

# Edit Distance

Given two strings, how many edits are needed to turn one string into another?

SNOWY vs SUNNY

# Edit Distance

Need:

- Strings – Snowy, Sunny
- Cost function - character misalignment = +1

What are the costs of these two different alignments?

S – N O W Y

S U N N – Y

cost = ?

- S N O W - Y

S U N - - N Y

cost = ?

# Edit Distance

Need:

- Strings – Snowy, Sunny
- Cost function - character misalignment = +1

What are the costs of these two different alignments?

S – N O W Y

S U N N – Y

cost = 3

- S N O W - Y

S U N - - N Y

cost = 5

**Edit distance = cheapest possible alignment.**

# Edit Distance

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- Strings – Snowy, Sunny
- Cost function - character misalignment = +1

What are the costs of these two different alignments?

S – N O W Y

S U N N – Y

cost = 3

- S N O W - Y

S U N - - N Y

cost = 5

**Does a brute force solution sound like a good idea?**

# Edit Distance

We want to align two strings,  $x = [x_1, \dots, x_n]$  and  $y = [y_1, \dots, y_m]$ .

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**Dynamic Programming?**

# Edit Distance

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$E(i, j)$  = optimal cost of aligning  $[x_1, \dots, x_i]$  and  $[y_1, \dots, y_j]$ .

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Can we say anything about optimal alignment of  $[x_1, \dots, x_i]$  and  $[y_1, \dots, y_j]$ ?

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**Specifically, how must the optimal alignments end?  
(three possibilities).**

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We want to align two strings,  $x = [x_1, \dots, x_n]$  and  $y = [y_1, \dots, y_m]$ .

$E(i, j)$  = optimal cost of aligning  $[x_1, \dots, x_i]$  and  $[y_1, \dots, y_j]$ .

Can we say anything about optimal alignment of  $[x_1, \dots, x_i]$  and  $[y_1, \dots, y_j]$ ?

Optimal alignments end in one of three ways:

$$\begin{array}{ccc} x_i & - & x_i \\ - & y_j & y_j \end{array}$$

Cost: 1    1    0,1

# Edit Distance

We want to align two strings,  $x = [x_1, \dots, x_n]$  and  $y = [y_1, \dots, y_m]$ .

$E(i, j)$  = optimal cost of aligning  $[x_1, \dots, x_i]$  and  $[y_1, \dots, y_j]$ .

Can we say anything about optimal alignment of  $[x_1, \dots, x_i]$  and  $[y_1, \dots, y_j]$ ?

Optimal alignments end in one of three ways:

$$\begin{array}{ccc} x_i & - & x_i \\ - & y_j & y_j \end{array}$$


**Need to align  $[x_1, \dots, x_{i-1}]$  with  $[y_1, \dots, y_{j-1}]$ :**  
 $E(i - 1, j - 1)$

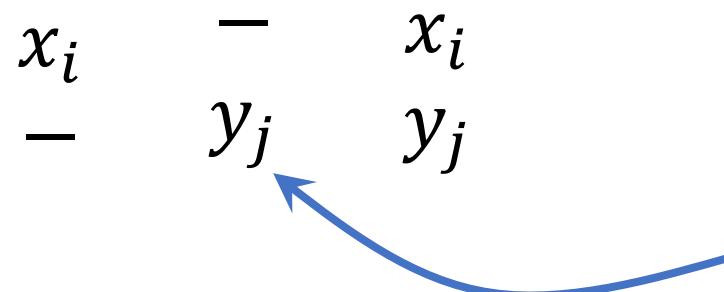
# Edit Distance

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$E(i, j)$  = optimal cost of aligning  $[x_1, \dots, x_i]$  and  $[y_1, \dots, y_j]$ .

Can we say anything about optimal alignment of  $[x_1, \dots, x_i]$  and  $[y_1, \dots, y_j]$ ?

Optimal alignments end in one of three ways:



**Need to align  $[x_1, \dots, x_i]$  with  $[y_1, \dots, y_{j-1}]$ :  $E(i, j - 1)$**

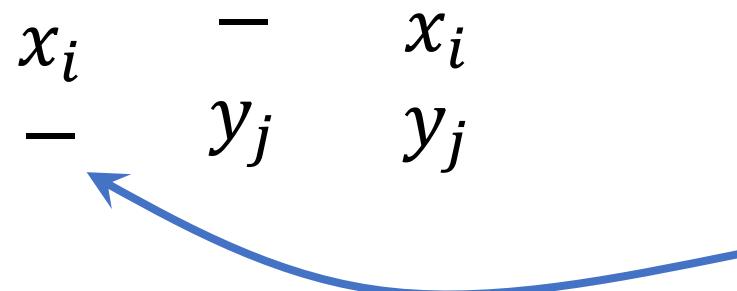
# Edit Distance

We want to align two strings,  $x = [x_1, \dots, x_n]$  and  $y = [y_1, \dots, y_m]$ .

