CSCI 132:
Basic Data Structures and Algorithms

Recursion (Part 3)

Reese Pearsall

Spring 2024

Announcements

No class on Wednesday (4/17)

Program 4 due next Friday

Gianforte Hall Groundbreaking Ceremony: 4/17 @ 2:00 PM

Change Making Problem

Given a set a coin denominations **D**, how can you represent **K** cents with the smallest number of coins?

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D = [1, 5, 10, 25]

K = 37

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D = [1, 5, 10, 25]

K = 37

Answer = 4

(Quarter, dime, two pennies)

Change Making Problem

Given a set a coin denominations **D**, how can you represent **K** cents with the smallest number of coins?

D = [1, 5, 10, 25]

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Answer = 4

(Quarter, dime, two pennies)

Algorithm?

Change Making Problem

Given a set a coin denominations **D**, how can you represent **K** cents with the smallest number of coins?

D = [1, 5, 10, 25]

K = 37

Answer = 4

(Quarter, dime, two pennies)

Use as many quarters as possible, then as many dimes as possible, …

Change Making Problem

Given a set a coin denominations **D**, how can you represent **K** cents with the smallest number of coins?

D = [1, 5, 10, 25]

K = 37

Answer = 4

(Quarter, dime, two pennies)

Use as many quarters as possible, then as many dimes as possible, …

This is known as the **greedy** approach

Change Making Problem

Given a set a coin denominations **D**, how can you represent **K** cents with the smallest number of coins?

D = [1, 5, 10, 25]

K = 37

Greedy Algorithm

Use as many quarters as possible, then as many dimes as possible, …

Change Making Problem

Given a set a coin denominations **D**, how can you represent **K** cents with the smallest number of coins?

D = [1, 5, 10, 18, 25]

K = 37

Greedy Algorithm

Use as many quarters as possible, then as many 18 cent pieces as possible, then dimes , …

What if there were also an 18-cent coin?

Change Making Problem

Given a set a coin denominations **D**, how can you represent **K** cents with the smallest number of coins?

D = [1, 5, 10, 18, 25]

K = 37

Greedy Algorithm

Use as many quarters as possible, then as many 18 cent pieces as possible, then dimes , …

25, 10, 1, 1 **(4 coins)**

What if there were also an 18-cent coin?

Change Making Problem

Given a set a coin denominations **D**, how can you represent **K** cents with the smallest number of coins?

Greedy Algorithm

D = [1, 5, 10, 18, 25]

K = 37

Use as many quarters as possible, then as many 18 cent pieces as possible, then dimes , …

25, 10, 1, 1 (4 coins)

What if there were also an 18-cent coin?

Real Answer = 18, 18, 1 (3 coins)

Change Making Problem

Given a set a coin denominations **D**, how can you represent **K** cents with the smallest number of coins?

D = [1, 5, 10, 18, 25]

K = 37

Greedy Algorithm

Use as many quarters as possible, then as many 18 cent pieces as possible, then dimes , …

25, 10, 1, 1 (4 coins)

What if there were also an 18-cent coin?

Real Answer = 18, 18, 1 (3 coins)

Lesson Learned: The Greedy approach works for the United States denominations, but not for a general set of denominations

Change Making Problem

Suppose I tell you that 2 quarters, 1 dime, and 3 pennies are the minimum number of coins needed to make **63 cents**

(We will assume we have the standard US denominations [1, 5, 10, 25] (NO 50 CENT PIECE)

$$25 + 25 + 10 + 1 + 1 + 1 = 63$$

What can you conclude?

Does this provide an answer to any other change making problems?

Change Making Problem

Suppose I tell you that 2 quarters, 1 dime, and 3 pennies are the minimum number of coins needed to make **63 cents**

(We will assume we have the standard US denominations [1, 5, 10, 25] (NO 50 CENT PIECE)

25 + 25 + 10 + 1 + 1 + 1 = 63

This is the minimum coins needed to make 38 cents

Change Making Problem

Suppose I tell you that 2 quarters, 1 dime, and 3 pennies are the minimum number of coins needed to make **63 cents**

(We will assume we have the standard US denominations [1, 5, 10, 25] (NO 50 CENT PIECE)

$$25 + 25 + 10 + 1 + 1 + 1 = 63$$

This is the minimum coins needed to make 13 cents

Change Making Problem

Suppose I tell you that 2 quarters, 1 dime, and 3 pennies are the minimum number of coins needed to make **63 cents**

(We will assume we have the standard US denominations [1, 5, 10, 25] (NO 50 CENT PIECE)

25 + 25 + 10 + 1 + 1 + 1 = 63

This is the minimum coins needed to make 3 cents

Change Making Problem

Suppose I tell you that 2 quarters, 1 dime, and 3 pennies are the minimum number of coins needed to make **63 cents**

(We will assume we have the standard US denominations [1, 5, 10, 25] (NO 50 CENT PIECE)

25 + 25 + 10 + 1 + 1 + 1 = 63

This is the minimum coins needed to make 2 cents

Change Making Problem

Suppose I tell you that 2 quarters, 1 dime, and 3 pennies are the minimum number of coins needed to make **63 cents**

(We will assume we have the standard US denominations [1, 5, 10, 25] (NO 50 CENT PIECE)

25 + 25 + 10 + 1 + 1 + 1 = 63

This is the minimum coins needed to make 1 cent

Change Making Problem

Suppose I tell you that 2 quarters, 1 dime, and 3 pennies are the minimum number of coins needed to make **63 cents**

(We will assume we have the standard US denominations [1, 5, 10, 25] (NO 50 CENT PIECE)

25 + 25 + 10 + 1 + 1 + 1 = 63

The solution to the change making problems consists of solutions to smaller change making problems

We can use **recursion** to solve this problem

Change Making Problem

In general, suppose a country has coins with denominations:

$1 = d_1 < d_2 < \cdots < d_k$ (US coins: $d_1 = 1, d_2 = 5, d_3 = 10, d_4 = 25$)

Algorithm: To make change for p cents, we are going to figure out change for every value $x < p$. We will build solution for p out of smaller solutions.

Change Making Problem

$C(p)$ – minimum number of coins to make p cents.

x – value (e.g. \$0.25) of a coin used in the optimal solution.

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x – value (e.g. \$0.25) of a coin used in the optimal solution.

$$C(p) = 1 + C(p - x).$$

C(37) = 1 + C(12)

We used one quarter

Now find the minimum number of coins needed to make 12 cents

$C(p)$ – minimum number of coins to make p cents.

x – value (e.g. \$0.25) of a coin used in the optimal solution.

$$C(p) = 1 + C(p - x).$$

C(37) = 1 + C(12)

C(12) = 1 + C(2)

We used one dime

Now find the minimum number of coins needed to make 2 cents

Change Making Problem

$C(p)$ – minimum number of coins to make p cents.

x – value (e.g. \$0.25) of a coin used in the optimal solution.

$$C(p) = 1 + C(p - x).$$

C(37) = 1 + C(12) 

C(12) = 1 + C(2) 

C(2) = 1 + C(1) 

C(1) = 1 + C(0) 

Change Making Problem

$C(p)$ – minimum number of coins to make p cents.

x – value (e.g. \$0.25) of a coin used in the optimal solution.

$$C(p) = 1 + C(p - x).$$

C(37) = 1 + C(12) 

C(12) = 1 + C(2) 

C(2) = 1 + C(1) 

C(1) = 1

Change Making Problem

$C(p)$ – minimum number of coins to make p cents.

x – value (e.g. \$0.25) of a coin used in the optimal solution.

$$C(p) = 1 + C(p - x).$$

C(37) = 1 + C(12) 

C(12) = 1 + C(2) 

C(2) = 1 + 1

Change Making Problem

$C(p)$ – minimum number of coins to make p cents.

x – value (e.g. \$0.25) of a coin used in the optimal solution.

$$C(p) = 1 + C(p - x).$$

C(37) = 1 + C(12) 

C(12) = 1 + C(2) 

C(2) = 2

Change Making Problem

$C(p)$ – minimum number of coins to make p cents.

x – value (e.g. \$0.25) of a coin used in the optimal solution.

$$C(p) = 1 + C(p - x).$$

C(37) = 1 + C(12) 

C(12) = 1 + 2 

Change Making Problem

$C(p)$ – minimum number of coins to make p cents.

x – value (e.g. \$0.25) of a coin used in the optimal solution.

$$C(p) = 1 + C(p - x).$$

C(37) = 1 + C(12) 

C(12) = 3

Change Making Problem

$C(p)$ – minimum number of coins to make p cents.

x – value (e.g. $0.25) of a coin used in the optimal solution.

$$C(p) = 1 + C(p - x).$$

C(37) = 1 + 3

Change Making Problem

$C(p)$ – minimum number of coins to make p cents.

x – value (e.g. \$0.25) of a coin used in the optimal solution.

$$C(p) = 1 + C(p - x).$$

C(37) = 4

The minimum number of coins needed to make 37 cents is 4

In general, suppose a country has coins with denominations:

$$1 = d_1 < d_2 < \cdots < d_k$$

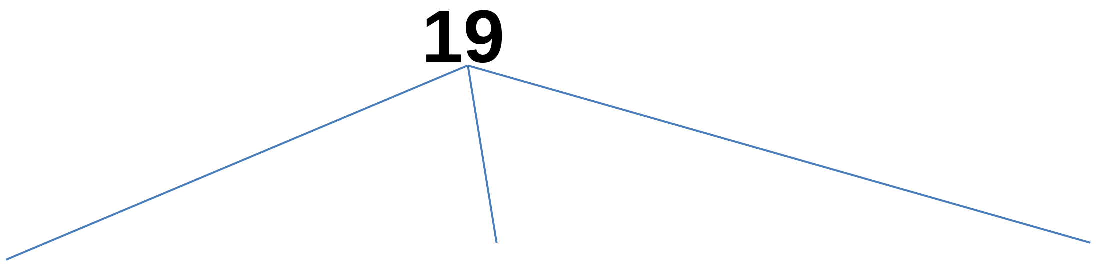
(US coins: $d_1 = 1, d_2 = 5, d_3 = 10, d_4 = 25$)

(This algorithm must work for ALL denominations)

Algorithm: To make change for p cents, we are going to figure out change for every value $x < p$. We will build solution for p out of smaller solutions.

Change Making Problem

Make $0.19 with $0.01, $0.05, $0.10



Change Making Problem

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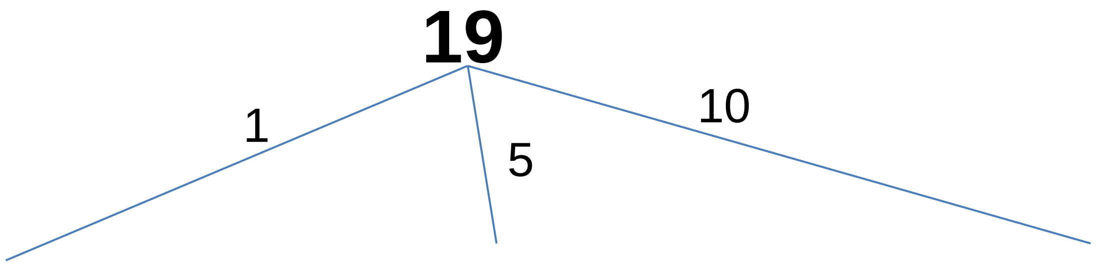
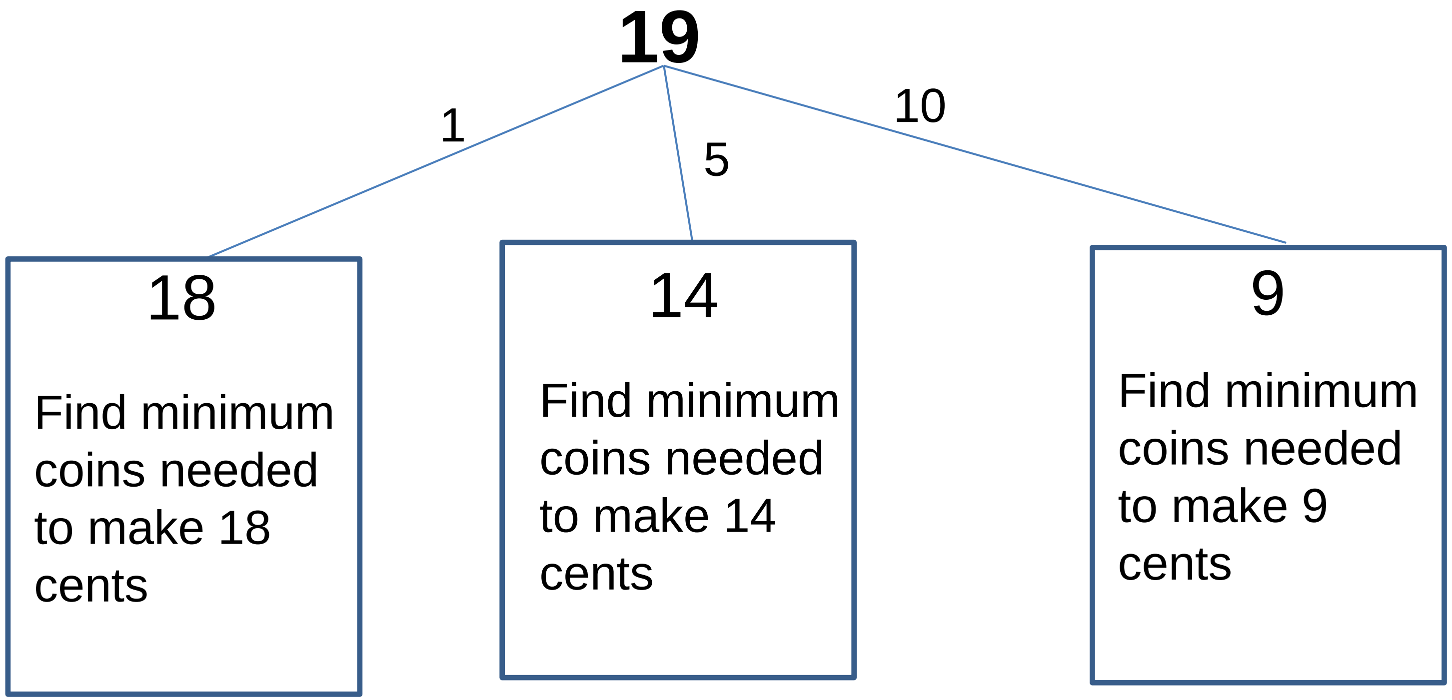


