CSCI 232:
Data Structures and Algorithms

Graphs (Representation)

Reese Pearsall

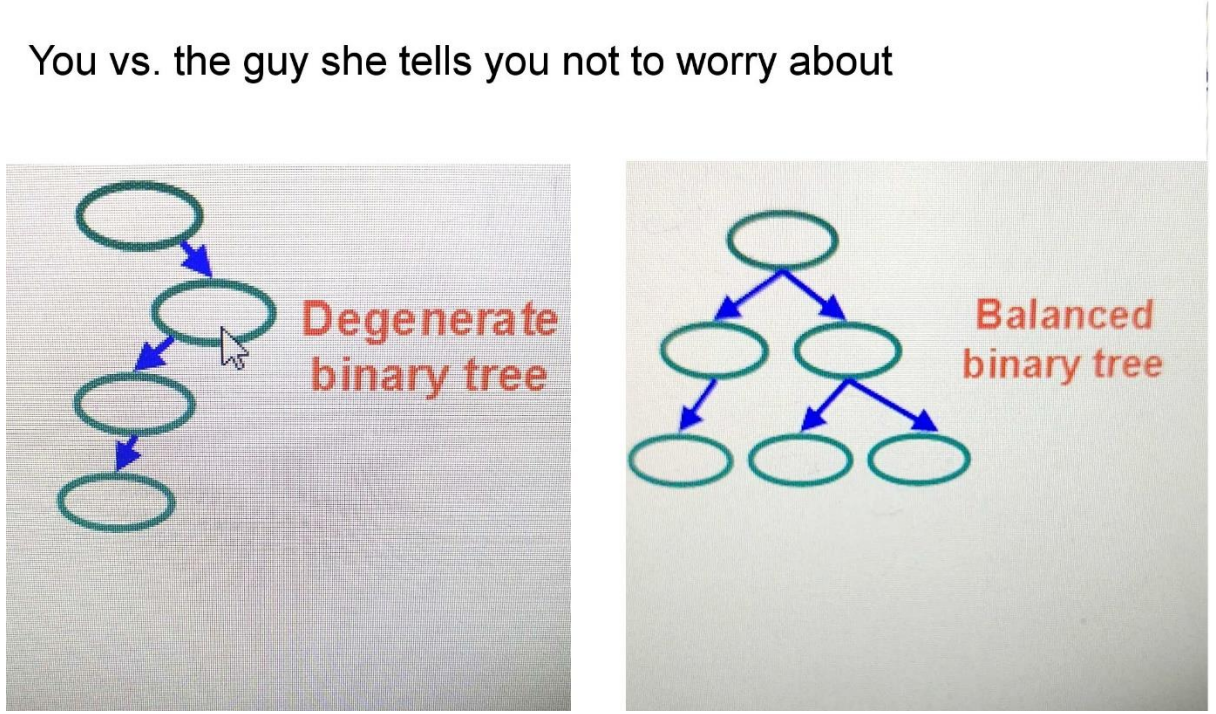
Spring 2024

Announcements

Lab 8 on Friday
→ Short survey… should be free points for you

Program 3 will be posted later this week

Next Tuesday (April 2nd) will be an asynchronous lecture (I'll post a lecture recording but no in-person lecture)



How could we visualize: The US Road Network?

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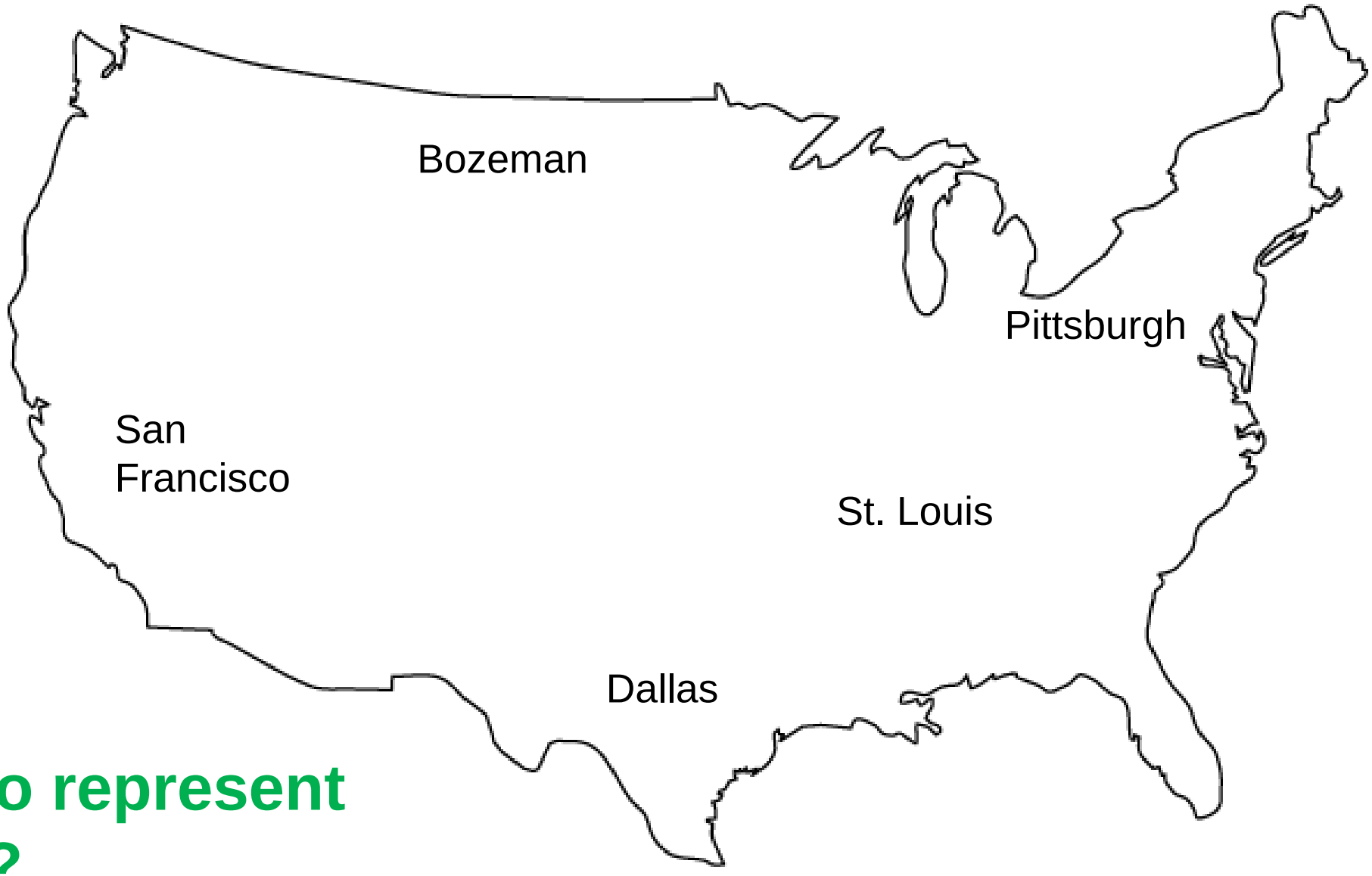

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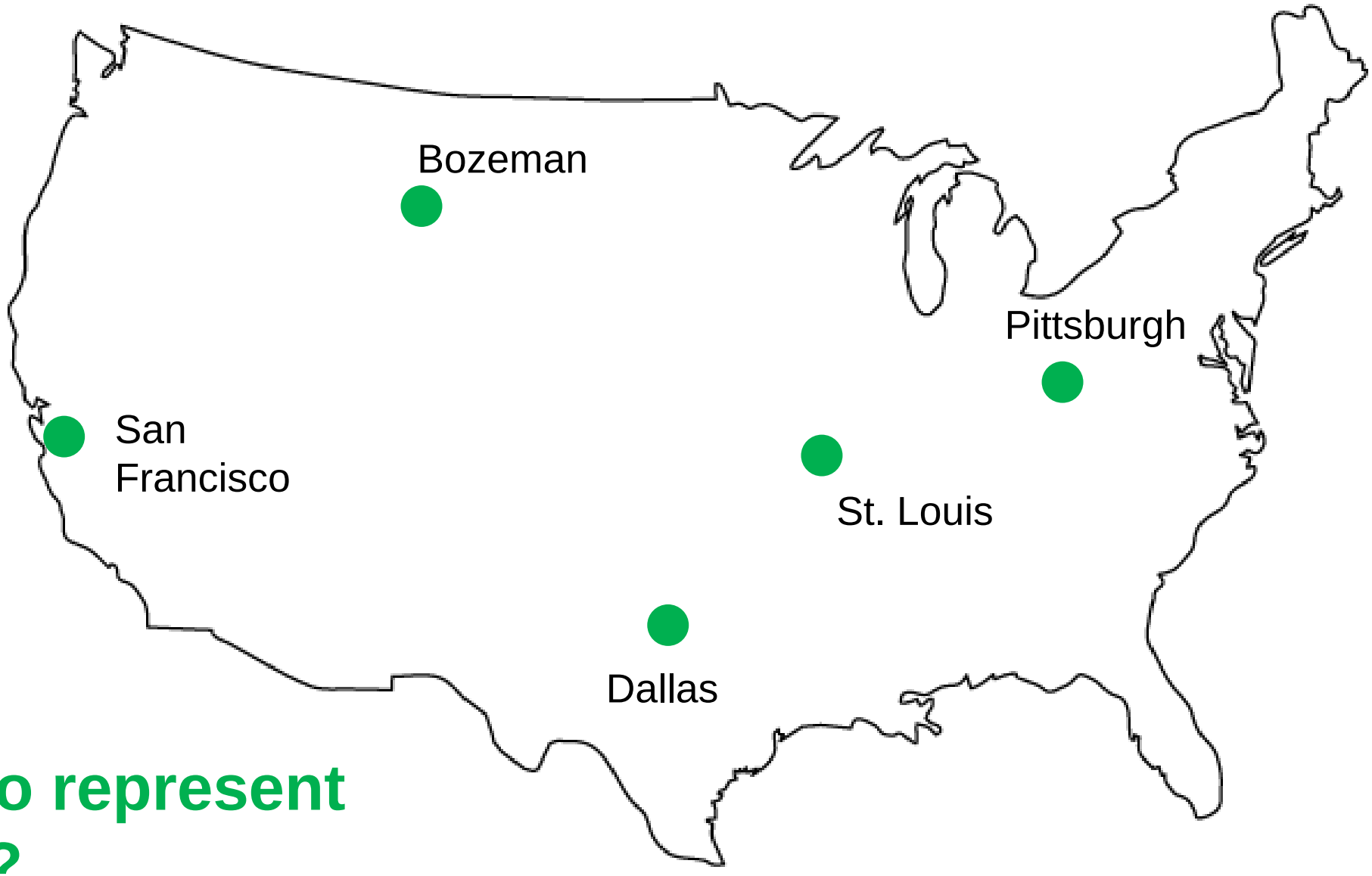

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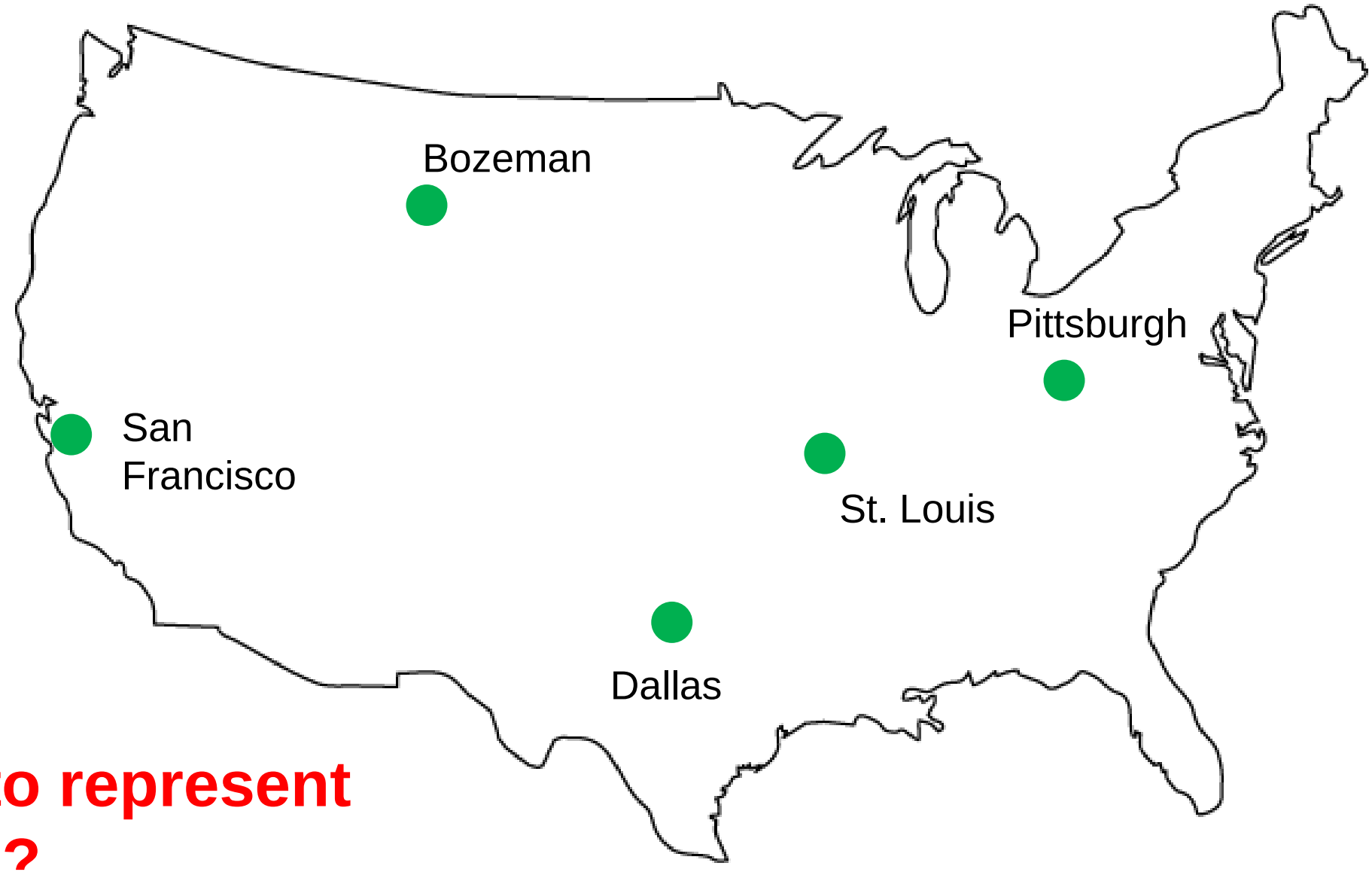

