CSCI 132:
Basic Data Structures and Algorithms

Searching (Binary Search)

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Fall 2024

Announcements

Lab 12 posted
→ You can reference a lot of the code from today

(jk, send a message on discord)

Searching

We store values in data structures, but we also need to retrieve/search for values!

Today, we will discuss techniques for how to search for a value in a data structure

(We will be using arrays, but these techniques could also be used on Linked Lists, queues, stacks, etc)

Searching

Option 1: Linear Search

Check every spot until one by one until we find what we are looking for

```
public int linear_search(int[] array, int s) {
      for(int i = 0; i < array.length; i++) {
                  if(array[i] == s)  {
                     return i;
            }
      }
      return -1;
}
```

Searching

Option 1: Linear Search

Check every spot until one by one until we find what we are looking for

Not efficient for large data structures. **O(n)** running time

```
public int linear_search(int[] array, int s) {
    for(int i = 0; i < array.length; i++) {
        if(array[i] == s) {
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```

Searching

Option 1: Linear Search

Check every spot until we find one until we f

Not efficient for large

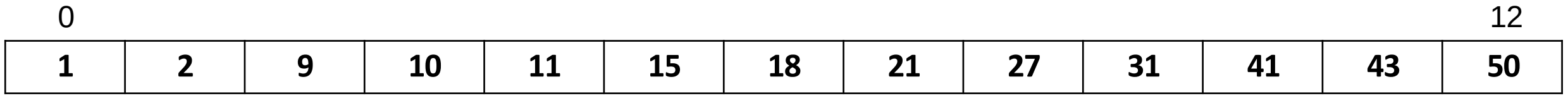
```
public
    for(
        }
    }
}
```

Can we do better?

0												12
1	**2**	**9**	**10**	**11**	**15**	**18**	**21**	**27**	**31**	**41**	**43**	**50**

What if our array is sorted?

