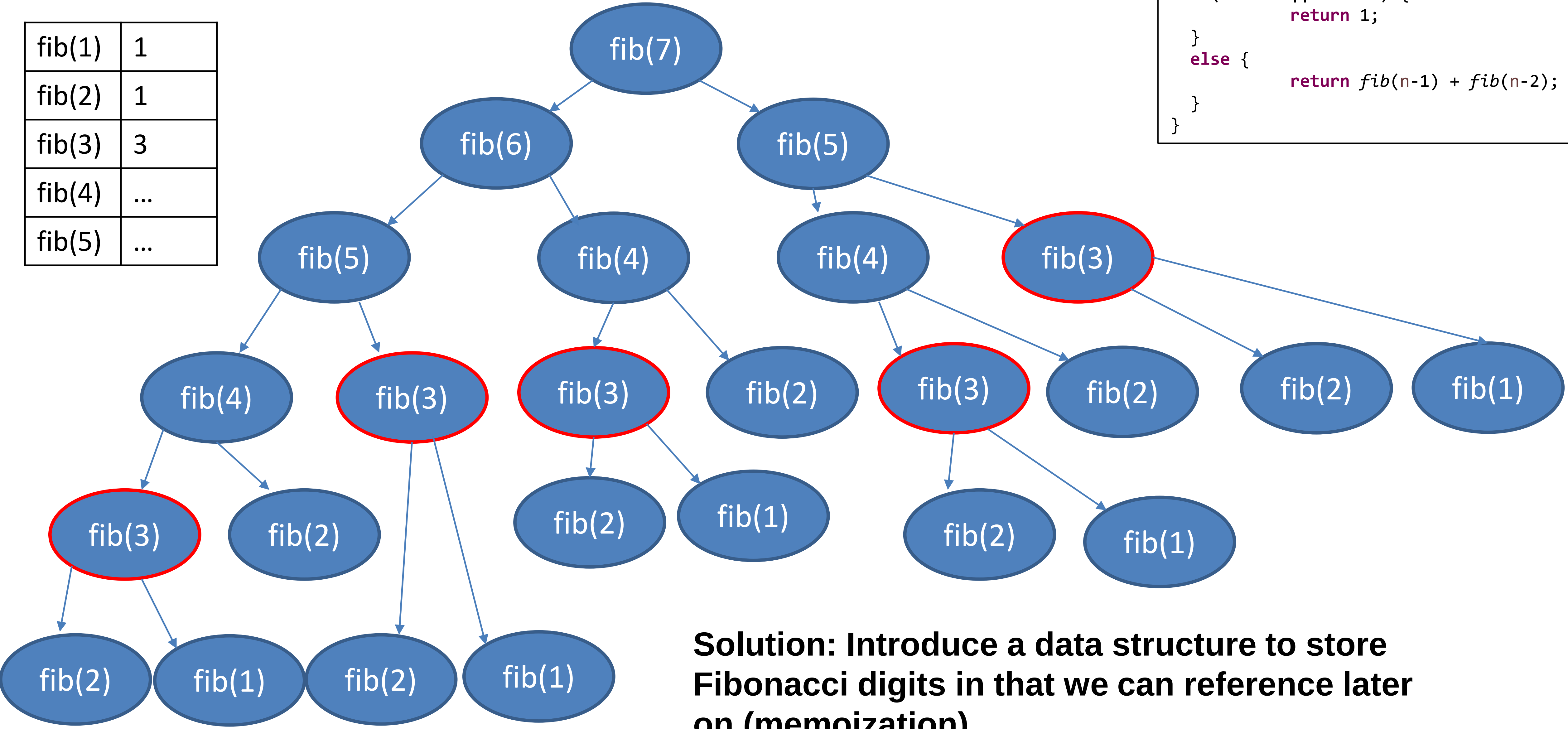
}
```



Solution: Introduce a data structure to store Fibonacci digits in that we can reference later on (memoization)

(These lookups happen in constant time!)

countX(“oxxo”)