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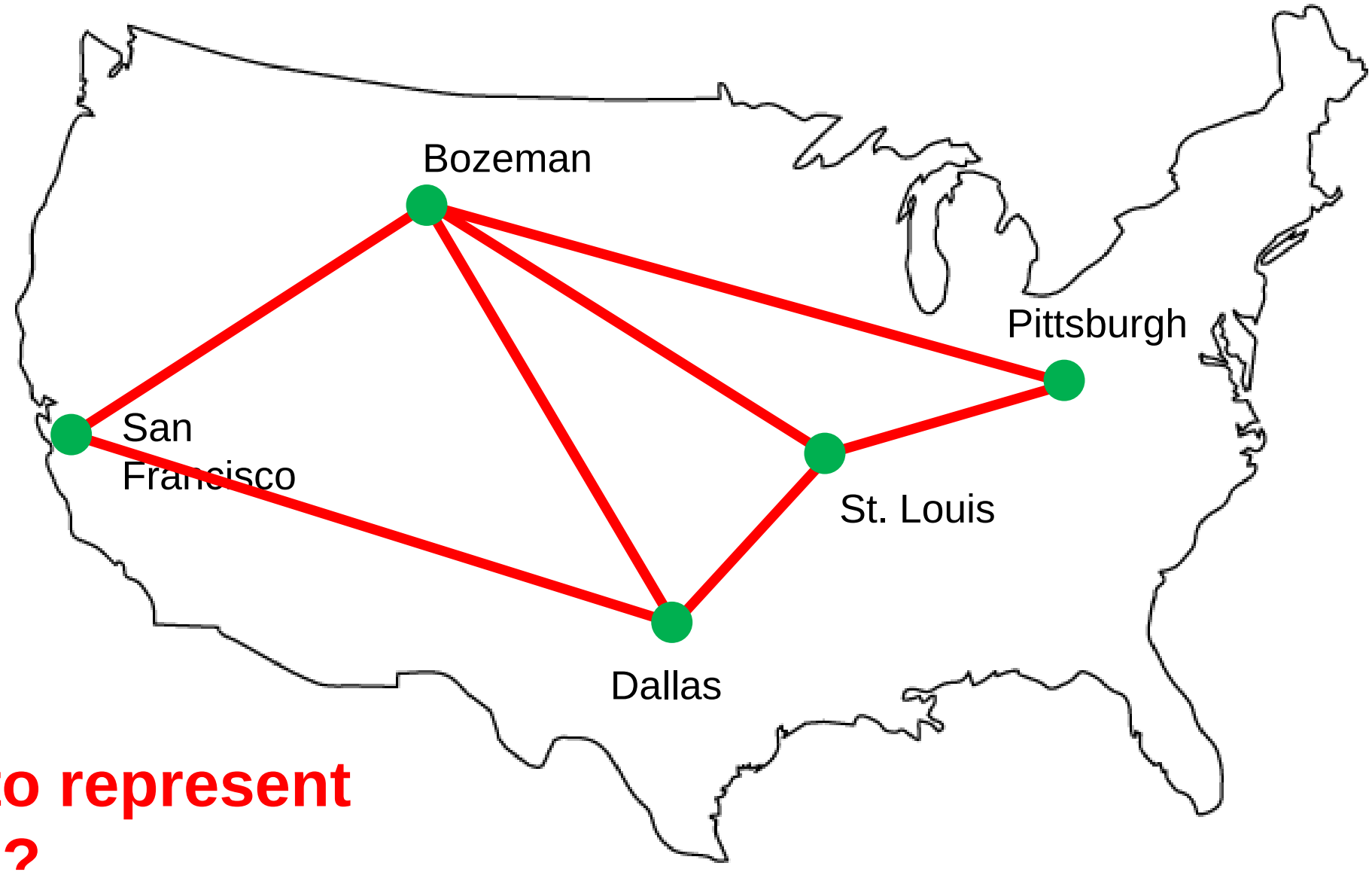

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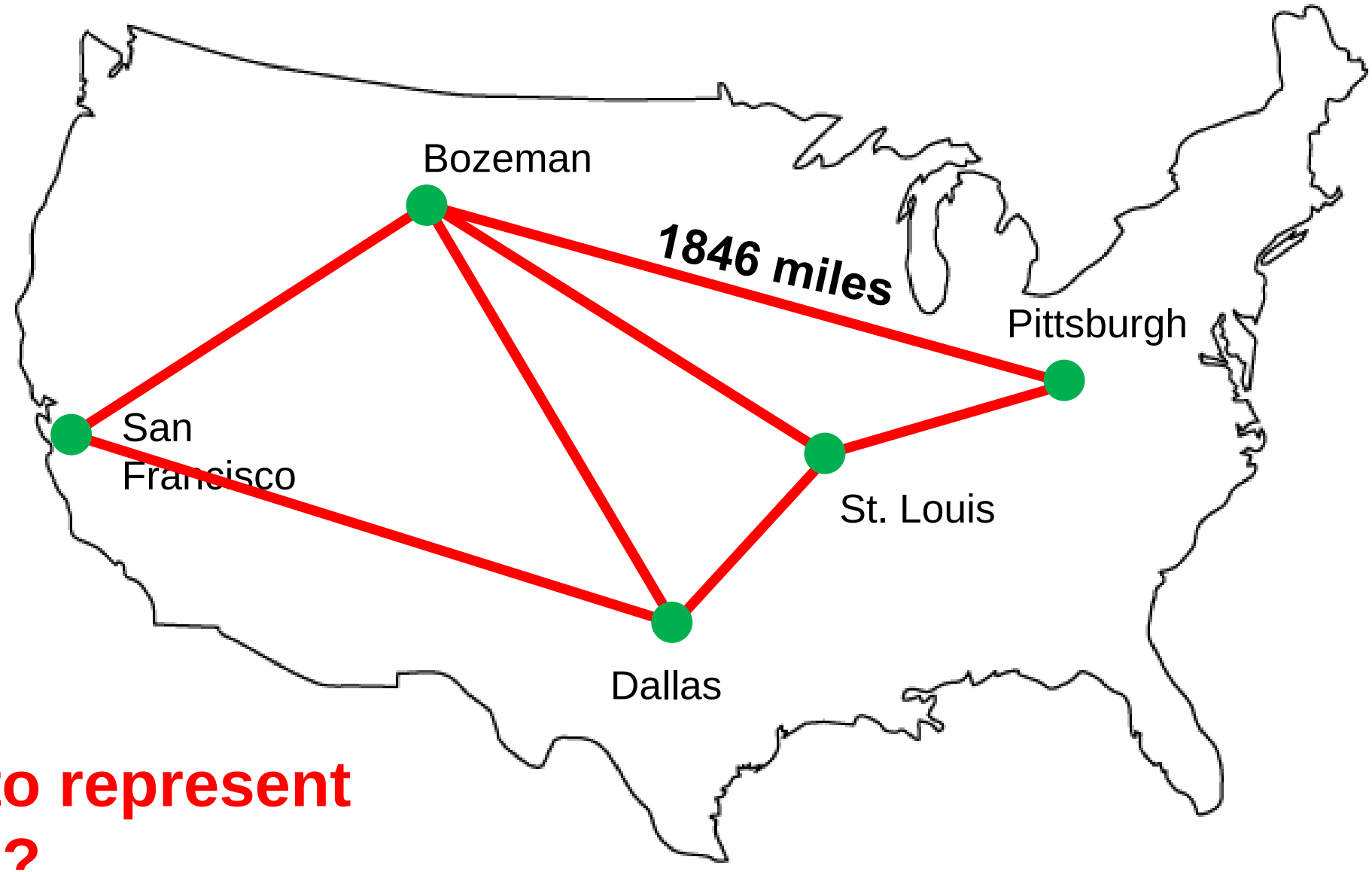

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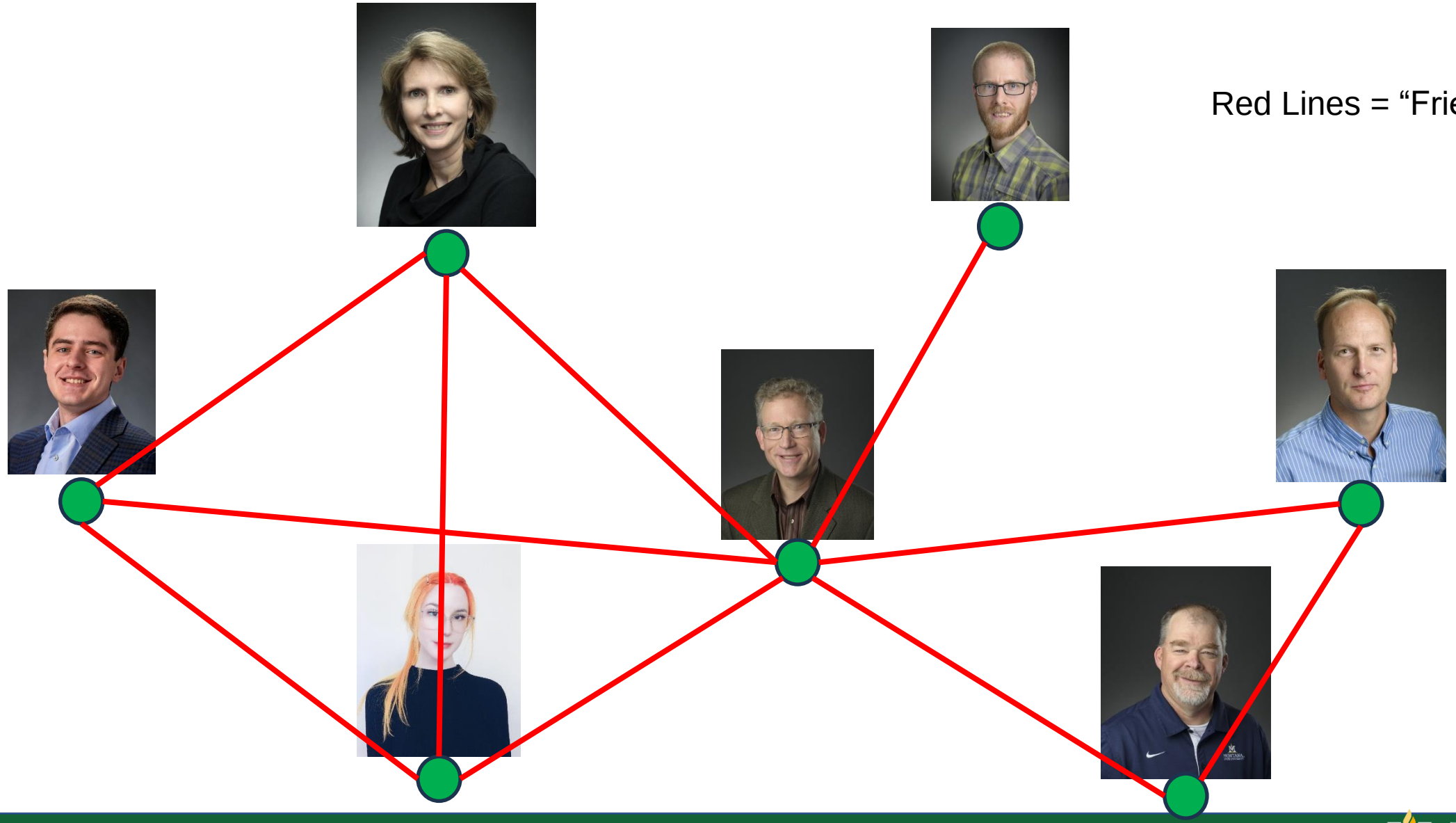

Red Lines = “Friends with”

