CSCI 132:
Basic Data Structures and Algorithms

Recursion (Part 1)

Reese Pearsall & Iliana Castillon

Fall 2024

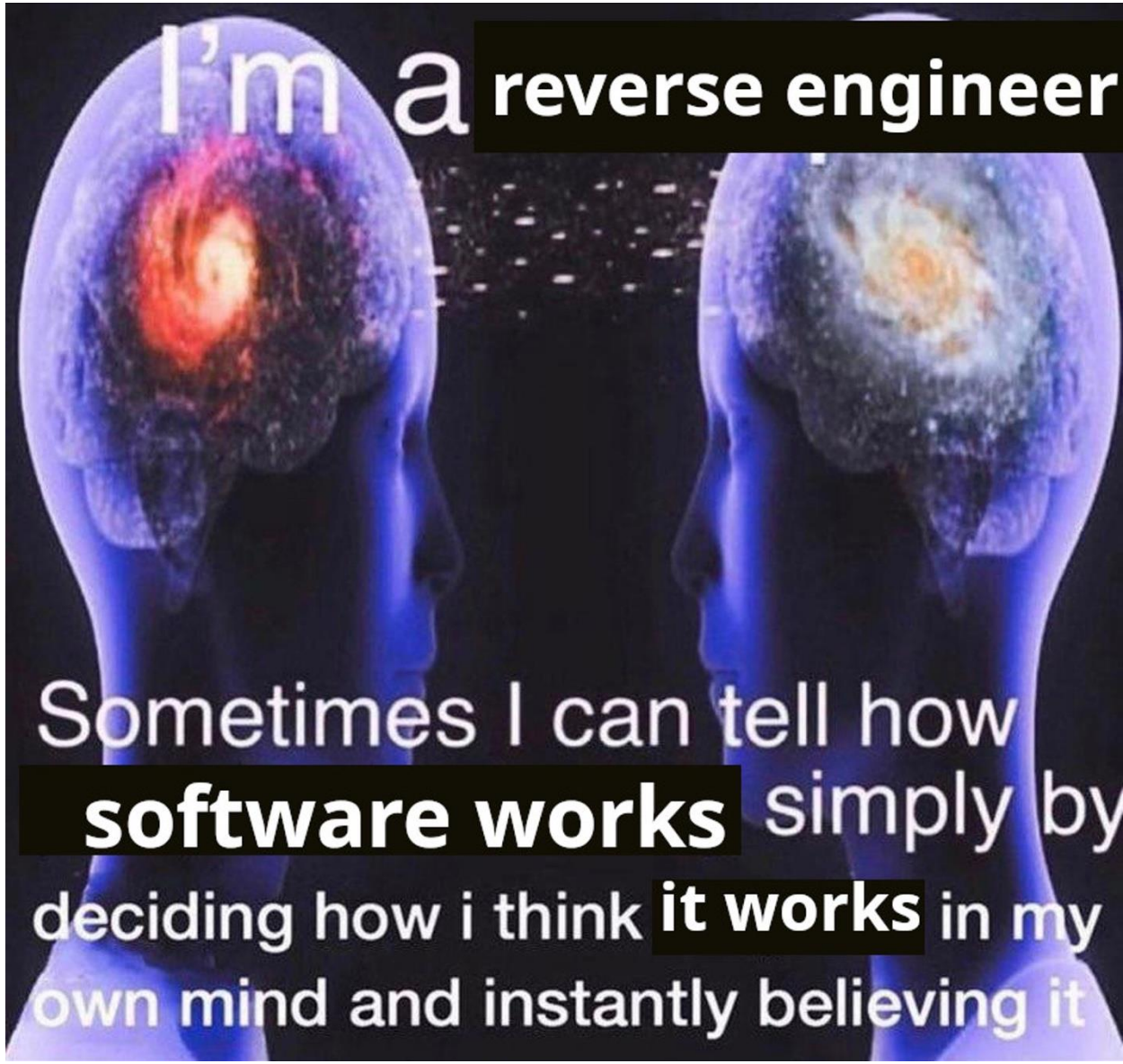
Announcements

Lab 9 due **tomorrow**

Program 3 due **Friday**

Margot Lee Shetterly talk today @ 6pm

Program 4 posted, due ~two weeks from now (November 15)