$E(i, j)$  = optimal cost of aligning  $[x_1, \dots, x_i]$  and  $[y_1, \dots, y_j]$ .

Can we say anything about optimal alignment of  $[x_1, \dots, x_i]$  and  $[y_1, \dots, y_j]$ ?

Optimal alignments end in one of three ways:



**Need to align  $[x_1, \dots, x_{i-1}]$  with  $[y_1, \dots, y_j]$ :  $E(i - 1, j)$**

# Edit Distance

$$E(i, j) = \min \left\{ \begin{array}{l} ? \end{array} \right.$$

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$$E(i, j) = \min \left\{ \begin{array}{l} \text{?} \end{array} \right.$$

$x_i$       -       $x_i$   
-       $y_j$        $y_j$

Need to align  $[x_1, \dots, x_{i-1}]$   
with  $[y_1, \dots, y_j]$ :  $E(i-1, j)$

# Edit Distance

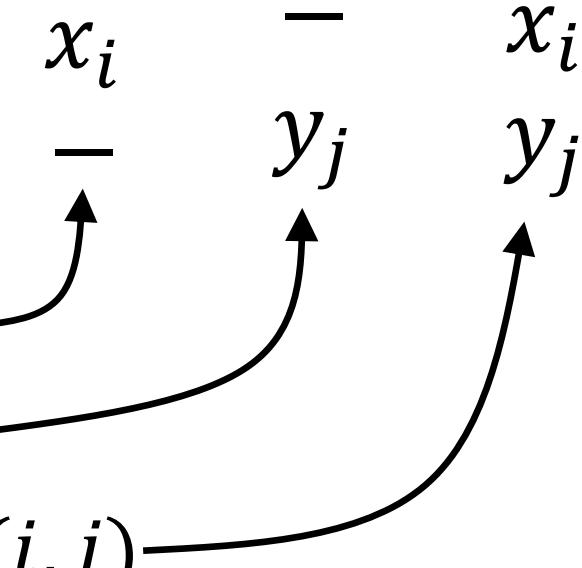
$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$

where  $\text{diff}(i, j) = \begin{cases} 0, & x_i = y_j \\ 1, & x_i \neq y_j \end{cases}$

The diagram shows two horizontal sequences of characters:  $x_i$ ,  $-$ ,  $x_i$  on top and  $y_j$ ,  $-$ ,  $y_j$  on the bottom. Three arrows originate from the bottom sequence and point to the top sequence: one arrow points from  $y_j$  to  $x_i$ , another from  $-$  to  $x_i$ , and a third from  $y_j$  to  $-$ .

# Edit Distance

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$



$$\text{where } \text{diff}(i, j) = \begin{cases} 0, & x_i = y_j \\ 1, & x_i \neq y_j \end{cases}$$

Finding  $E(n, m)$  requires finding all the other  $E$ 's, which can be represented in a 2d table with the strings along the axes.

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0					
1	S					
2	N					
3	O					
4	W					
5	Y					

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$
$$\text{diff}(i, j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & x[i] \neq y[j] \end{cases}$$

$E(3, 4)$

Where can we start?

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0					
1	S					
2	N					
3	O					
4	W					
5	Y					

$$E(i,j) = \min \begin{cases} E(i-1,j) + 1 \\ E(i,j-1) + 1 \\ E(i-1,j-1) + \text{diff}(i,j) \end{cases}$$
$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & x[i] \neq y[j] \end{cases}$$

$E(3, 4)$

Where can we start?  
 $E(0,1)$  or  $E(1,0)$

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0					
1	S					
2	N					
3	O					
4	W					
5	Y					

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$
$$\text{diff}(i, j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(0, 1) = \min \begin{cases} E(-1, 1) + 1 \\ E(0, 0) + 1 \\ E(-1, 0) + 1 \end{cases} = ?$$