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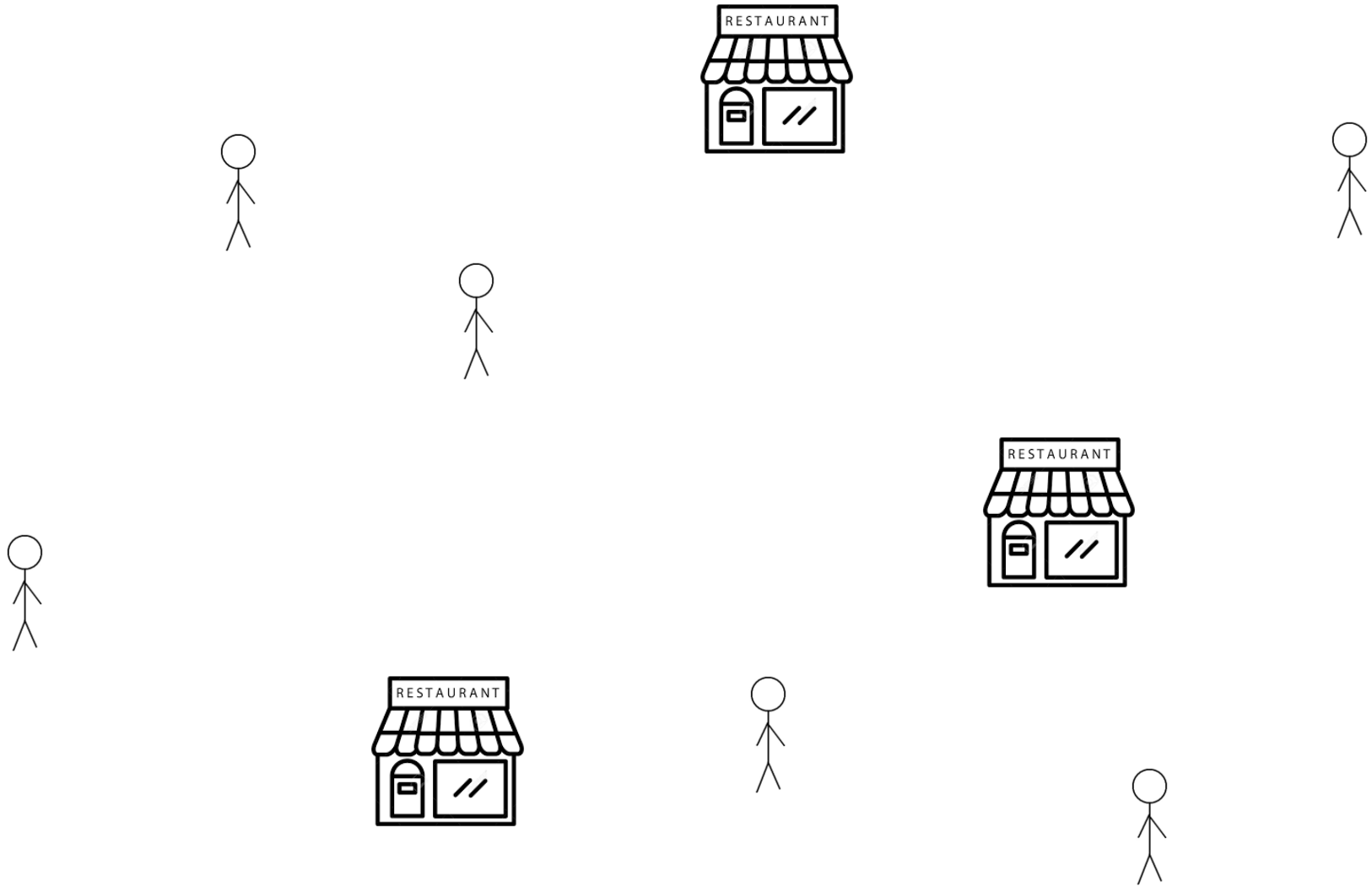

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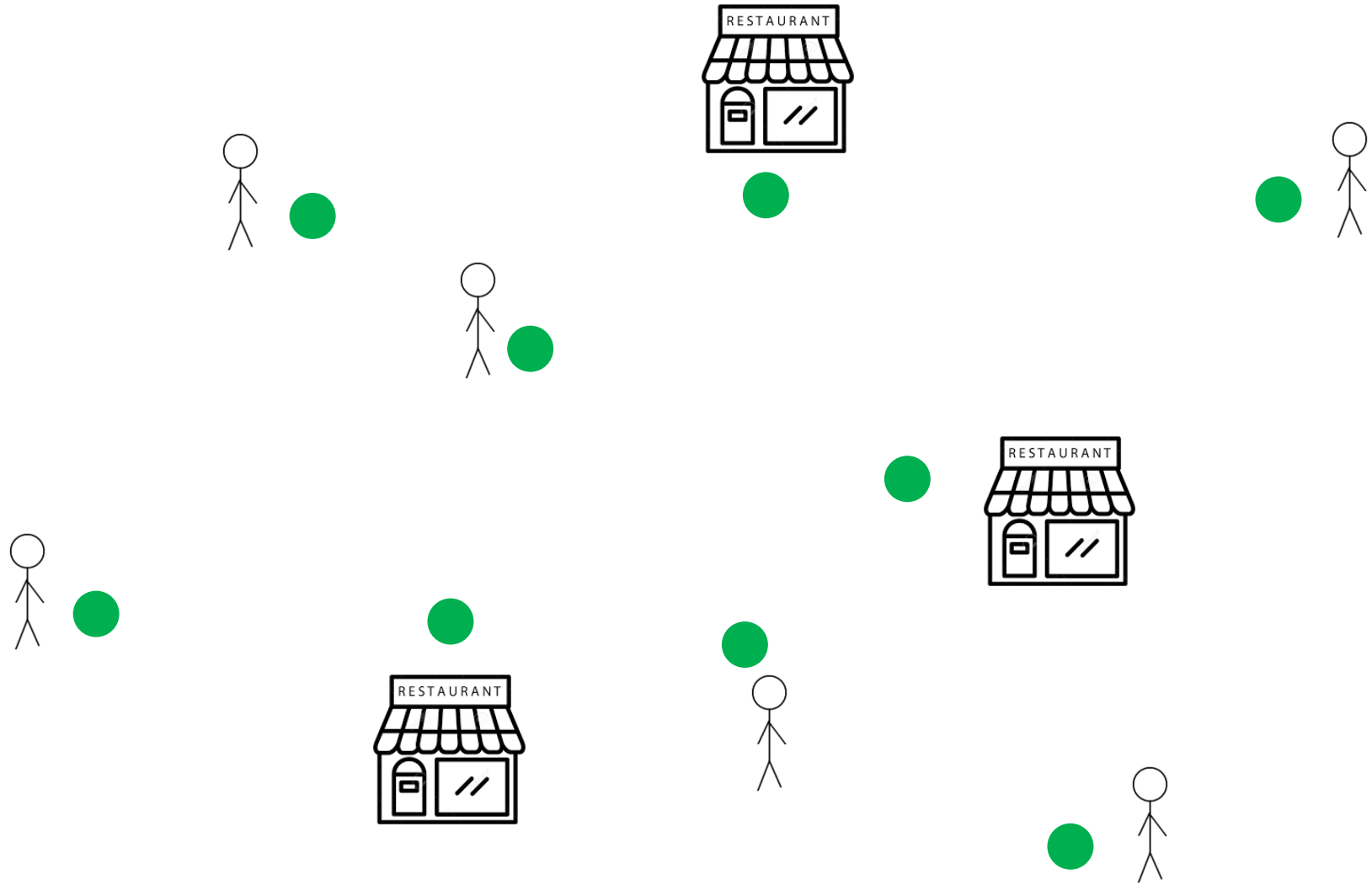

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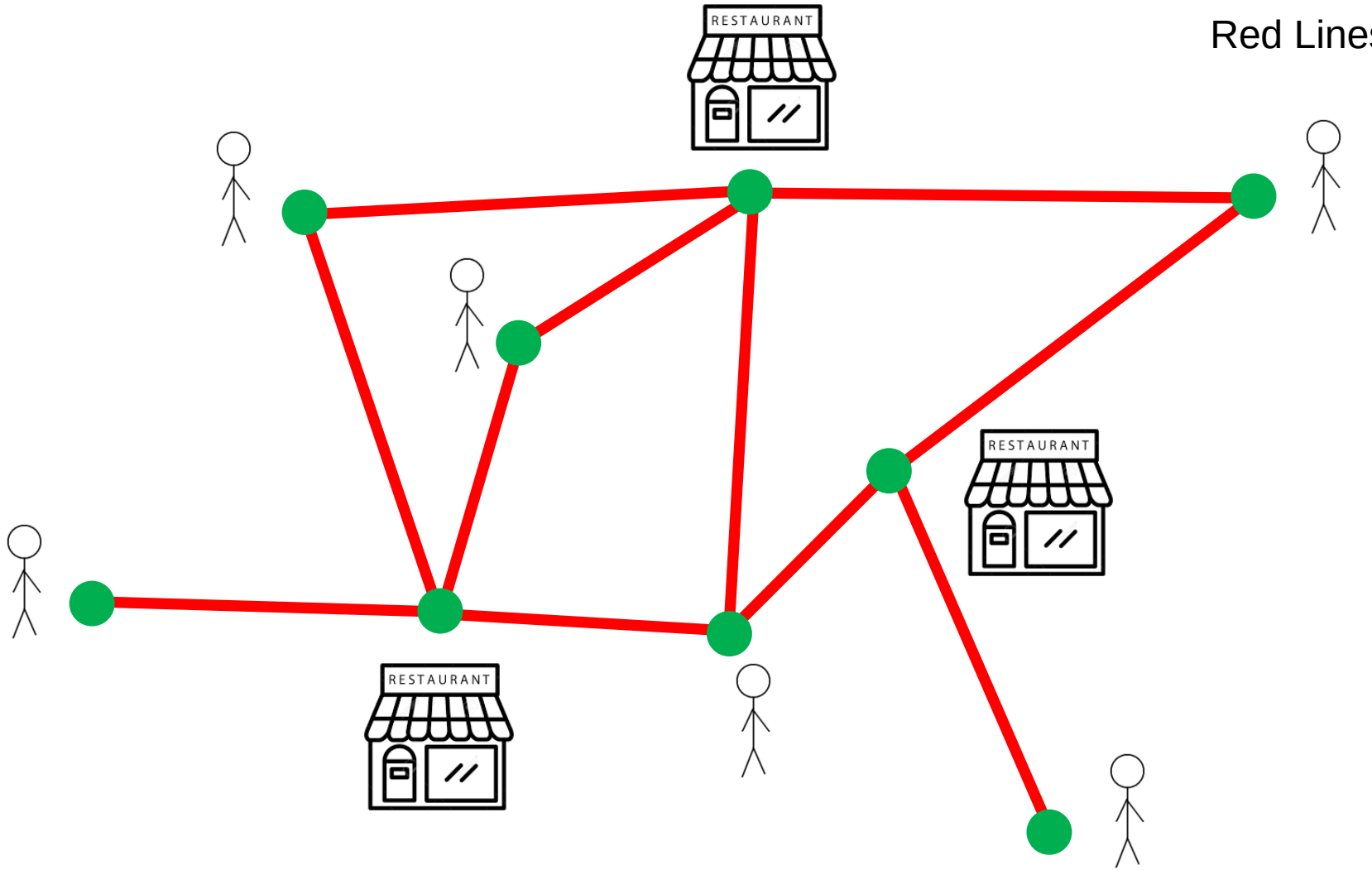
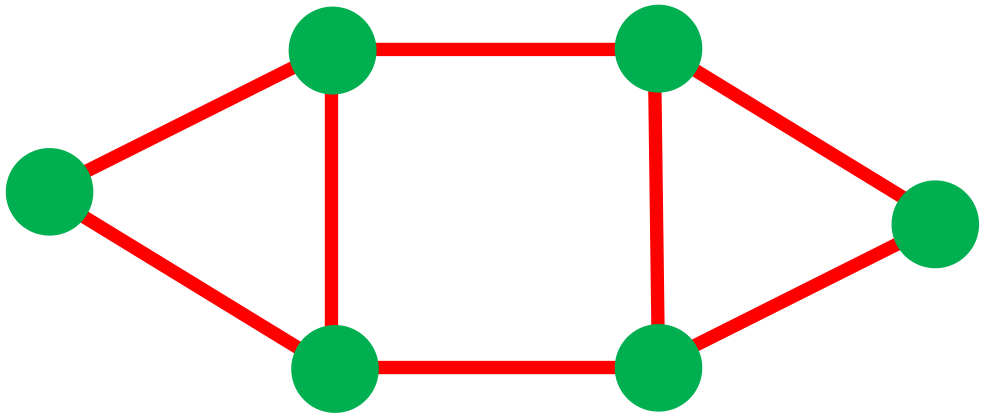


Graphs



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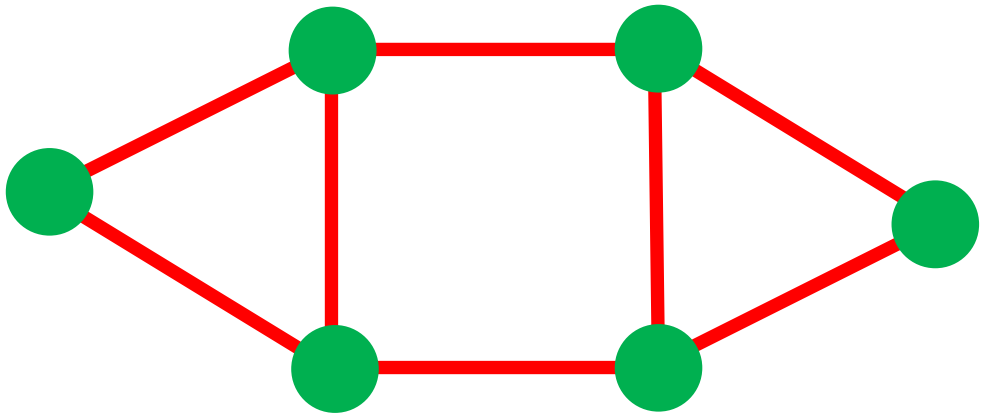