Change Making Problem

Make $0.19 with $0.01, $0.05, $0.10

k = # denominations

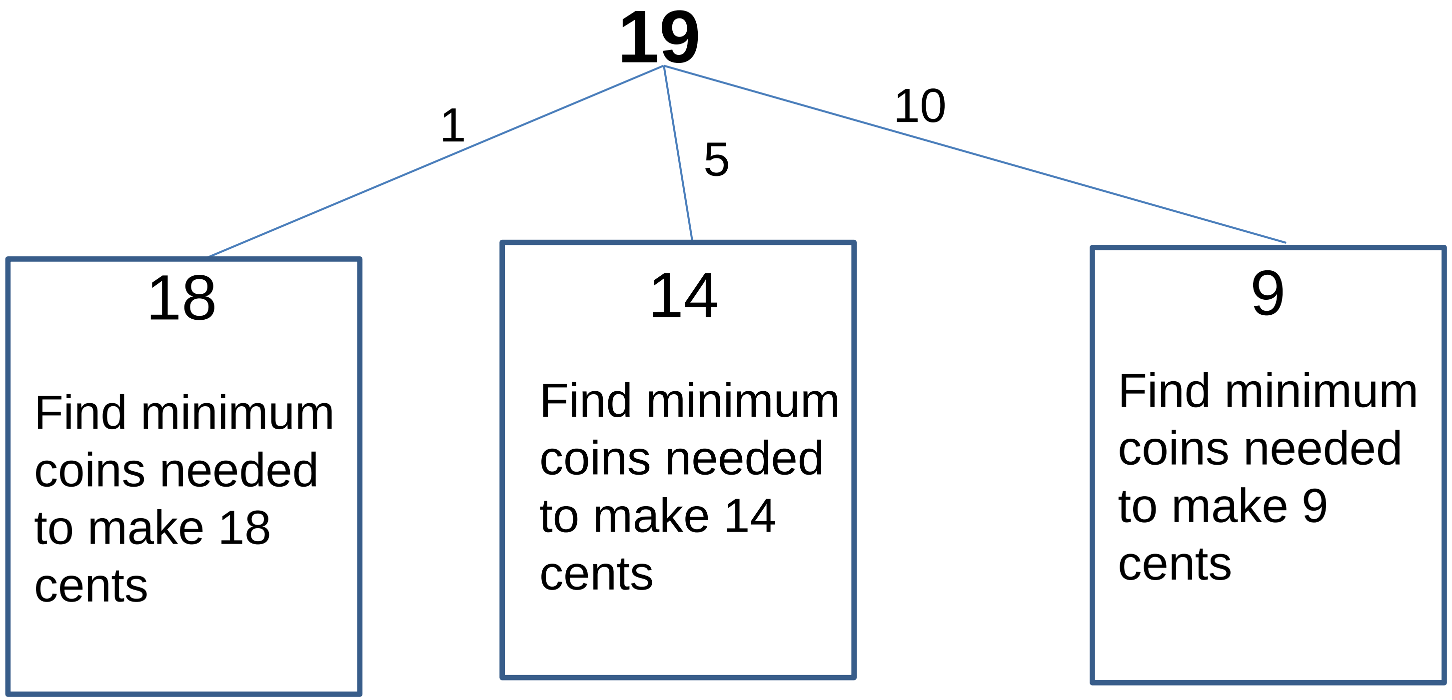


To find the minimum number of coins needed to create 19 cents, we generate **k** subproblems

Change Making Problem

Make \$0.19 with \$0.01, \$0.05, \$0.10

k = # denominations

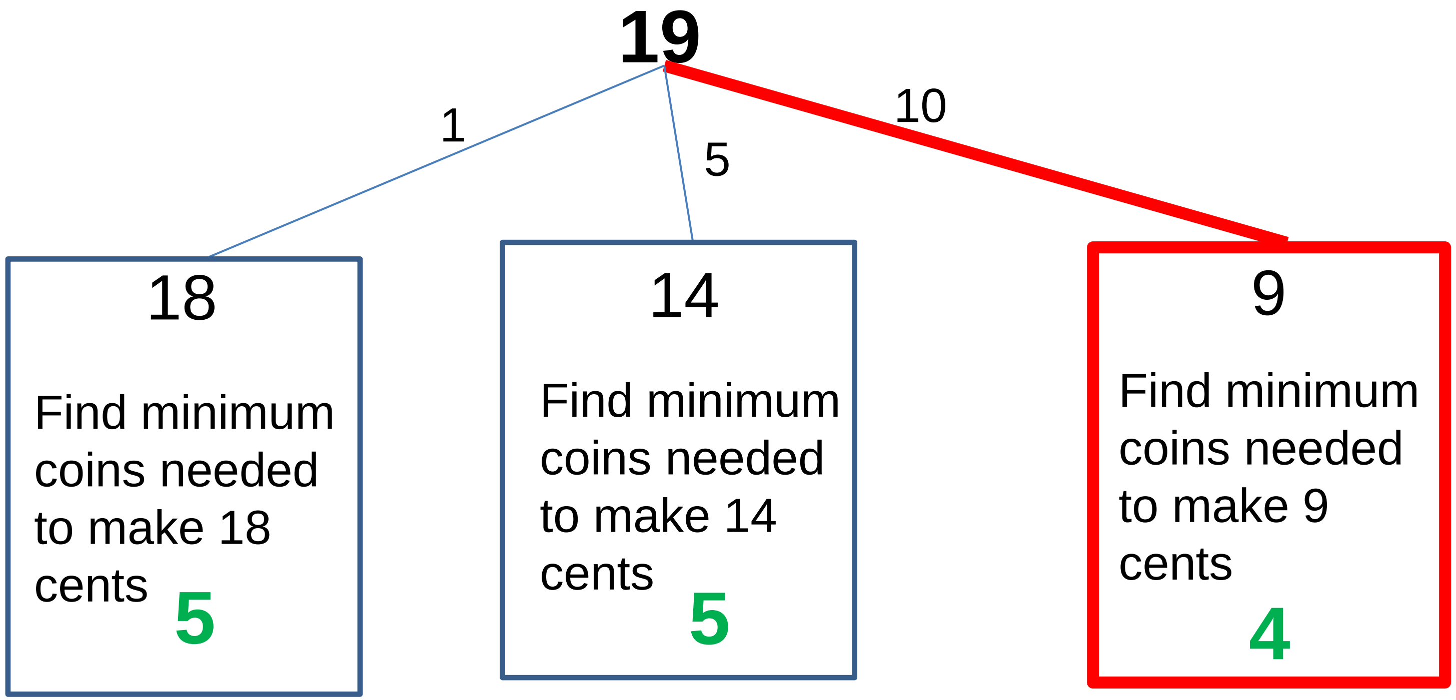


We want to select the **minimum** solution of these three subproblems

Change Making Problem

Make \$0.19 with \$0.01, \$0.05, \$0.10

k = # denominations

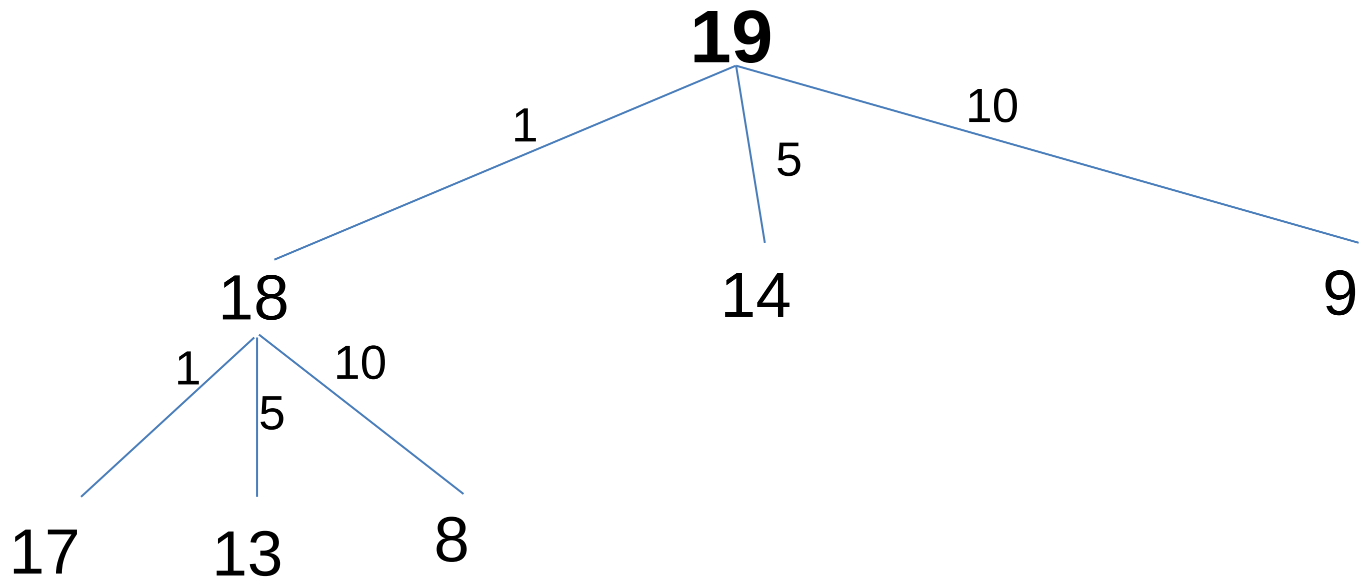


For the solution of our original problem (19), we want to select this branch (one dime used)

Change Making Problem

Make \$0.19 with \$0.01, \$0.05, \$0.10

k = # denominations



Find minimum coins needed to make 17 cents

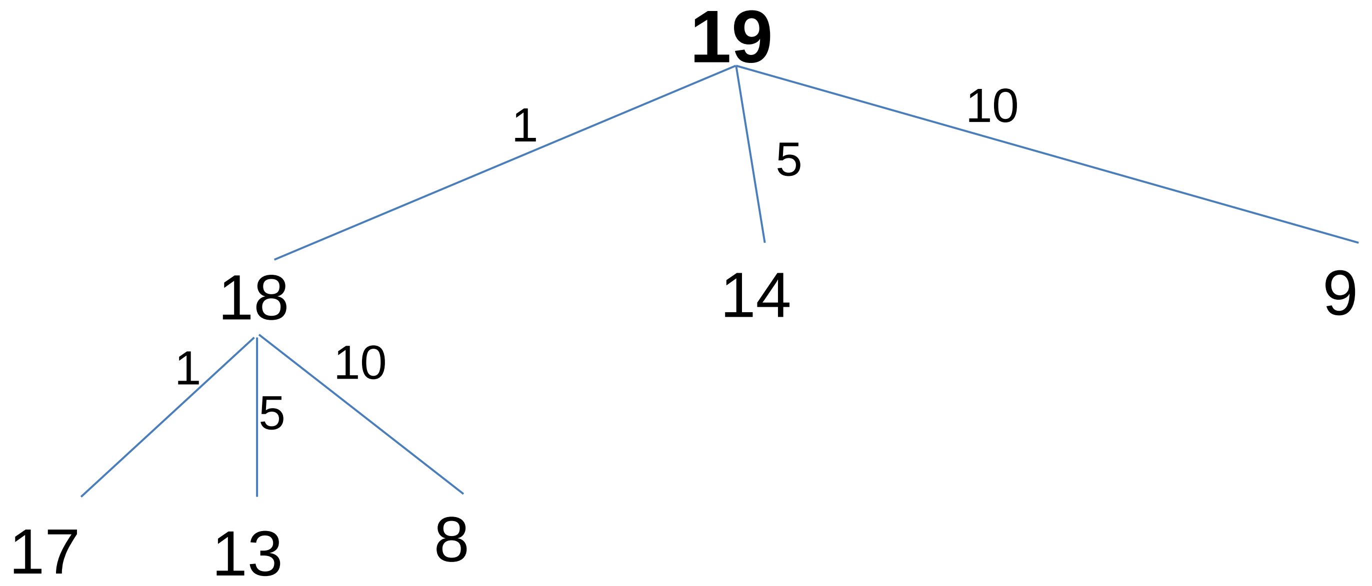
Find minimum coins needed to make 13 cents

Find minimum coins needed to make 8 cents

Change Making Problem

Make \$0.19 with \$0.01, \$0.05, \$0.10

k = # denominations



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Make $0.19 with $0.01, $0.05, $0.10