```
Target Value: 27
```



We can leverage the fact that this array is sorted to make searching more efficient

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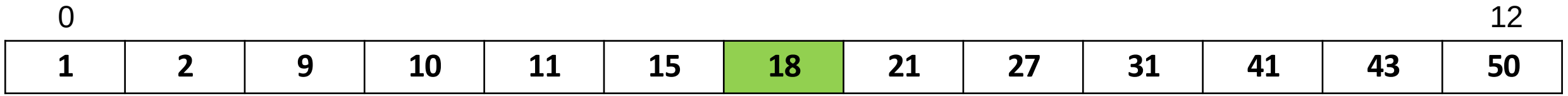

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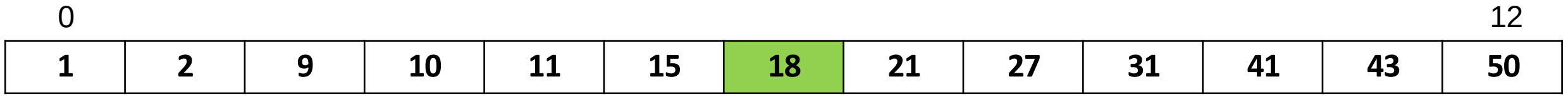

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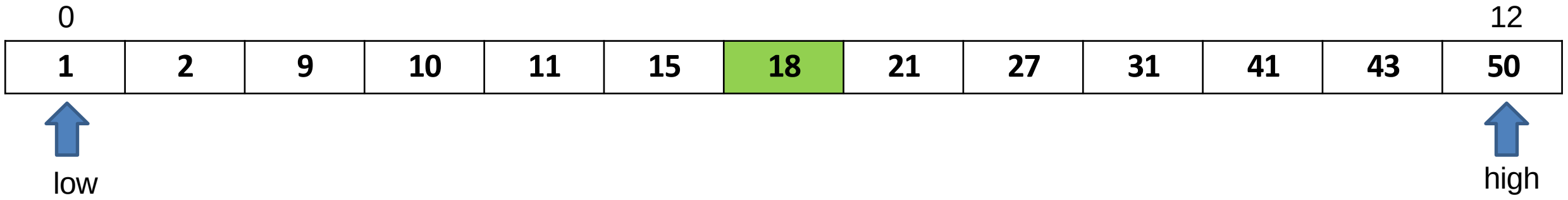

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We will define two pointers, `low` and `high` that point to the possible bounds of the target value

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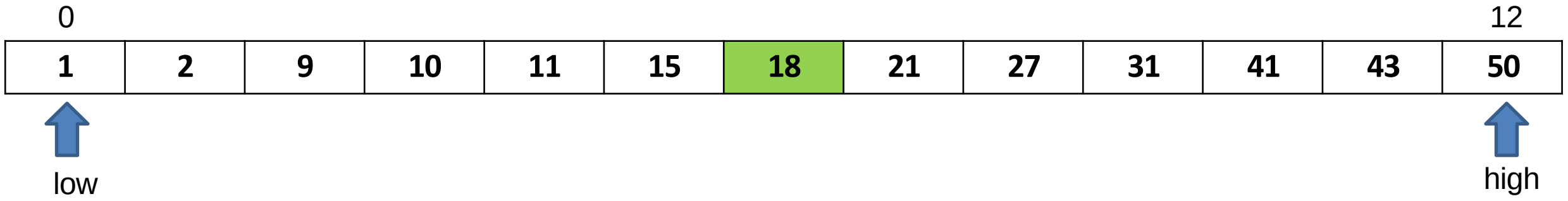

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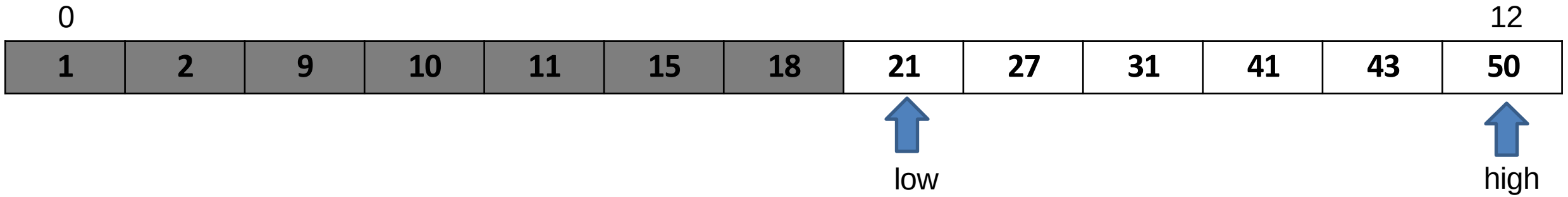

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→ If the target value is less than the middle, discard the "right section" of the array (move the high pointer)