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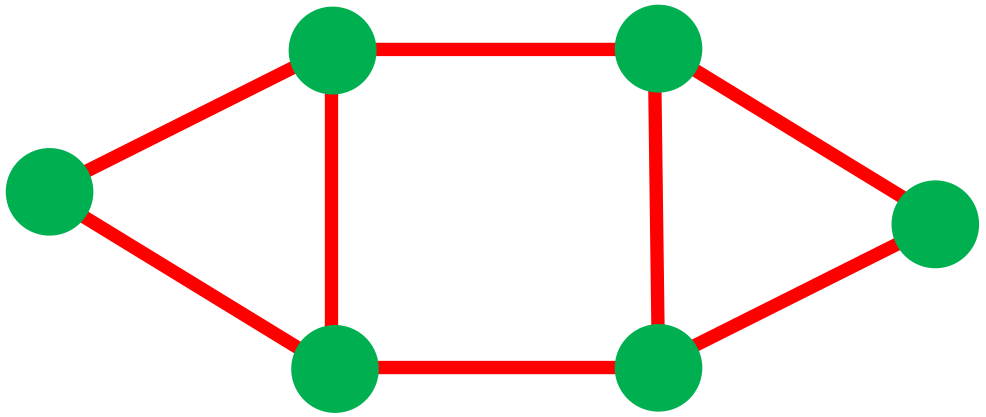

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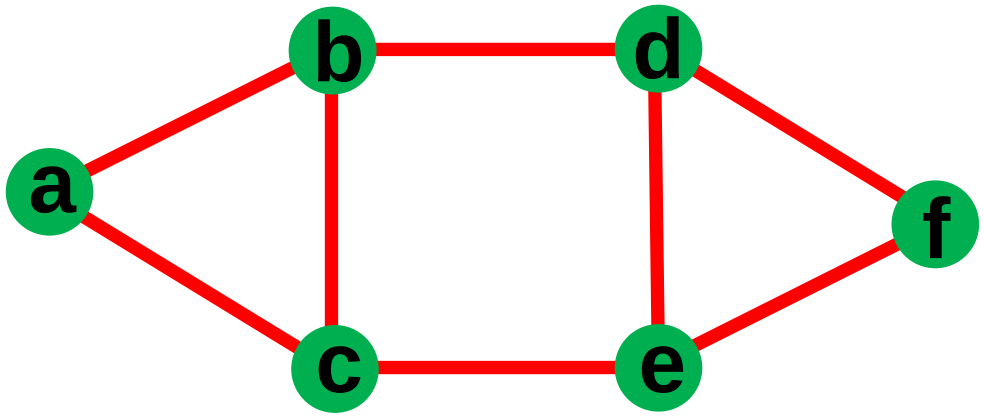

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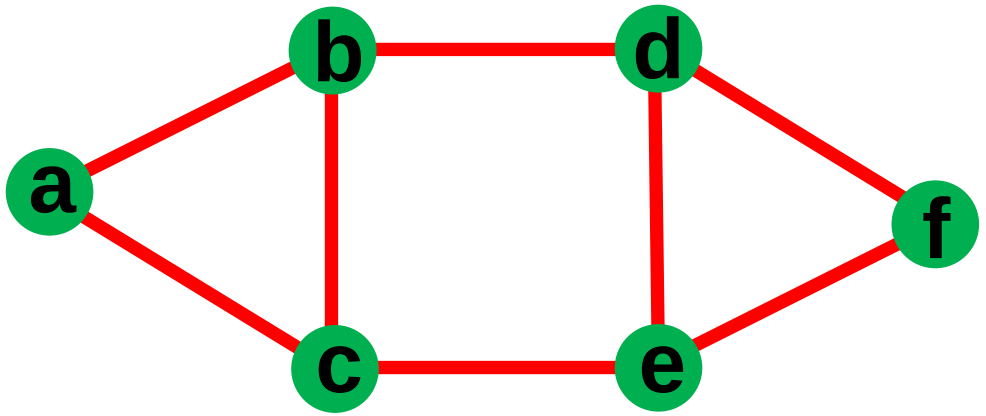

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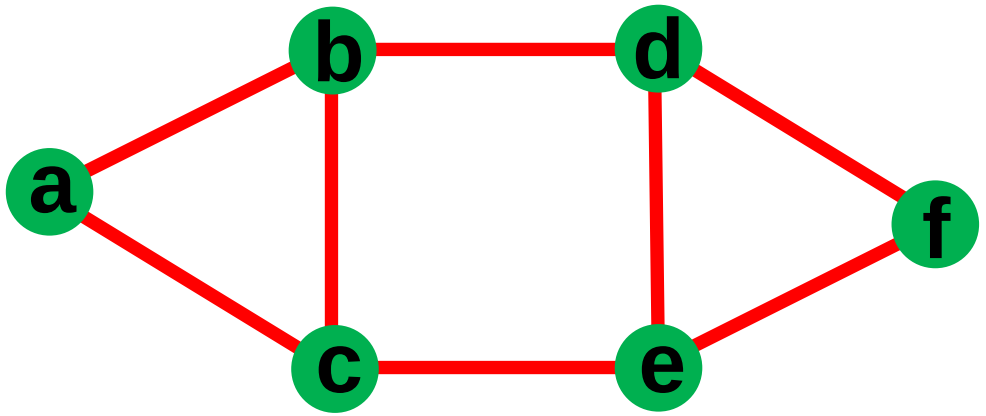

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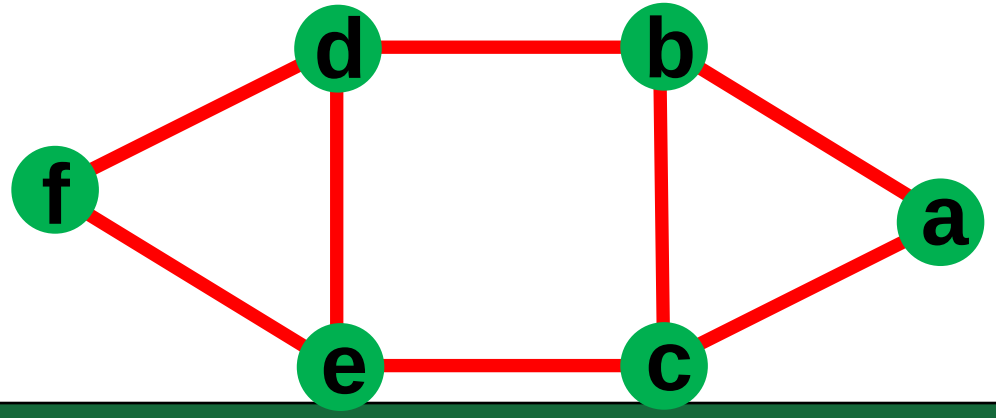

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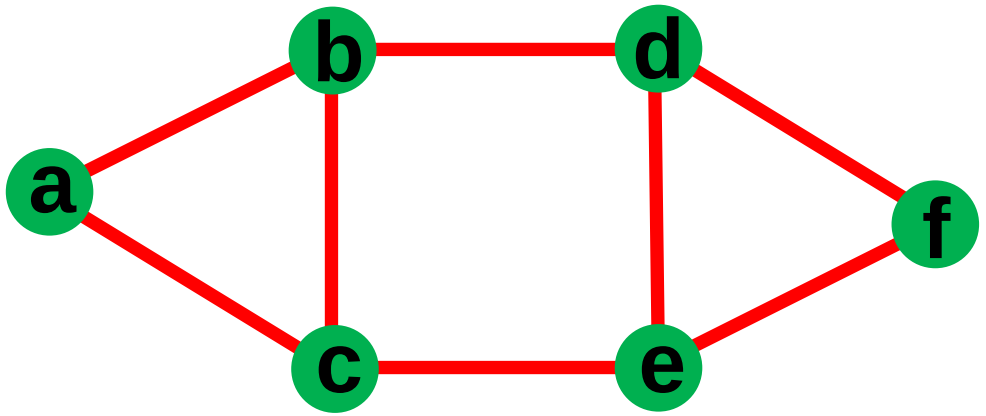

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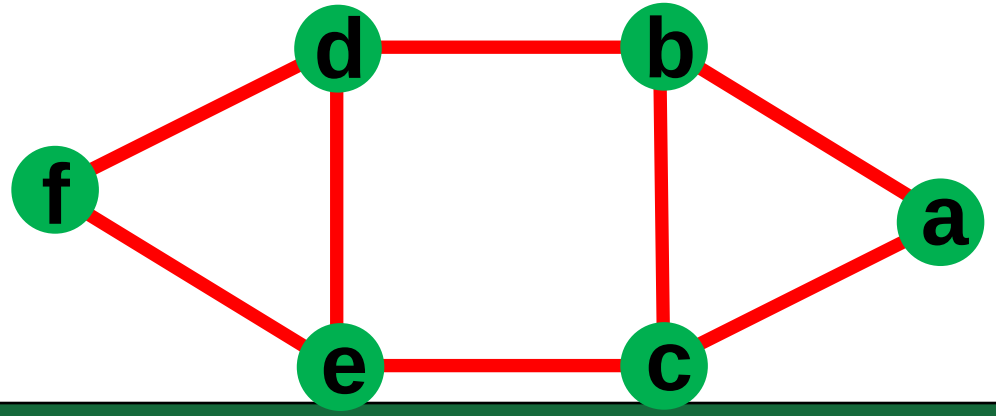

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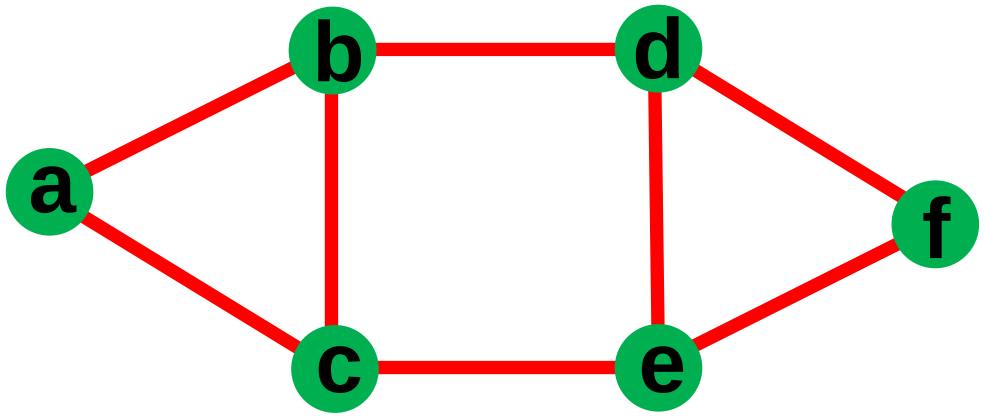

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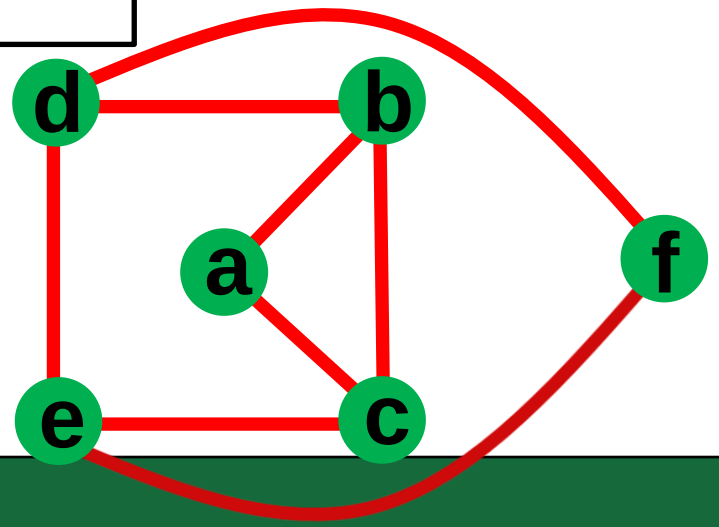

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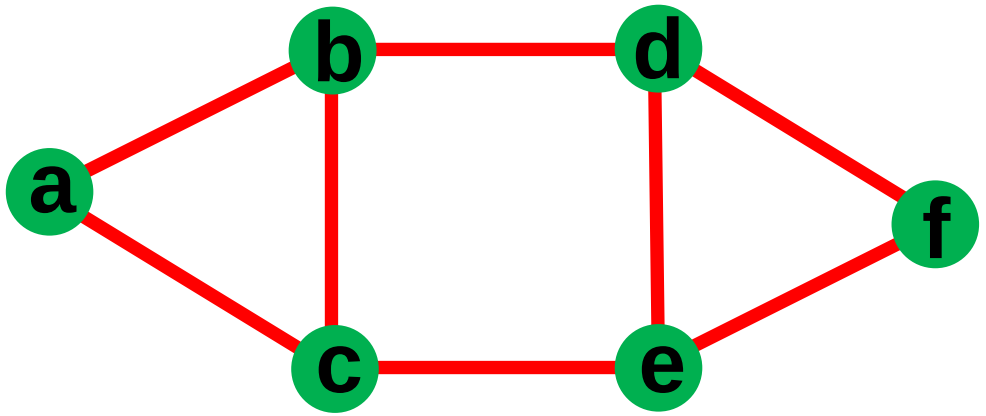

