CSCI 232:
Data Structures and Algorithms

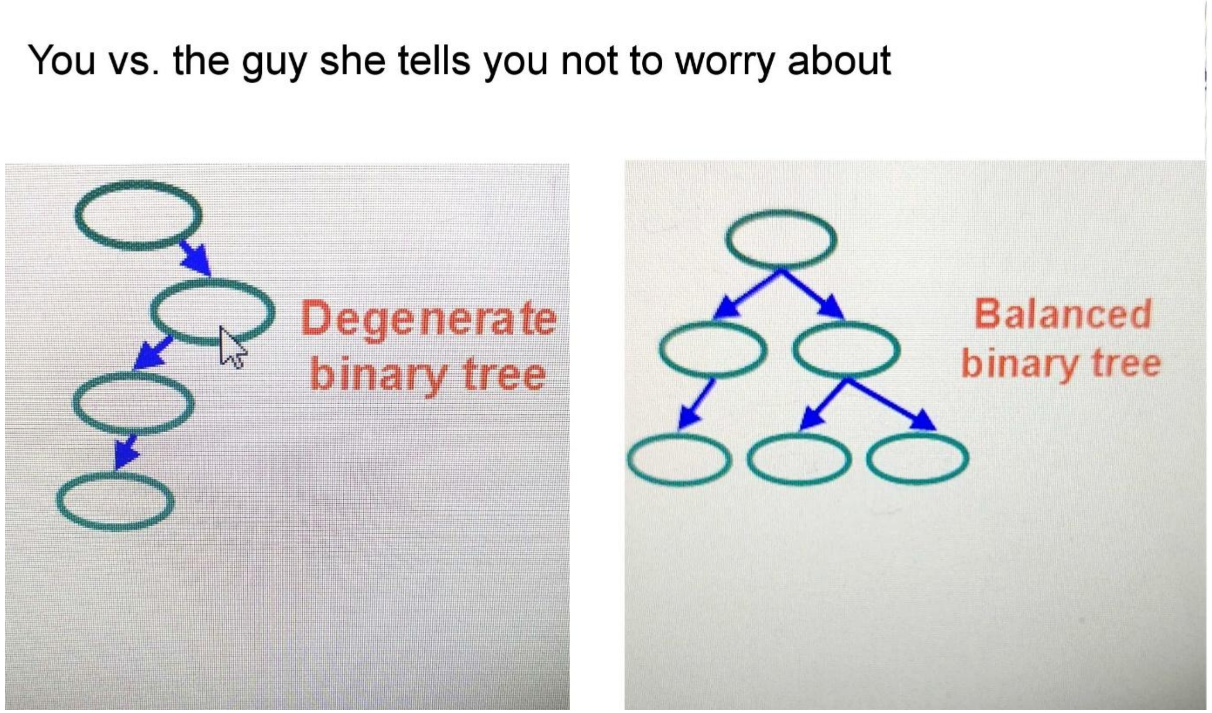
Graphs (Representation)

Reese Pearsall

Spring 2025

Announcements

Lab 7 due on Friday
→ Really easy



Registration

Next Classes (You can register for these anytime in the next couple years):

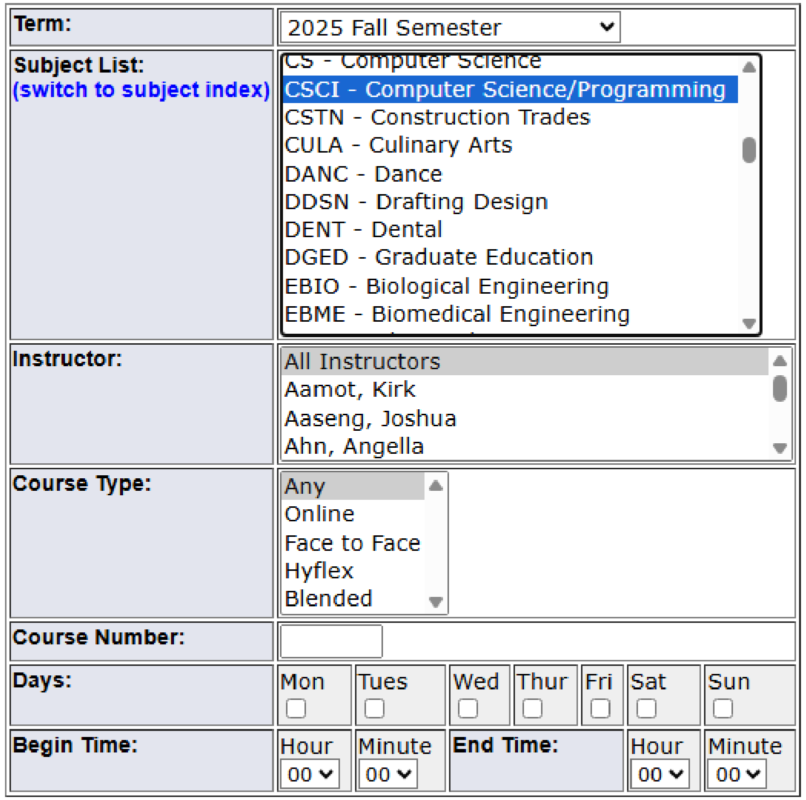
CSCI 366- Computer Systems
ESOF 322 – Software Engineering
CSCI 305 – Concepts of Programming Languages
CSCI 338 – Computer Science Theory

If you have not already:

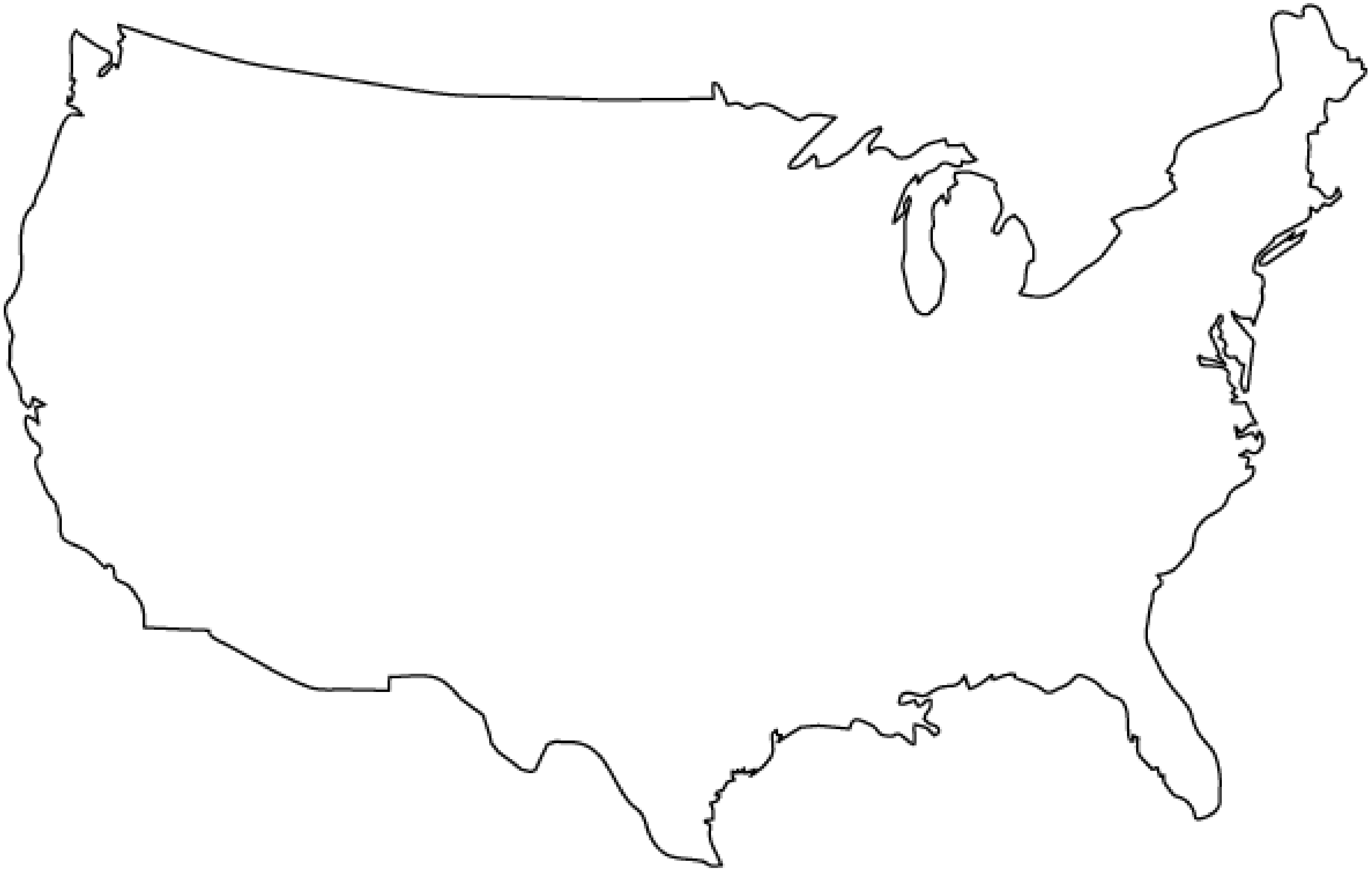
CSCI 246 – Discrete Structures
CSCI 112- Programming in C
MART 145- Web Design

Other Classes that may be of interest

CSCI 252- Intro to Data Science
CSCI 331- Web Development
CSCI 351 – System Administration
CSCI 440 – Database Systems
CSCI 443 – User Interface Design
CSCI 451 – Computational Biology
CSCI 446 – Artificial Intelligence*
CSCI 460 – Operating Systems
CSCI 466 – Networks
CSCI 476 – Computer Security



How could we visualize: The US Road Network?