```
public static int countX(String str) {
    if(str.length() == 0){
        return 0;
    }
    if(str.charAt(0) == 'x’){
        return 1 + countX(str.substring(1));
    }
    else{
        return 0 + countX(str.substring(1));
    }
}
```

countX(“**o**xxo”)

0 + countX(“xxo”)

```java
public static int countX(String str) {
    if(str.length() == 0){
        return 0;
    }
    if(str.charAt(0) == 'x’){
        return 1 + countX(str.substring(1));
    }
    else{
        return 0 + countX(str.substring(1));
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}
```

countX(“**o**xxo”)

0 + countX(“**x**xo”)

1 + countX(“xo”)

```java
public static int countX(String str) {
    if(str.length() == 0){
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    }
    if(str.charAt(0) == 'x’){
        return 1 + countX(str.substring(1));
    }
    else{
        return 0 + countX(str.substring(1));
    }
}
```

countX(“**o**xxo”)

0 + countX(“**x**xo”)

1 + countX(“**x**o”)

1 + countX(“o”)

```java
public static int countX(String str) {
    if(str.length() == 0){
        return 0;
    }
    if(str.charAt(0) == 'x’){
        return 1 + countX(str.substring(1));
    }
    else{
        return 0 + countX(str.substring(1));
    }
}
```

countX("**o**xxo")

0 + countX("**x**xo")

1 + countX("**x**o")

1 + countX("o")

0 + countX("")

```java
public static int countX(String str) {
    if(str.length() == 0){
        return 0;
    }
    if(str.charAt(0) == 'x'){
        return 1 + countX(str.substring(1));
    }
    else{
        return 0 + countX(str.substring(1));
    }
}
```

countX(“**o**xxo”)

0 + countX(“**x**xo”)

1 + countX(“**x**o”)

1 + countX(“o”)

0 + countX(“”)

0

```
public static int countX(String str) {
    if(str.length() == 0){
        return 0;
    }
    if(str.charAt(0) == 'x’){
        return 1 + countX(str.substring(1));
    }
    else{
        return 0 + countX(str.substring(1));
    }
}
```

countX(“**o**xxo”)

0 + countX(“**x**xo”)

1 + countX(“**x**o”)

1 + countX(“o”)

0 + 0

```java
public static int countX(String str) {
	if(str.length() == 0){
		return 0;
	}
	if(str.charAt(0) == 'x'){
		return 1 + countX(str.substring(1));
	}
	else{
		return 0 + countX(str.substring(1));
	}
}
```

countX(“**o**xxo”)

0 + countX(“**x**xo”)

1 + countX(“**x**o”)

1 + 0

```java
public static int countX(String str) {
    if(str.length() == 0){
        return 0;
    }
    if(str.charAt(0) == 'x’){
        return 1 + countX(str.substring(1));
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    else{
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    }
}
```

countX(“**o**xxo”)

0 + countX(“**x**xo”)

1 + countX(“**x**o”)

1 + 0

```java
public static int countX(String str) {
    if(str.length() == 0){
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    if(str.charAt(0) == 'x'){
        return 1 + countX(str.substring(1));
    }
    else{
        return 0 + countX(str.substring(1));
    }
}
```

countX(“**o**xxo”)

0 + countX(“**x**xo”)

1 + 1

```java
public static int countX(String str) {
    if(str.length() == 0){
        return 0;
    }
    if(str.charAt(0) == 'x’){
        return 1 + countX(str.substring(1));
    }
    else{
        return 0 + countX(str.substring(1));
    }
}
```

countX(“**o**xxo”)

0 + 2

```java
public static int countX(String str) {
    if(str.length() == 0){
        return 0;
    }
    if(str.charAt(0) == 'x’){
        return 1 + countX(str.substring(1));
    }
    else{
        return 0 + countX(str.substring(1));
    }
}
```

Final answer = 2

```
public static int countX(String str) {
    if(str.length() == 0){
        return 0;
    }
    if(str.charAt(0) == 'x’){
        return 1 + countX(str.substring(1));
    }
    else{
        return 0 + countX(str.substring(1));
    }
}
```

Limitations of recursion?