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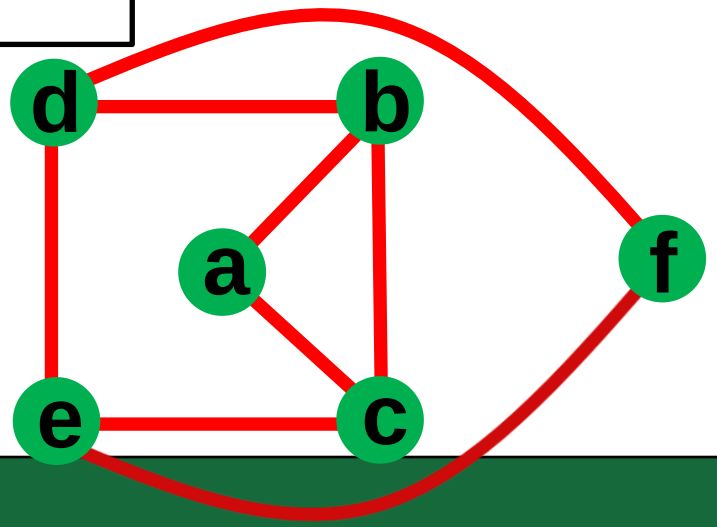

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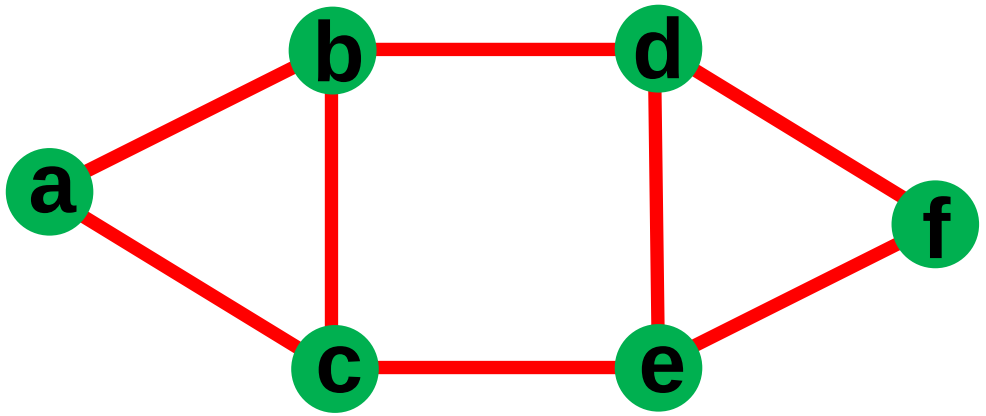

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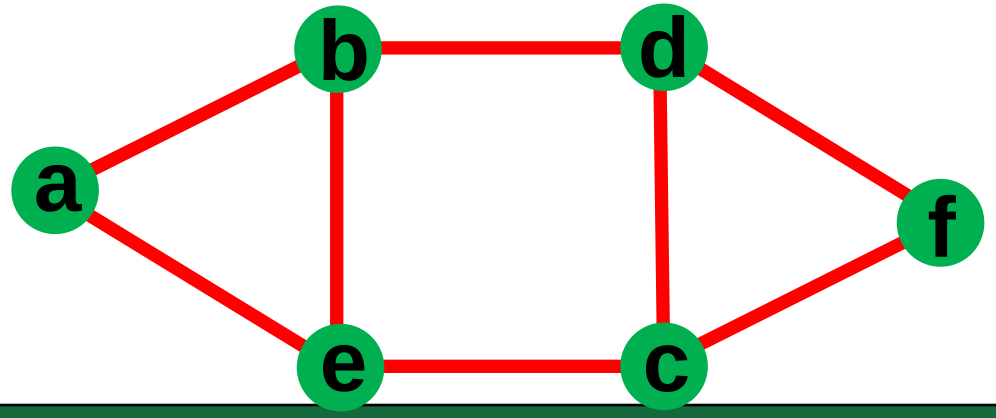

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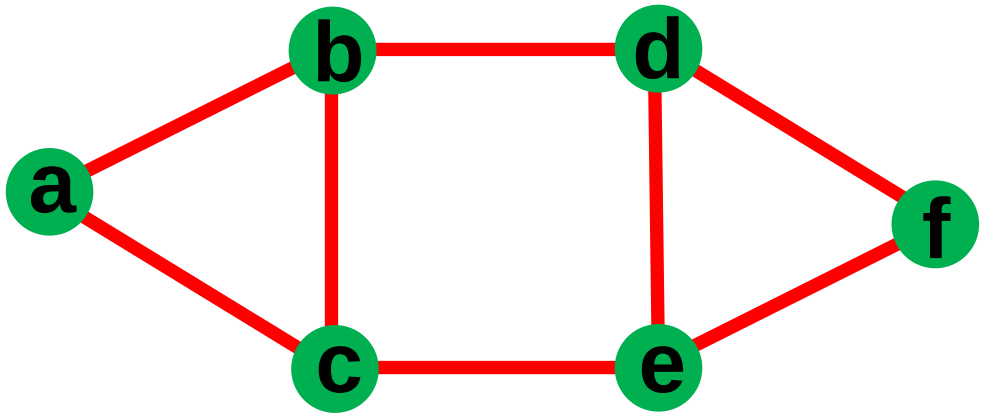

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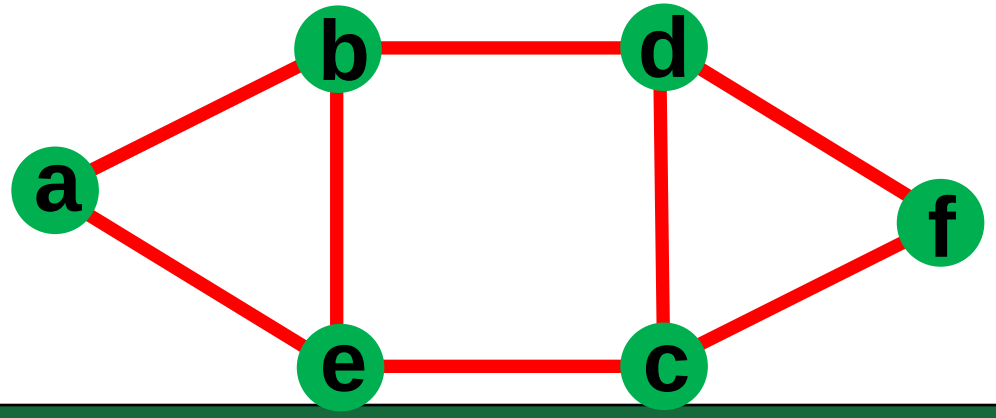

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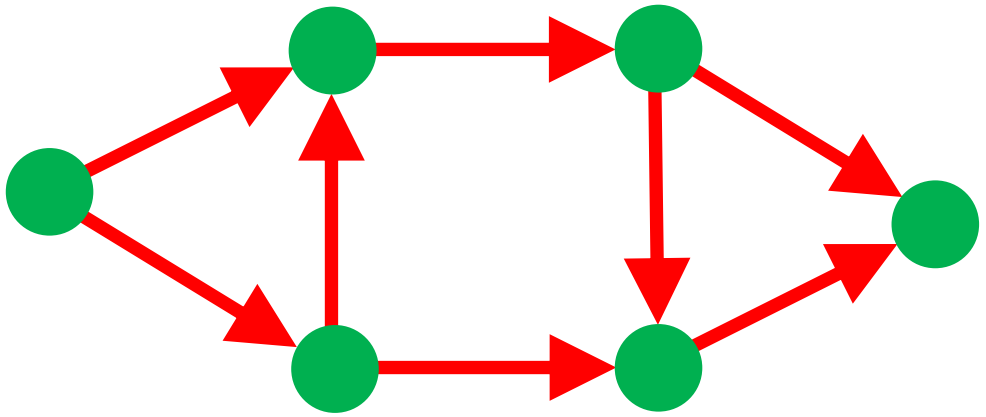
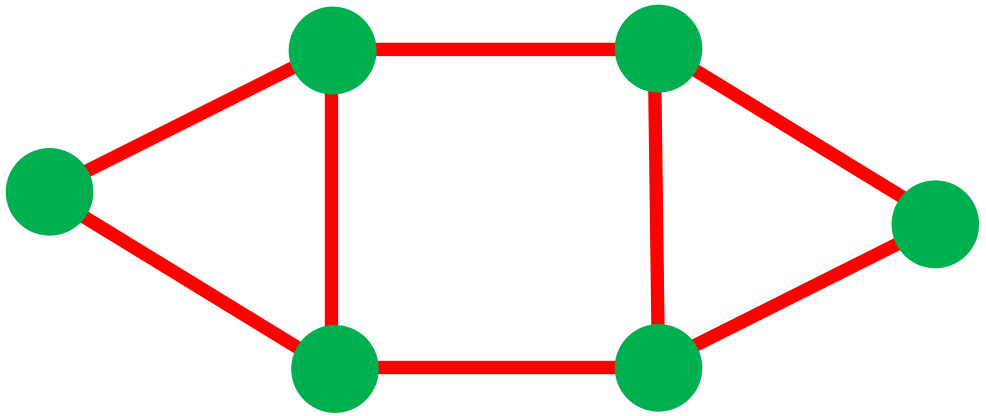


Vertices (or Nodes)
Edges

$$G = (V, E)$$

- Edges can be directed…

Graphs



Vertices (or Nodes)
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- Edges can be directed or undirected.

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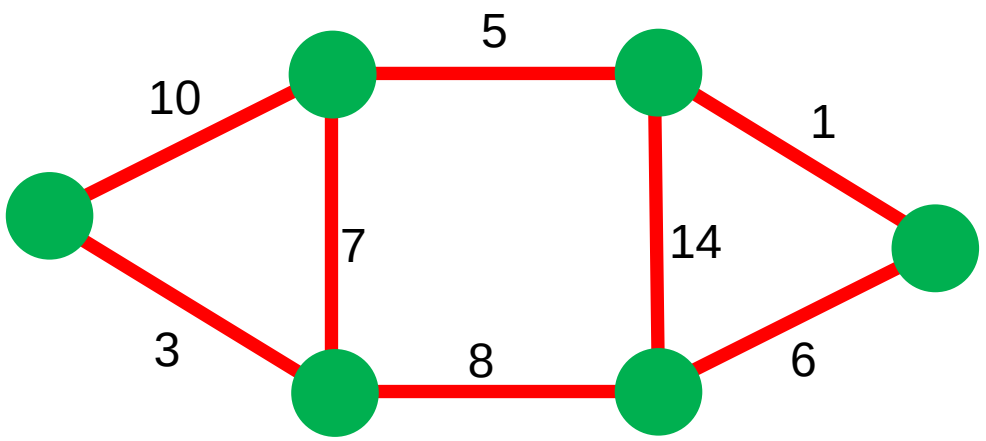

Vertices (or Nodes)
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$$G = (V, E)$$

- Edges can be directed or undirected.
- Edges can have **weights**

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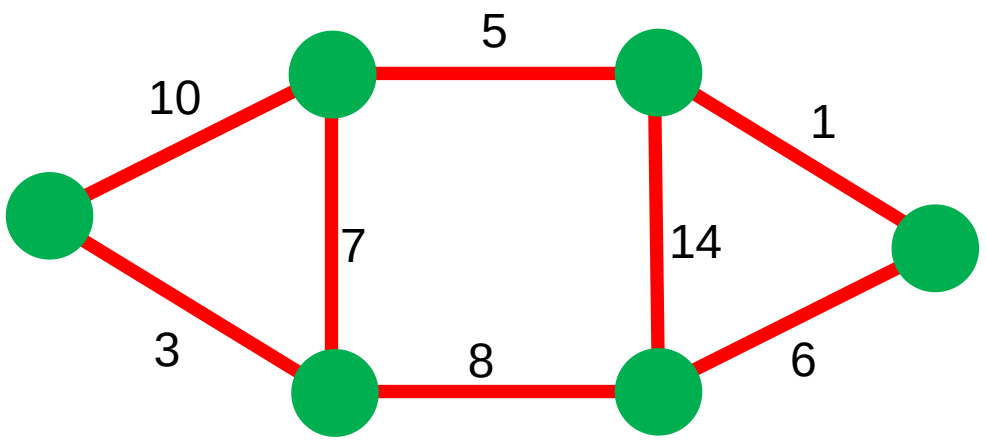