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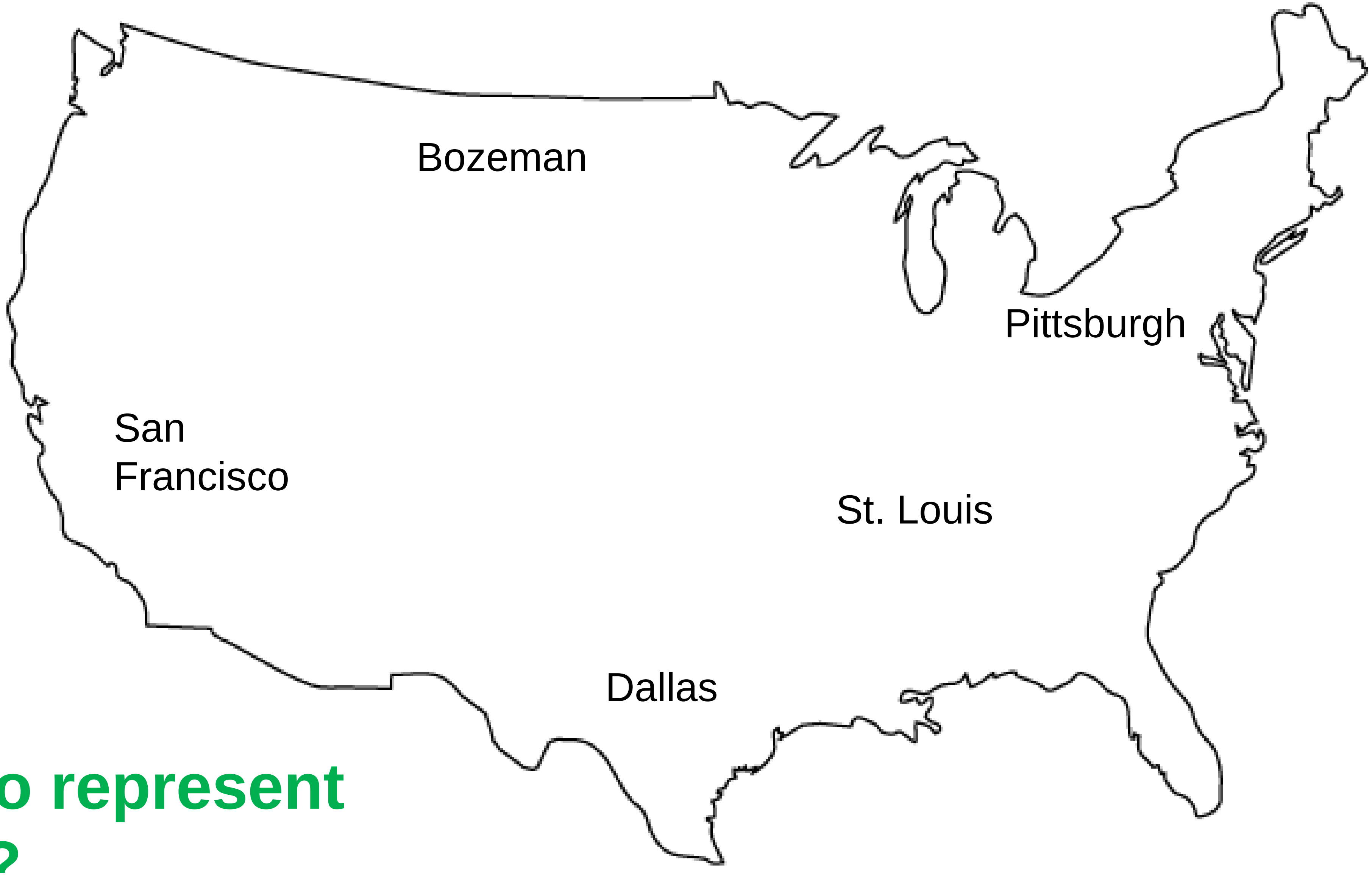

How to represent cities?

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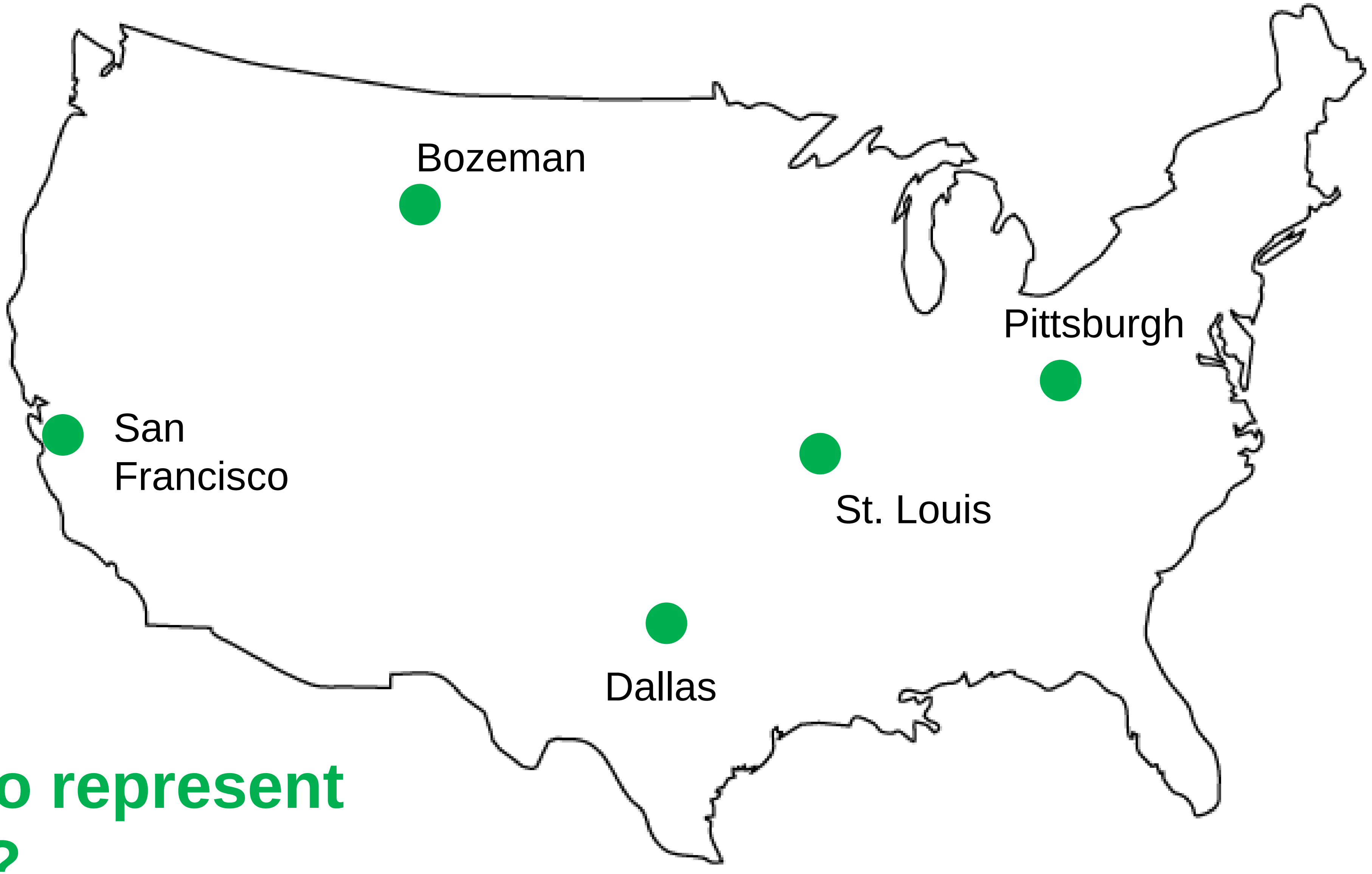

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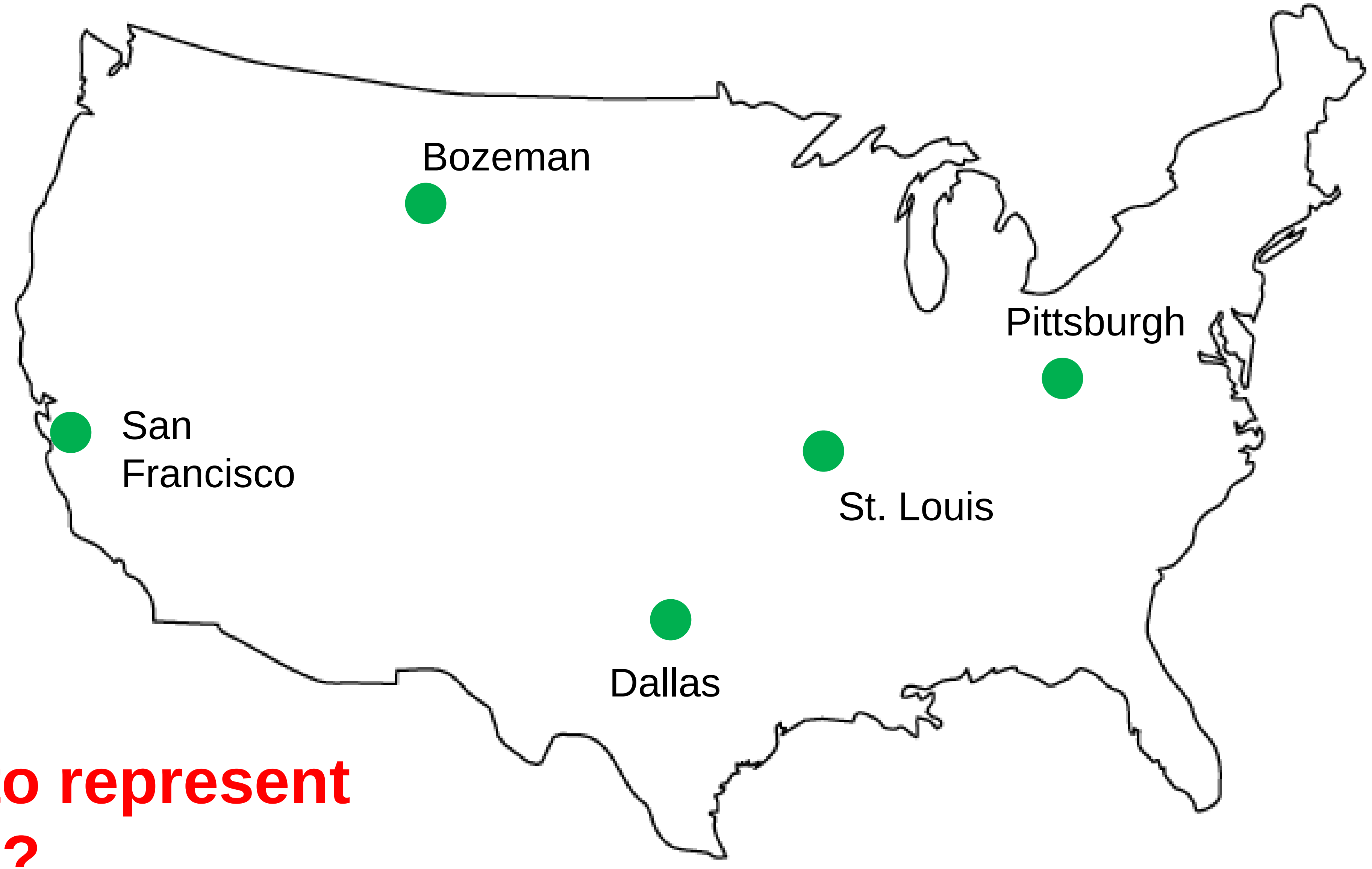