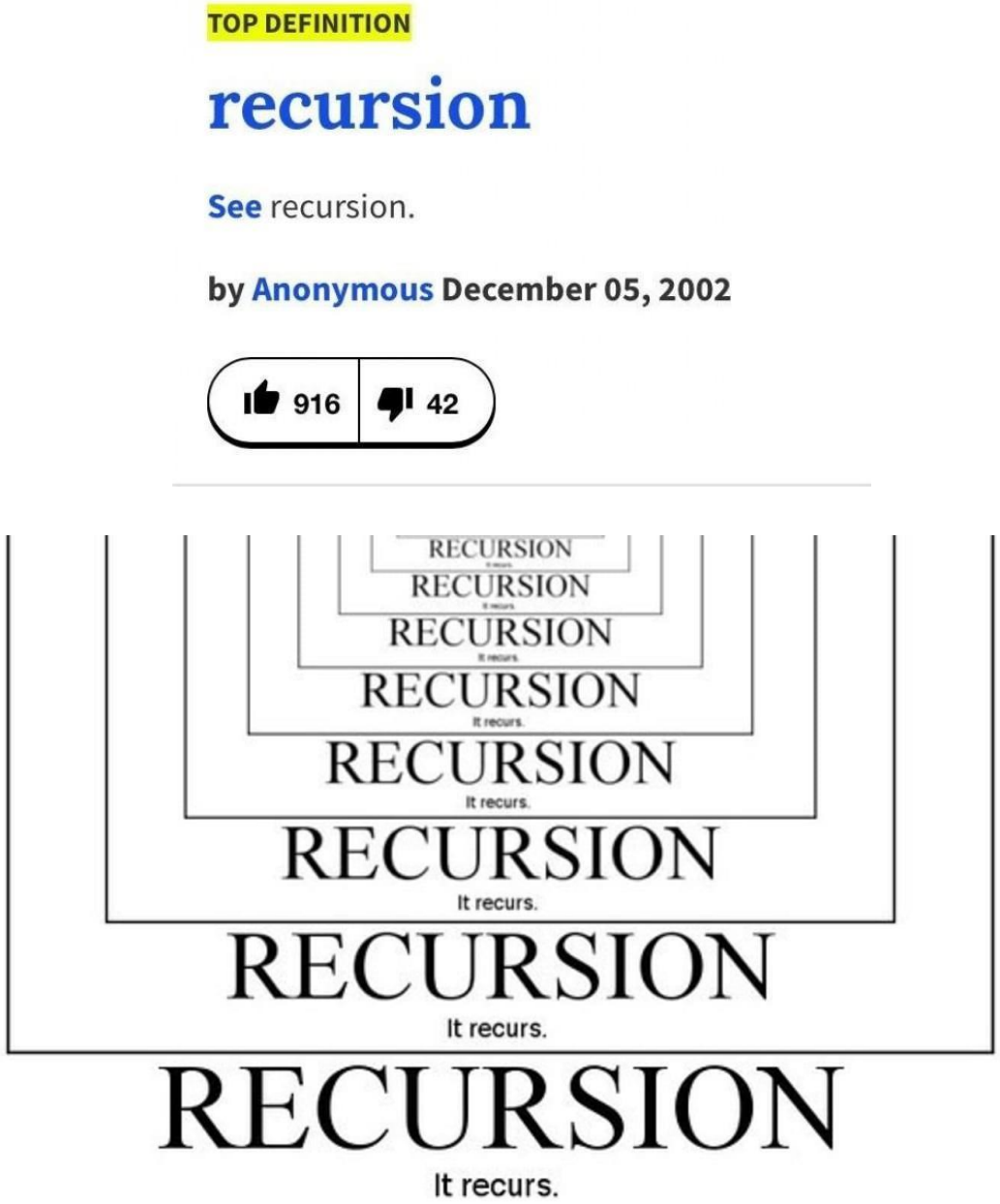


Program 4

Recursion is a problem-solving technique that involves a *method calling itself* to solve some smaller problem

```
static int factorial(int n)
    {
        if (n == 0)
            return 1;

        return n * factorial(n - 1);
    }
```



```
static int factorial(int n)
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```
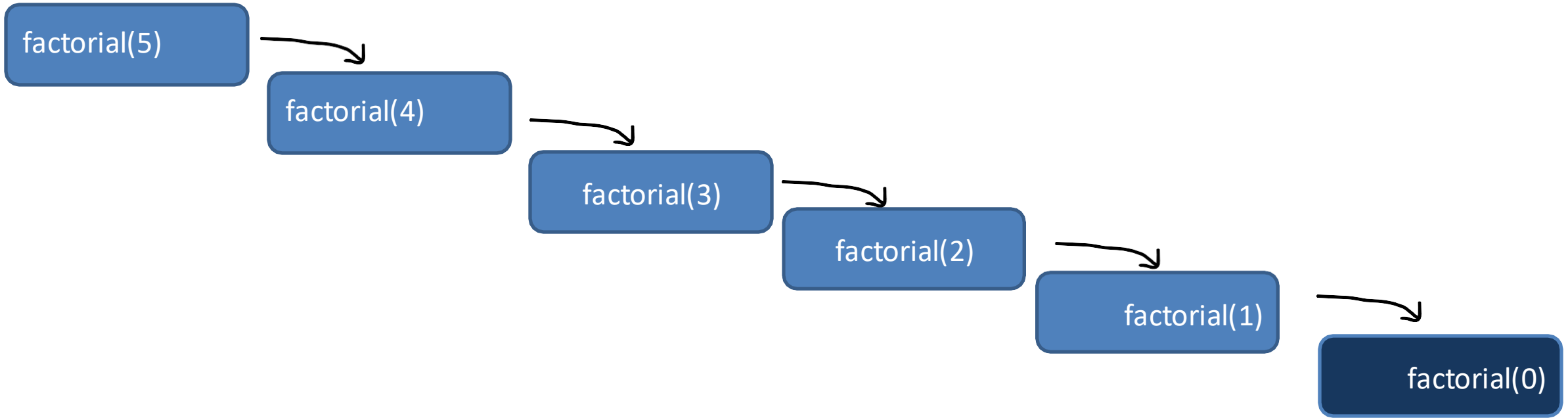

We can solve the factorial for n by solving smaller problems (factorial of n-1) !



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```

(base case)

(recursive case)

factorial(5)

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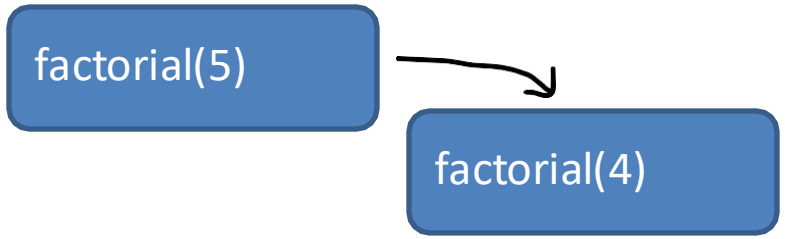
(recursive case)



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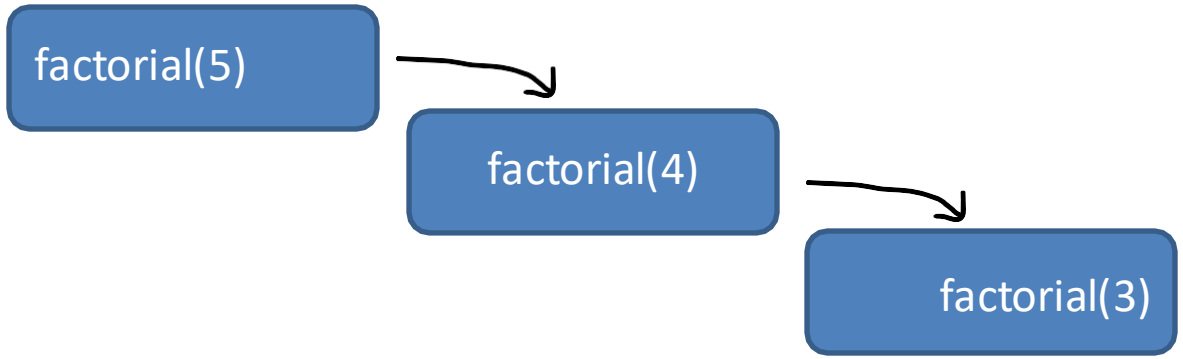

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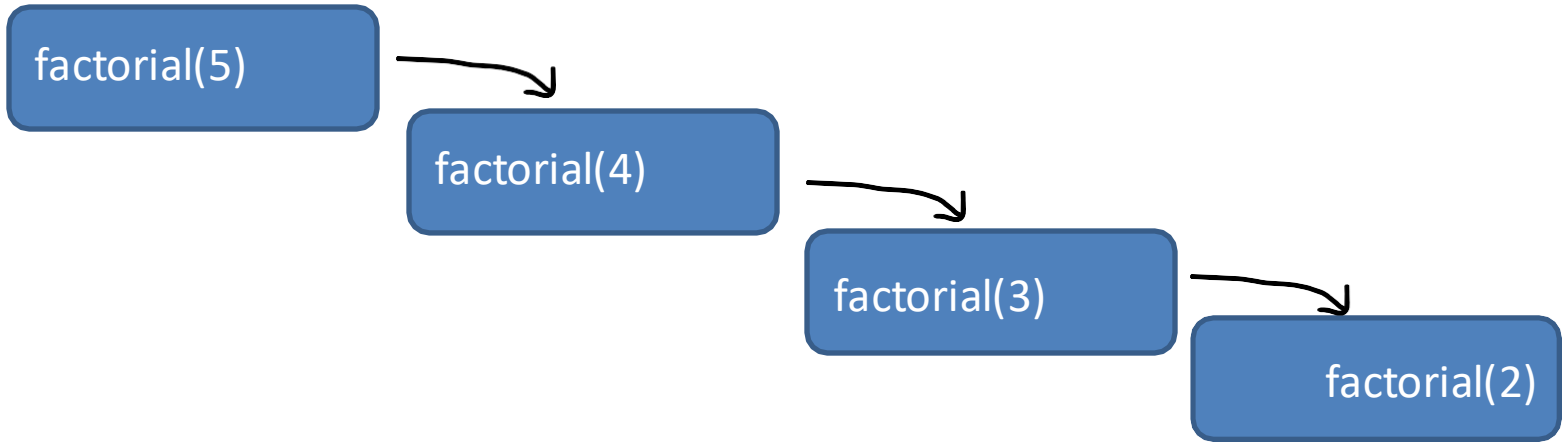

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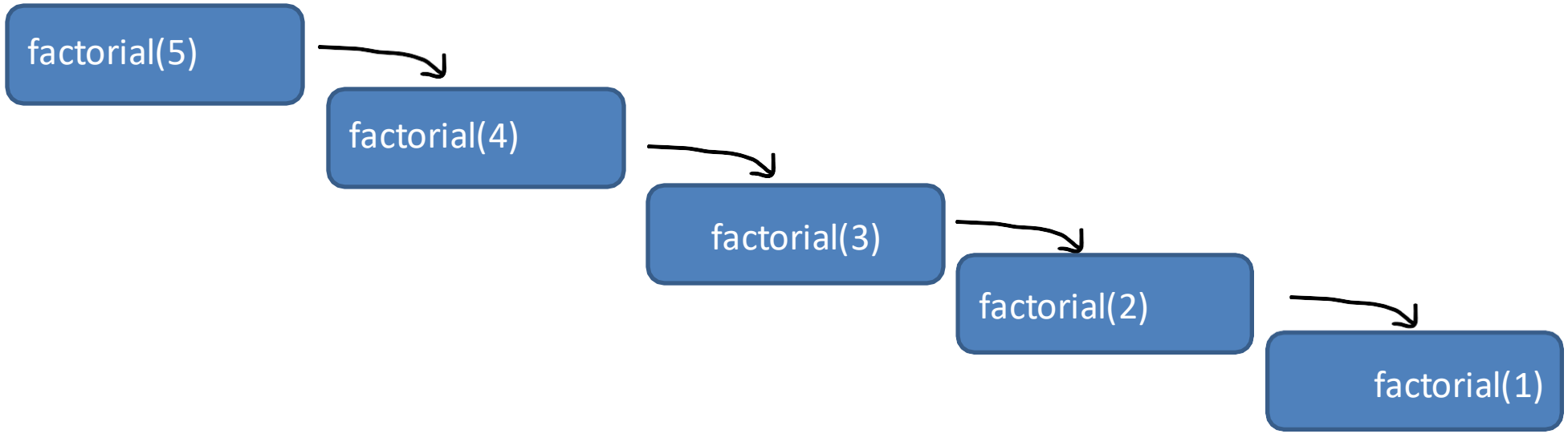

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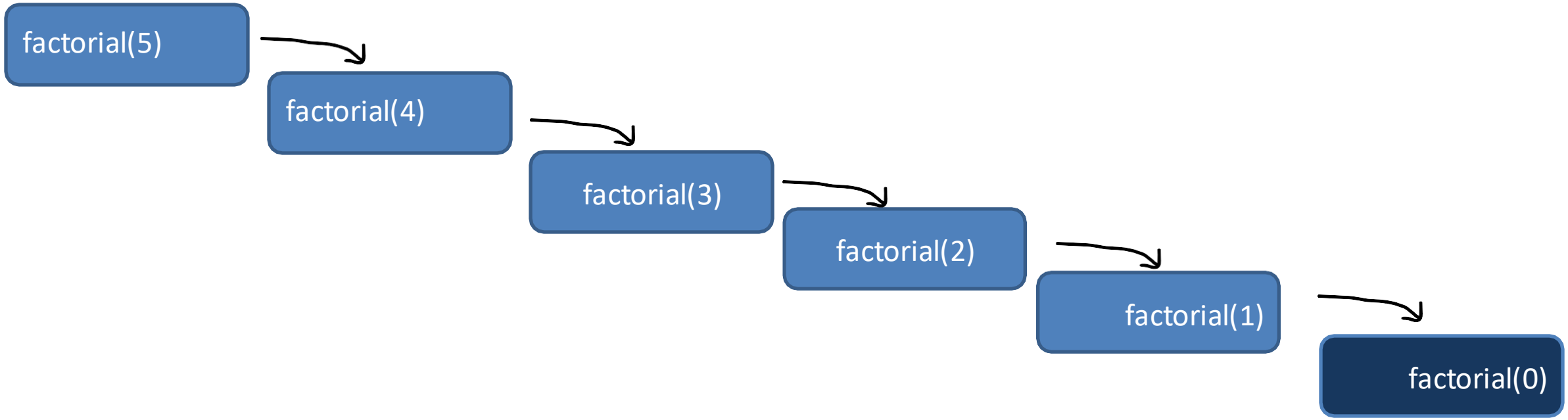

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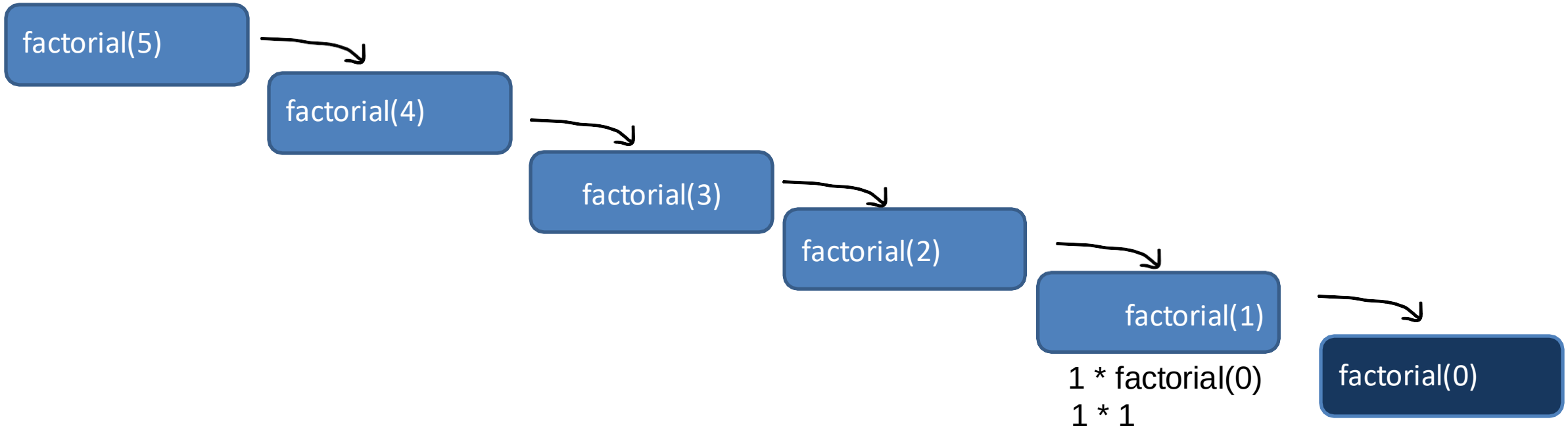

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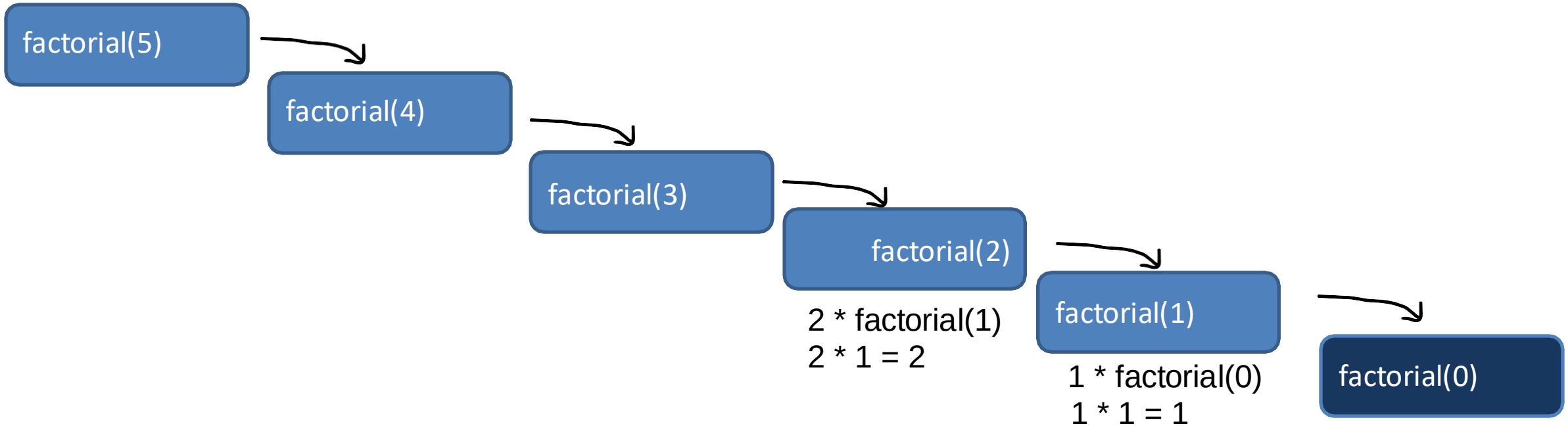

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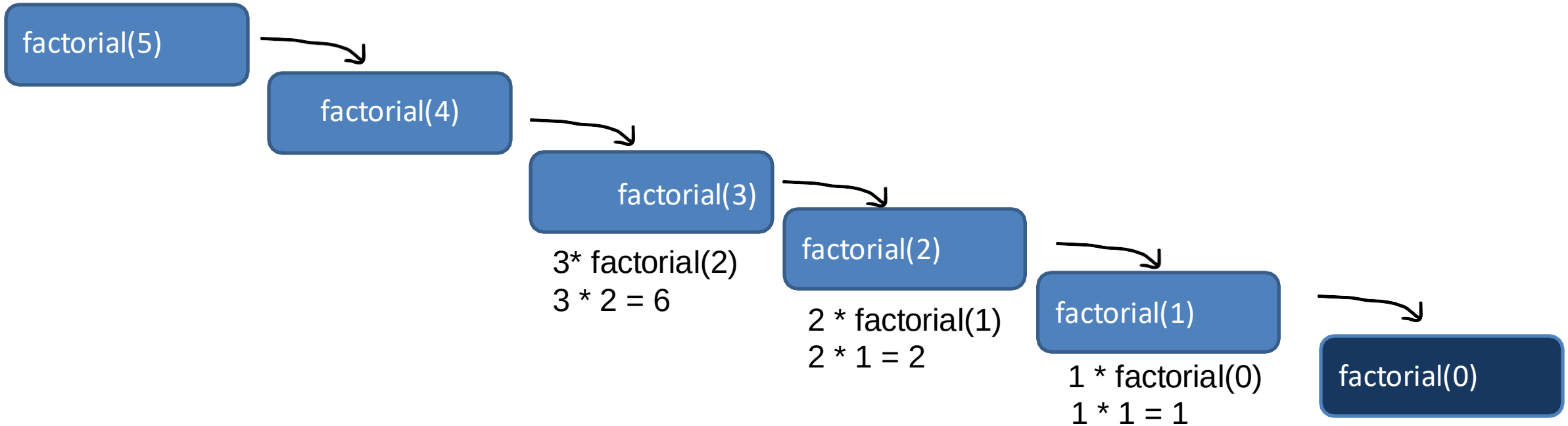

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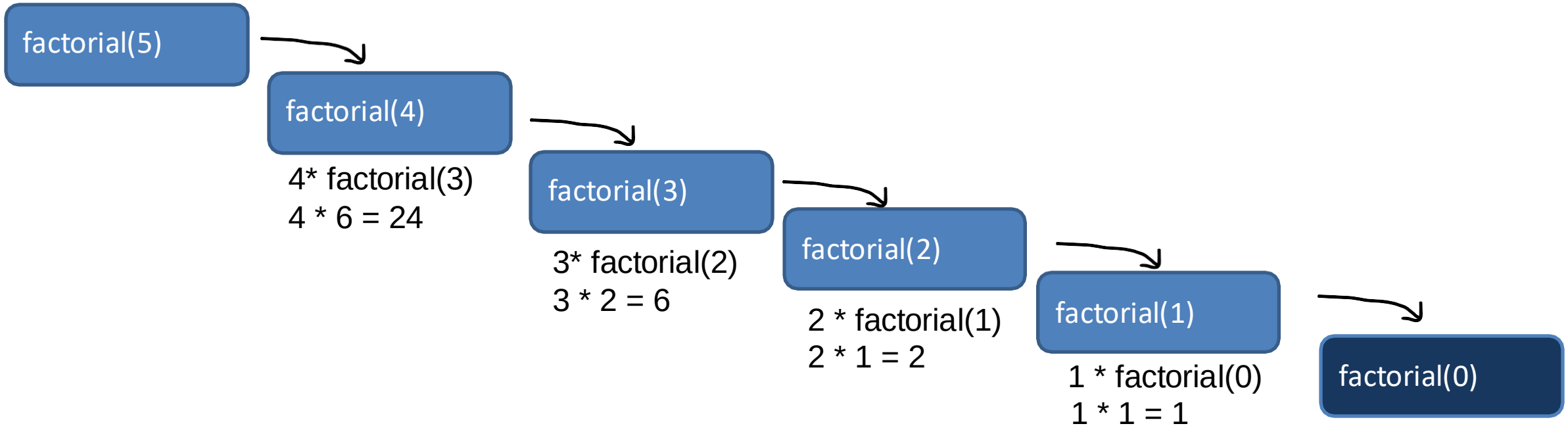