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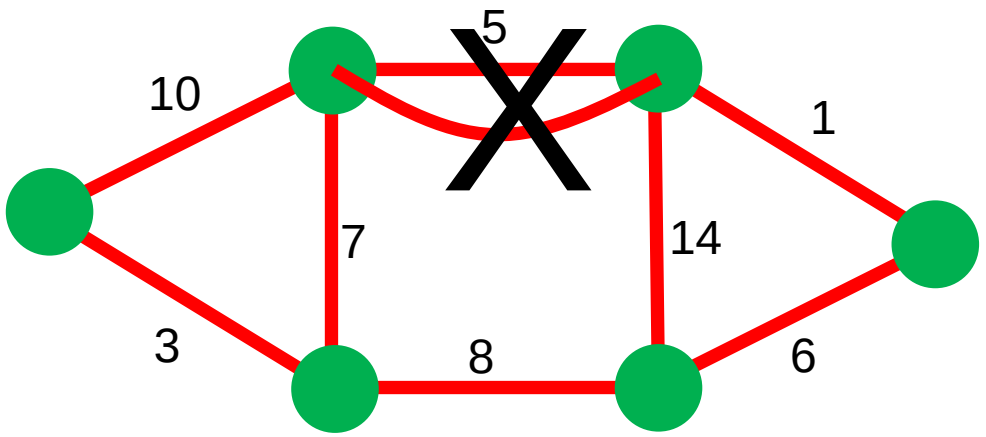

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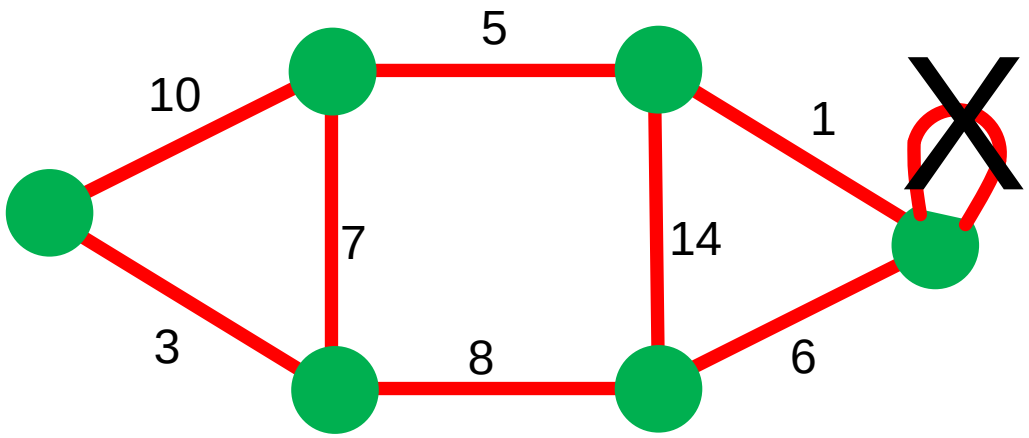

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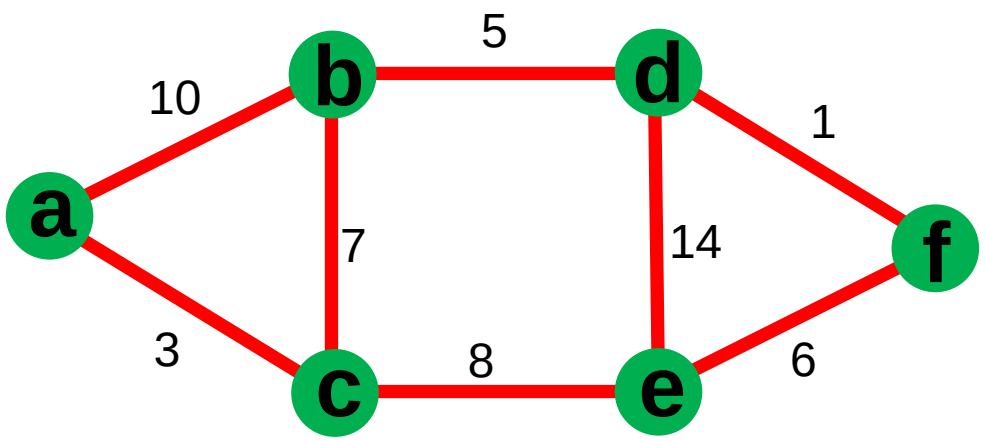

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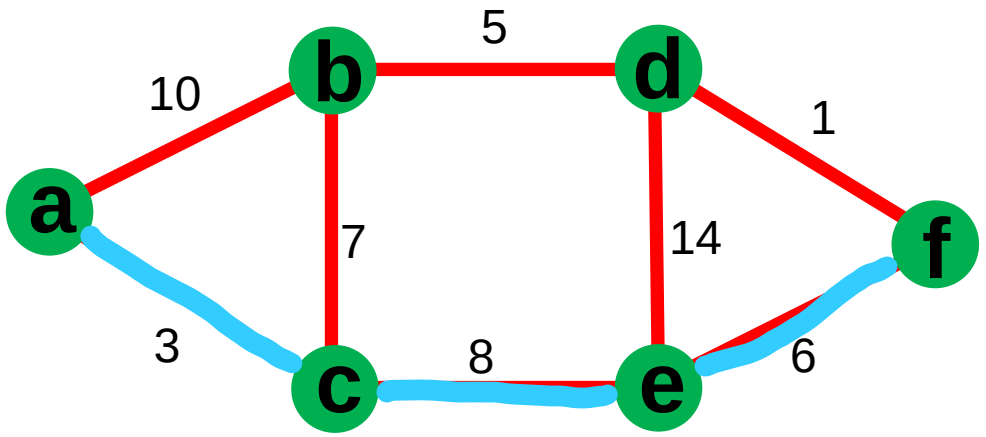

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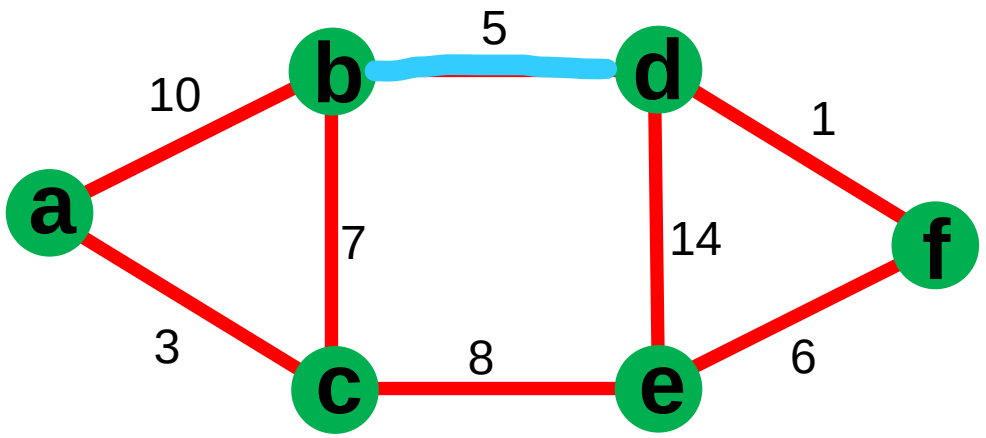
a,c,e,f ✓ "cost" of path = 17

Graphs



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b,d ✓

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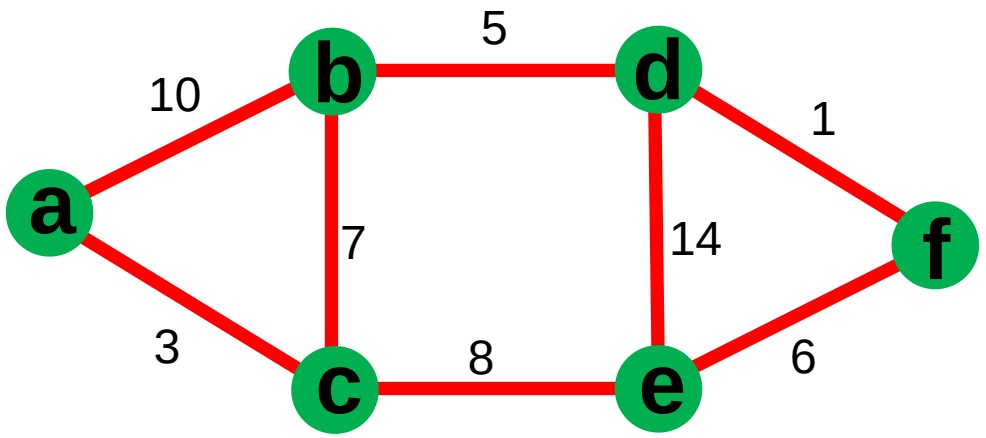

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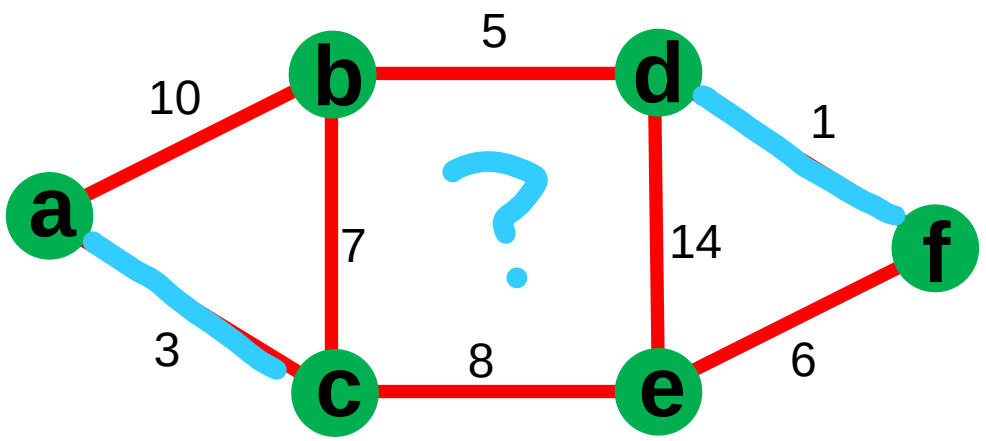

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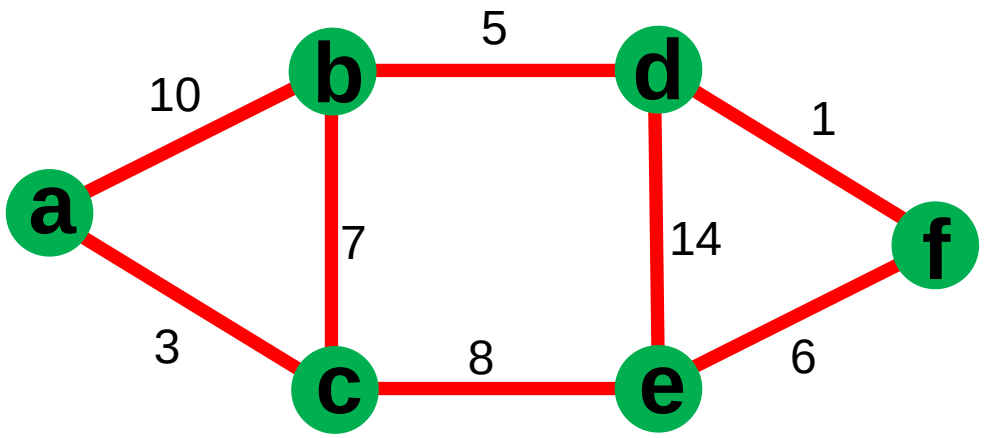

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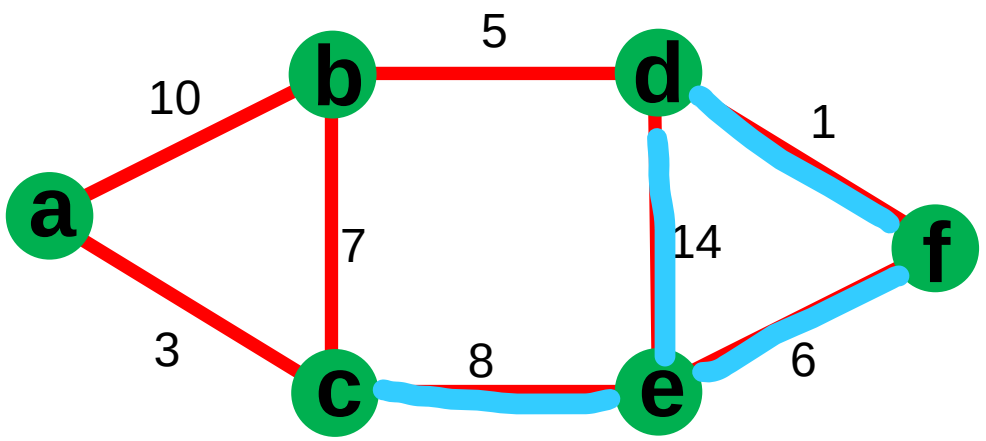
c,e,d,f,e