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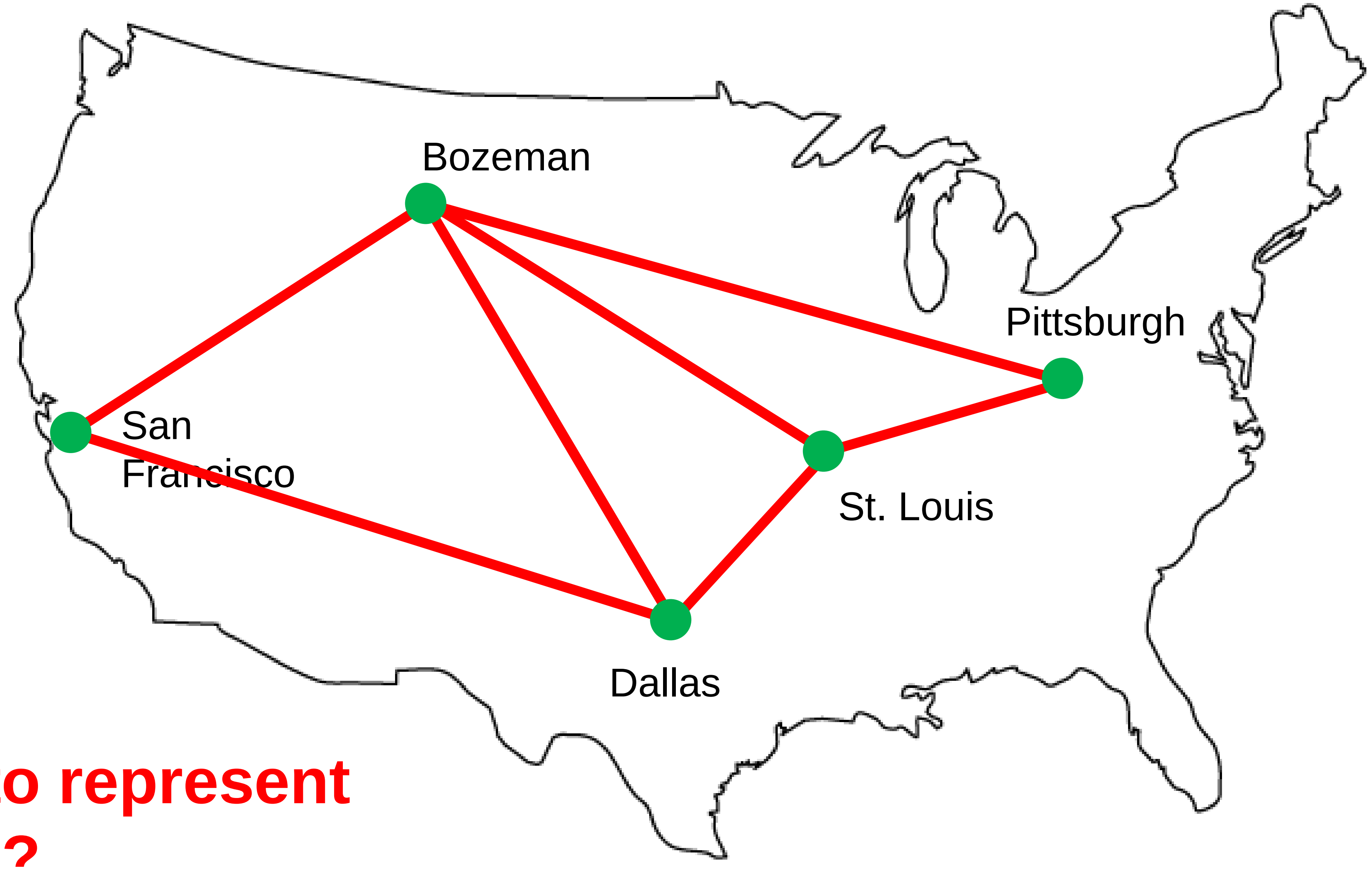

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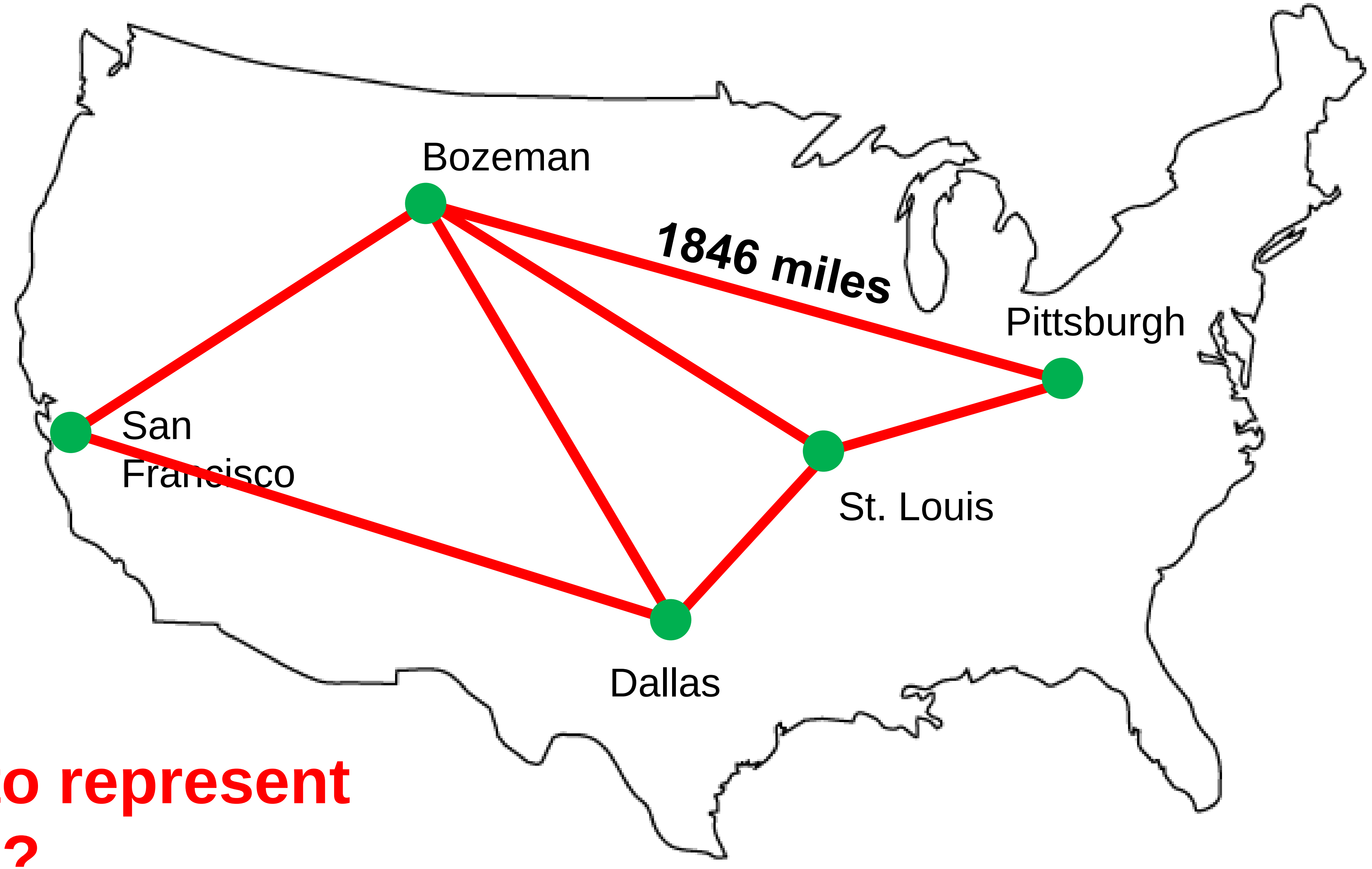

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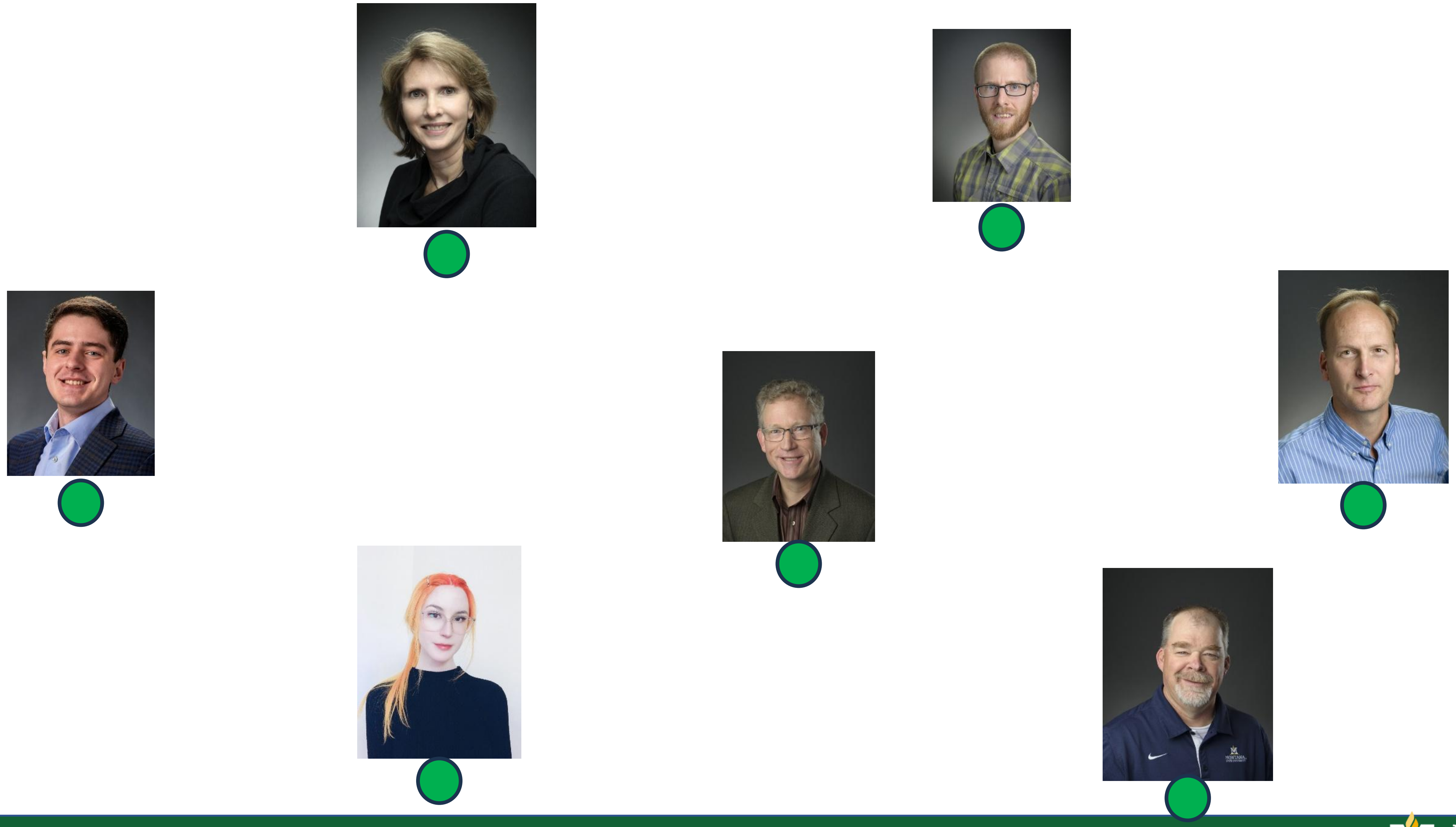