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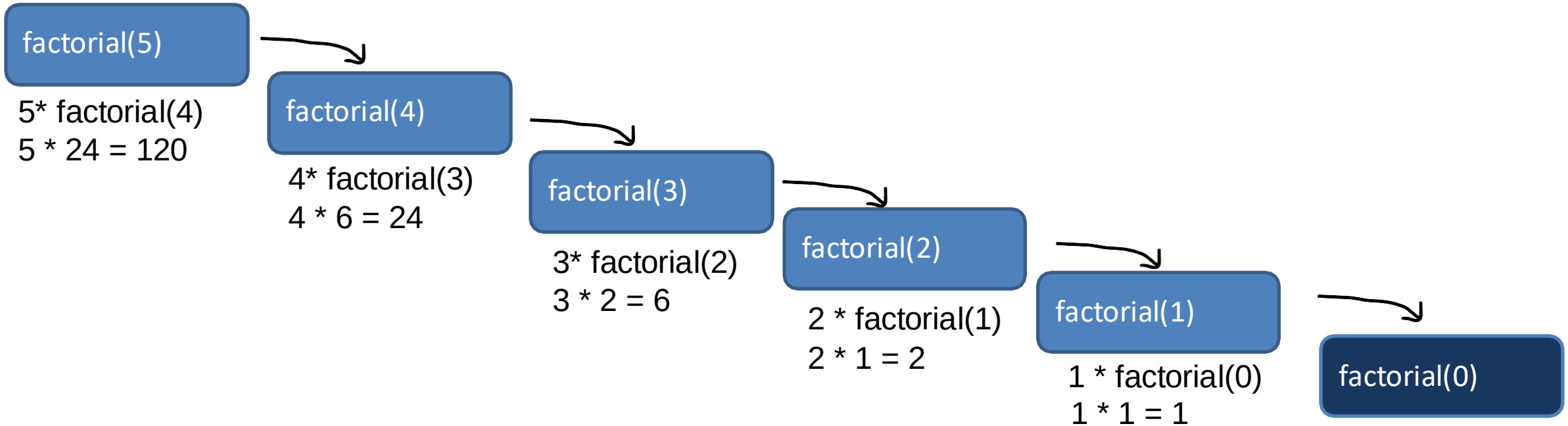

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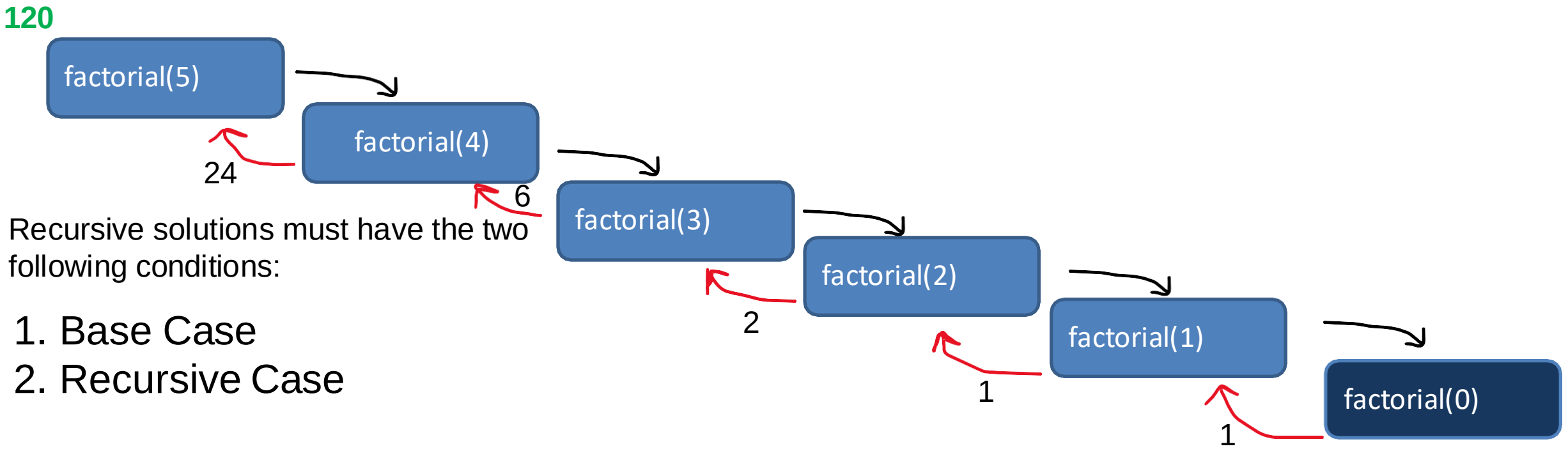

Recursive solutions must have the two following conditions:

1. Base Case
2. Recursive Case

The Fibonacci sequence is a sequence in which each number is the sum of the two preceding ones

So, the N^{th} digit of the Fibonacci Sequence = f(N-1) + f(N-2)

The Fibonacci Sequence

1,1,2,3,5,8,13,21,34,55,89,144,233,377…

1+1=2
1+2=3
2+3=5
3+5=8
5+8=13
8+13=21
13+21=34
21+34=55
34+55=89
55+89=144
89+144=233
144+233=377

Because the solution to some problem can be expressed in terms of some smaller problem(s), recursion may be a good fit here

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So, the N^{th} digit of the Fibonacci Sequence = f(N-1) + f(N-2)

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Base Case?

Recursive Case?

Calculate

The Fibonacci sequence is a sequence in which each number is the sum of the two preceding ones

So, the N^{th} digit of the Fibonacci Sequence = f(N-1) + f(N-2)

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1,1,2,3,5,8,13,21,34,55,89,144,233,377…

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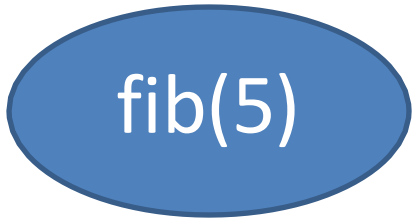
Base Case?

If finding the 1^{st} or 2^{nd} digit, return 1

Recursive Case?

Calculate the previous two digits, f(n-1), f(n-2)