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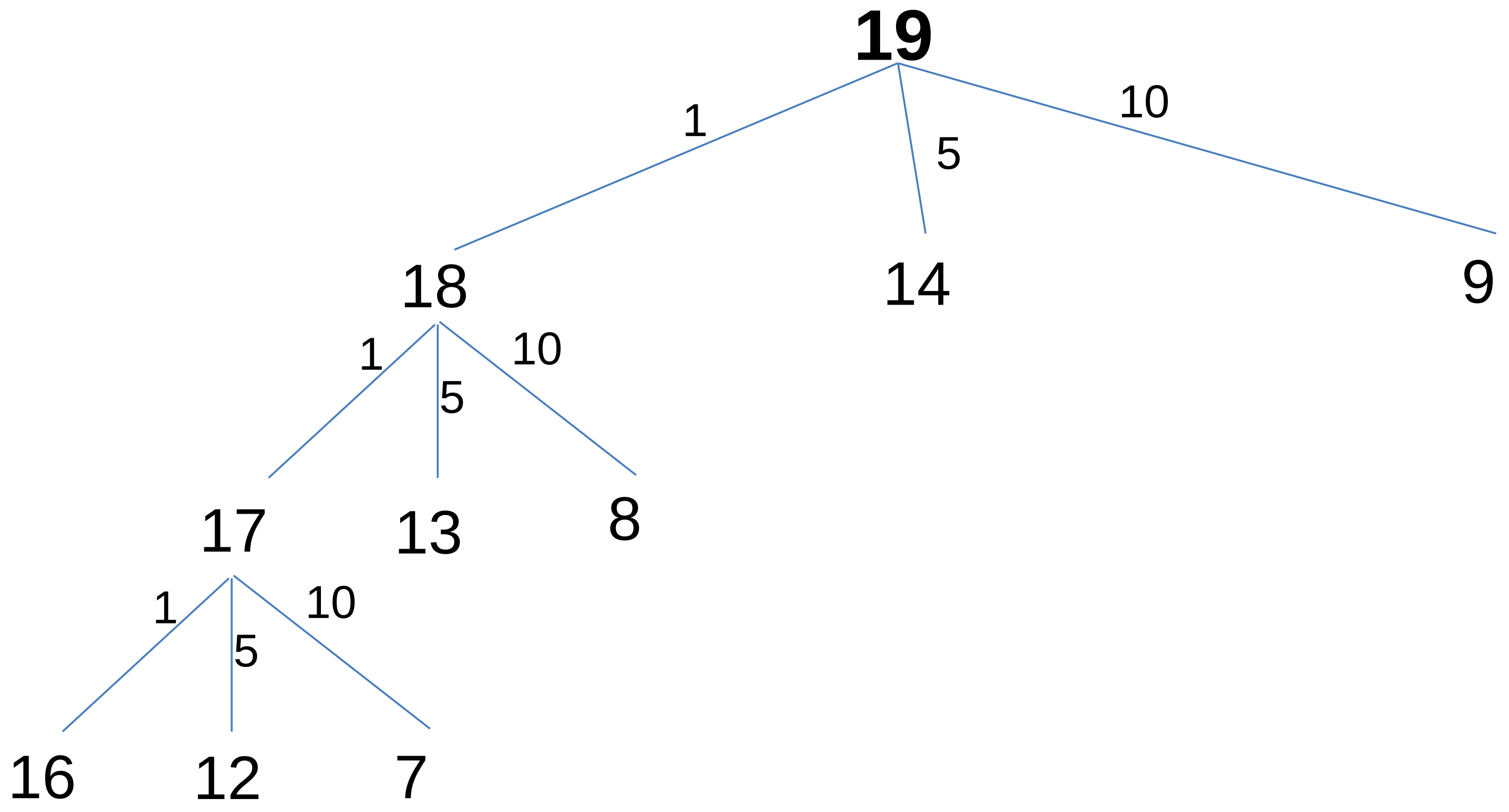

Change Making Problem

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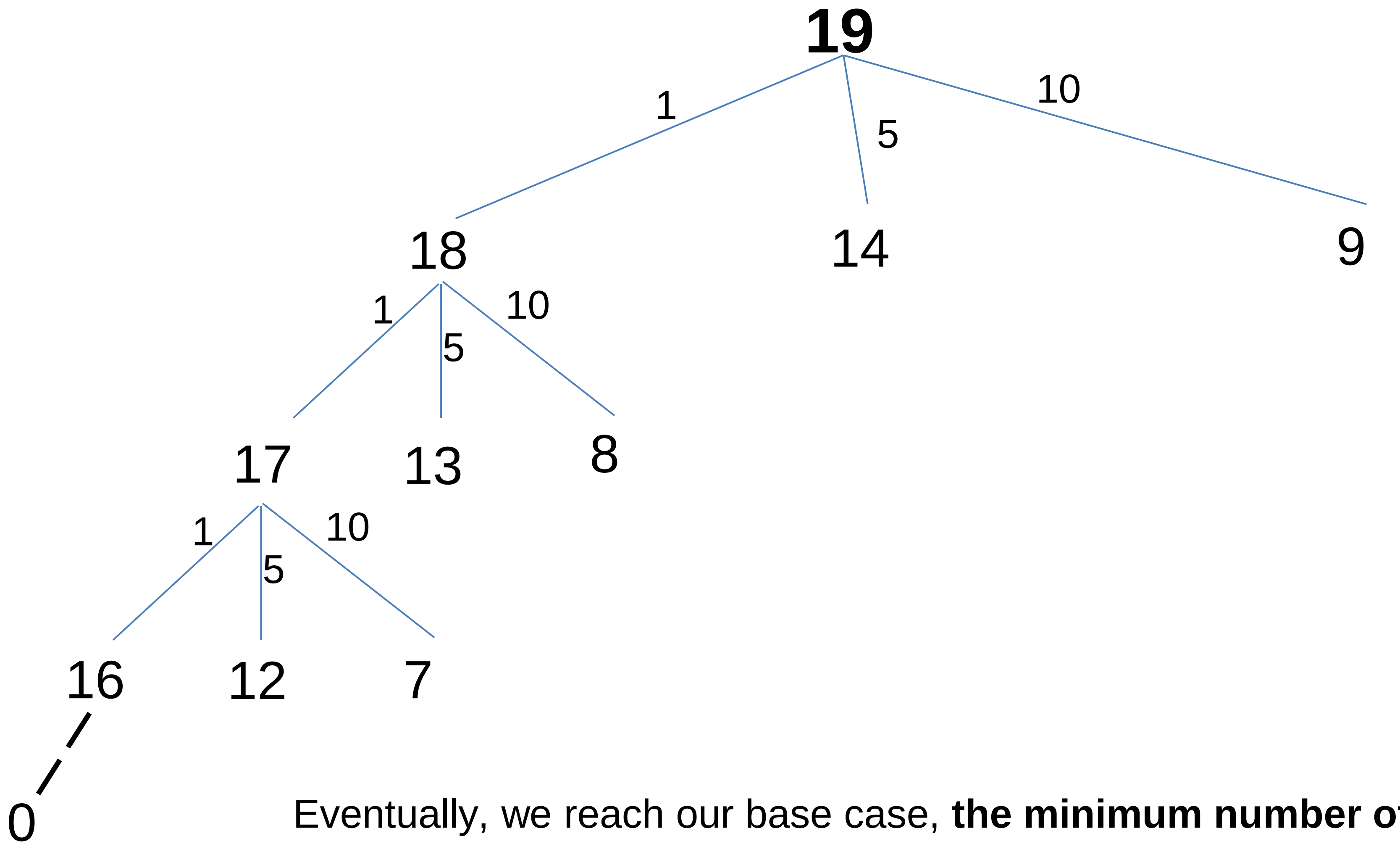
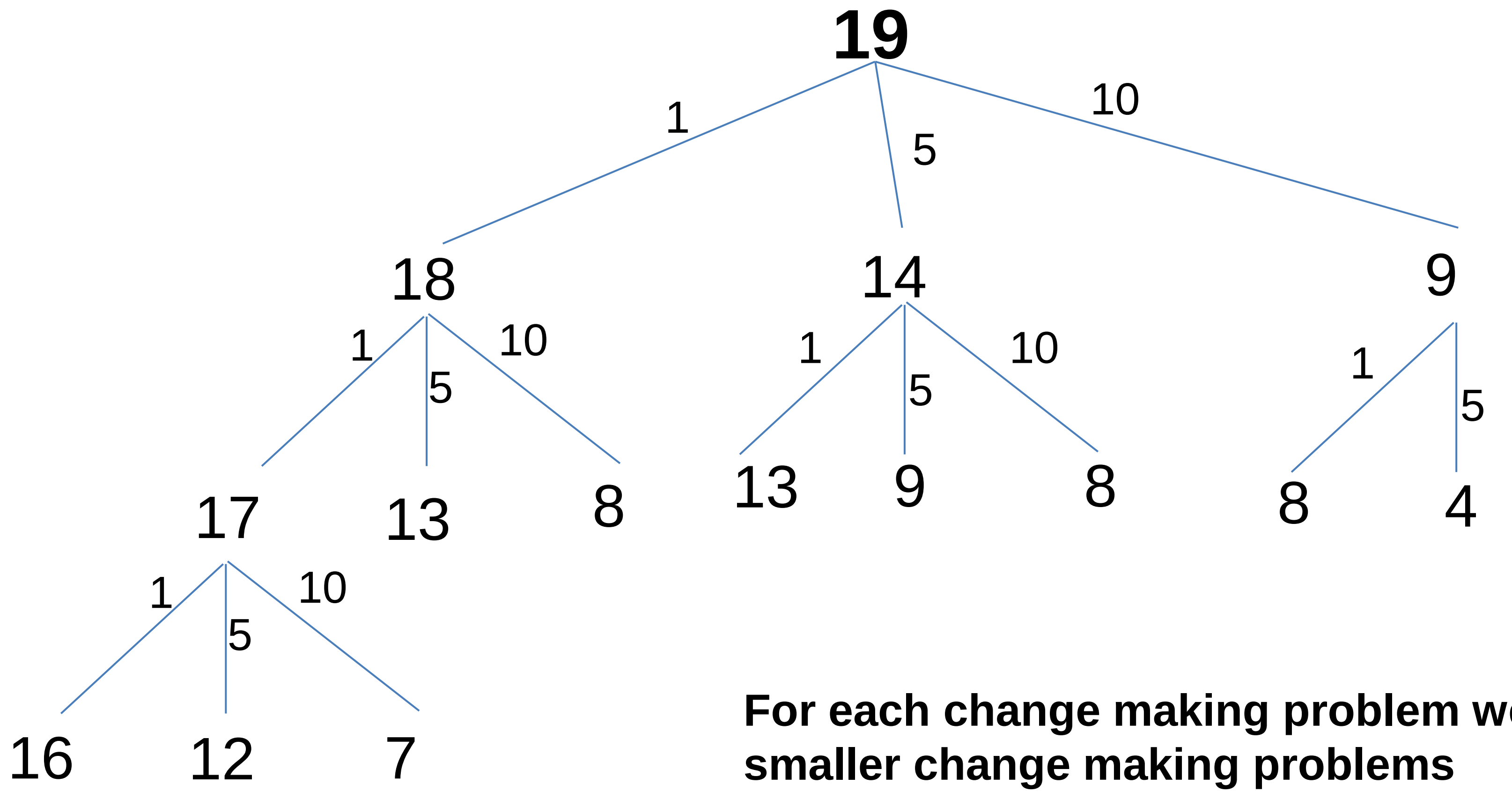


Eventually, we reach our base case, **the minimum number of coins needed to make 0 cents**

Change Making Problem

Make $0.19 with $0.01, $0.05, $0.10

k = # denominations



When C(9), we cant use a 10 cent piece…

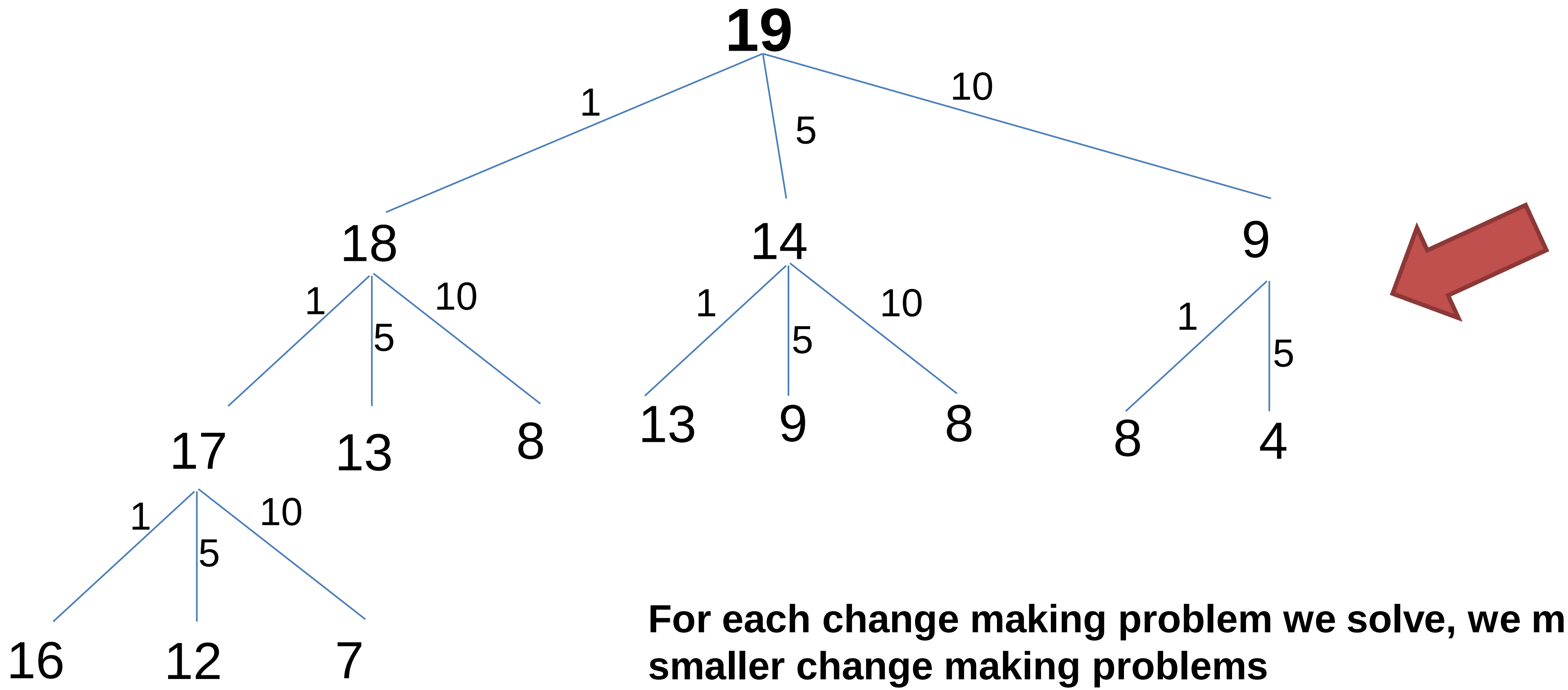
For each change making problem we solve, we must solve at most 3 smaller change making problems