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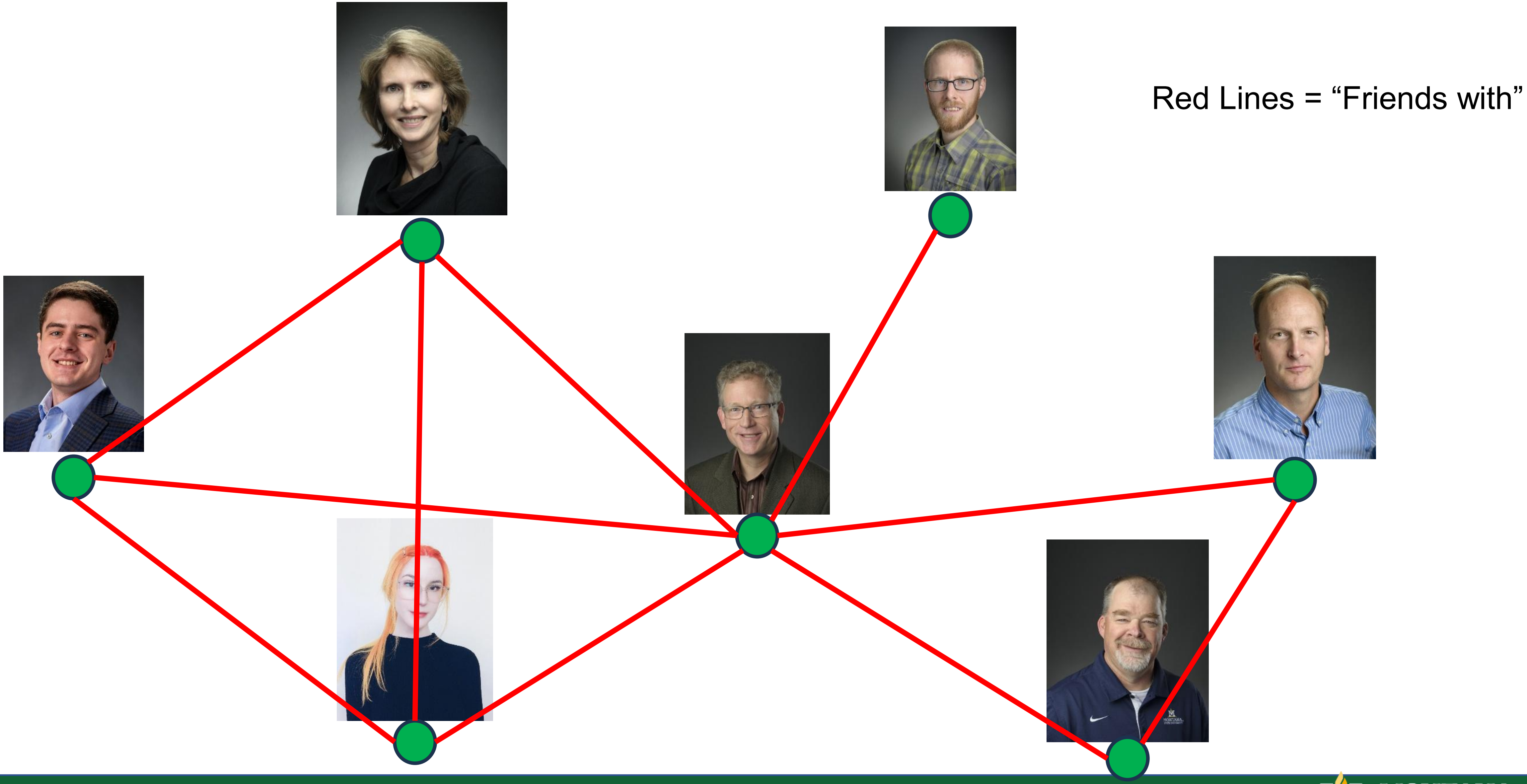
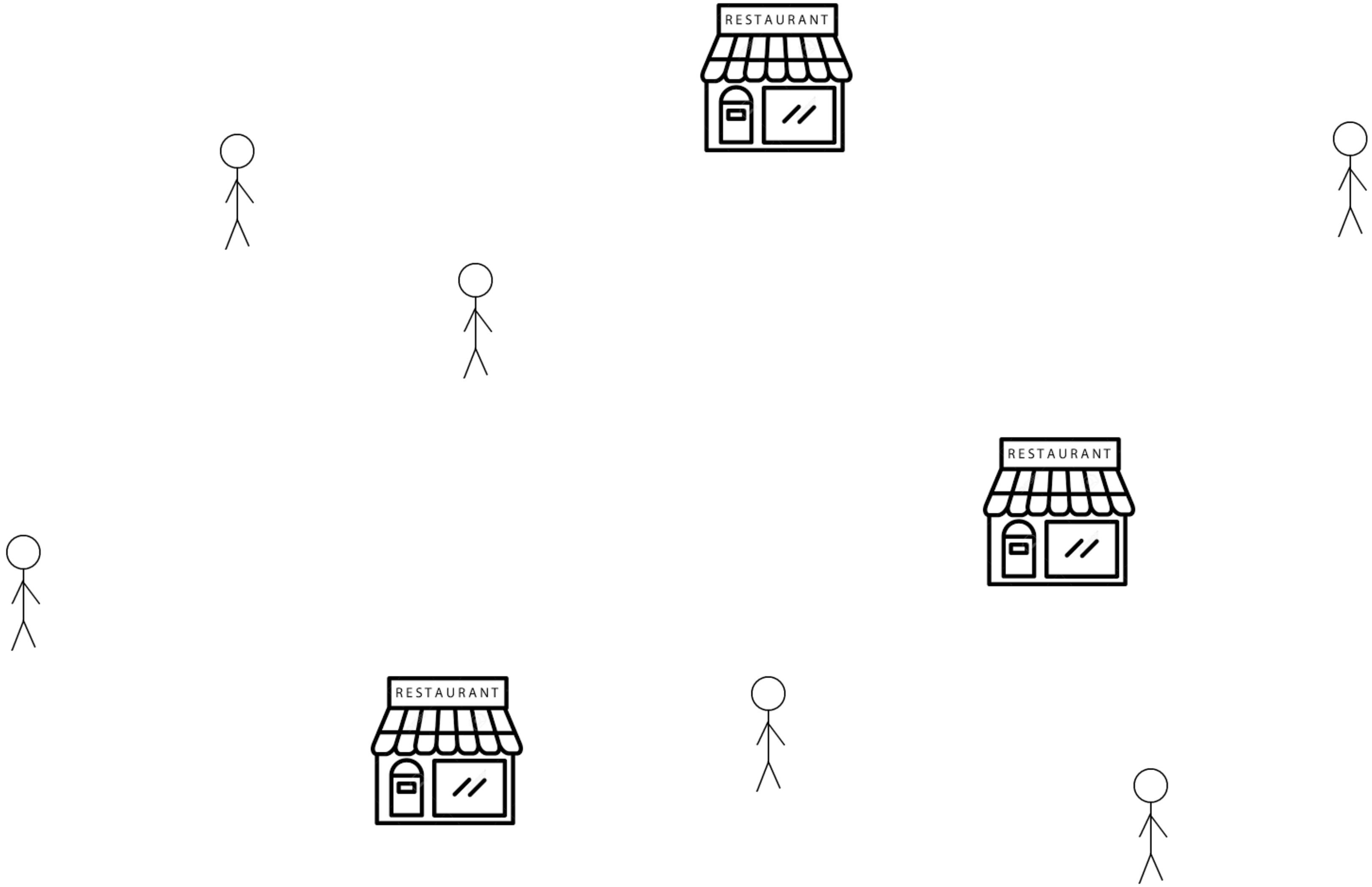


How could we visualize: Restaurants and Potential Customers?