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0					
1	S					
2	N					
3	O					
4	W					
5	Y					

$$E(i,j) = \min \begin{cases} E(i-1,j) + 1 \\ E(i,j-1) + 1 \\ E(i-1,j-1) + \text{diff}(i,j) \end{cases}$$
$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(0,1) = \min \begin{cases} \cancel{E(-1,1) + 1} \\ E(0,0) + 1 = ? \\ \cancel{E(-1,0) + 1} \end{cases}$$

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0	1				
1	S					
2	N					
3	O					
4	W					
5	Y					

$$E(i,j) = \min \begin{cases} E(i-1,j) + 1 \\ E(i,j-1) + 1 \\ E(i-1,j-1) + \text{diff}(i,j) \end{cases}$$
$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(0,1) = \min \begin{cases} \cancel{E(-1,1) + 1} \\ E(0,0) + 1 = 1 \\ \cancel{E(-1,0) + 1} \end{cases}$$

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0	1				
1	S					
2	N					
3	O					
4	W					
5	Y					

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$
$$\text{diff}(i, j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(1, 1) = \min \begin{cases} E(0, 1) + 1 \\ E(1, 0) + 1 = ? \\ E(0, 0) + 0 \end{cases}$$

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0	1				
1	S					
2	N					
3	O					
4	W					
5	Y					

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$
$$\text{diff}(i, j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(1, 1) = \min \begin{cases} E(0, 1) + 1 \\ \textcolor{red}{E(1, 0)} + 1 = ? \\ E(0, 0) + 0 \end{cases}$$

**Not calculated yet!**

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0	1				
1	S					
2	N					
3	O					
4	W					
5	Y					

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$
$$\text{diff}(i, j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

**Need upper left hand corner filled out before we can progress.**

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0	1	2			
1	S					
2	N					
3	O					
4	W					
5	Y					

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$
$$\text{diff}(i, j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(0, 2) = \min \begin{cases} E(-1, 2) + 1 \\ E(0, 1) + 1 \\ E(-1, 1) + 1 \end{cases} = 2$$

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0	1	2			
1	S	1				
2	N					
3	O					
4	W					
5	Y					

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$
$$\text{diff}(i, j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(1, 0) = \min \begin{cases} E(0, 0) + 1 \\ E(1, -1) + 1 = 1 \\ E(0, -1) + 1 \end{cases}$$

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	Y	
0	0	1	2			
1	S	1	0			
2	N					
3	O					
4	W					
5	Y					

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$
$$\text{diff}(i, j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(1, 1) = \min \begin{cases} E(0, 1) + 1 \\ E(1, 0) + 1 = 0 \\ E(0, 0) + 0 \end{cases}$$

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$
$$\text{diff}(i, j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

Running Time?

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$
$$\text{diff}(i, j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

Running Time?

Fill out  $n \times m$  table with constant operations:  $O(nm)$

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

Edit distance = **3**.

How can we recreate the actual alignments?

Backtracking.

Ask the question: “How did we get here?”

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

How did we get to  $E(5,5)$ ? ?

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

How did we get to  $E(5,5)$ ?  
From  $E(5,4)$ ?

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

How did we get to  $E(5,5)$ ?

From  $E(5,4)$ ? – No. Can never go down in cost.

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

How did we get to  $E(5,5)$ ?

From  $E(5,4)$ ? – No. Can never go down in cost.

From  $E(4,5)$ ?

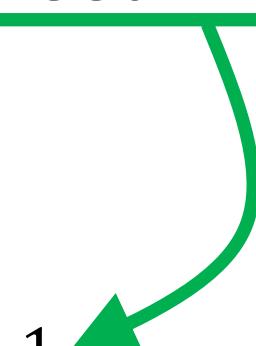
# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

How did we get to  $E(5,5)$ ?

From  $E(5,4)$ ? – No. Can never go down in cost.

From  $E(4,5)$ ? – No. Need +1 to move that direction.

$$E(i,j) = \min \begin{cases} E(i-1,j) + 1 \\ E(i,j-1) + 1 \\ E(i-1,j-1) + \text{diff}(i,j) \end{cases}$$


# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

How did we get to  $E(5,5)$ ?

From  $E(5,4)$ ? – No. Can never go down in cost.

From  $E(4,5)$ ? – No. Need +1 to move that direction.

From  $E(4,4)$ ?

$$E(i,j) = \min \begin{cases} E(i-1,j) + 1 \\ E(i,j-1) + 1 \\ E(i-1,j-1) + \text{diff}(i,j) \end{cases}$$

# Edit Distance

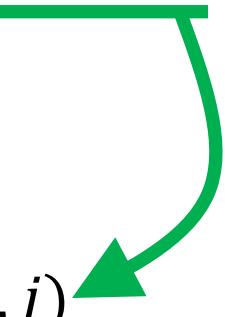
$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

How did we get to  $E(5,5)$ ?