```
private static int fib(int n) {
  if(n == 1 || n == 2) {
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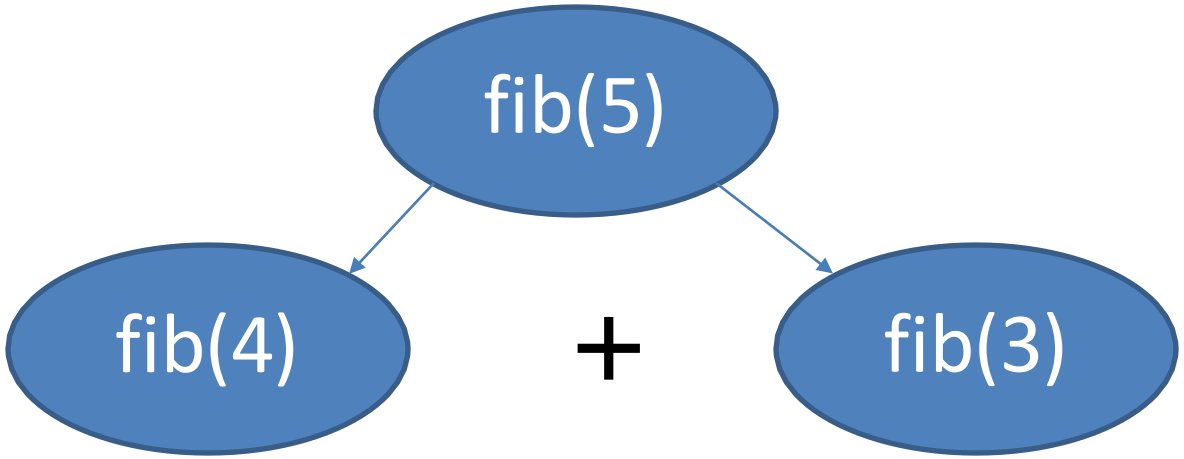