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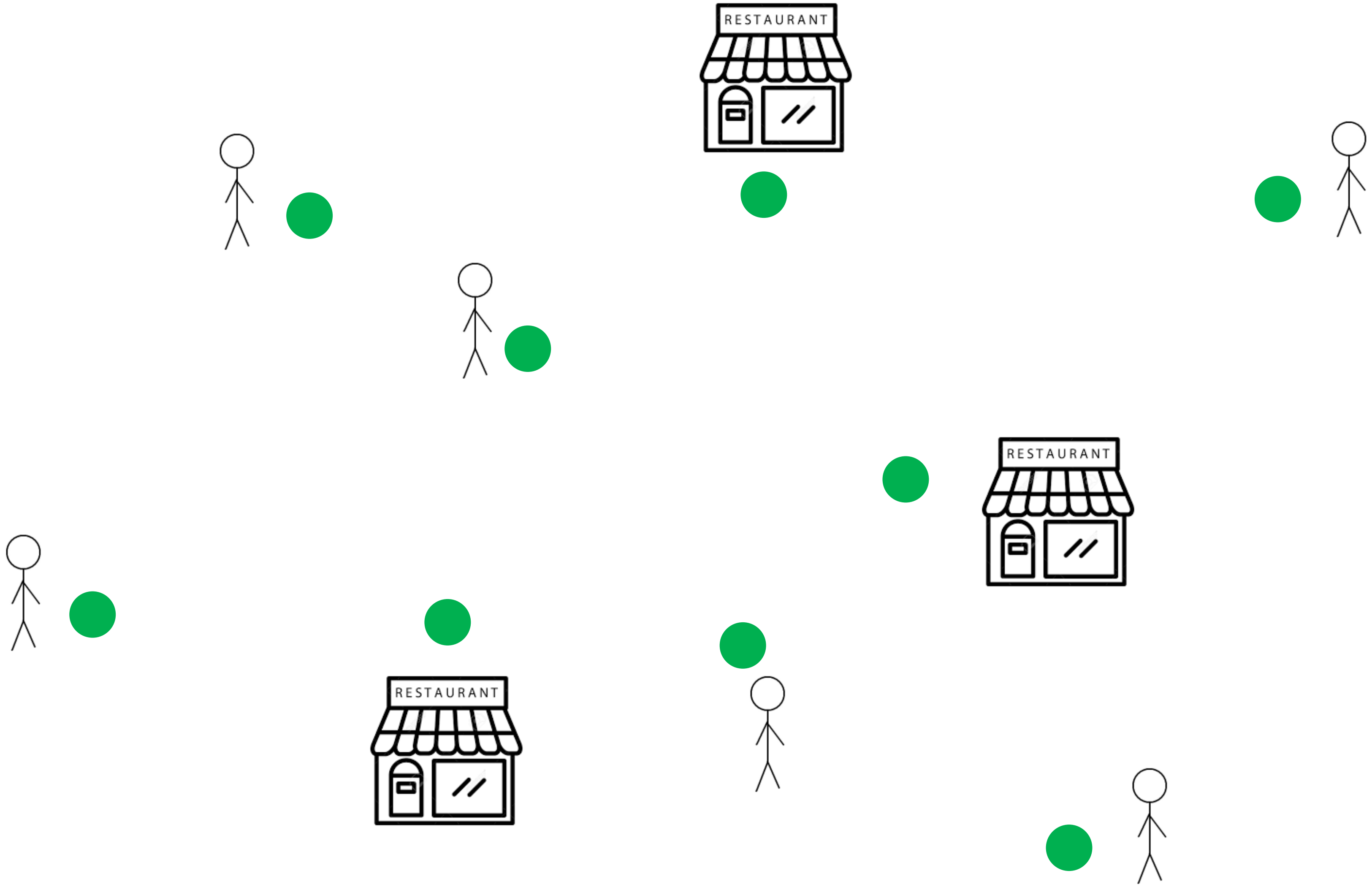
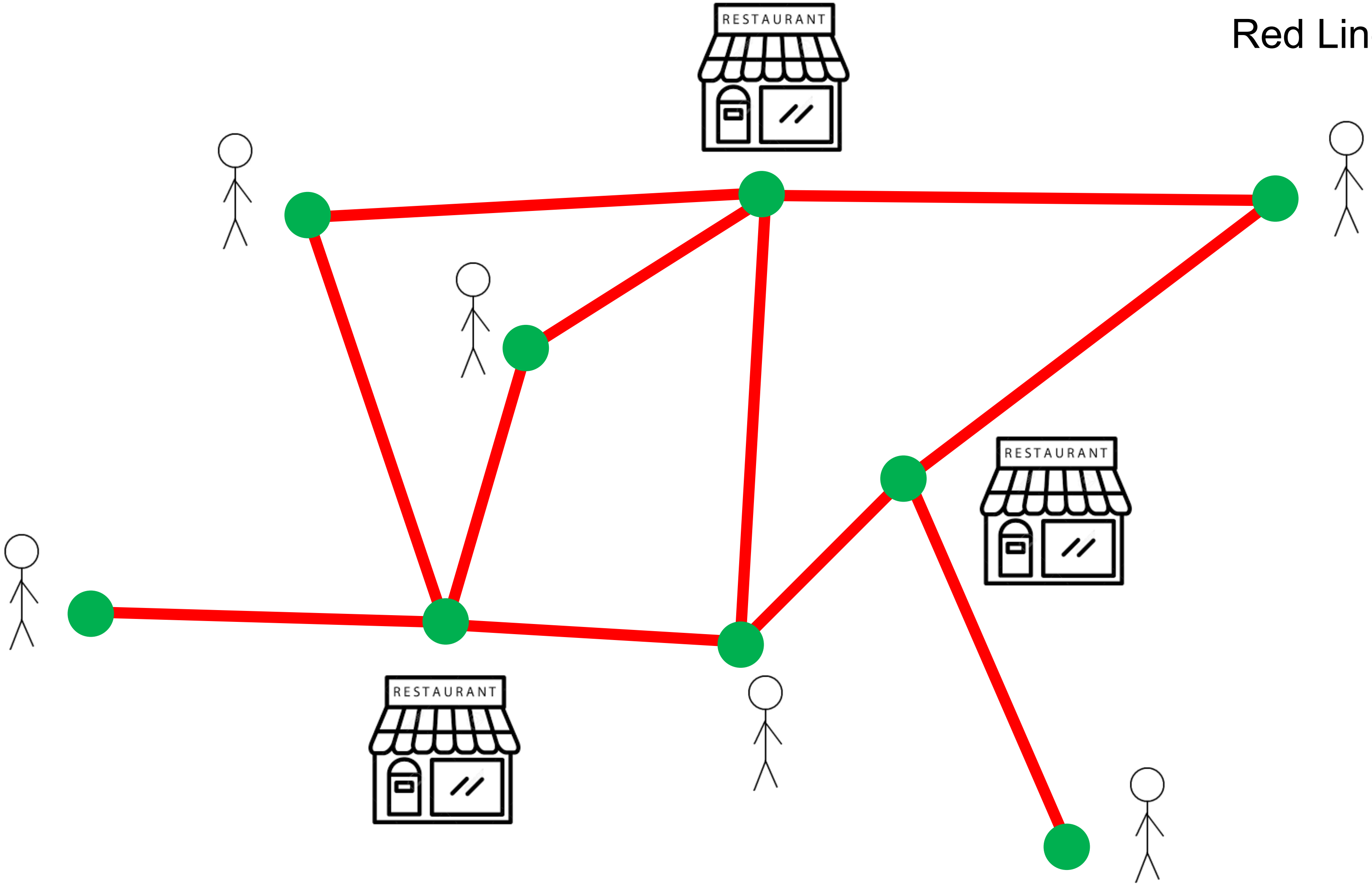
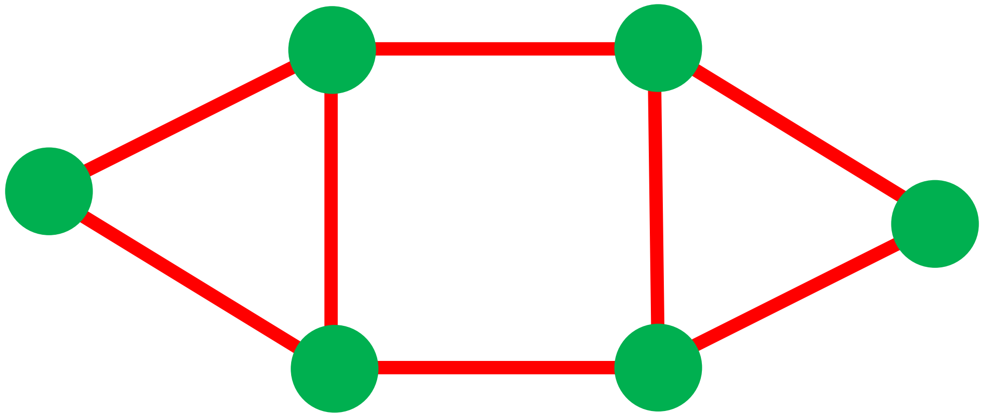


How could we visualize: Restaurants and Potential Customers?

Red Lines = "Would dine at"

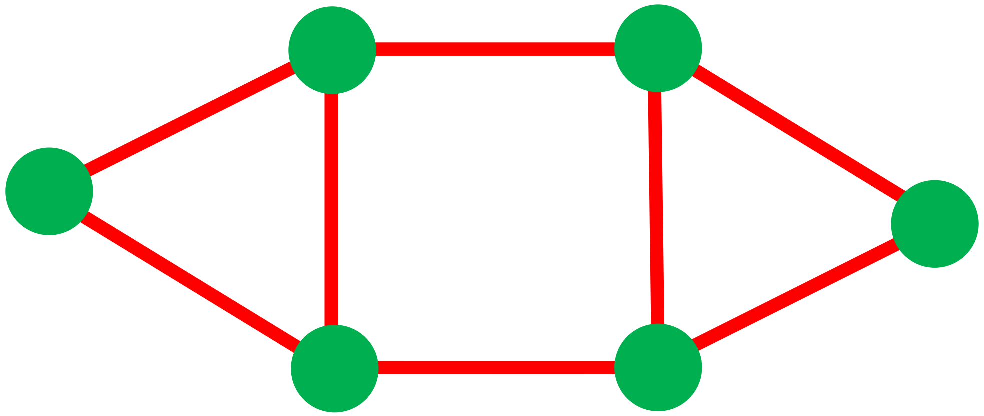


Graphs



Vertices (or Nodes)

Graphs



Vertices (or Nodes)
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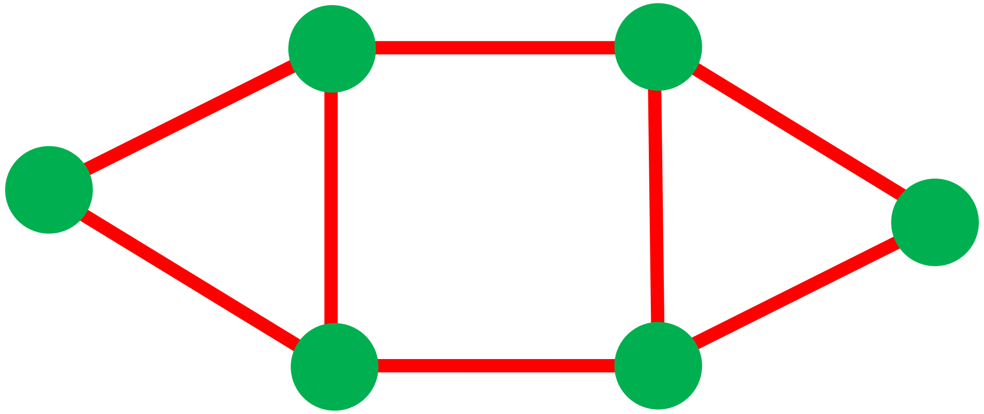