From  $E(5,4)$ ? – No. Can never go down in cost.

From  $E(4,5)$ ? – No. Need +1 to move that direction.

From  $E(4,4)$ ? – Yes. Match Y's.

$$E(i,j) = \min \begin{cases} E(i-1,j) + 1 \\ E(i,j-1) + 1 \\ E(i-1,j-1) + \text{diff}(i,j) \end{cases}$$


# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

Continuing the process yields all of the optimal solutions.

Diagonal move indicates ?

Vertical move indicates ?

Horizontal move indicates ?

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

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Diagonal move indicates match.

Vertical move indicates ?

Horizontal move indicates ?

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

Continuing the process yields all of the optimal solutions.

Diagonal move indicates match.

Vertical move indicates space inserted in  $j$ .

Horizontal move indicates ?

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

Continuing the process yields all of the optimal solutions.

Diagonal move indicates match.

Vertical move indicates space inserted in  $j$ .

Horizontal move indicates space inserted in  $i$ .

$$E(i, j) = \min \begin{cases} E(i - 1, j) + 1 \\ E(i, j - 1) + 1 \\ E(i - 1, j - 1) + \text{diff}(i, j) \end{cases}$$

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	N	Y
0	0	1	2	3	4	5
1	S	1	0	1	2	3
2	N	2	1	1	1	2
3	O	3	2	2	2	2
4	W	4	3	3	3	3
5	Y	5	4	4	4	3

The diagram shows a grid of numbers representing edit distances between two strings. Arrows indicate the operations: green arrows for diagonal moves (match), blue arrows for vertical moves (insert space in j), and red arrows for horizontal moves (insert space in i).

Diagonal move indicates match.

Vertical move indicates space inserted in  $j$ .

Horizontal move indicates space inserted in  $i$ .

S - N O W Y

S U N N - Y

# Edit Distance

$j$	0	1	2	3	4	5	
$i$	S	U	N	N	Y		
0	0	1	2	3	4	5	
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

The diagram shows a 7x7 edit distance matrix. The columns are labeled  $j$  (0 to 5) and the rows are labeled  $i$  (0 to 5). The first row and column are headers. The matrix values are: (0,0)=0, (0,1)=1, (0,2)=2, (0,3)=3, (0,4)=4, (0,5)=5; (1,0)=1, (1,1)=0, (1,2)=1, (1,3)=2, (1,4)=3, (1,5)=4; (2,0)=2, (2,1)=1, (2,2)=1, (2,3)=1, (2,4)=2, (2,5)=3; (3,0)=3, (3,1)=2, (3,2)=2, (3,3)=2, (3,4)=2, (3,5)=3; (4,0)=4, (4,1)=3, (4,2)=3, (4,3)=3, (4,4)=3, (4,5)=3; (5,0)=5, (5,1)=4, (5,2)=4, (5,3)=4, (5,4)=4, (5,5)=3. Arrows indicate moves: green arrows for diagonal moves (matches), blue arrows for vertical moves (insertions in  $j$ ), and red arrows for horizontal moves (insertions in  $i$ ). For example, from (0,0) to (1,1) is a green arrow, from (1,1) to (2,0) is a blue arrow, and from (2,0) to (3,1) is a red arrow.

Diagonal move indicates match.

Vertical move indicates space inserted in  $j$ .

Horizontal move indicates space inserted in  $i$ .

Alignment?

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	N	Y
0	0	1	2	3	4	5
1	S	1	0	1	2	3
2	N	2	1	1	1	2
3	O	3	2	2	2	2
4	W	4	3	3	3	3
5	Y	5	4	4	4	3

Diagonal move indicates match.

Vertical move indicates space inserted in  $j$ .

Horizontal move indicates space inserted in  $i$ .

S - N O W Y

S U N - N Y

# Edit Distance

$j$	0	1	2	3	4	5
$i$	S	U	N	N	N	Y
0	0	1	2	3	4	5
1	S	1	0	1	2	3
2	N	2	1	1	1	2
3	O	3	2	2	2	2
4	W	4	3	3	3	3
5	Y	5	4	4	4	3

Diagonal move indicates match.

Vertical move indicates space inserted in  $j$ .

Horizontal move indicates space inserted in  $i$ .

S N O W Y

S U N N Y