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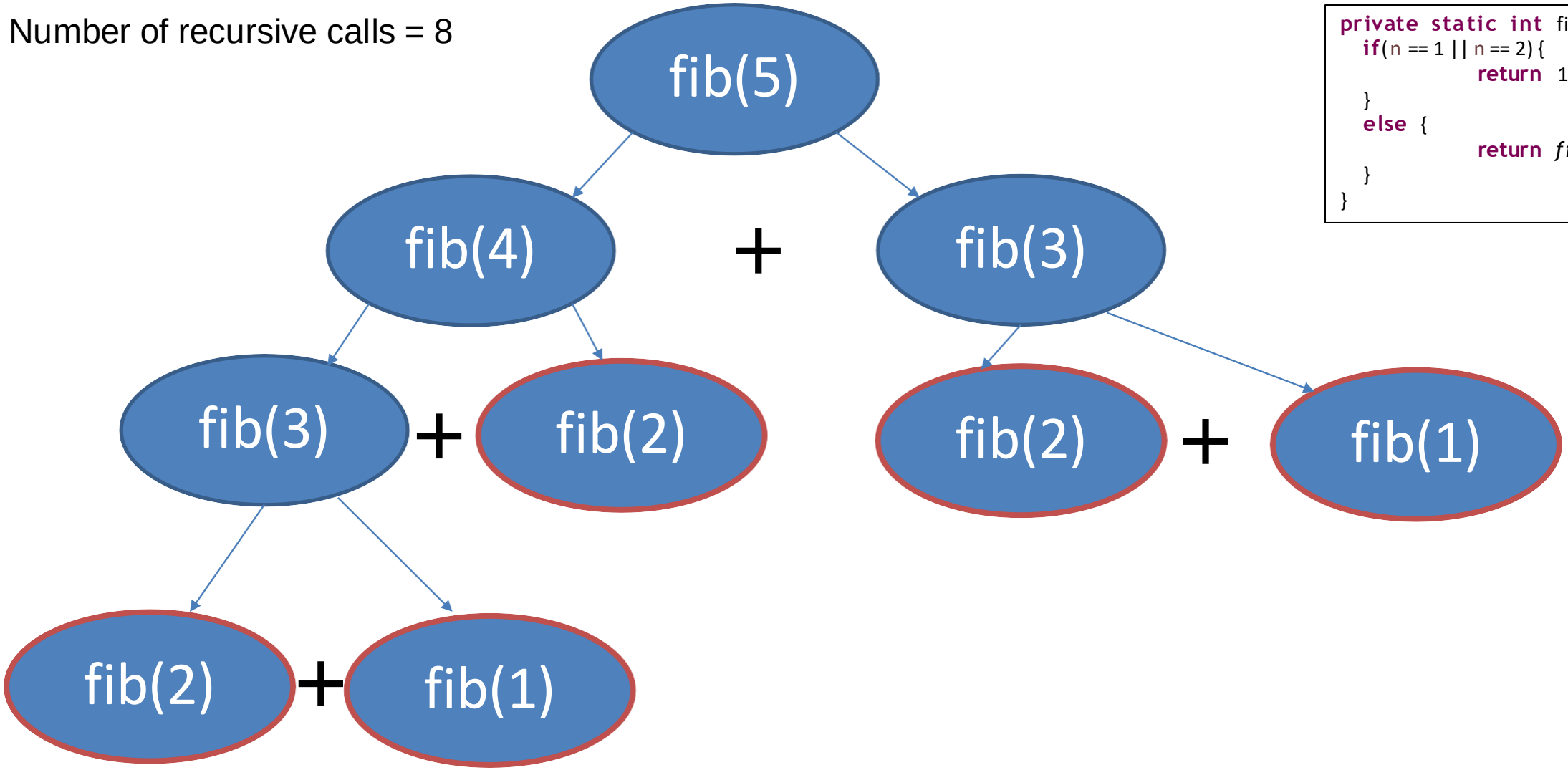
```

Number of recursive calls = 8

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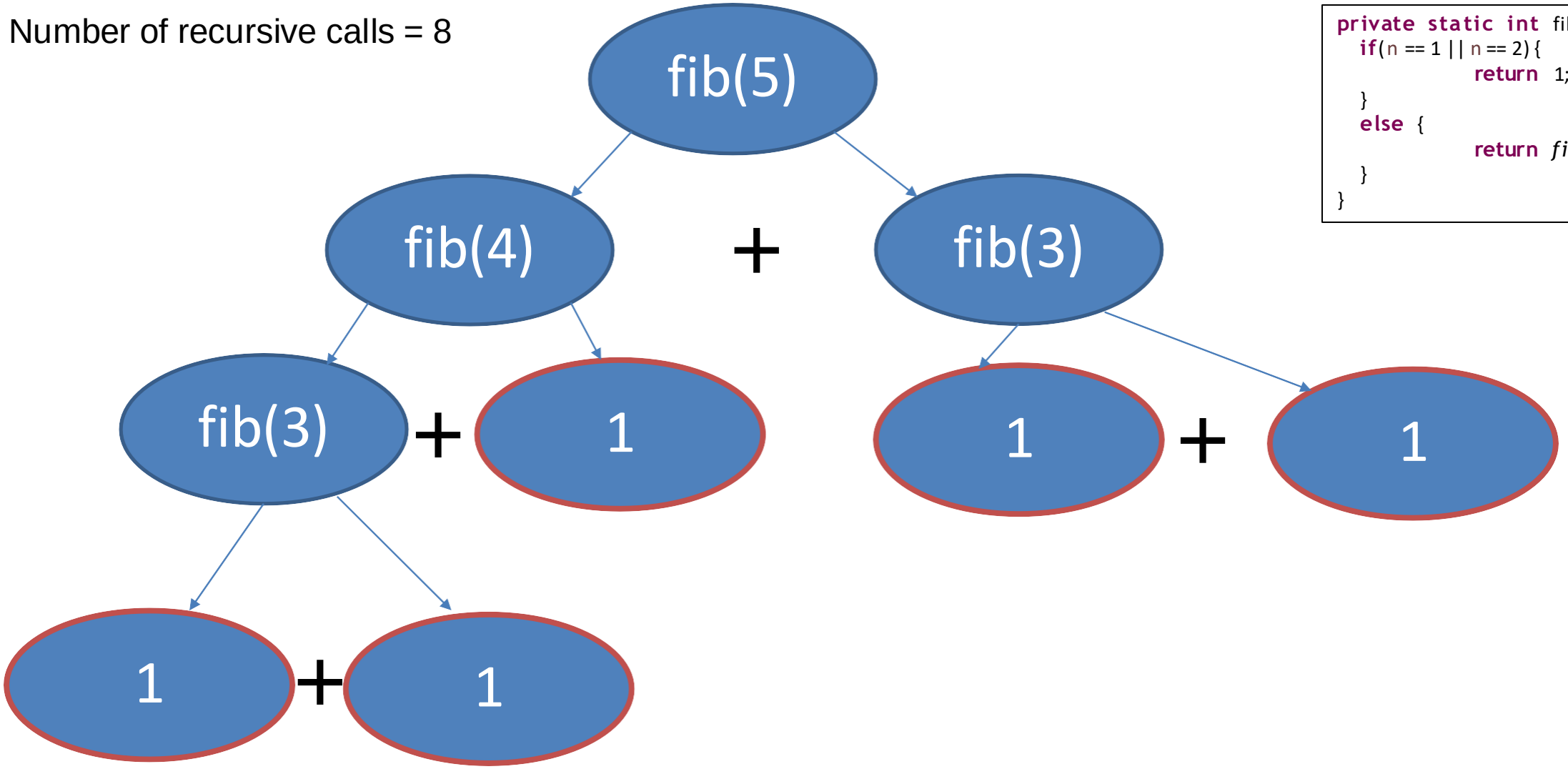


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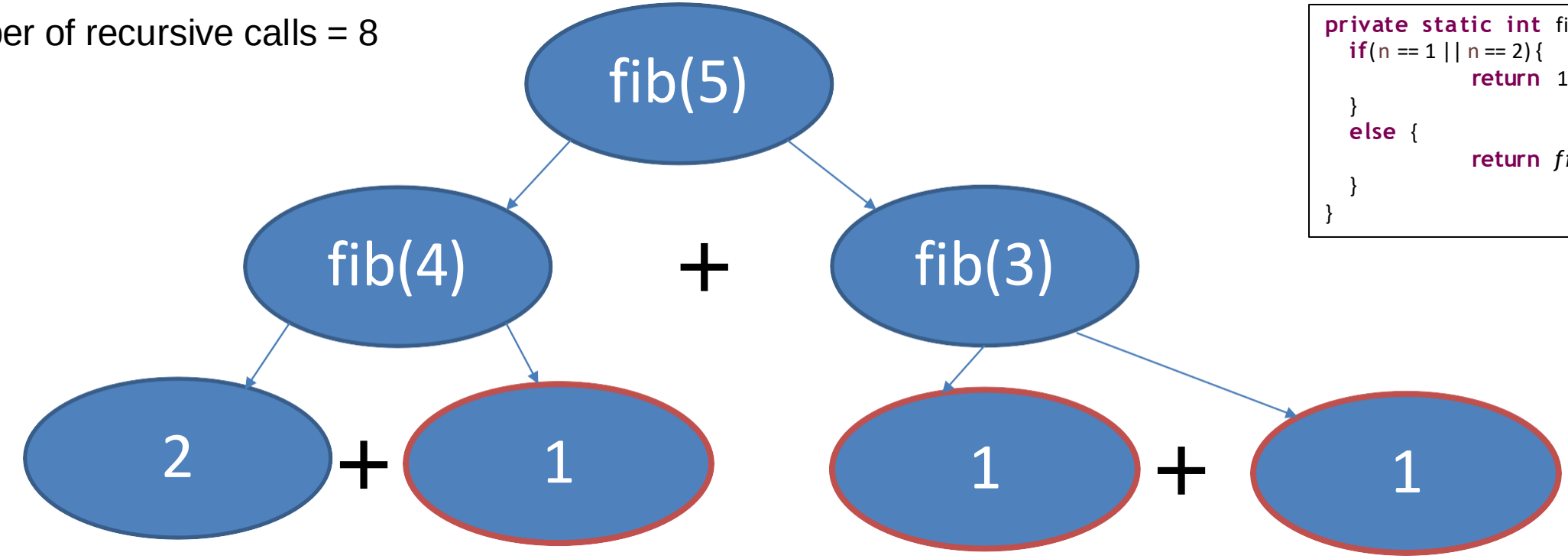


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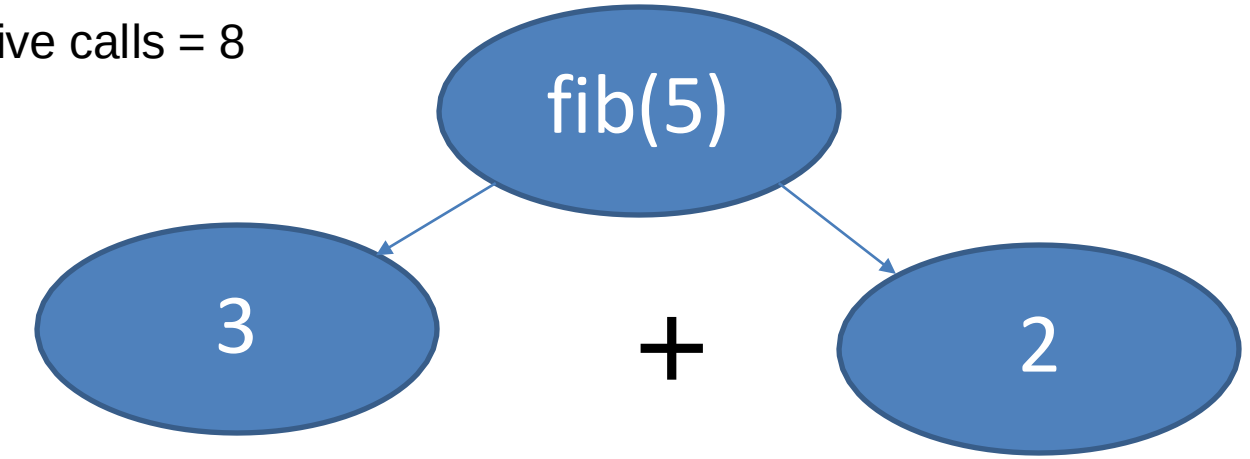


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Number of recursive calls = 8

Final answer!

```
private static int fib(int n) {
  if(n == 1 || n == 2) {
            return 1;
  }
  else {
            return fib(n-1) + fib(n-2);
  }
}
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```
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   }
   else {
         return fib(n-1) + fib(n-2);
   }
}
```

Running Time?

```
private static int fib(int n) {
  if(n == 1 || n == 2) {O(1)
      return 1;O(1)
  }
  else {          O(1)      O(1)
      return fib(n-1) + fib(n-2);
  }
}
```

Running Time?

O(1) ?