Graphs



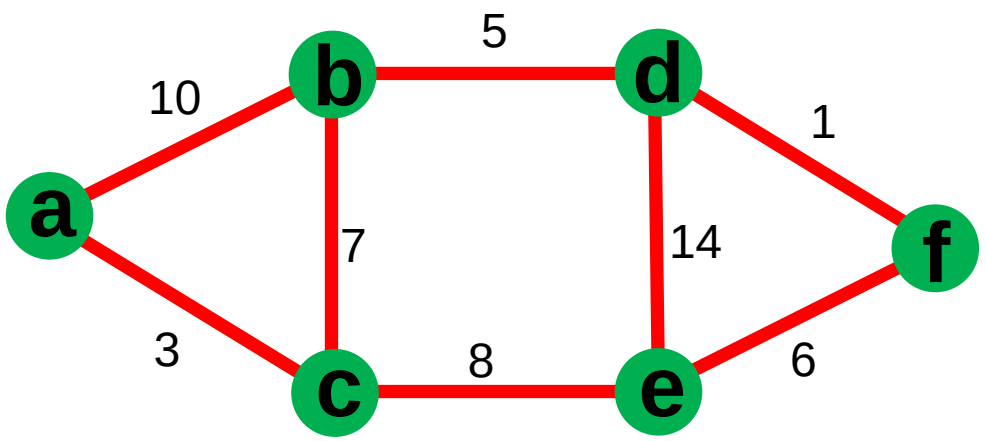
Vertices (or Nodes)
Edges

$$G = (V, E)$$

- Edges can be directed or undirected.
- Edges can have **weights**
- Simple graph = At most one edge between pair of vertices and no edges that start and end at same vertex.
- Path = Sequence of vertices connected by edges without loops.

c,e,d,f,e ✗

Graphs



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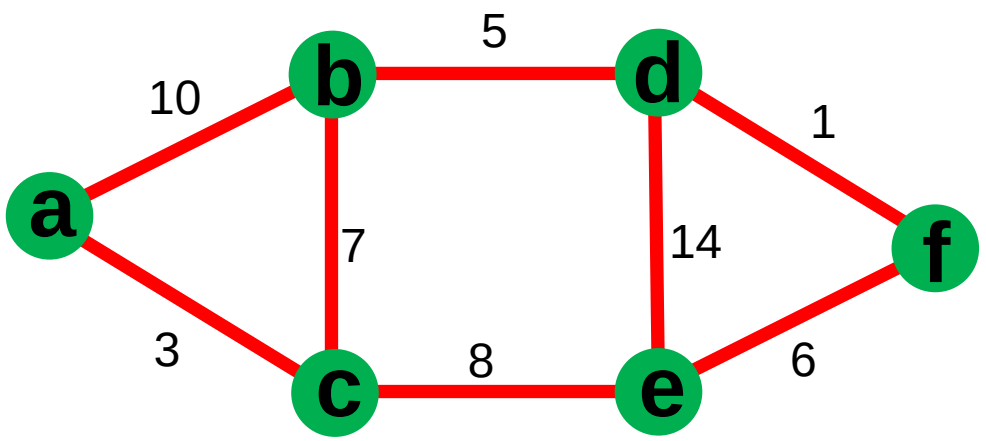

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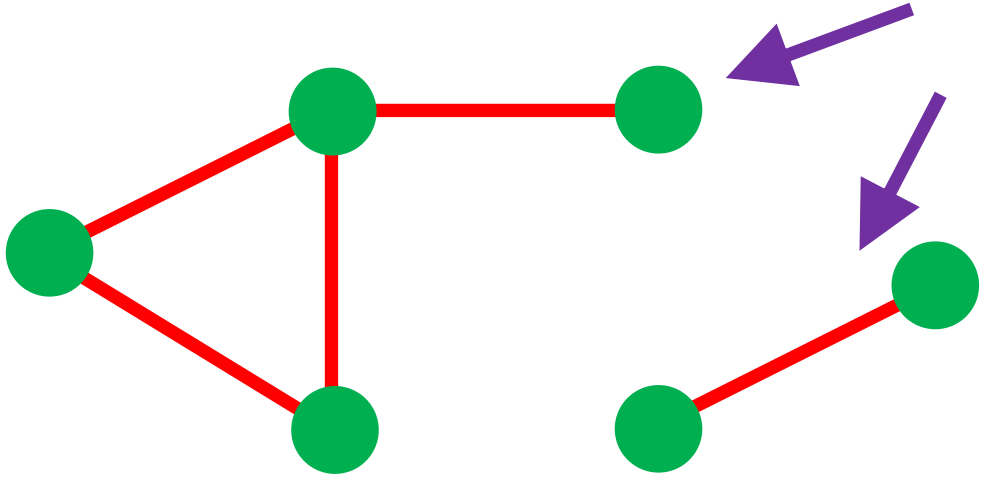

Two Connected Components

Vertices (or Nodes)

Edges

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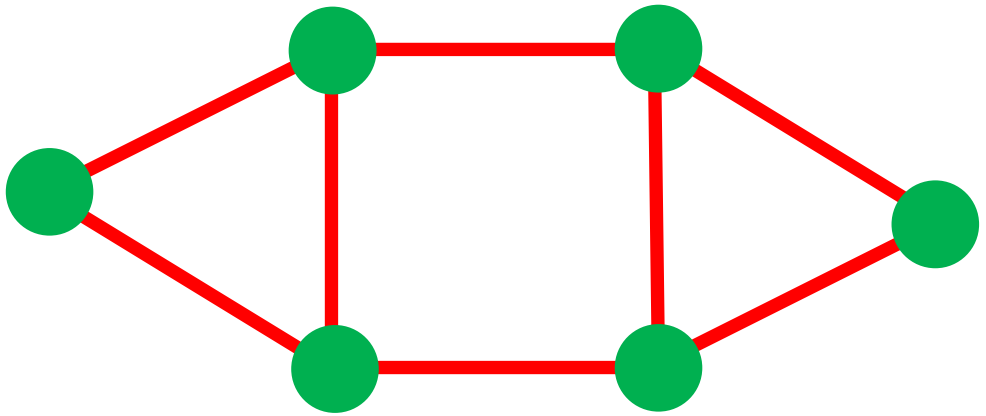
Graphs



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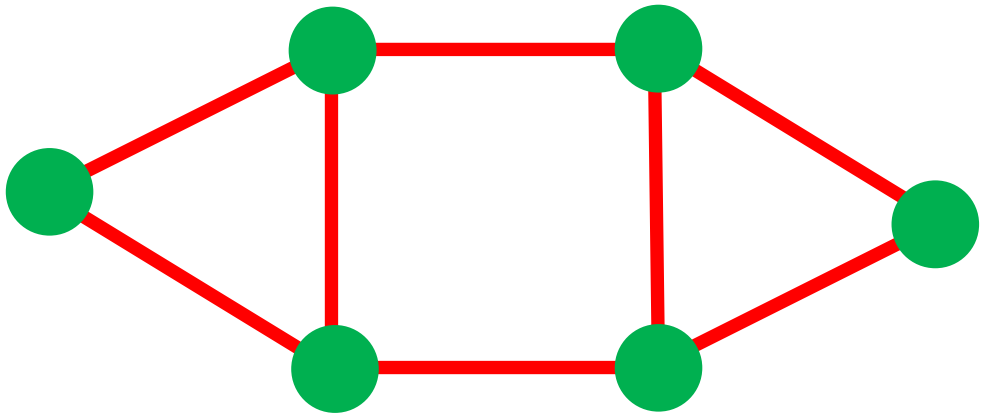
- Degree of a vertex = $\deg(v)$ = # of edges touching it (undirected).

Graphs



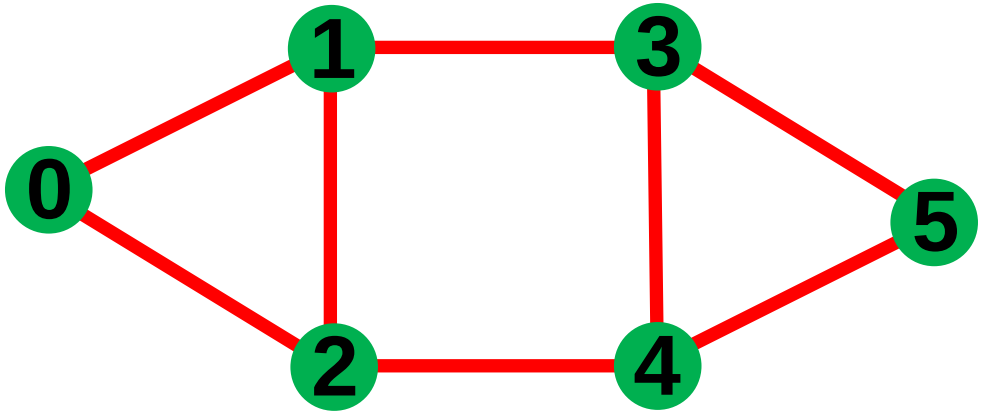
Vertices (or Nodes)
Edges

$$G = (V, E)$$

What are some operations we may want to perform on a graph?

- Add vertices/edges.
- Find path between vertex pair.
- Is graph connected?
- Find degree of vertex.
- Is the graph simple?
- Get number of vertices/edges.
- Get neighbors of vertex.
- Is there a cycle?
- Find max degree of graph.

How can we represent a graph in a computer?



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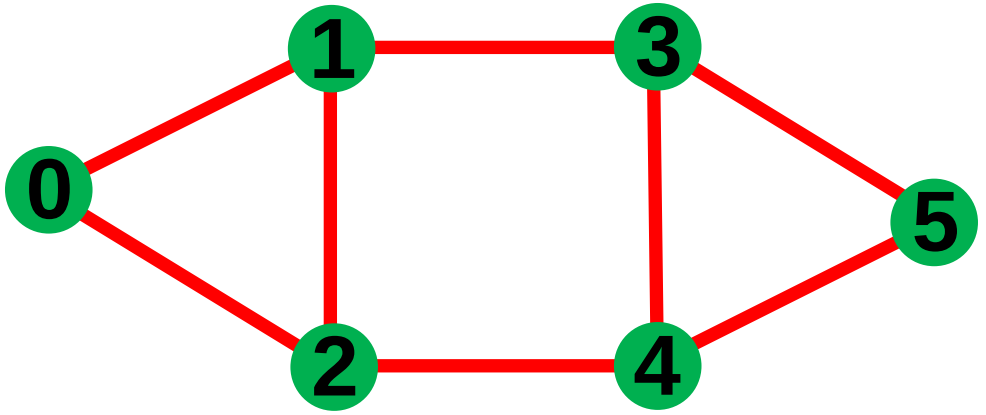

1. Adjacency List

0	→	{1,2}
1	→	{0,2,3}
2	→	{0,1,4}
3	→	{1,4,5}
4	→	{2,3,5}
5	→	{3,4}

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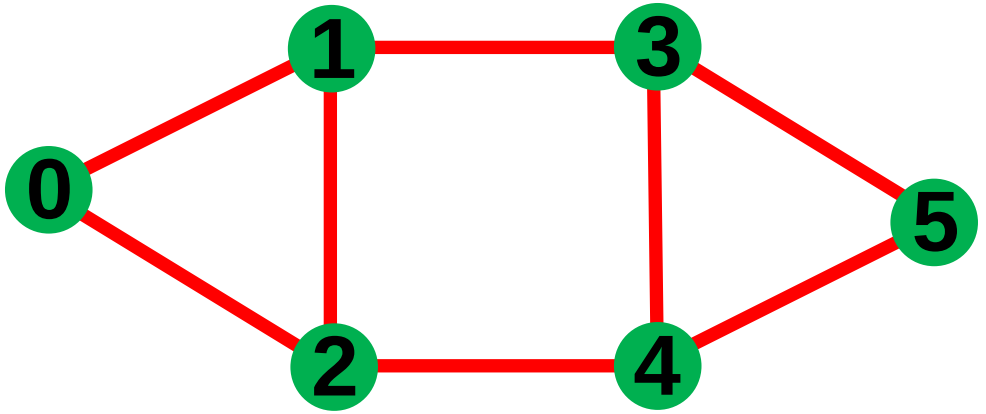

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2. Adjacency Matrix

	0	1	2	3	4	5
0	F	T	T	F	F	F
1	T	F	T	T	F	F
2	T	T	F	F	T	F
3	F	T	F	F	T	T
4	F	F	T	T	F	T
5	F	F	F	T	T	F

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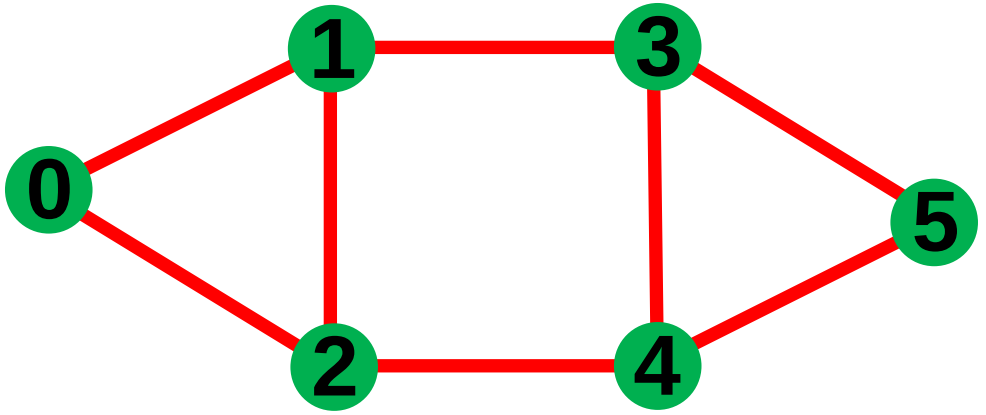

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3	F	T	F	F	T	T
4	F	F	T	T	F	T
5	F	F	F	T	T	F