Once we solve the smaller problems, we must select the branch that has the minimum value

Change Making Problem

Make \$0.19 with \$0.01, \$0.05, \$0.10

k = # denominations



When C(9), we cant use a 10 cent piece…

For each change making problem we solve, we must solve at most 3 smaller change making problems

Once we solve the smaller problems, we must select the branch that has the minimum value

Change Making Problem

$$C(p) = \begin{cases} \min_{i: d_i \le p} C(p - d_i) + 1, p > 0 \\ 0, p = 0 \end{cases}$$

Least change for 19 cents = minimum of:

- least change for 19-10 = 9 cents
- least change for 19-5 = 14 cents
- least change for 19-1 = 18 cents

For each problem P, we will solve the problem for (P – d), where d represents each possible denomination

Change Making Problem

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For each problem P, we will solve the problem for (P – d), where d represents each possible denomination

We want to select only the branch the yields the minimum value

Change Making Problem

$$C(p) = \begin{cases} \min_{i:d_i \le p} C(p - d_i) + 1, p > 0 \\ 0, p = 0 \end{cases}$$

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- least change for 19-10 = 9 cents
- least change for 19-5 = 14 cents
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For each problem P, we will solve the problem for (P – d), where d represents each possible denomination

If we ever need to make change for 0 cents, return 0

We want to select only the branch the yields the minimum value

Change Making Problem

$$C(p) = \begin{cases} \min_{i:d_i \le p} C(p - d_i) + 1, p > 0 \\ 0, p = 0 \end{cases}$$

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For each problem P, we will solve the problem for (P – d), where d represents each possible denomination

If we ever need to make change for 0 cents, return 0

We want to select only the branch the yields the minimum value

Change Making Problem

D = array of denominations [1, 5, 10, 18, 25]
p = desired change (37)

```
min_coins(D, p)
```

Change Making Problem

D = array of denominations [1, 5, 10, 18, 25]
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```
min_coins(D, p)

    if p == 0
        return 0;
```

Base Case

Change Making Problem

D = array of denominations [1, 5, 10, 18, 25]
p = desired change (37)

```
min_coins(D, p)

   if p == 0
      return 0;

   else
      min = ∞
      a = ∞
```

Base Case

```
int min = Integer.MAX_VALUE;
int a = Integer.MAX_VALUE;;
```

Change Making Problem

D = array of denominations [1, 5, 10, 18, 25]
p = desired change (37)

```
min_coins(D, p)

  if p == 0
      return 0;

  else
      min = ∞
      a = ∞

      for each d_i in D
          if (p - d_i) >= 0
              a = min_coins(D, p - d_i )
```

Base Case

```
int min = Integer.MAX_VALUE;
int a = Integer.MAX_VALUE;;
```

Recurse, and find the minimum number of coins needed using each valid denomination

Change Making Problem

D = array of denominations [1, 5, 10, 18, 25]
p = desired change (37)

```
min_coins(D, p)

  if p == 0
      return 0;

  else
     min = ∞
     a = ∞

     for each d_i in D
         if (p - d_i) >= 0
             a = min_coins(D, p - d_i )
         if a < min
             min = a
```

Base Case

```
int min = Integer.MAX_VALUE;
int a = Integer.MAX_VALUE;;
```

Recurse, and find the minimum number of coins needed using each valid denomination

Select the branch that has the minimum value

Change Making Problem

D = array of denominations [1, 5, 10, 18, 25]
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min_coins(D, p)
    if p == 0
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    else
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Change Making Problem

D = array of denominations [1, 5, 10, 18, 25]
p = desired change (37)

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min_coins(D, p)
    if p == 0
        return 0;
    else
        min = ∞
        a = ∞

        for each d_i in D
            if (p - d_i) >= 0
                a = min_coins(D, p - d_i )
            if a < min
                min = a

    return 1 + min
```

Base Case

```
int min = Integer.MAX_VALUE;
int a = Integer.MAX_VALUE;;
```

Recurse, and find the minimum number of coins needed using each valid denomination

Select the branch that has the minimum value

Once, our for loop finishes, we should know the branch that had the minimum, so return (1 + min), 1 because one coin was used in the current method call

Change Making Problem

```
min_coins(D, p)
   if p == 0
       return 0;
   else
      min = ∞
      a = ∞

      for each d_i in D
         if (p - d_i) >= 0
            a = min_coins(D, p - d_i )
         if a < min
            min = a

     return 1 + min
```

Change Making Problem

```
min_coins(D, p)
    if p == 0
        return 0;
    else
        min = ∞
        a = ∞

        for each di in D
            if (p - di) >= 0
                a = min_coins(D, p - di )
            if a < min
                min = a

        return 1 + min
```

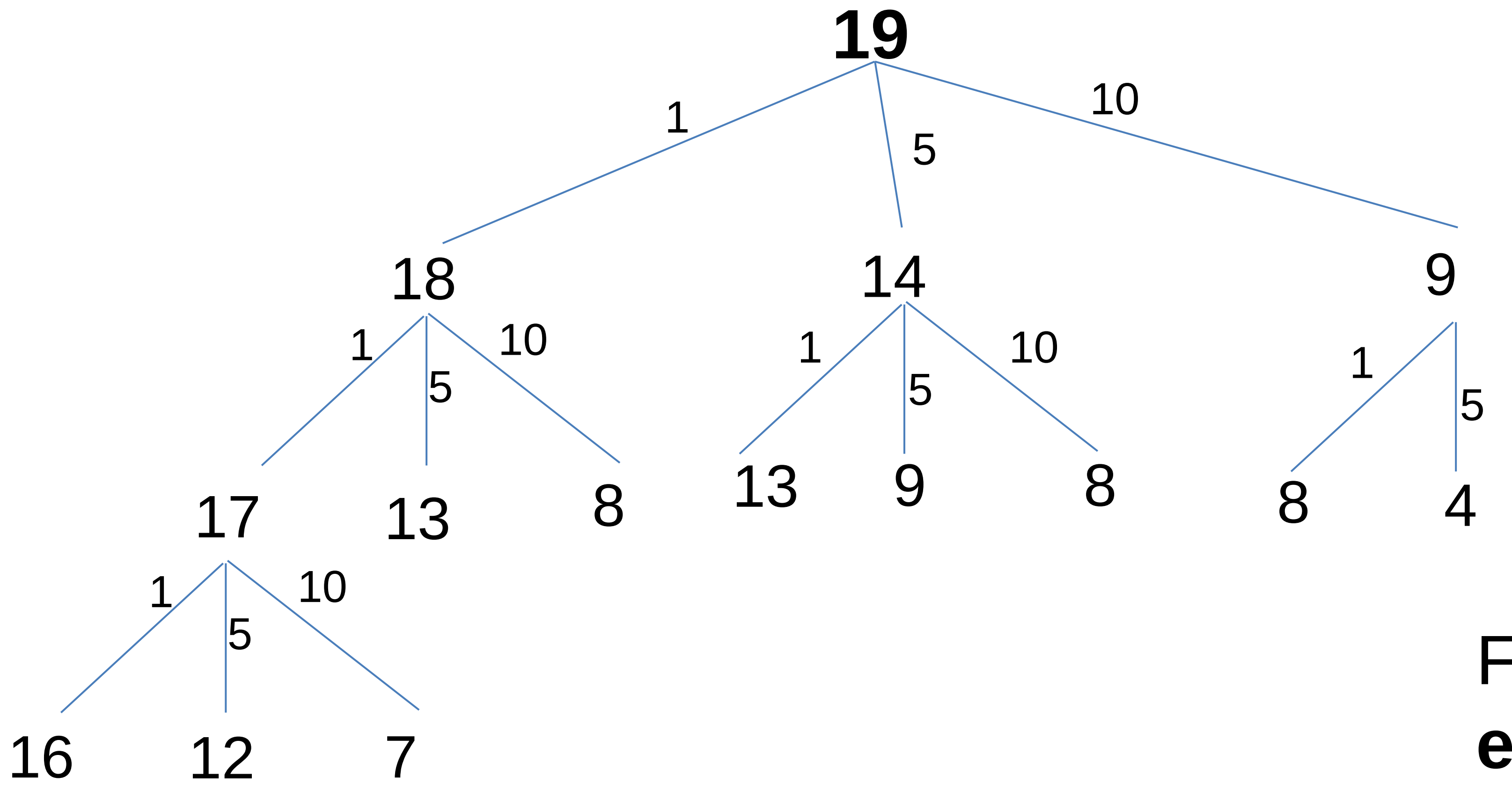
Running time?

Change Making Problem

Make $0.19 with $0.01, $0.05, $0.10

k = # denominations

p = value to make change for



For sufficiently large p, **every permutation** of denominations is included.

Change Making Problem

Make $0.19 with $0.01, $0.05, $0.10

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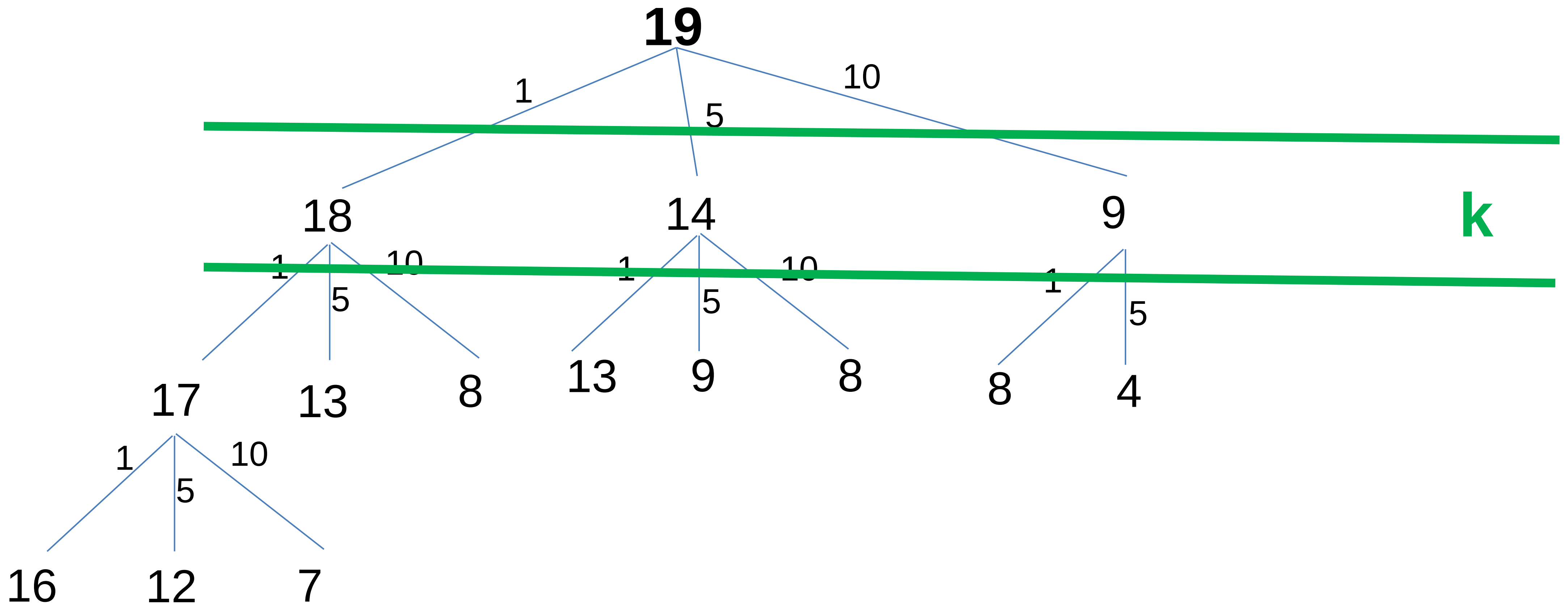

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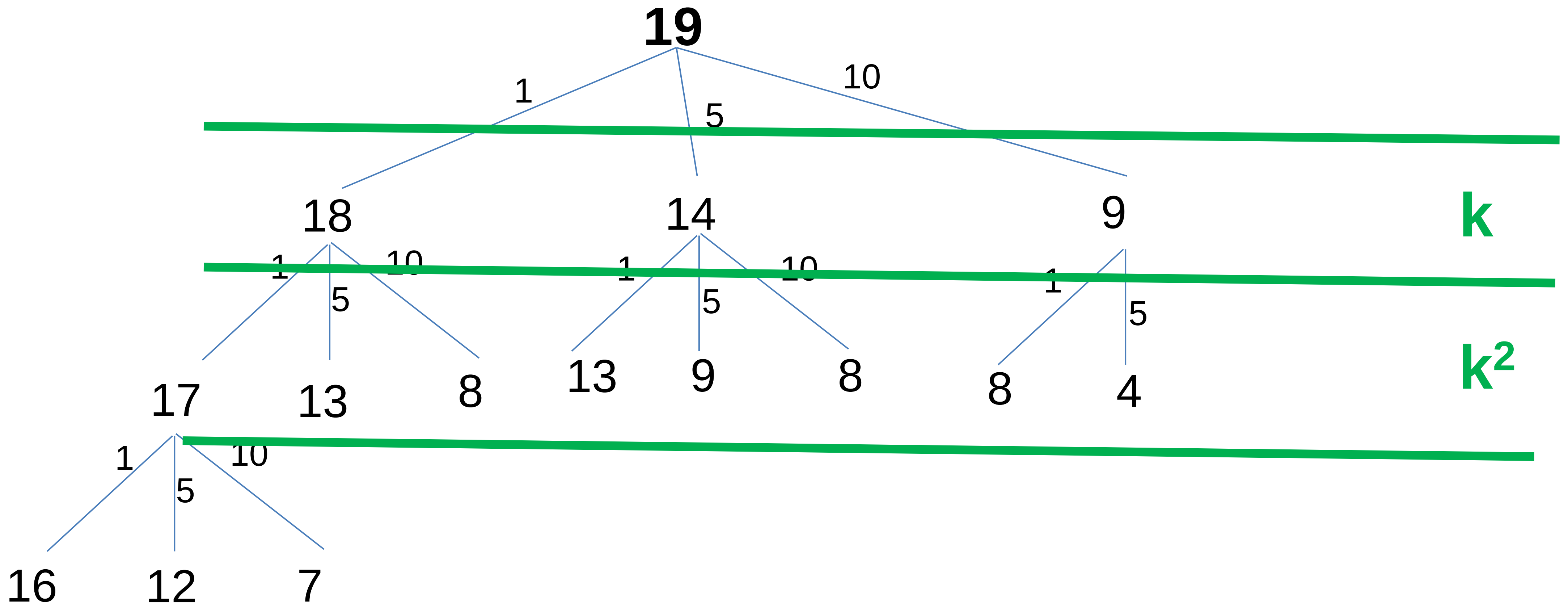