```
private static int fib(int n) {
  if(n == 1 || n == 2) {O(1)
      return 1;O(1)
  }
  else {          O(1)      O(1)
      return fib(n-1) + fib(n-2);
  }
}
```

Running Time?

~~O(1) ?~~

No!

When we are analyzing recursive algorithms, we have to calculate running time slightly different

```
private static int fib(int n) {
   if(n == 1 || n == 2) {
        return 1;
   }
   else {
        return fib(n-1) + fib(n-2);
   }
}
```

Generally speaking, we can compute the running time of a recursive algorithm by using the following formula:

Running time = # of recursive calls made * amount of work done in each call

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private static int fib(int n) {
  if(n == 1 || n == 2) {O(1)
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Generally speaking, we can compute the running time of a recursive algorithm by using the following formula:

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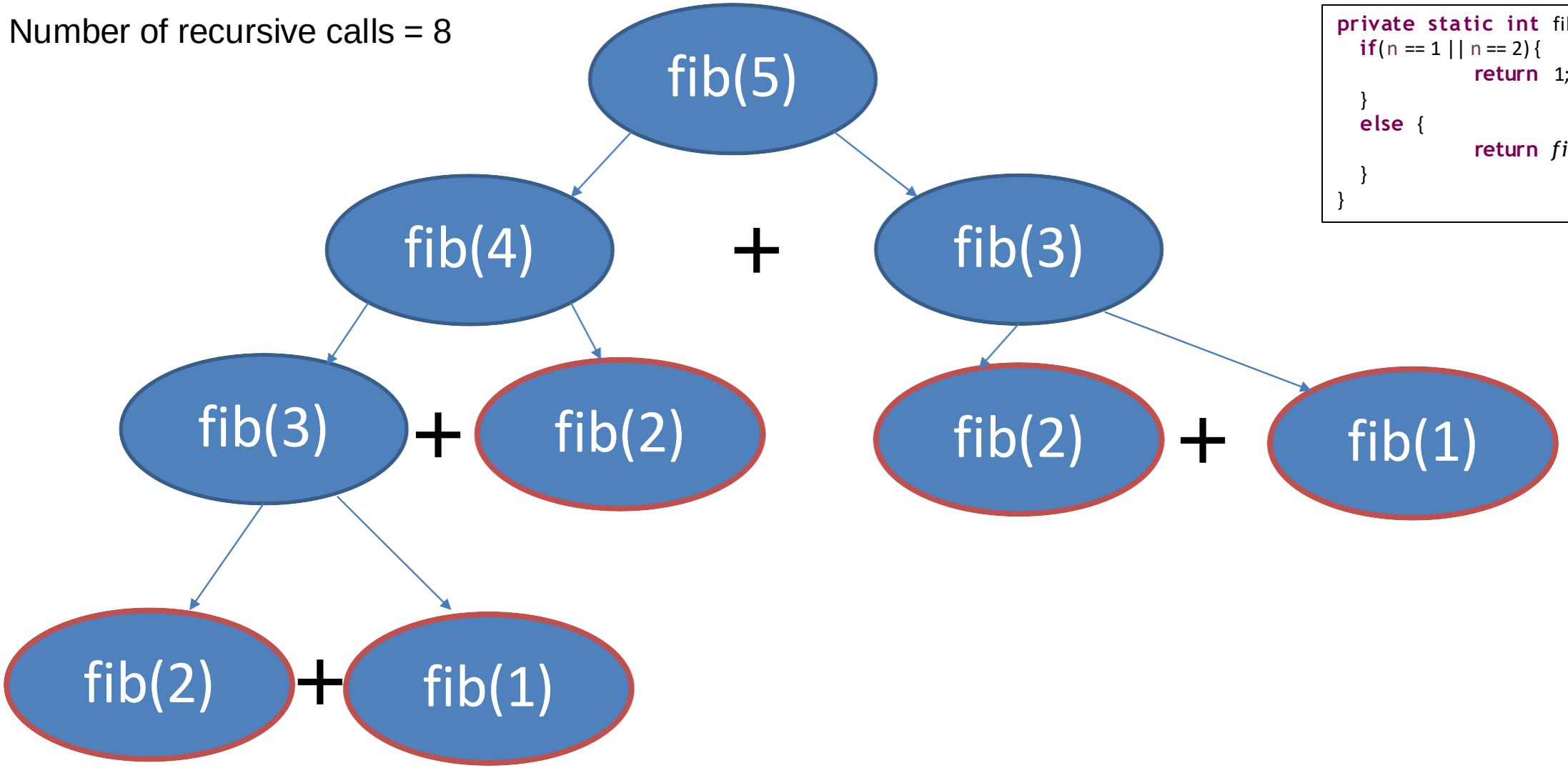
Running time = ??? * **O(1)**

Number of recursive calls = 8

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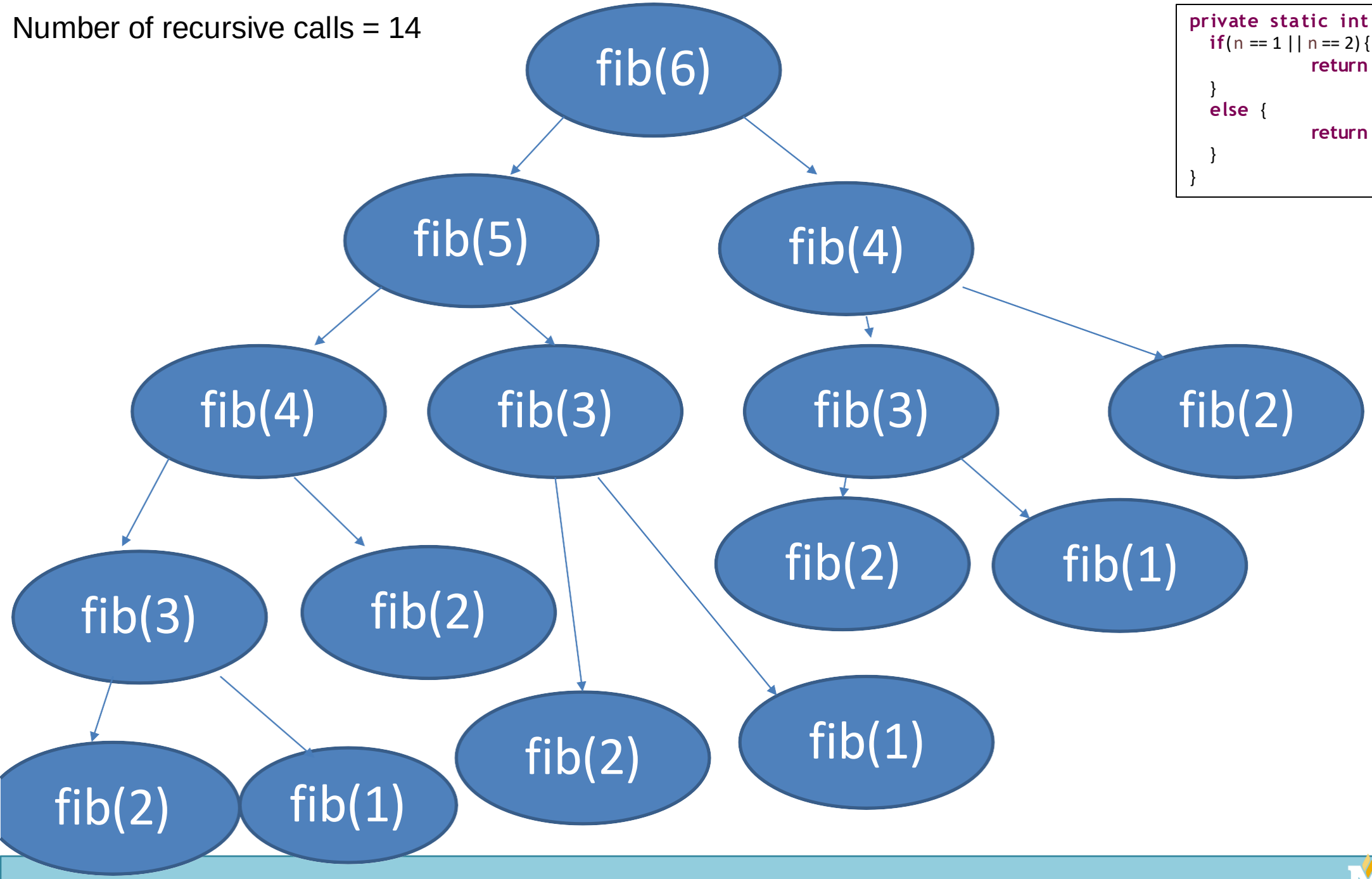
  }
}
```

Number of recursive calls = 14

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  else {
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  }
}
```

Number of recursive calls = 24