Because we know the array is sorted, and the target value is greater than our mid point, then we know the target value must be located somewhere to the right.

We can eliminate half of the array!!!

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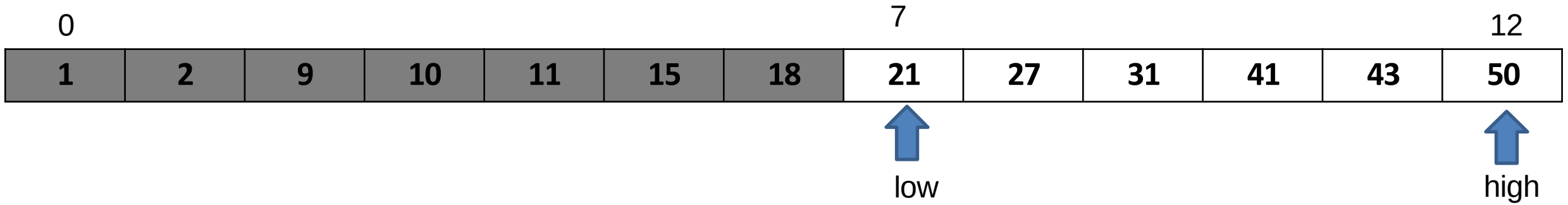

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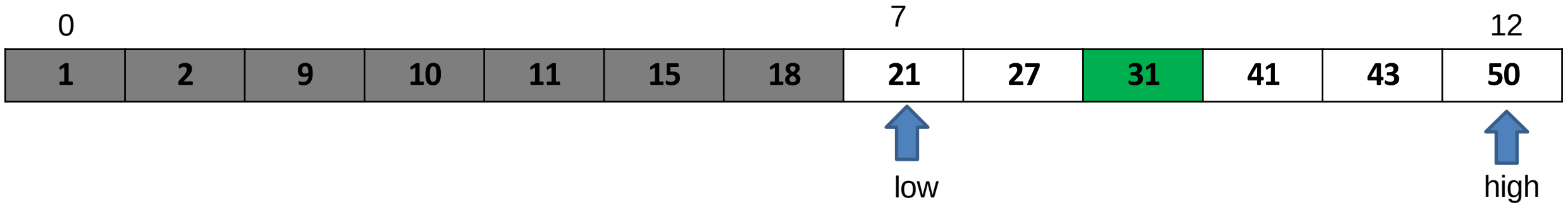

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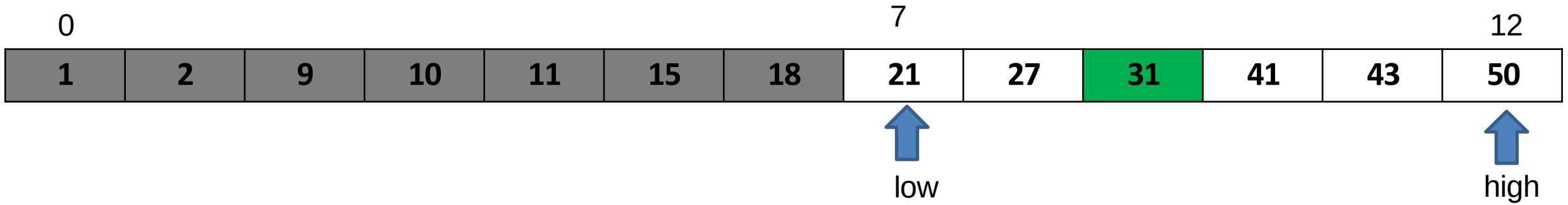

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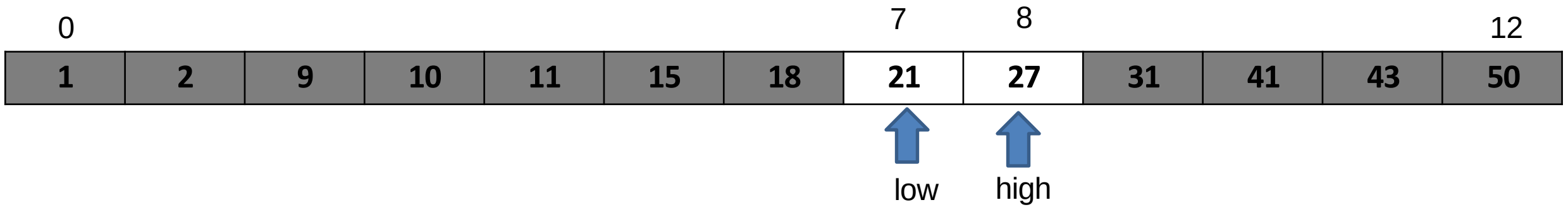

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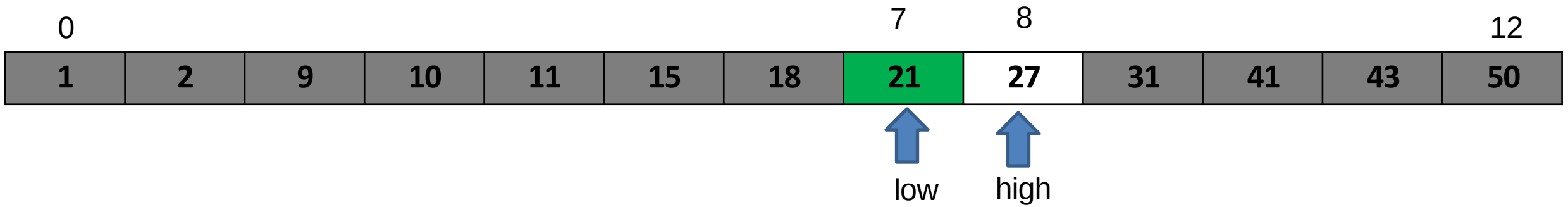

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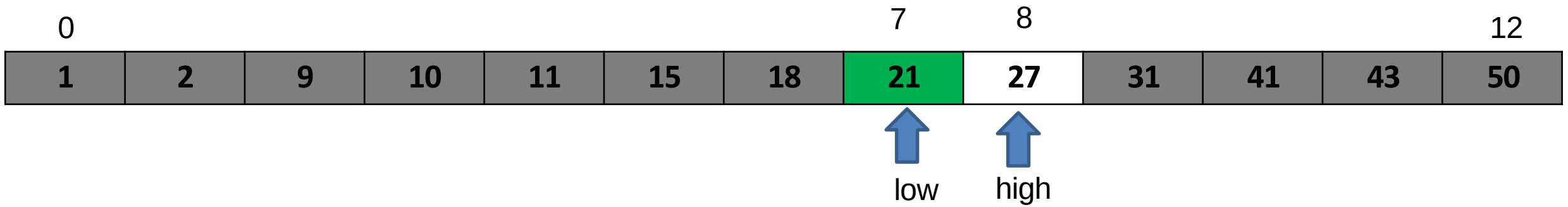