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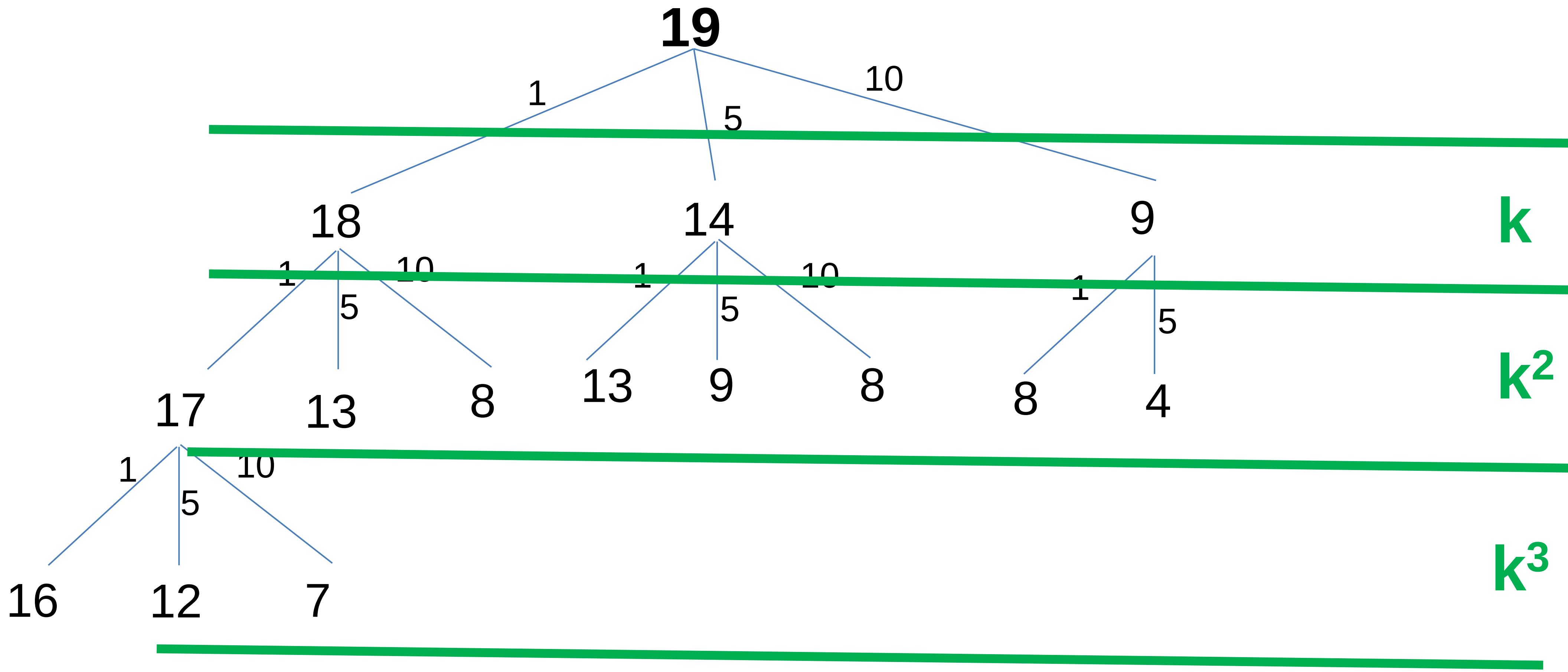

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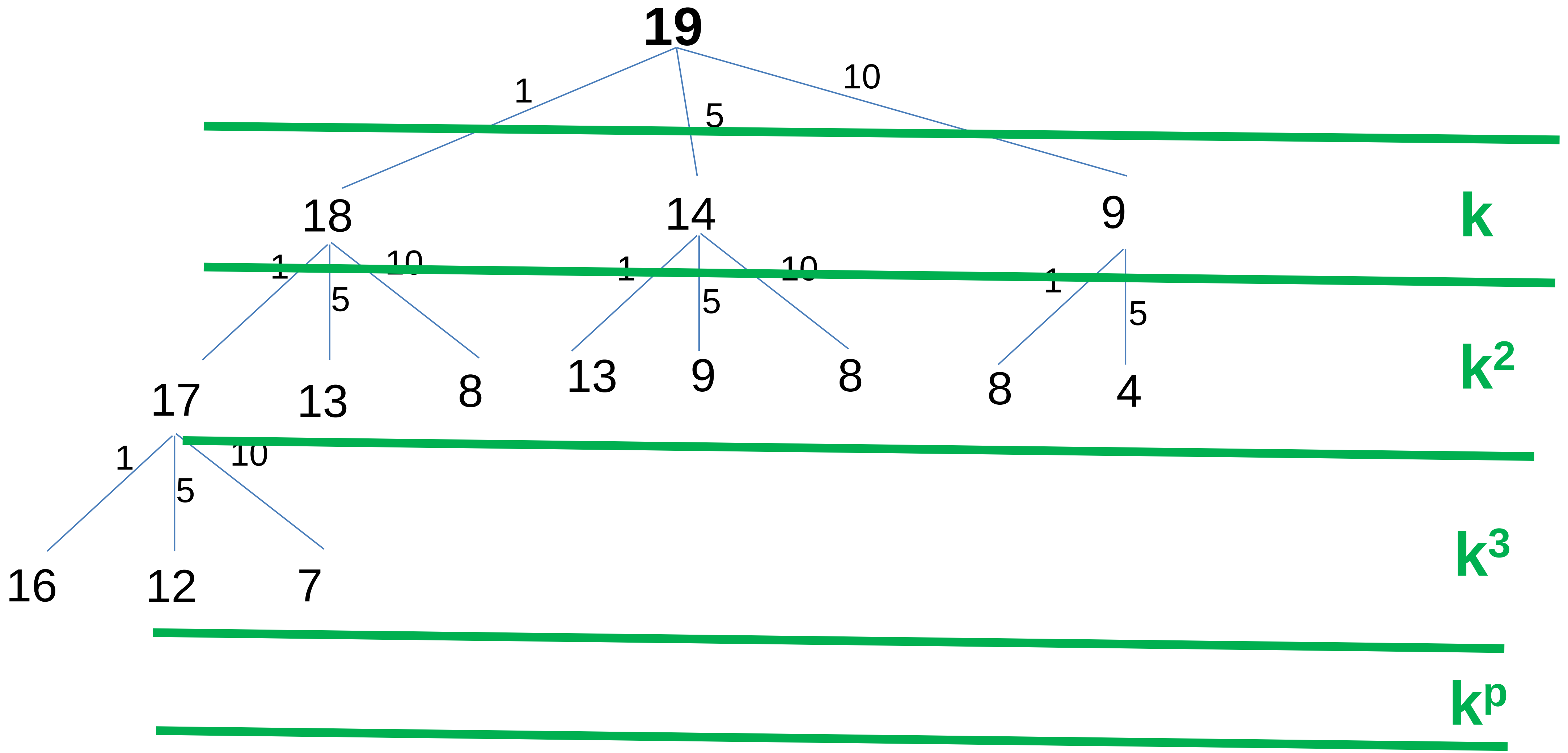


Make \$0.19 with \$0.01, \$0.05, \$0.10

k = # denominations

p = value to make change for

As k and p both grow, the number of recursive calls being made will grow **exponentially**

If we have a lot of coin denominations, we will have **a lot** of branching

k

k^2

k^3

k^p

Change Making Problem

Make $0.19 with $0.01, $0.05, $0.10

k = # denominations

p = value to make change for

19

As k and p both grow, the number of recursive calls being made will grow **exponentially**

Running time: O(💀)

k

k^2

k^3

k^p

Change Making Problem

Make \$0.19 with \$0.01, \$0.05, \$0.10

k = # denominations

p = value to make change for

19

As k and p both grow, the number of recursive calls being made will grow **exponentially**

Running time: probably $O(k^p)$ or $O(k!)$

For a large set of denominations, or a large p, this algorithm will take a long time to run

k

k^2

k^3

k^p

Change Making Problem

Combinations for 7 cents
Using D = [1, 5, 10]

[1, 1, 5]
[1, 1, 1, 1, 1, 1, 1]

[1, 1, 5] and [5, 1, 1] is the same combination…

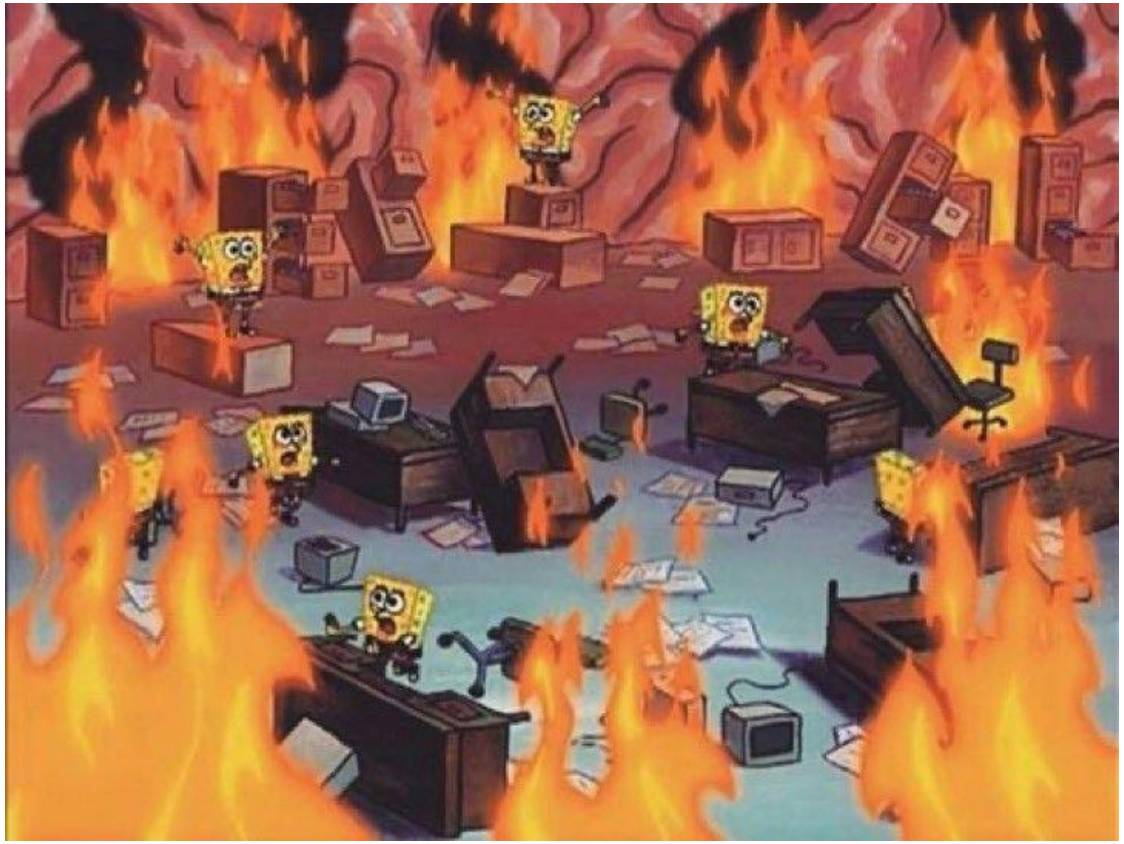
Permutations for 7 cents
Using D = [1, 5, 10]

[1, 1, 5]
[5, 1, 1]
[1, 5, 1]
…
[1, 1, 1, 1, 1, 1, 1]

Order does not matter.
[1, 1, 5] and [5, 1, 1] are considered different solutions

In our change making algorithm, we are calculating every possible permutation (bad)

Let’s try 81 cents!