```
private static int fib(int n) {
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            return 1;
  }
  else {
            return fib(n-1) + fib(n-2);
  }
}
```

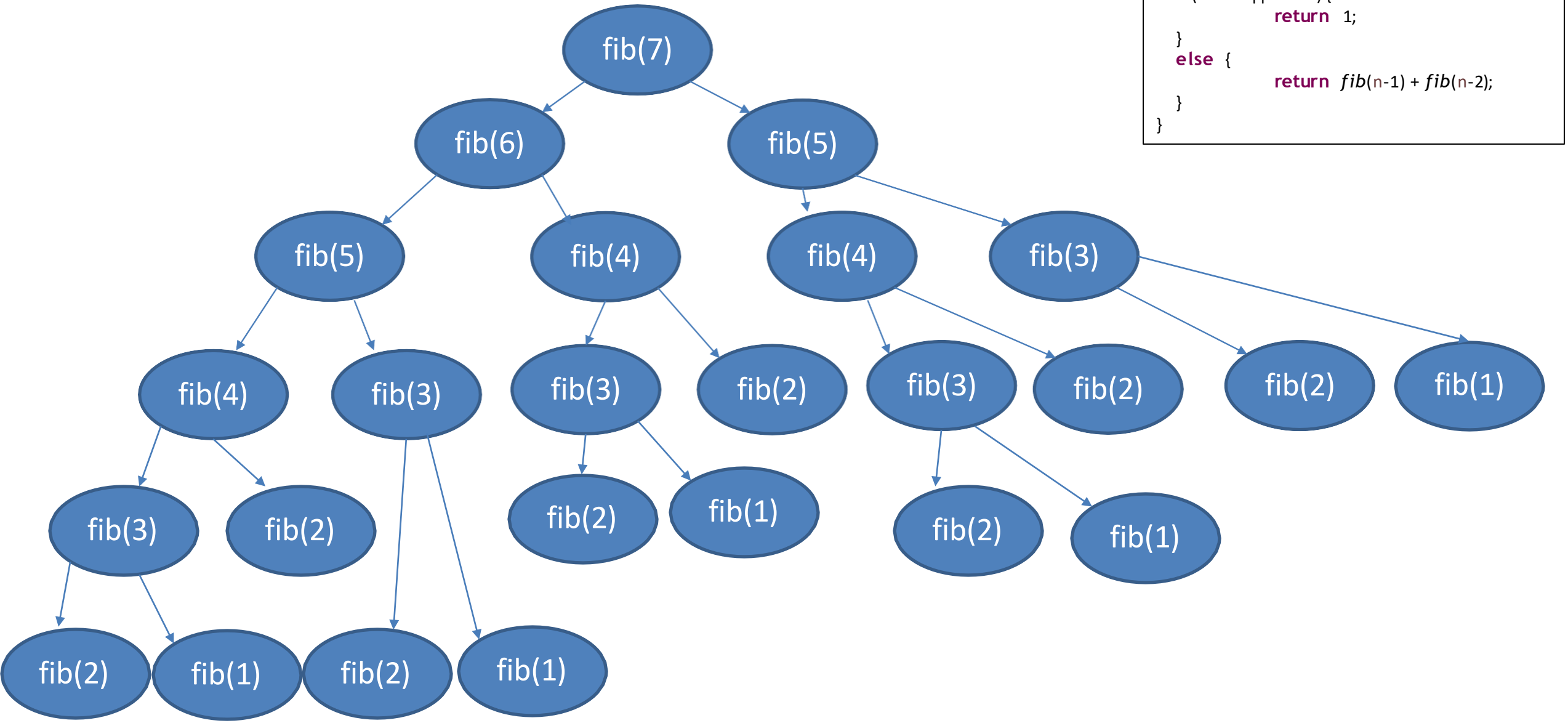

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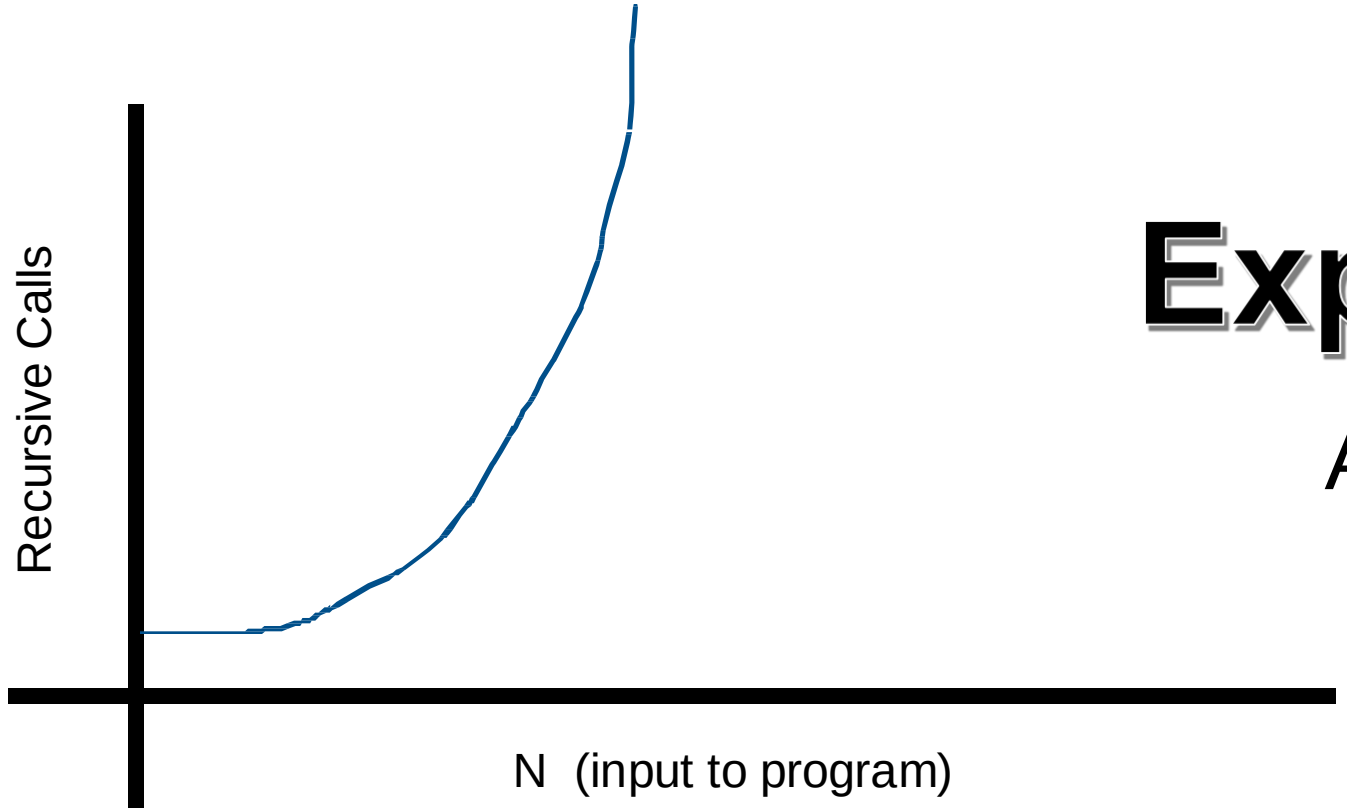
```

If we were to plot the number of recursive calls made as n increases, it would look something like his:



Exponential

Aka. $O(2^n)$

```
private static int fib(int n) {
   if(n == 1 || n == 2) {O(1)
         return 1;   O(1)
   }
   else {            O(1)      O(1)
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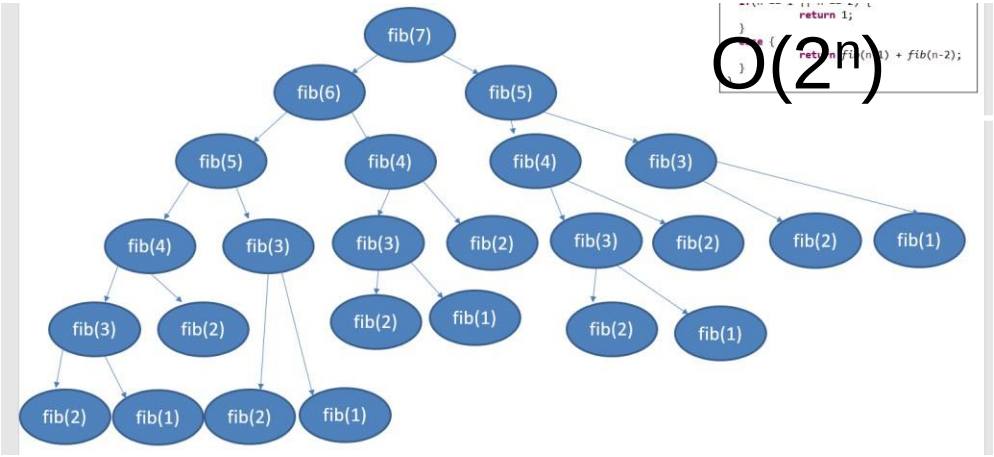

$O(2^n)$

Generally speaking, we can compute the running time of a recursive algorithm by using the following formula:

Running time = # of recursive calls made * amount of work done in each call

Running time = **$O(2^n)$ * O(1)**

Total running time = **$O(2^n)$**

n = requested Fibonacci digit

$O(2^n)$ is very bad…

Number of recursive calls = 24