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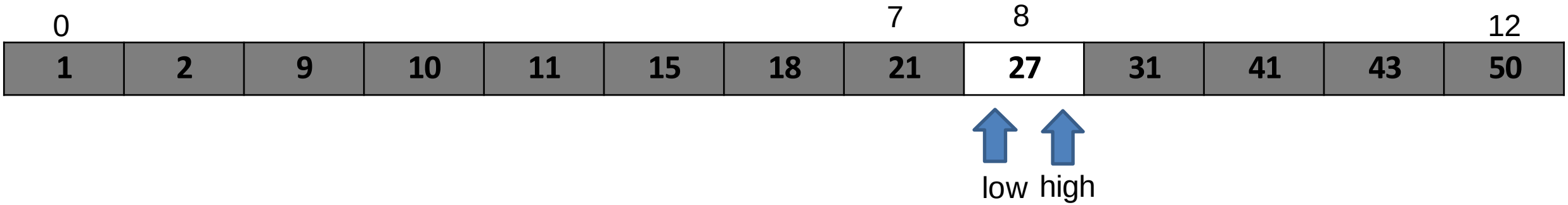

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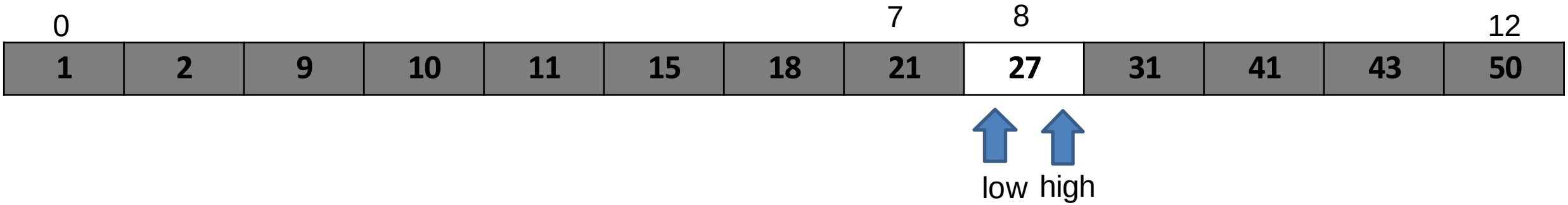

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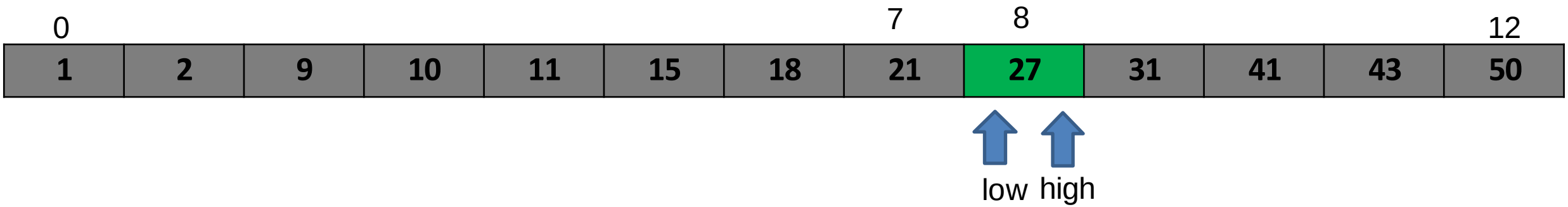

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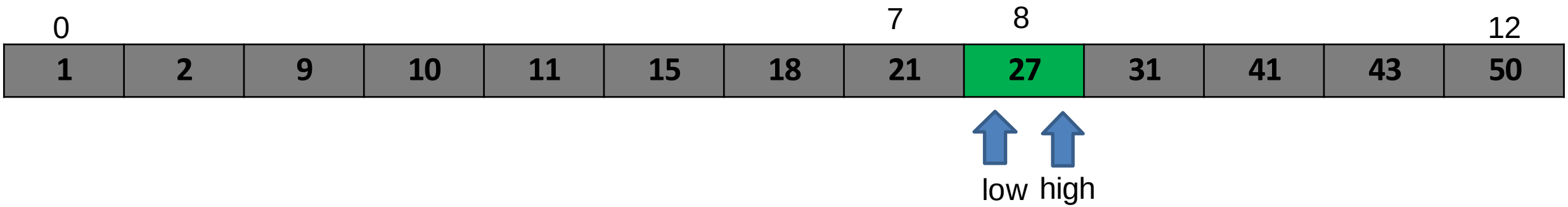

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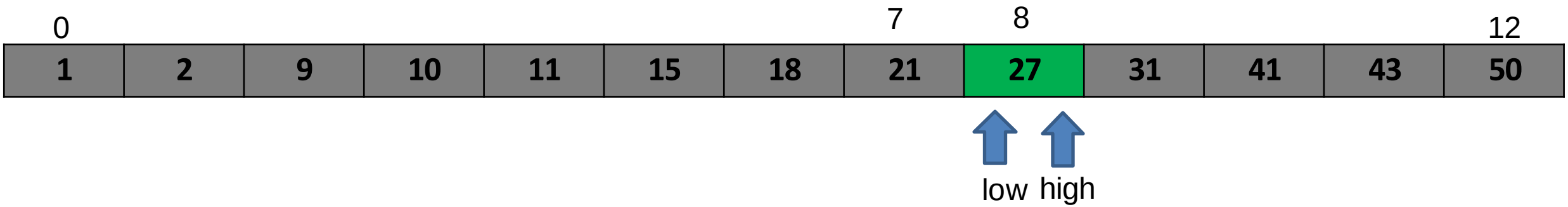

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This algorithm is known as **Binary Search**

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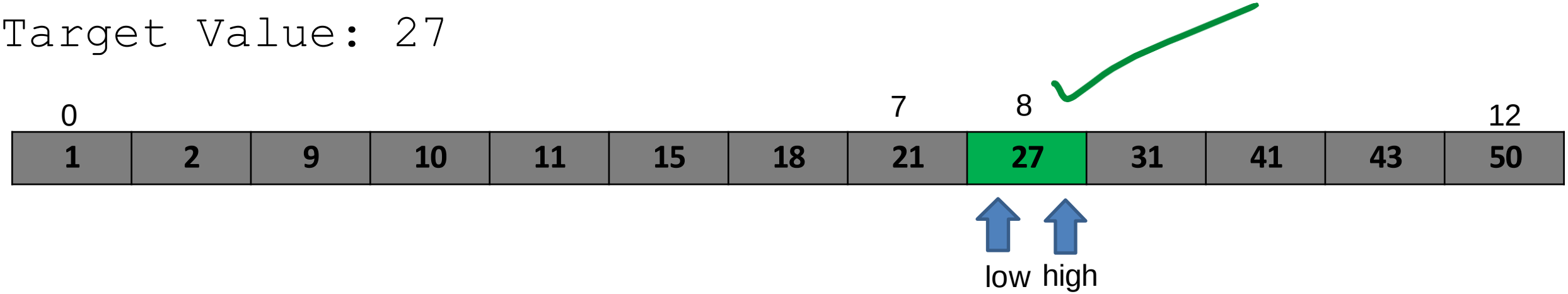

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How to calculate the mid point?

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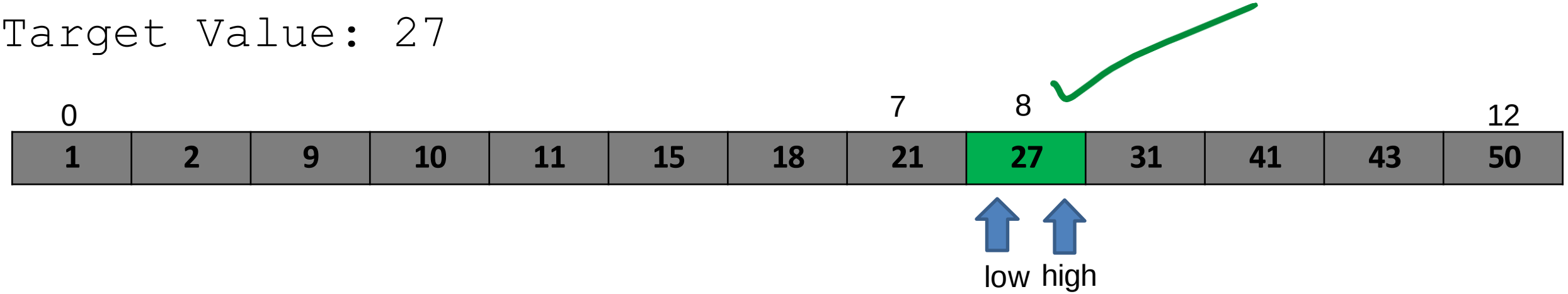