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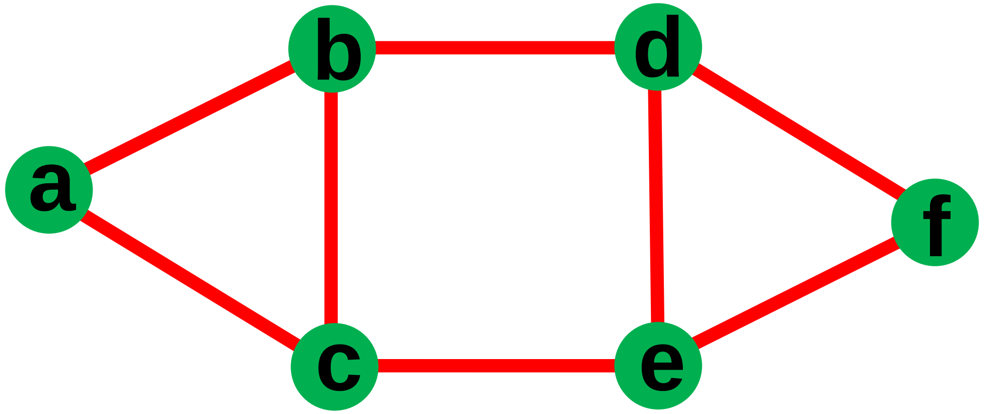

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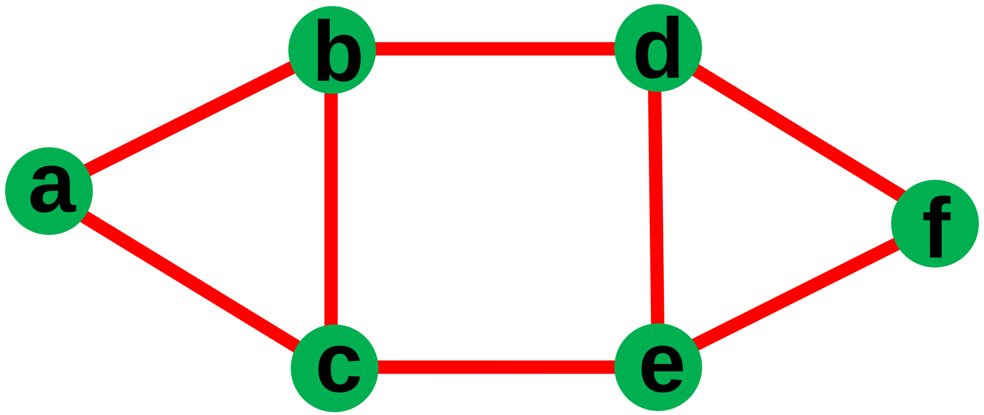
Graphs



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$\Big\}\ \boldsymbol{G = (V, E)}$

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If and only if $V_1 = V_2$ and $E_1 = E_2$.

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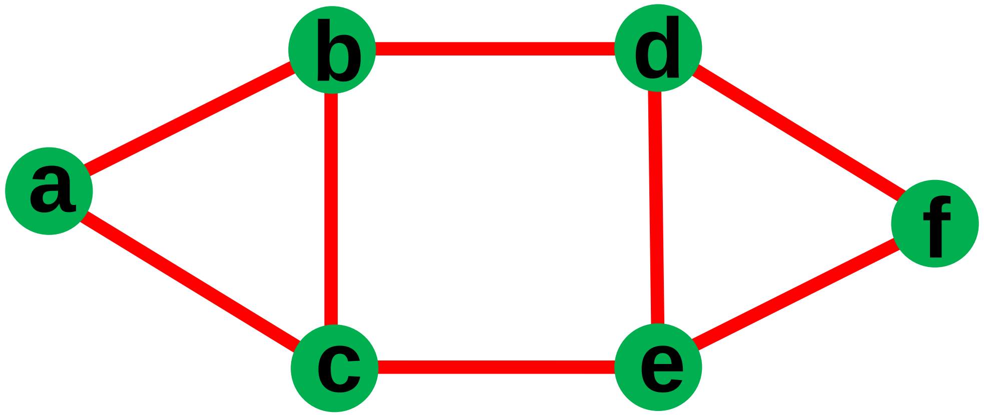

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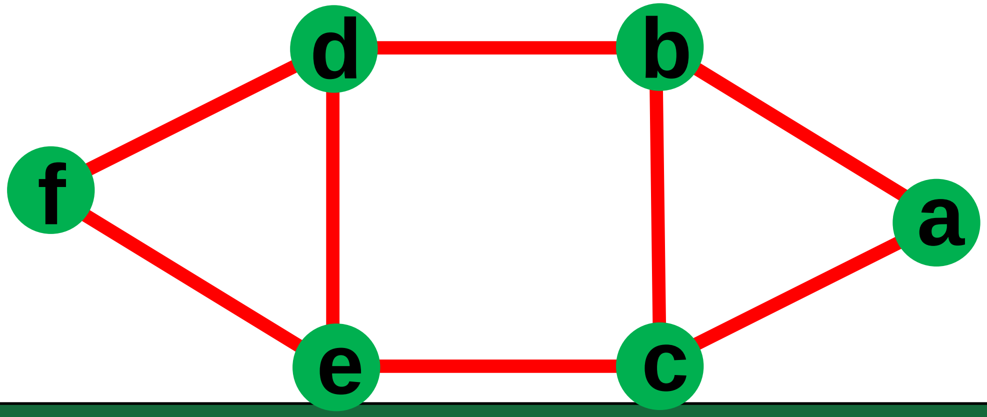
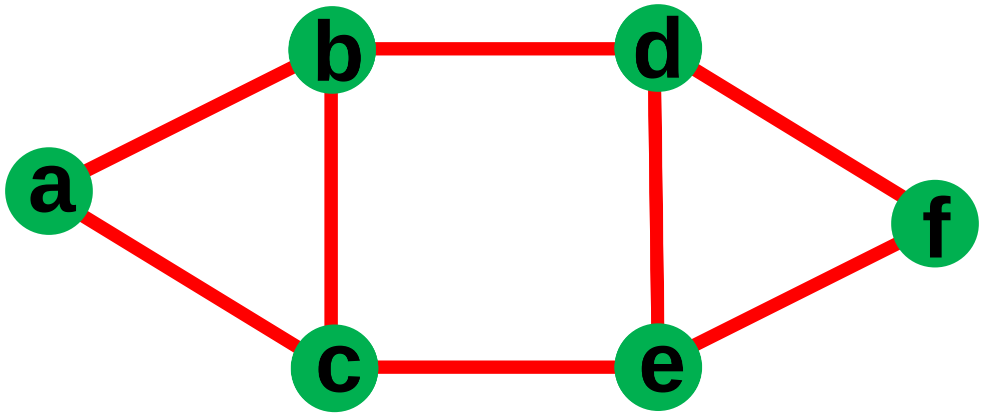


Same graph? ✓

Graphs



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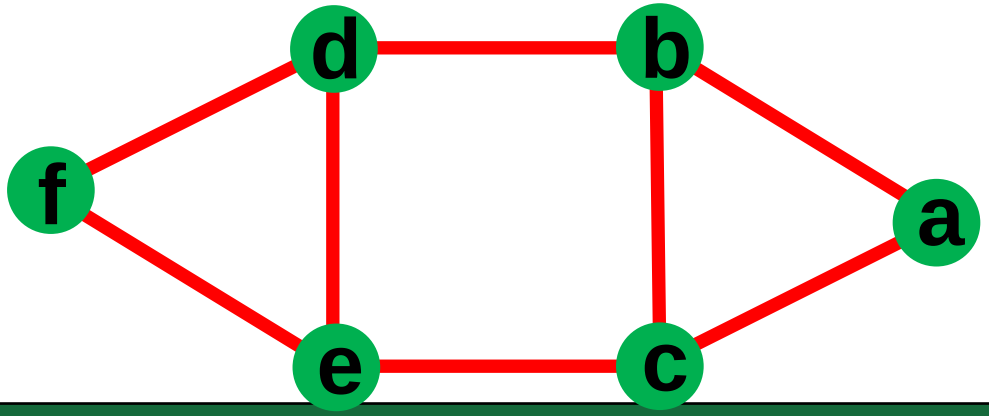

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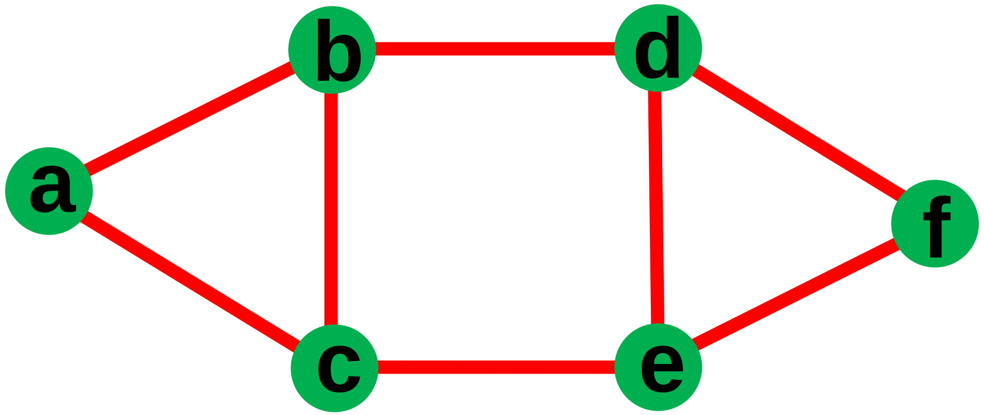

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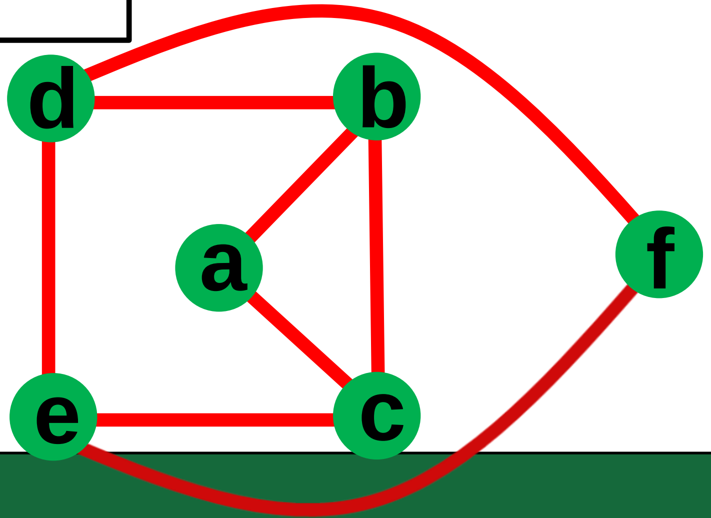