```
private static int fib(int n) {
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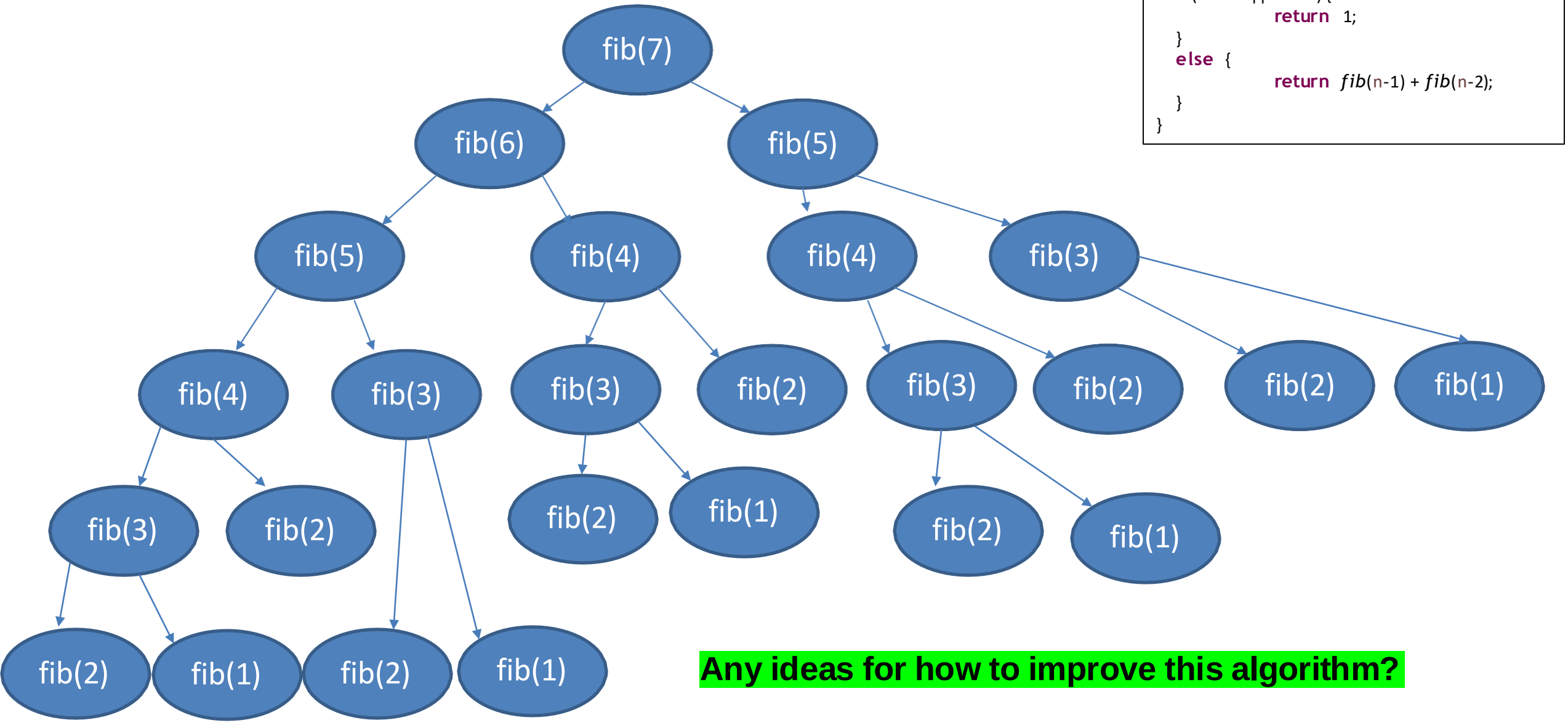

Any ideas for how to improve this algorithm?

Number of recursive calls = 24

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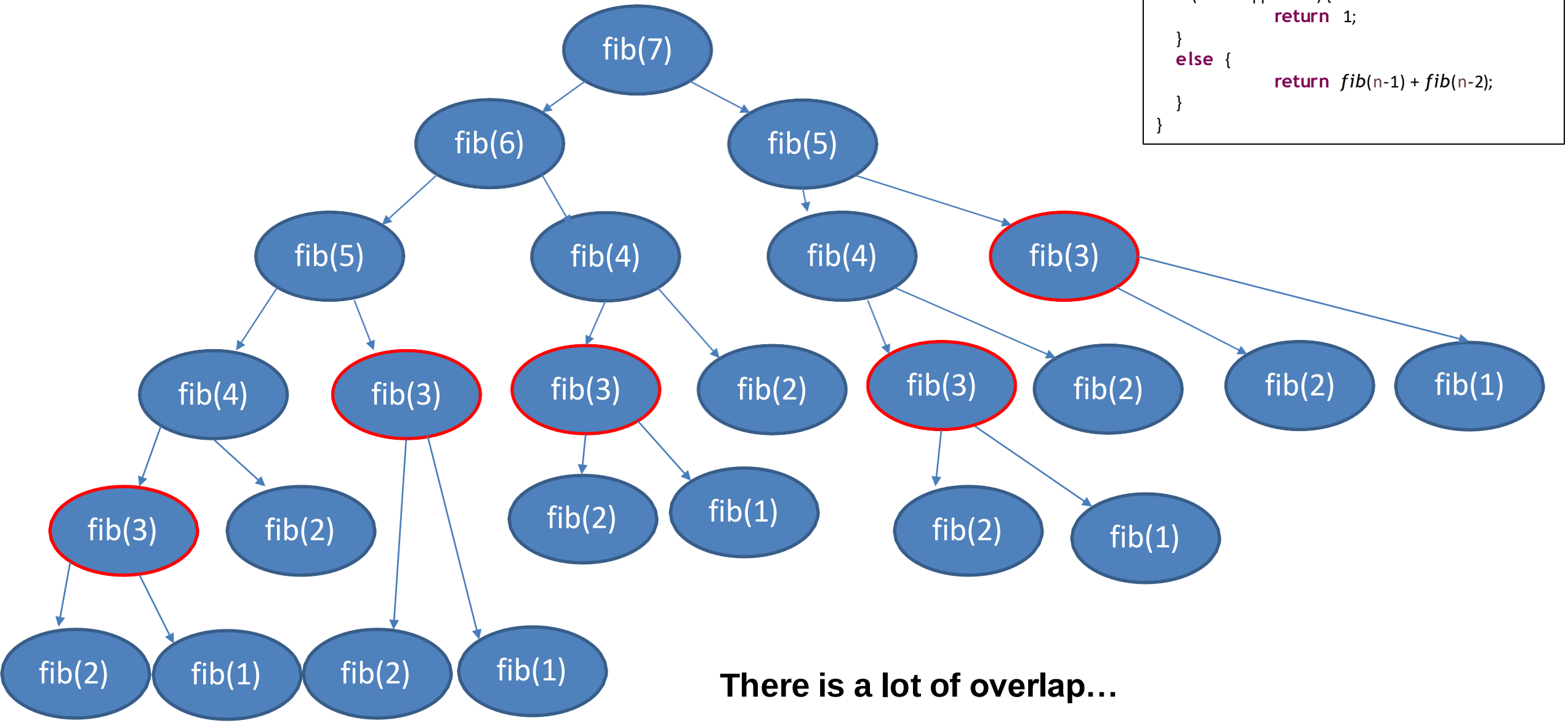

There is a lot of overlap…

Number of recursive calls = 24

fib(1)	1
fib(2)	1
fib(3)	3
fib(4)	…
fib(5)	…

```
private static int fib(int n) {
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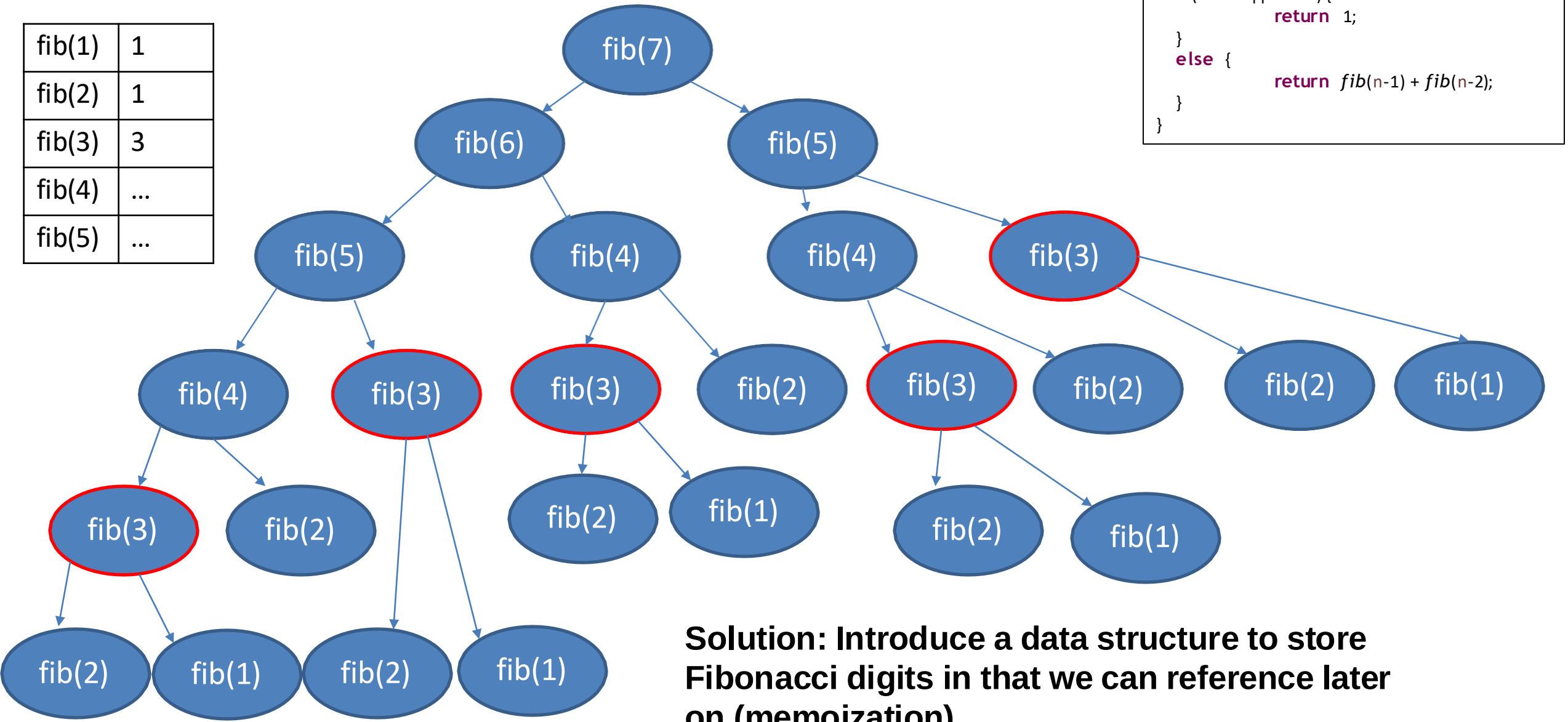

Solution: Introduce a data structure to store Fibonacci digits in that we can reference later on (memoization)

(These lookups happen in constant time!)

Limitations of recursion?