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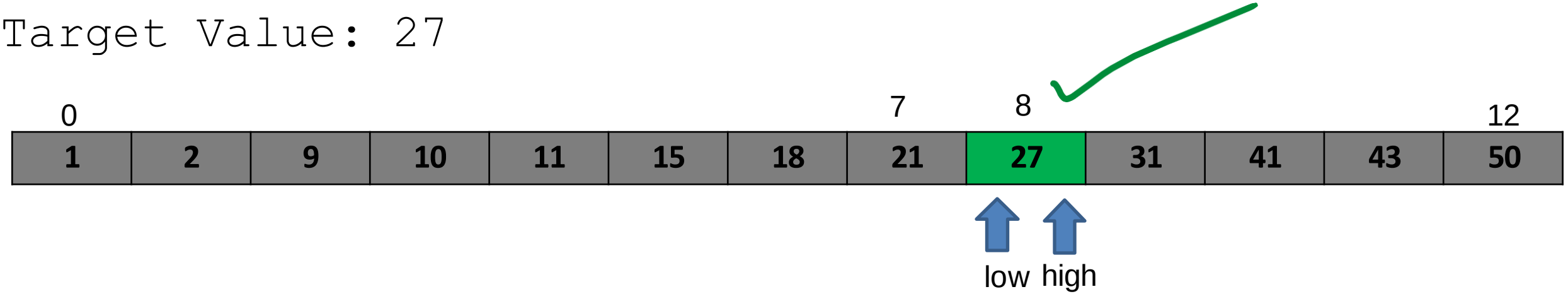
How to calculate the mid point? `(low + high) / 2`

Target Value: 27



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How do we know when to stop looping?

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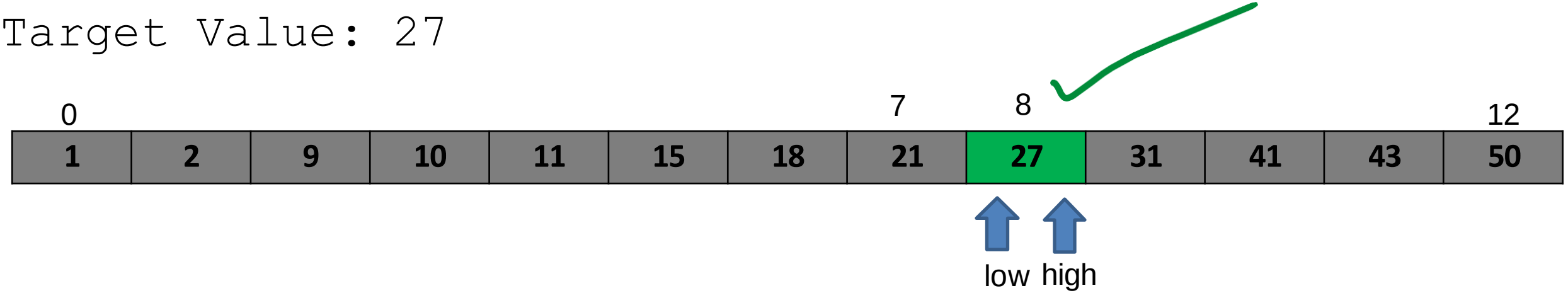

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How do we know when to stop looping? If we find the target value, or if `low` and `high` cross each other (low > high)

Target Value: 27

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→ If
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sectio
→ If
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3. Re
target value is found

How do we know when to stop looping? If we find the target value, or if `low` and `high` cross each other (low > high)