Change Making Problem

This algorithm returns the minimum number of coins needed (ie 4), but it does not tell us what coins were used in that solution

Change Making Problem

This algorithm returns the minimum number of coins needed (ie 4), but it does not tell us what coins were used in that solution

D = Array of coin denominations [1, 5, 10, 25]
p = value to make change for
n = minimum number of coins used to make p cents

Goal: Find an **n-length** combination of coins from **D** that were used to make **p**

Change Making Problem

This algorithm returns the minimum number of coins needed (ie 4), but it does not tell us what coins were used in that solution

D = Array of coin denominations [1, 5, 10, 25]
p = value to make change for
n = minimum number of coins used to make p cents

Goal: Find an **n-length** combination of coins from **D** that were used to make **p**

To do this, we will compute **all n-length combinations**, but only return the combinations that add up to be **p** (not very efficient)

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

For **n=3**, these are the combinations to be generated:

- [1, 1, 1]
- [1, 1, 5]
- [1, 1, 10]
- [1, 5, 5]
- [1, 5, 10]
- [1, 10, 10]
- [5, 5, 5]
- [5, 5, 10]
- [5, 10, 10]
- [10, 10, 10]

Note:

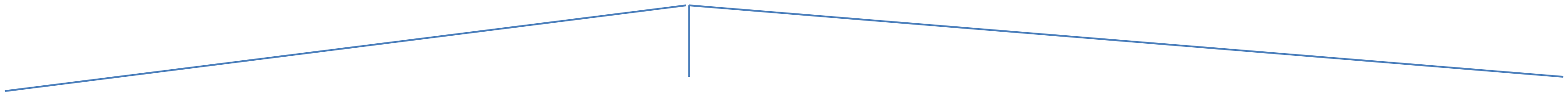
[5, 1, 1] is not a “valid” combination, because it is the same thing as [1, 1, 5]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

[___, ___, ___,]



Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]
n = 3

[__, __, __,]

[1, __, __,]

[5, __, __,]

[10, __, __,]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]
n = 3

[__, __, __,]

D = [1, 5, 10]
[1, __, __,]

D = [5, 10]
[5, __, __,]

D = [10]
[10, __, __,]

This branch does not get 1 included in the denomination set, because all the combinations involving 1 will be handled by the left tree

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

[__, __, __,]

D = [1, 5, 10]

[1, __, __,]

D = [5, 10]

[5, __, __,]

D = [10]

[10, __, __,]

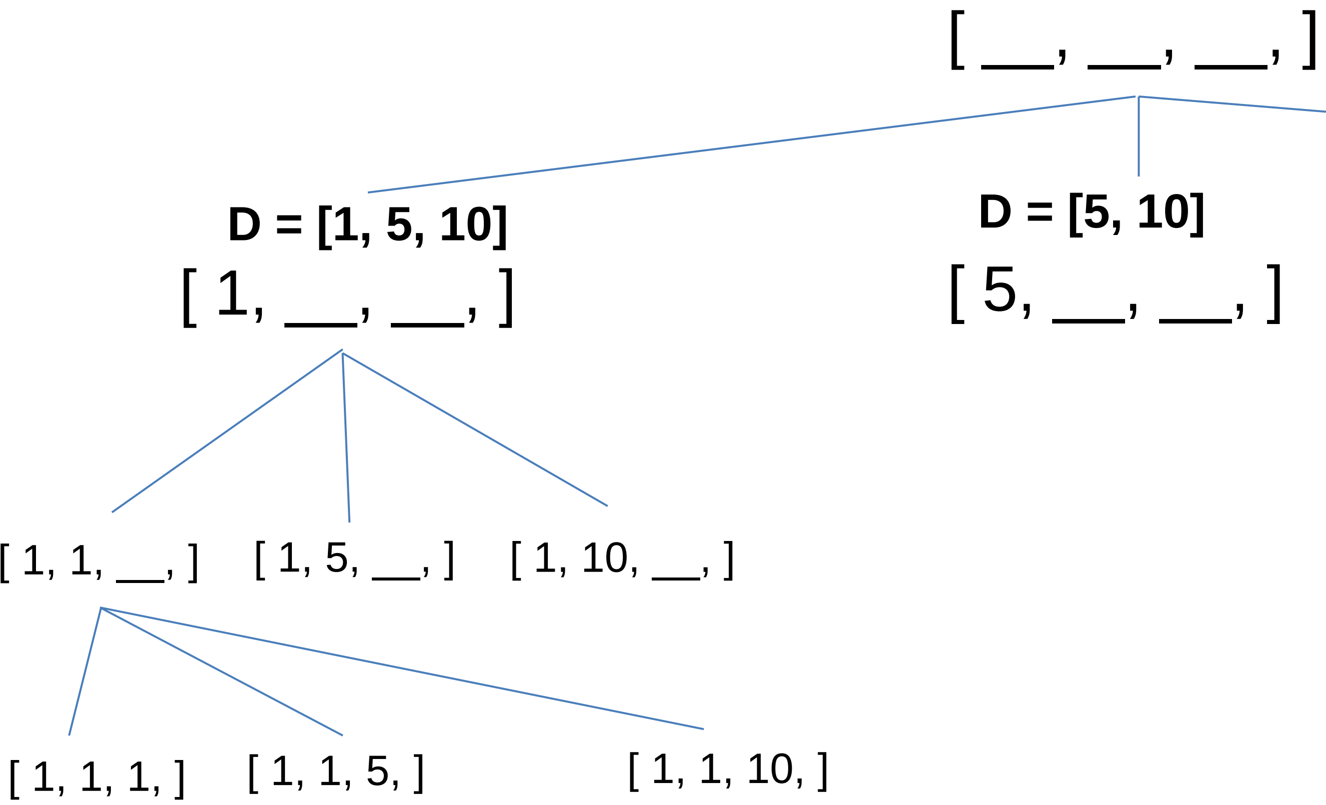
[1, 1, __,]

[1, 5, __,]

[1, 10, __,]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]
n = 3

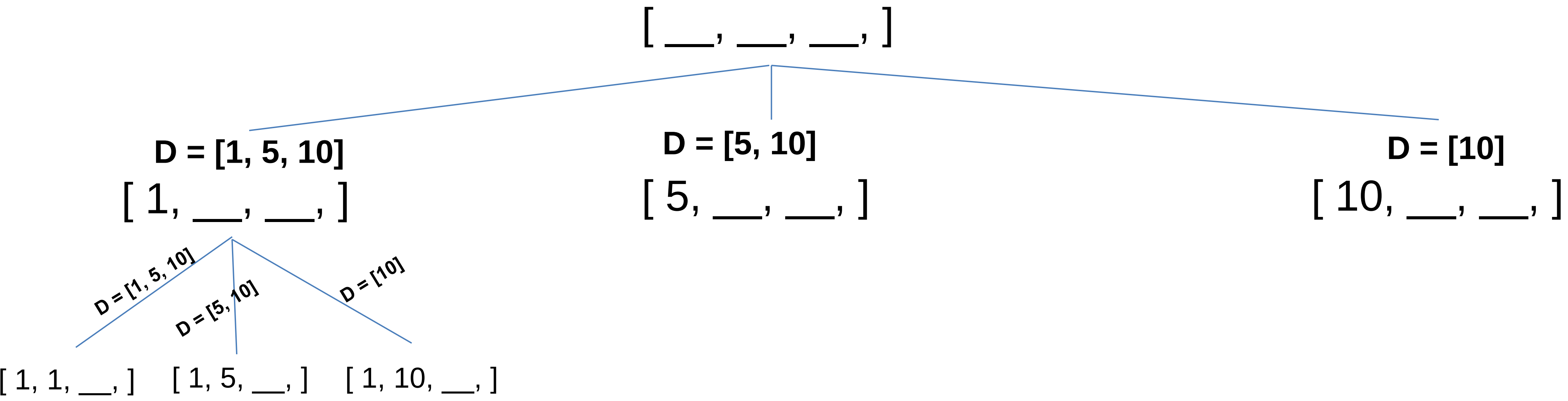


(Base case) When are combinations reach a length of 3, we will stop recursing

[1, 1, 1] [1, 1, 5] [1, 1, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]
n = 3