3. Objects

```
public class Node {
   private Set<Node> neighbors;

   …
}
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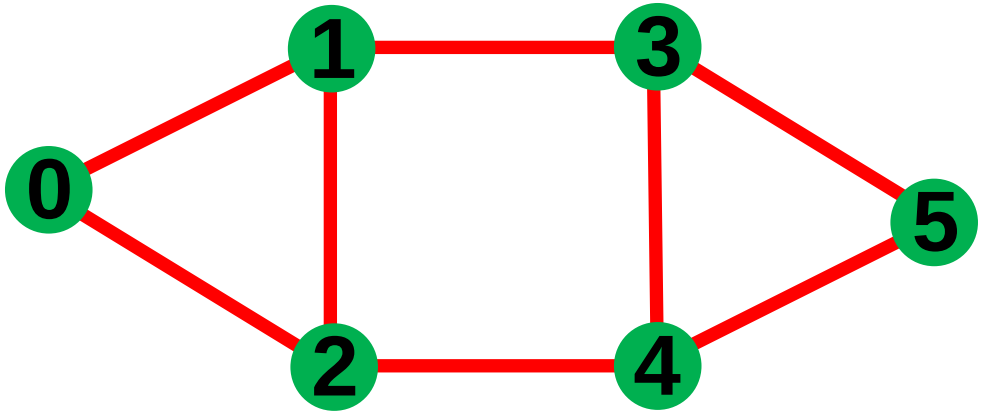

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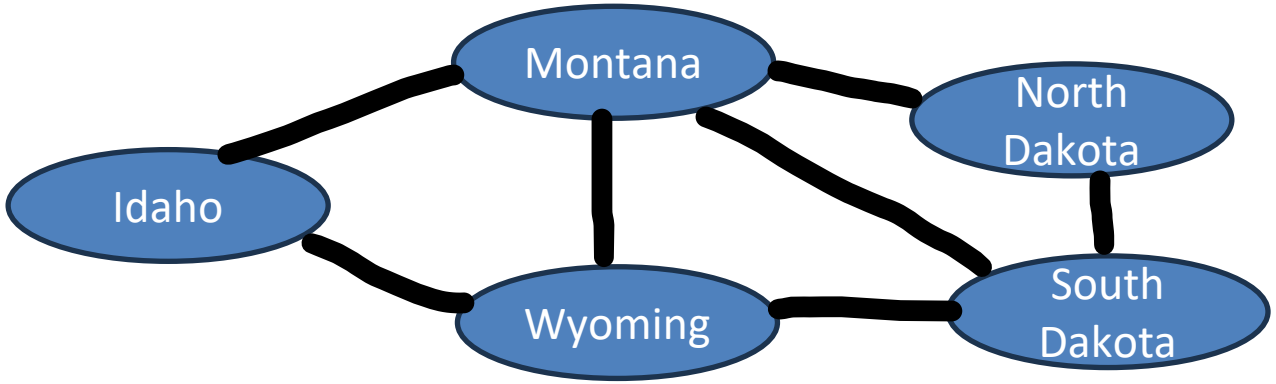

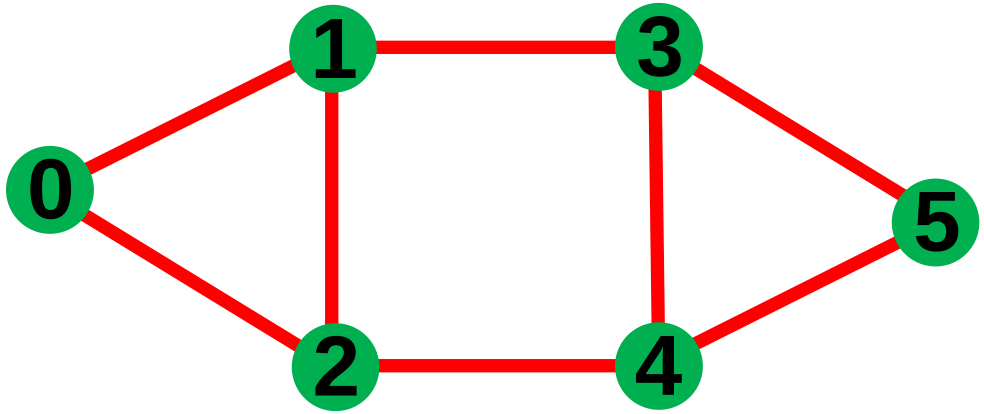

Montana: [North Dakota, South Dakota, Wyoming, Idaho]
Idaho: [Montana, Wyoming]
Wyoming: [Idaho, Montana, South Dakota]
North Dakota: [Montana, South Dakota]
South Dakota: [Wyoming, Montana, North Dakota]

?

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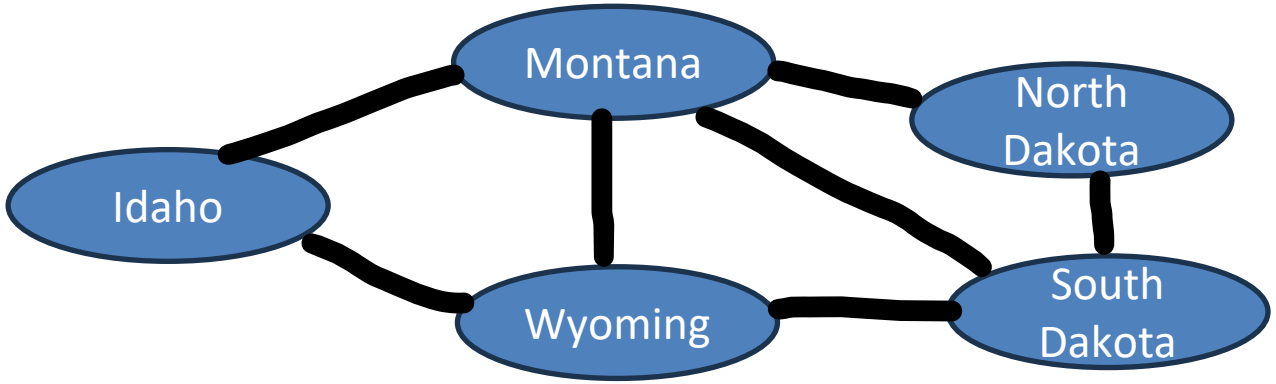

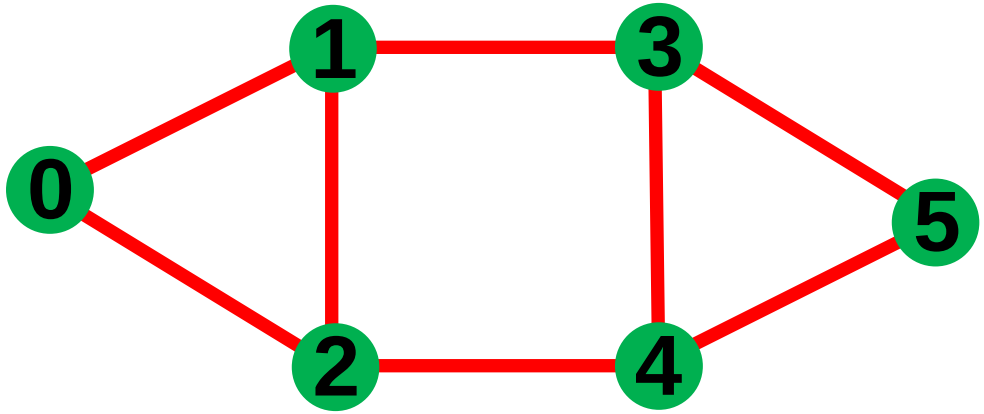

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HashMap<String, LinkedList<String>>

HashMap<String, LinkedList<Edge>>

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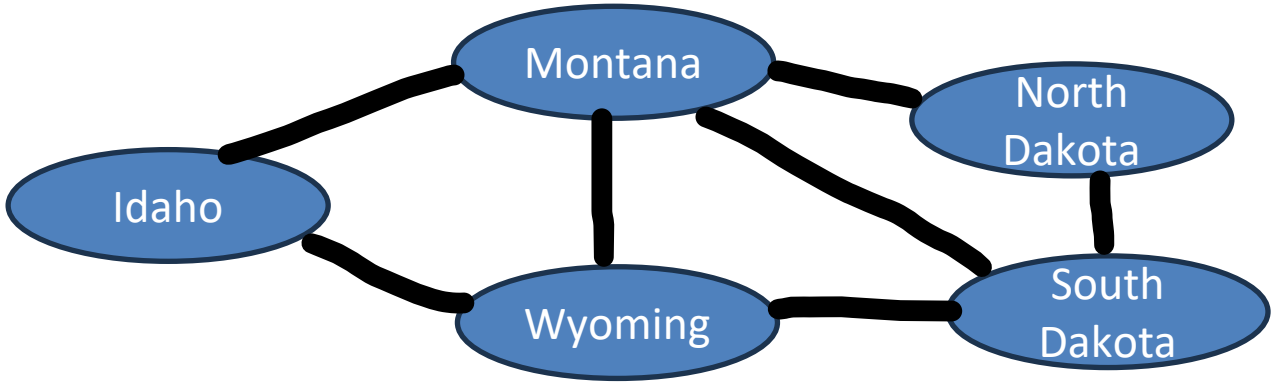

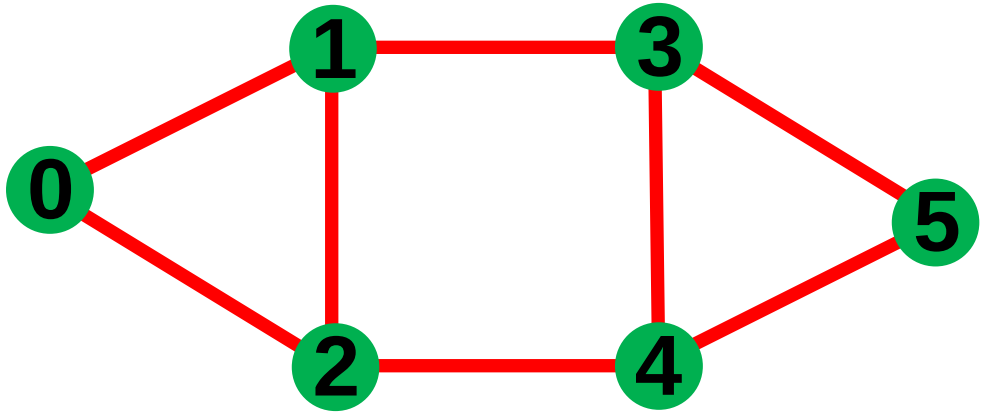

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HashMap<String, LinkedList<String>>
HashMap<String, LinkedList<Edge>>

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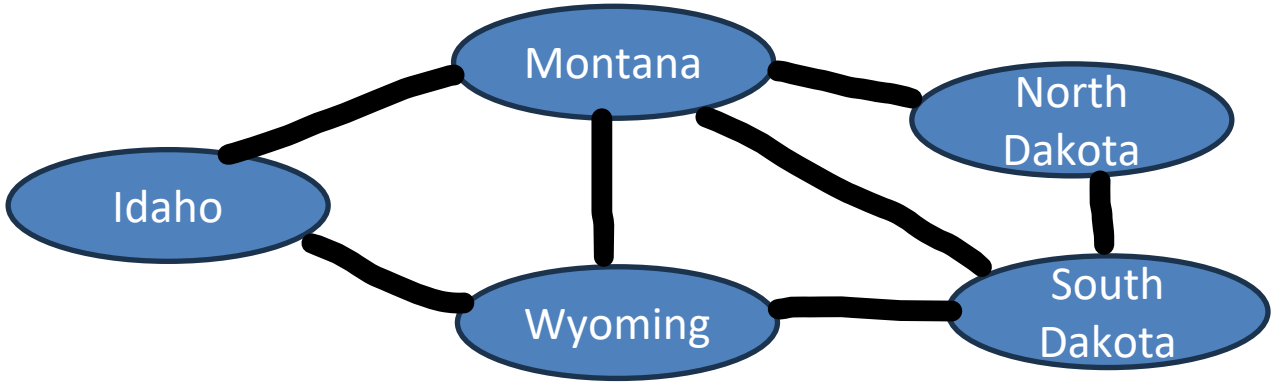

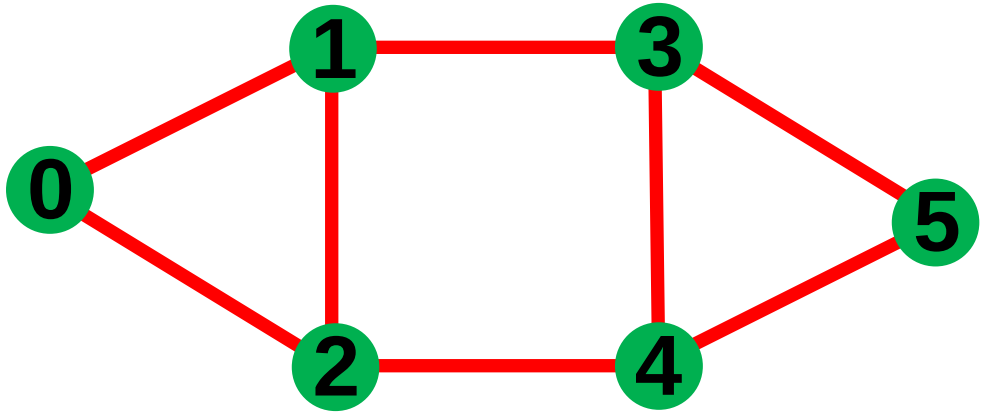

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Array of Linked Lists