```
private static int binary_search(int[] array, int n) {
        int low = 0;
        int high = array.length - 1;
        while(low <= high) {
                int mid = (low + high) / 2;
                if(n == array[mid]) {
                        return mid;
                }
                else if(n > array[mid]) {
                        low = mid + 1;
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                else {
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Running time?

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Running time? Each time we loop, we eliminate **half** the array

Running time?

Initial length of array $= n$

Iteration 1 - Length of array $= n/2$

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Iteration 2 - Length of array $= (n/2)/2 = n/2^2$

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Iteration k - Length of array $= n/2^k$

Running time?

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Iteration 2 - Length of array $= (n/2)/2 = n/2^2$

Iteration k - Length of array $= n/2^k$

After k iterations, eventually our array has been reduced to one element

Length of array $= n/2^k = 1$

$$n = 2^k$$

"Two to what power makes n??"

Running time?

After k iterations, eventually our array has been reduced to one element

$$\text{Length of array} = n/2^k = 1$$

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$$log_2(n) = log_2(2^k)$$

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$$log_2(n) = log_2(2^k)$$

$$log_2(n) = k * log_2 2$$

Running time?

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"Two to what power makes n??"

$$log_2(n) = log_2(2^k)$$

$$log_2(n) = k * \cancel{log_2 2}$$

$$log_2(n) = k$$

After K iterations, we will have done log(n) divisions

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                if(n == array[mid]) {
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                }
                else if(n > array[mid]) {
                        low = mid + 1;
                }
                else {
                        high = mid - 1;
                }
        }
        return -1;
}
```

Running time?

Generally speaking, whenever we eliminate half of the problem each iteration, that will give us **O(logn)** running time