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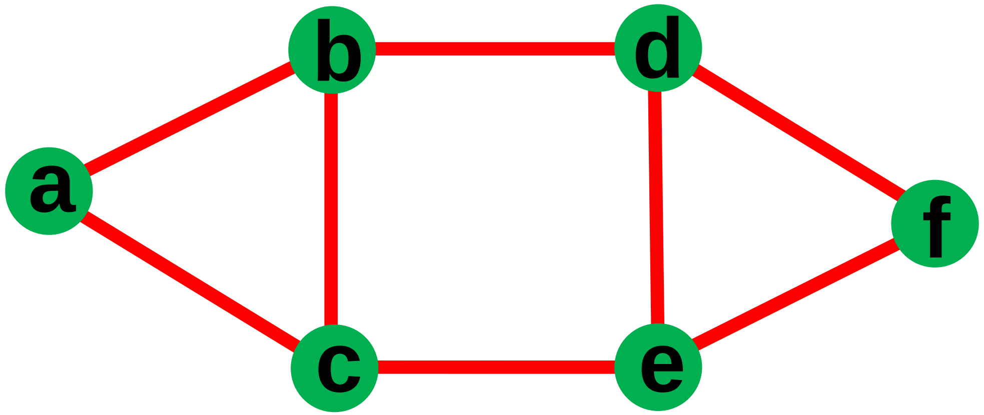

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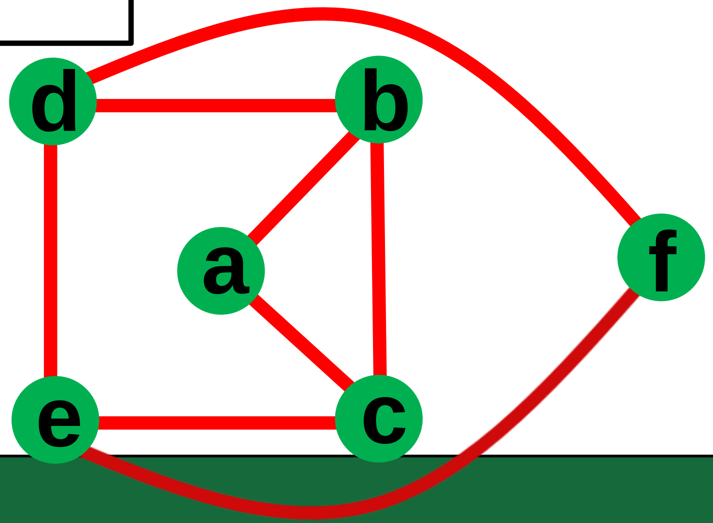

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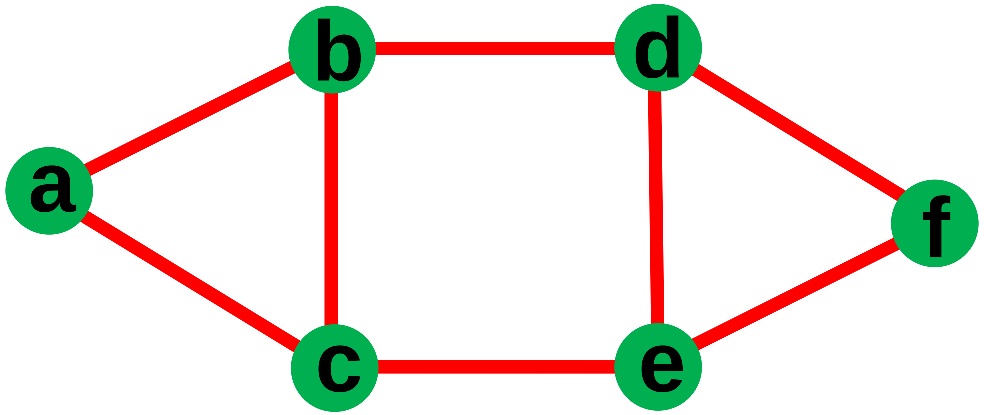

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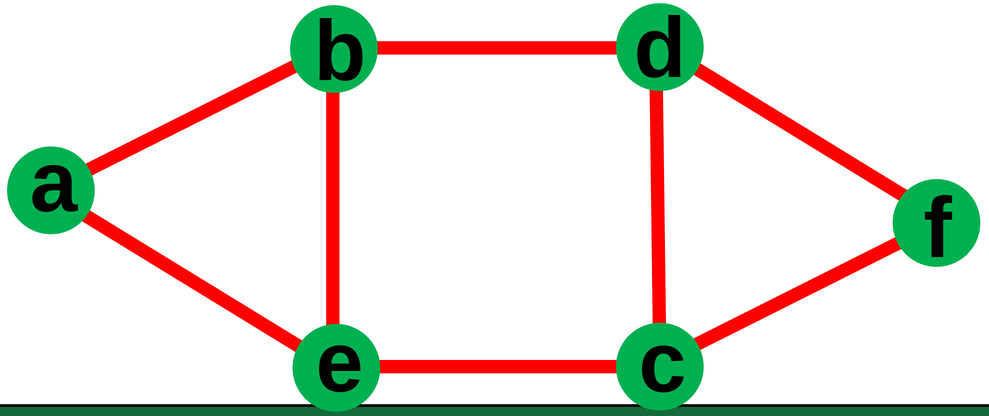

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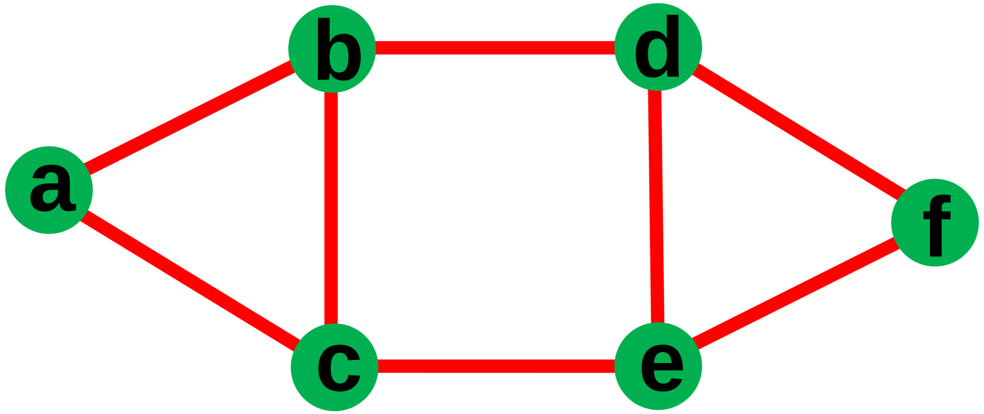

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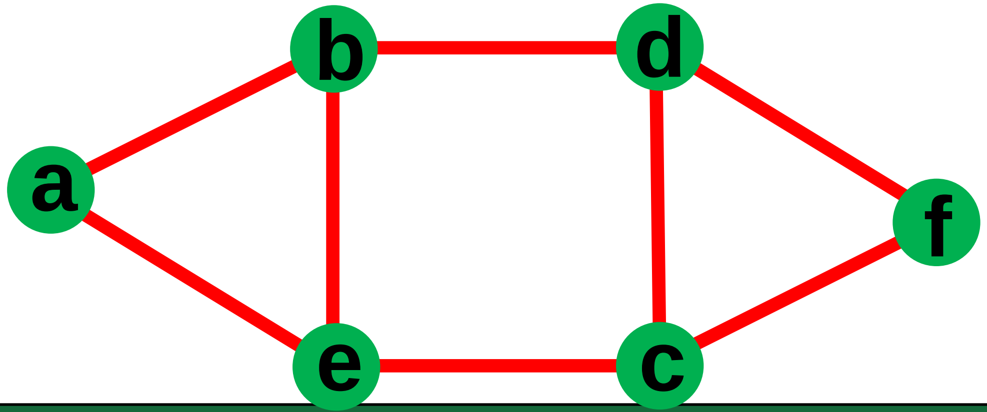

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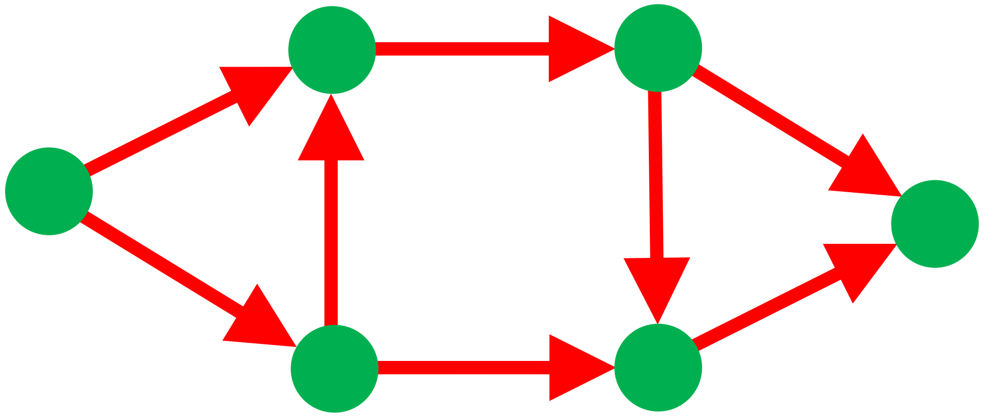

Vertices (or Nodes)
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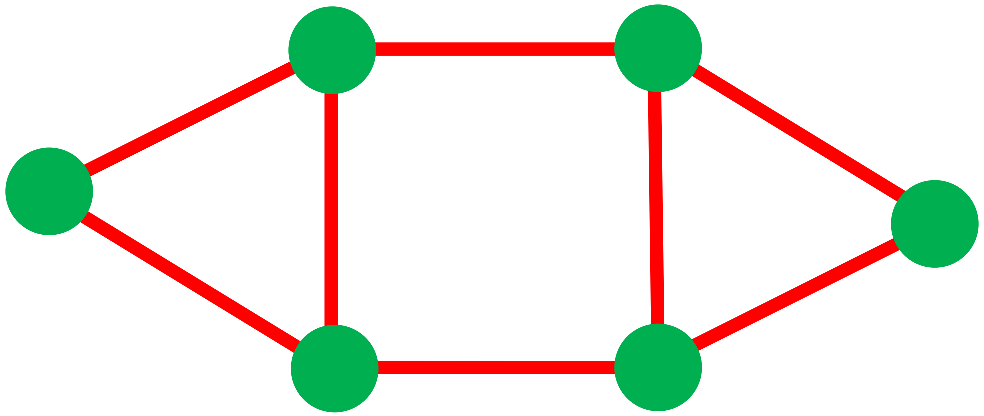
$G = (V, E)$