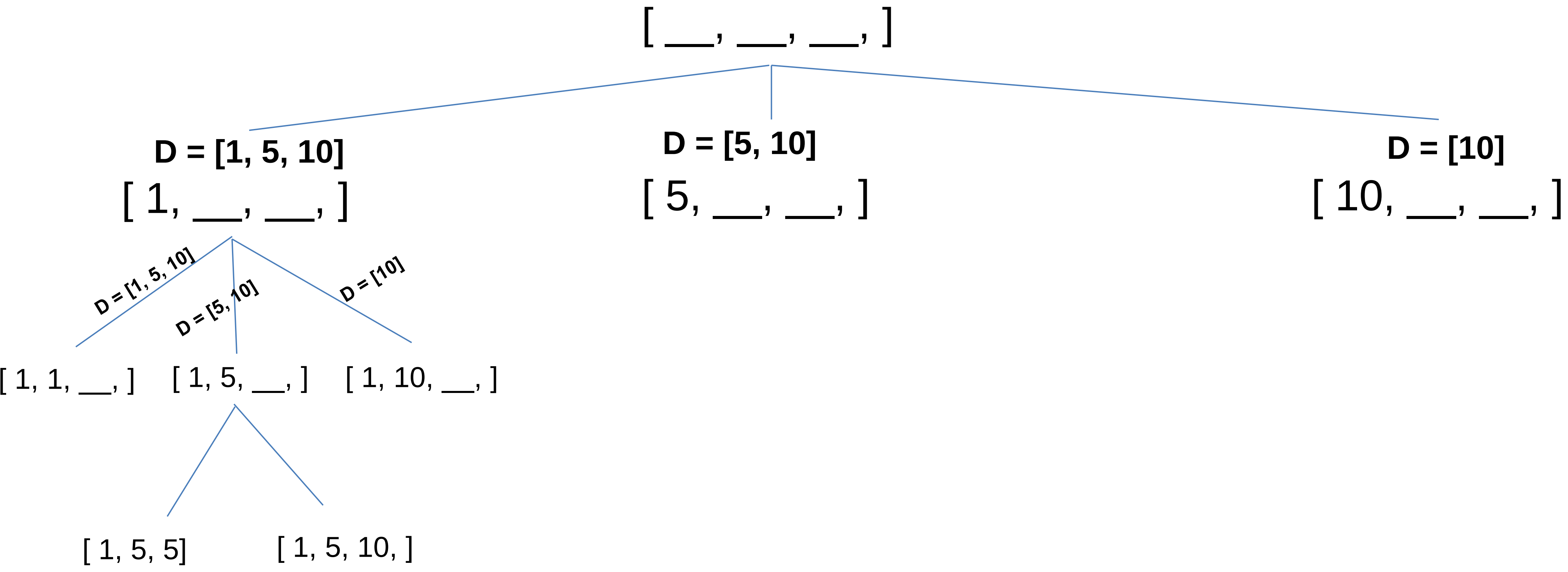


[1, 1, 1] [1, 1, 5] [1, 1, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

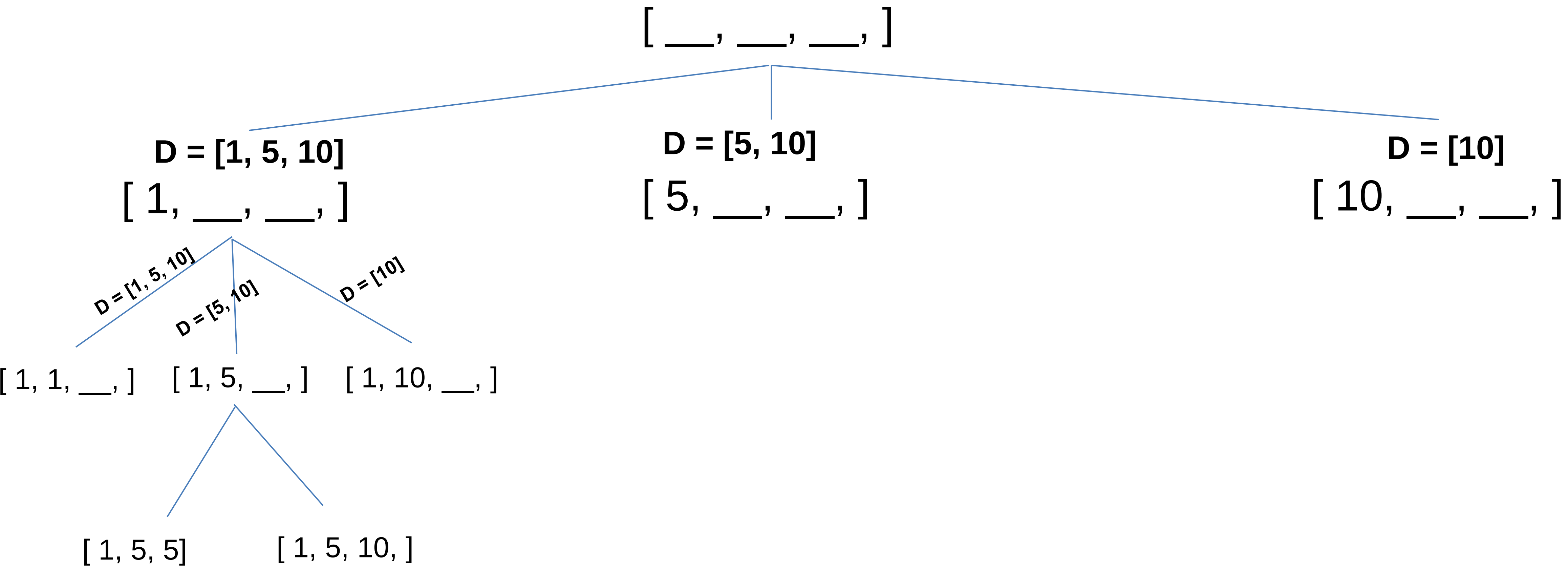


[1, 1, 1] [1, 1, 5] [1, 1, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

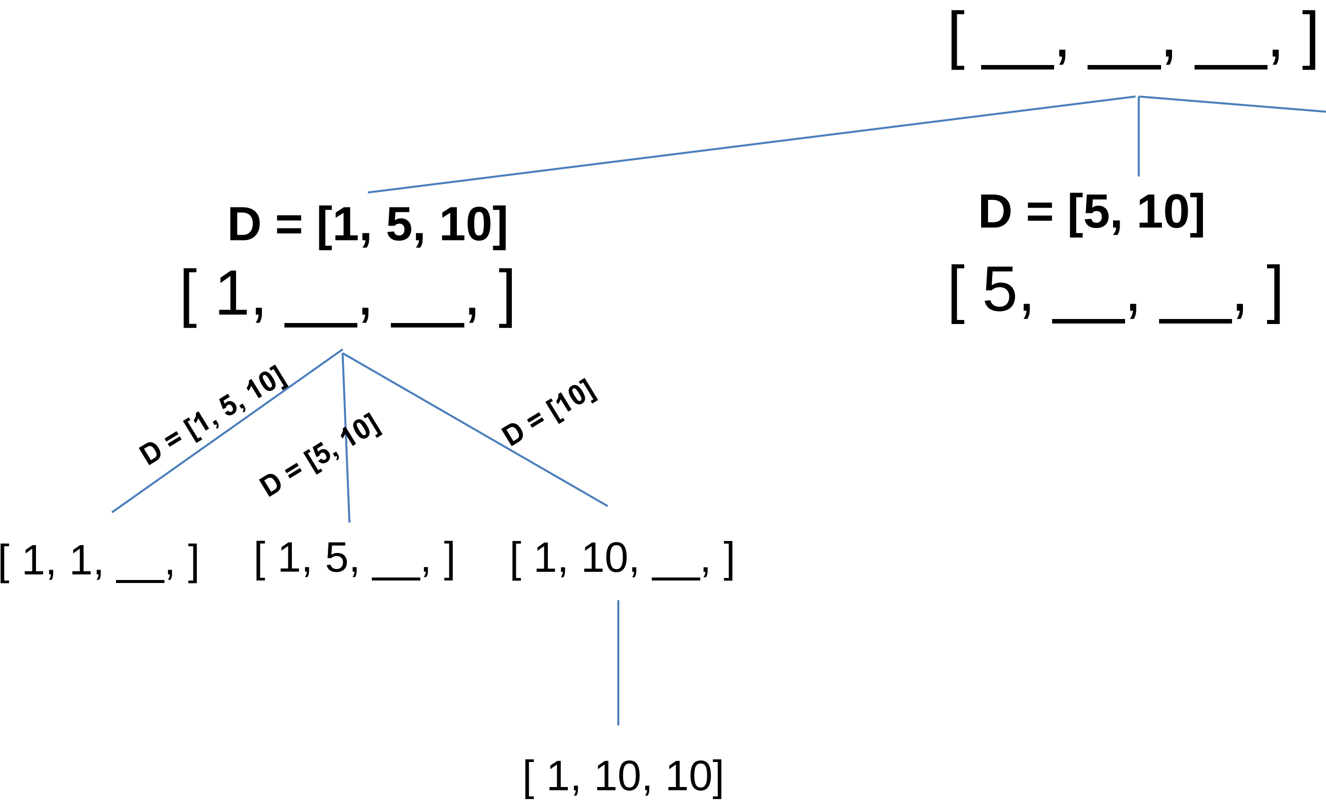


[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

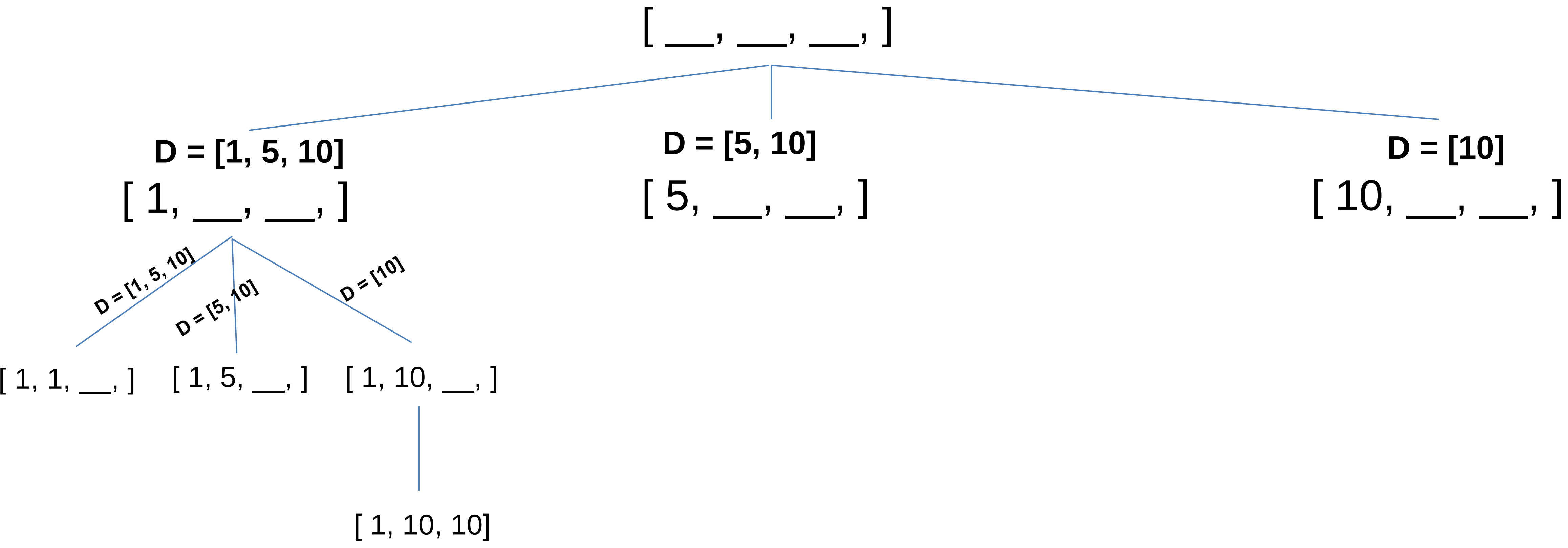
n = 3



[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]
n = 3



[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10] [1, 10, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]
n = 3

[__, __, __,]

D = [1, 5, 10]
[1, __, __,]

D = [5, 10]
[5, __, __,]

D = [10]
[10, __, __,]

D = [5, 10]
[5, 5, __,]

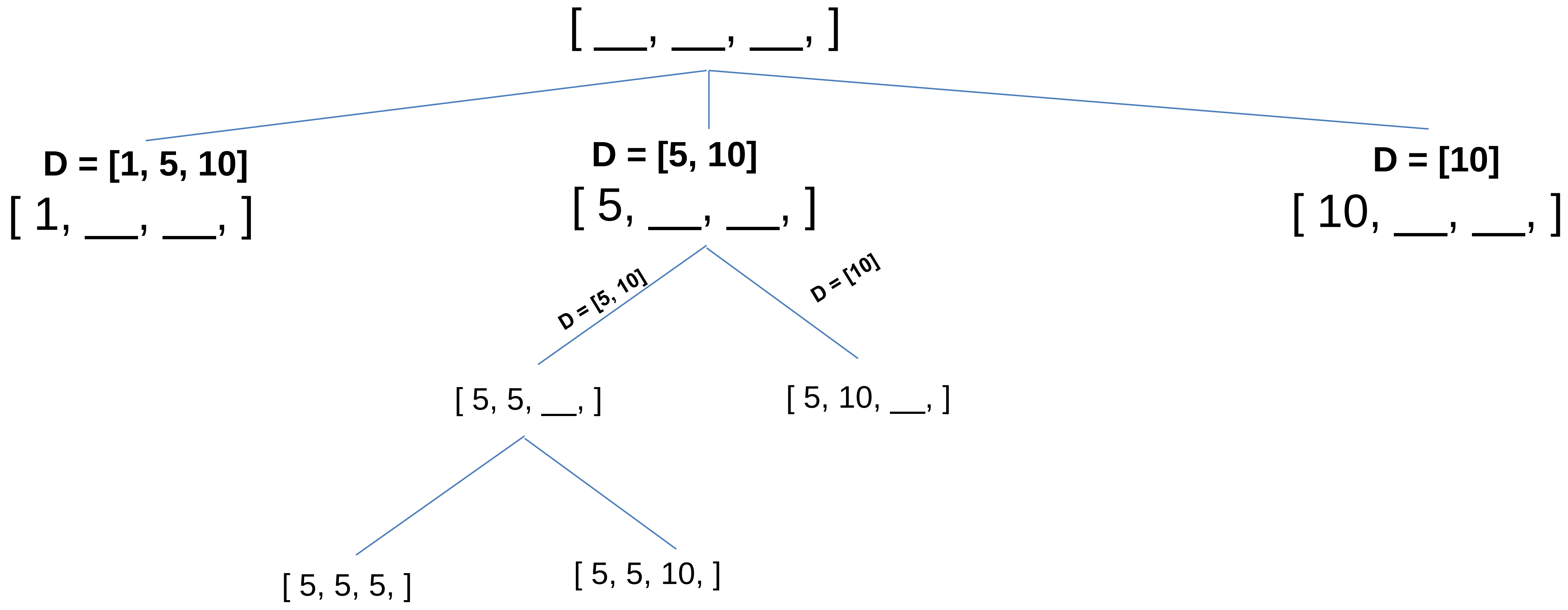
D = [10]
[5, 10, __,]

[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10] [1, 10, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

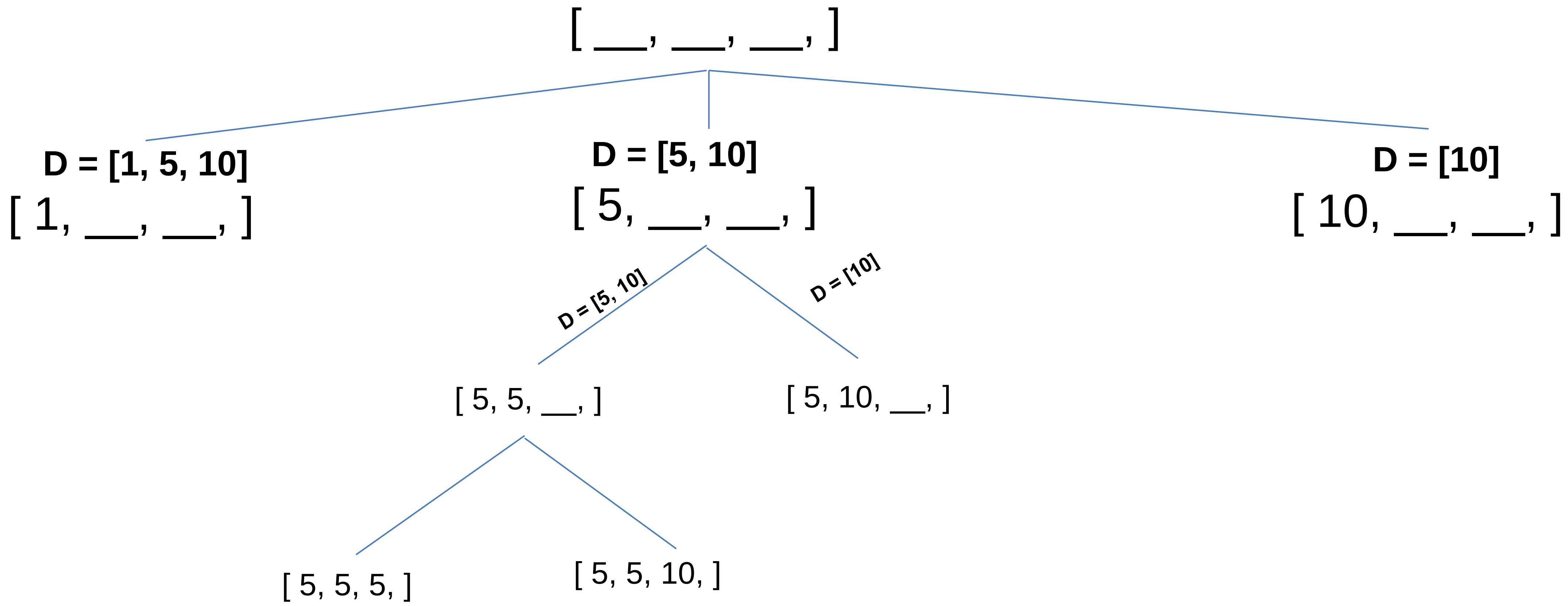


[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10] [1, 10, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

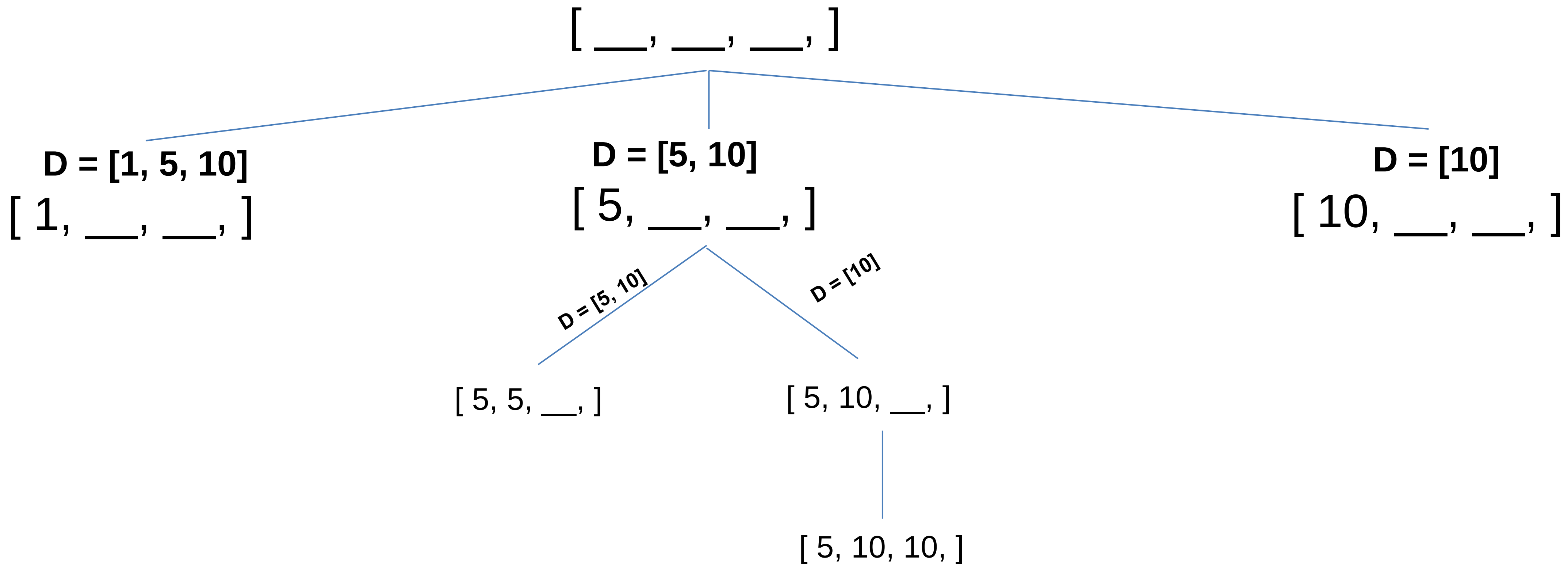
n = 3



[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10] [1, 10, 10] [5, 5, 5] [5, 5, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]
n = 3