```
private static int binary_search(int[] array, int n) {
        int low = 0;O(1)
        int high = array.length - 1;O(1)
        while(low <= high) { O(log n)
                int mid = (low + high) / 2;  O(1)
                if(n == array[mid]) {O(1)
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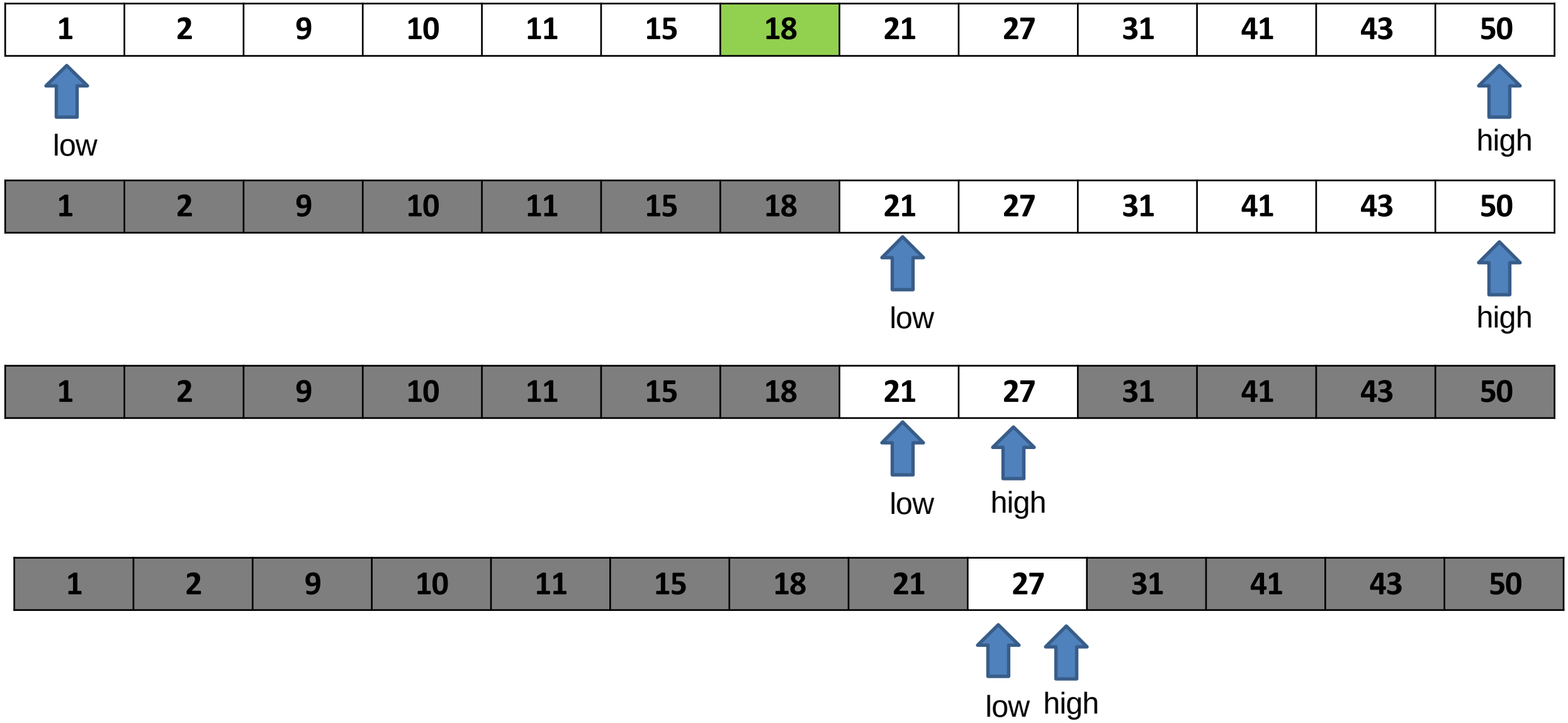
Running time? O(log n)

Why O(log n) ?


1 2 9 10 11 15 18 21 27 31 41 43 50
low
high
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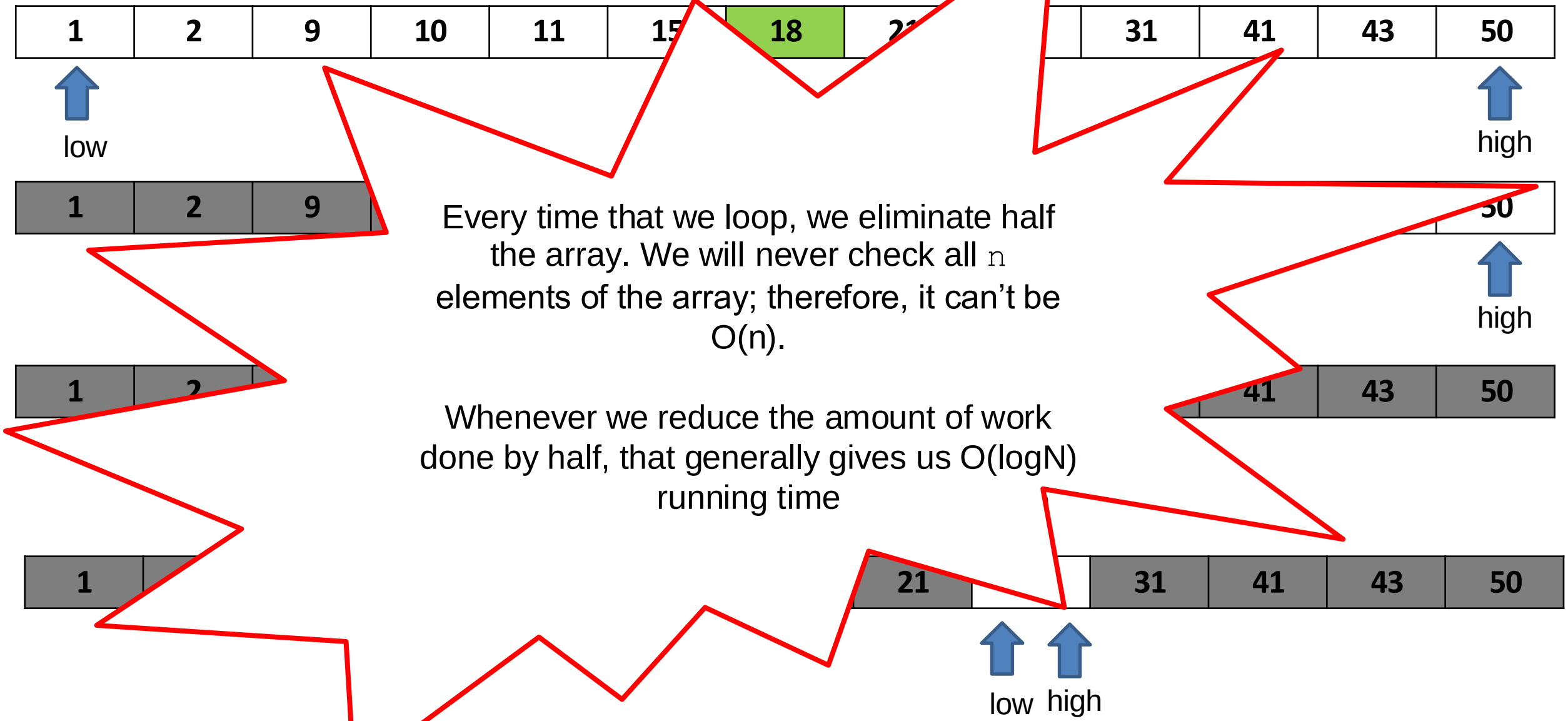
Why O(log n) ?



Every time that we loop, we eliminate half the array. We will never check all `n` elements of the array; therefore, it can't be O(n).

Whenever we reduce the amount of work done by half, that generally gives us O(logN) running time

```
private static int binary_search(int[] array, int n) {
        int low = 0;
        int high = array.length - 1;
        while(low <= high) {
                int mid = (low + high) / 2;

                int result = x.compareTo(array[mid])

                if(result = 0) {
                        return mid;
                }
                else if(result > 0){
                        low = mid + 1;
                }
                else {
                        high = mid - 1;
                }
        }
        return -1;
}
```

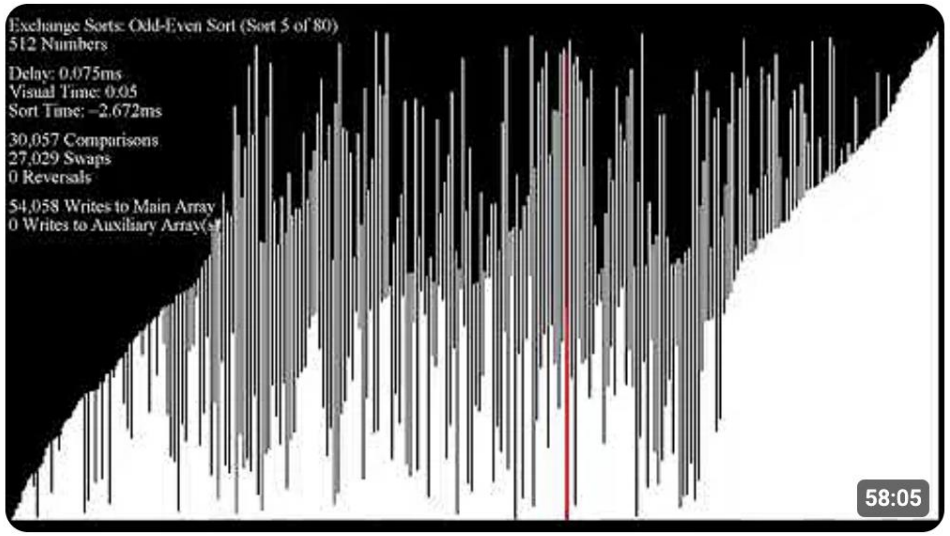
We can do binary search on an array of Strings using the `compareTo()` method

```
public static int binary_search_recursive(???????????) {
        if (low > high) {
                return -1;
        }
        int mid = (low + high) / 2;
        if (array[mid] == n) {
                return mid;
        }
        else if (n > array[mid]) {
                return binary_search_recursive(???????????); // right half
        }
        else {
                return binary_search_recursive(???????????); // left half
        }
}
```

Binary Search can also be implemented using recursion

```java
public static int binary_search_recursive(int[] array, int n, int low, int high) {
        if (low > high) {
                return -1; // base case
        }
        int mid = (low + high) / 2;
        if (array[mid] == n) {
                return mid; // found n
        }
        else if (n > array[mid]) {
                return binary_search_recursive(array, n, mid + 1, high); // right half
        }
        else {
                return binary_search_recursive(array, n, low, mid - 1); // left half
        }
}
```

Binary Search can also be implemented using recursion



sorting algorithms to relax/study to

2.9M views • 2 years ago

Musicombo

Shout-out to Control for this idea :P Check out the NEW home for

Lab 12