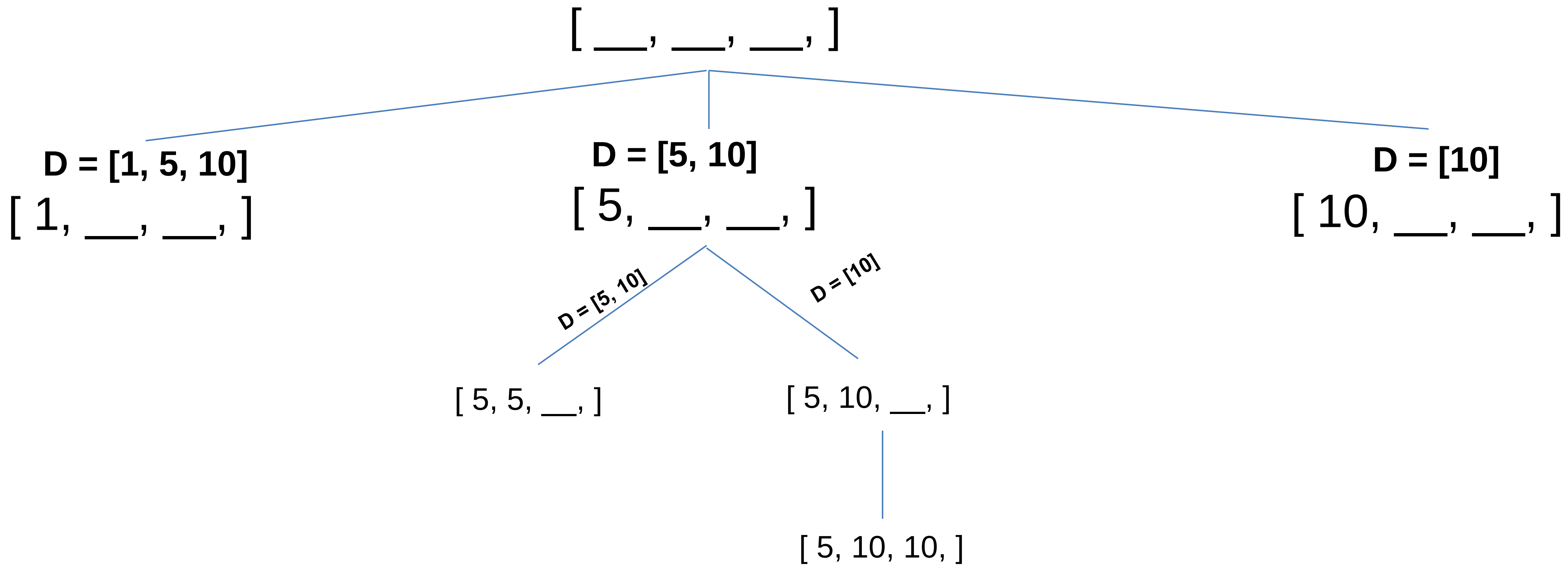


[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10] [1, 10, 10] [5, 5, 5] [5, 5, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3



[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10] [1, 10, 10] [5, 5, 5] [5, 5, 10] [5, 10, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]
n = 3

[__, __, __,]

D = [1, 5, 10]
[1, __, __,]

D = [5, 10]
[5, __, __,]

D = [10]
[10, __, __,]

D = [10]

[10, 10, __,]

[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10] [1, 10, 10] [5, 5, 5] [5, 5, 10] [5, 10, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

[__, __, __,]

D = [1, 5, 10]

[1, __, __,]

D = [5, 10]

[5, __, __,]

D = [10]

[10, __, __,]

D = [10]

[10, 10, __,]

[10, 10, 10,]

[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10] [1, 10, 10] [5, 5, 5] [5, 5, 10] [5, 10, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

[__, __, __,]

D = [1, 5, 10]

[1, __, __,]

D = [5, 10]

[5, __, __,]

D = [10]

[10, __, __,]

D = [10]

[10, 10, __,]

[10, 10, 10,]

[1, 1, 1] [1, 1, 5] [1, 1, 10] [1, 5, 5] [1, 5, 10] [1, 10, 10] [5, 5, 5] [5, 5, 10] [5, 10, 10] [10, 10, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

1. [1, 1, 1]
2. [1, 1, 5]
3. [1, 1, 10]
4. [1, 5, 5]
5. [1, 5, 10]
6. [1, 10, 10]
7. [5, 5, 5]
8. [5, 5, 10]
9. [5, 10, 10]
10. [10, 10, 10]

We've generated all combinations of length 3

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

1. [1, 1, 1]
2. [1, 1, 5]
3. [1, 1, 10]
4. [1, 5, 5]
5. [1, 5, 10]
6. [1, 10, 10]
7. [5, 5, 5]
8. [5, 5, 10]
9. [5, 10, 10]
10. [10, 10, 10]

We've generated all combinations of length 3

Now, we only want to print out the combinations that add up to **K**

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

K = 16

1. [1, 1, 1]
2. [1, 1, 5]
3. [1, 1, 10]
4. [1, 5, 5]
5. [1, 5, 10]
6. [1, 10, 10]
7. [5, 5, 5]
8. [5, 5, 10]
9. [5, 10, 10]
10. [10, 10, 10]

We've generated all combinations of length 3

Now, we only want to print out the combinations that add up to **K**

Suppose K = 16 (a minimum of 3 coins is needed to make 16 cents)

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

K = 16

1. [1, 1, 1]
2. [1, 1, 5]
3. [1, 1, 10]
4. [1, 5, 5]
5. [1, 5, 10]
6. [1, 10, 10]
7. [5, 5, 5]
8. [5, 5, 10]
9. [5, 10, 10]
10. [10, 10, 10]

We've generated all combinations of length 3

Now, we only want to print out the combinations that add up to **K**

Suppose K = 16 (a minimum of 3 coins is needed to make 16 cents)

Answer = [1, 5, 10]

Generating Combinations for finding coins

Denominations (D) = [1, 5, 10]

n = 3

LET'S CODE THIS!!

If you don't fully understand this code, that is fine.

o to **K**

16 cents)

10.[10, 10, 10]

Answer = [1, 5, 10]

```java
private static void find_coins(int[] d, int k, int n) {
        int chosen[] = new int[n + 1];
        calculate_combinations(chosen, d, 0, n, 0, d.length - 1, k);
}
```

```java
void calculate_combinations(int[] chosen, int[] arr, int index, int r, int start, int end, int target) {
```

Array that we build up over time. Holds **indices** of currently selected denominations for some combination

[1,__, __]

[1,1, __]

[1,1, 5]

```java
private static void find_coins(int[] d, int k, int n) {
        int chosen[] = new int[n + 1];
        calculate_combinations(chosen, d, 0, n, 0, d.length - 1, k);
}
```

```java
void calculate_combinations(int[] chosen, int[] arr, int index, int r, int start, int end, int target) {
```

Array of denominations we pass for each recursive call

[__, __, __,]

D = [1, 5, 10]
[1, __, __,]

D = [5, 10]
[5, __, __,]

D = [10]
[10, __, __,]

```java
private static void find_coins(int[] d, int k, int n) {
        int chosen[] = new int[n + 1];
        calculate_combinations(chosen, d, 0, n, 0, d.length - 1, k);
}
```

```java
void calculate_combinations(int[] chosen, int[] arr, int index, int r, int start, int end, int target) {
```

The next index that we need to insert at for `chosen` array

[__, __, __]   [1, __, __]   [1,1, __]   [1,1, 5]

index   index   index

```
private static void find_coins(int[] d, int k, int n) {
        int chosen[] = new int[n + 1];
        calculate_combinations(chosen, d, 0, n, 0, d.length - 1, k);
}
```

```
void calculate_combinations(int[] chosen, int[] arr, int index, int r, int start, int end, int target) {
```

The desired size of the combination. When `index == r`, we have reached the desired combination size

[__, __, __]
index = 0

[1, __, __]
index = 1

[1,1, __]
index = 2

[1,1, 5]
index = 3

We've hit our desired size!

```java
void calculate_combinations(int[] chosen, int[] arr, int index, int r, int start, int end, int target) {
        if (index == r) {
                int counter = 0;
                ArrayList<Integer> coins = new ArrayList<Integer>();
                for (int i = 0; i < r; i++) {
                        counter += arr[chosen[i]];
                        coins.add(arr[chosen[i]]);
                }
                if(counter == target) {
                        System.out.println(coins);
                }
                return;
        }
        for (int i = start; i <= end; i++) {
                chosen[index] = i;
                calculate_combinations(chosen, arr, index + 1, r, i, end, target);
        }
        return;
}
```

```java
void calculate_combinations(int[] chosen, int[] arr, int index, int r, int start, int end, int target) {
	if (index == r) {
		int counter = 0;
		ArrayList<Integer> coins = new ArrayList<Integer>();
		for (int i = 0; i < r; i++) {
			counter += arr[chosen[i]];
			coins.add(arr[chosen[i]]);
		}
		if(counter == target) {
			System.out.println(coins);
		}
		return;
	}
	for (int i = start; i <= end; i++) {
		chosen[index] = i;
		calculate_combinations(chosen, arr, index + 1, r, i, end, target);
	}
	return;
}
```

Base case

If we hit our base case, we know we have N things, so put them in an ArrayList and add them up

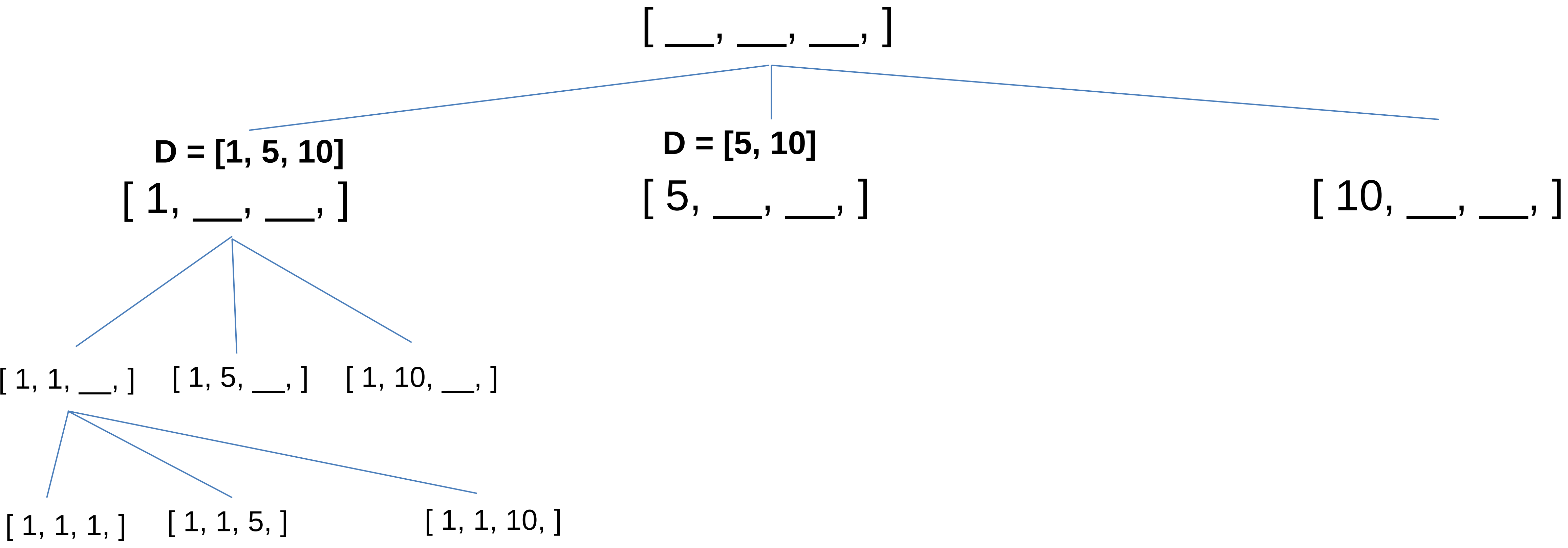
Only print out the combination if it adds up to `target`

```java
void calculate_combinations(int[] chosen, int[] arr, int index, int r, int start, int end, int target) {
        if (index == r) {
                int counter = 0;
                ArrayList<Integer> coins = new ArrayList<Integer>();
                for (int i = 0; i < r; i++) {
                        counter += arr[chosen[i]];
                        coins.add(arr[chosen[i]]);
                }
                if(counter == target) {
                        System.out.println(coins);
                }
                return;
        }
        for (int i = start; i <= end; i++) {
                chosen[index] = i;
                calculate_combinations(chosen, arr, index + 1, r, i, end, target);
        }
        return;
}
```

Recursive Case

Otherwise, insert selected coin into the `chosen` array

create `(end-start)` branches, and give it a smaller section of `D`


[__, __, __,]
D = [1, 5, 10]
[1, __, __,]
D = [5, 10]
[5, __, __,]
[10, __, __,]
[1, 1, __,]
[1, 5, __,]
[1, 10, __,]
[1, 1, 1,]
[1, 1, 5,]
[1, 1, 10,]