- Edges can be directed…

Graphs



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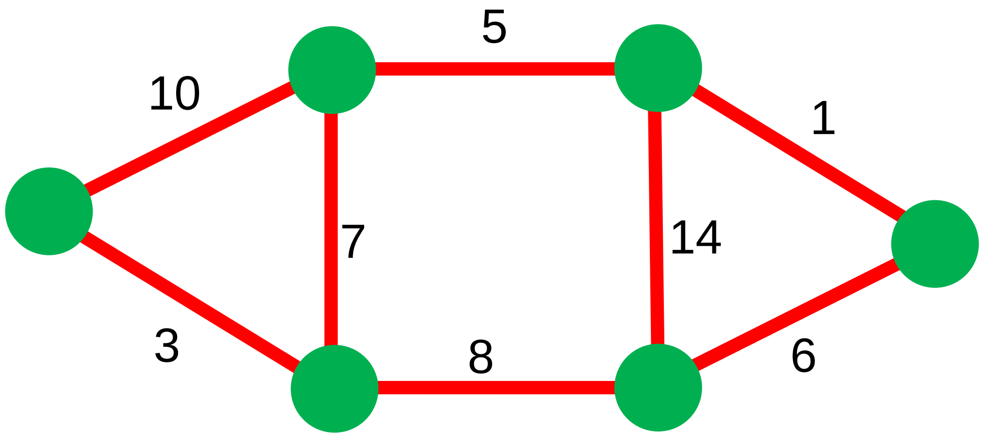
- Edges can be directed or <u>undirected</u>.

Graphs

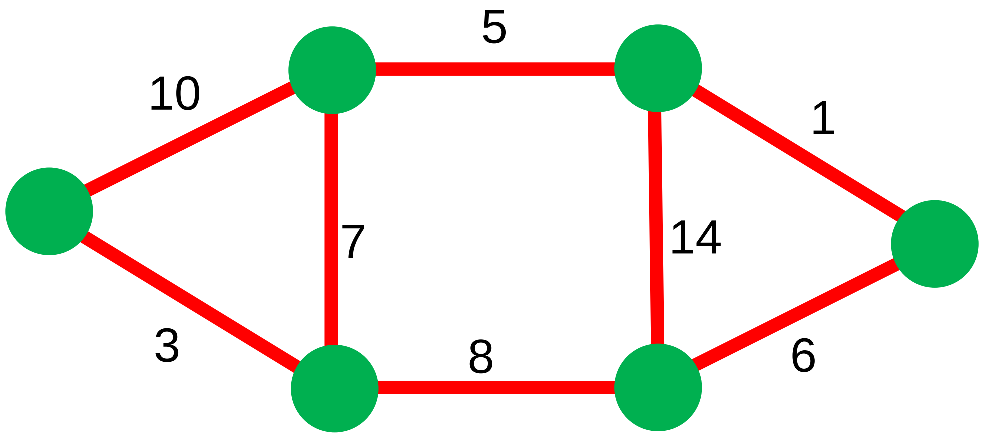


Vertices (or Nodes)
Edges

$$G = (V, E)$$

- Edges can be directed or undirected.
- Edges can have **weights**

Graphs



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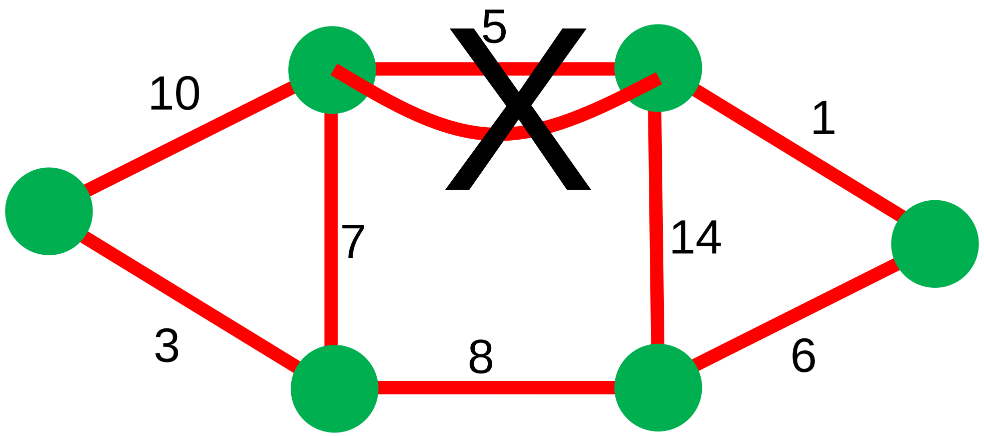

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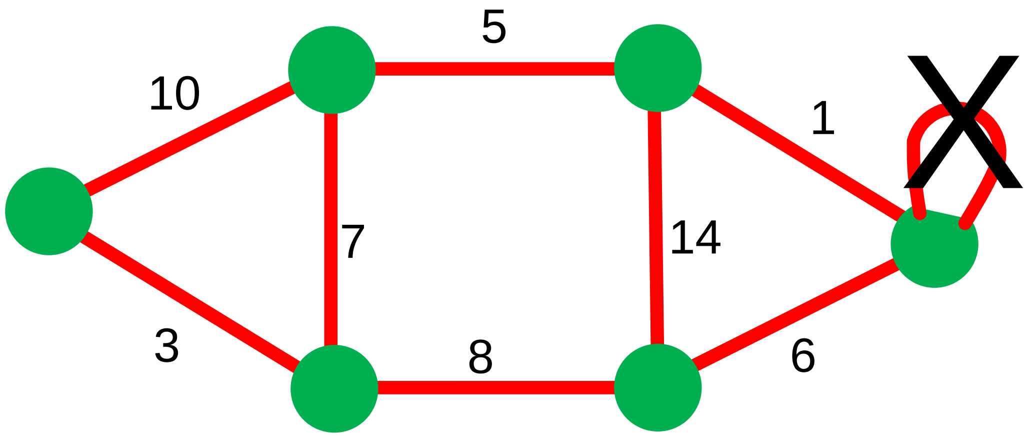

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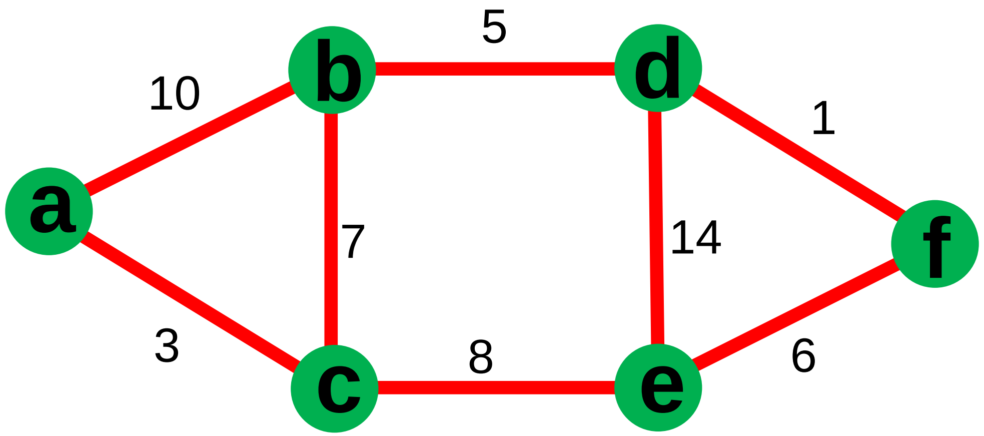

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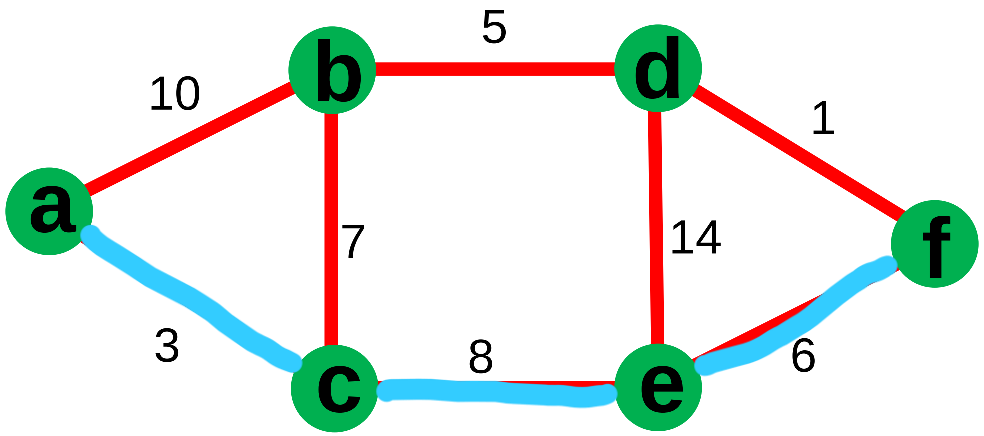

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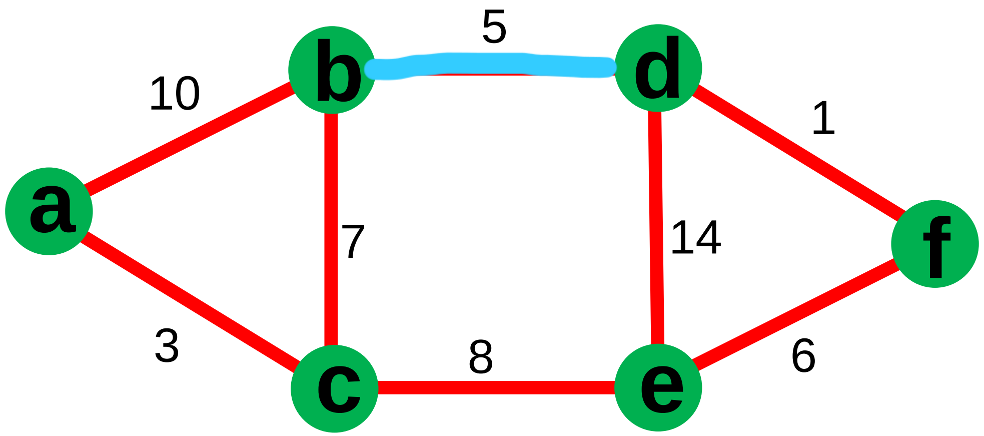
a,c,e,f ✓ "cost" of path = 17

Graphs



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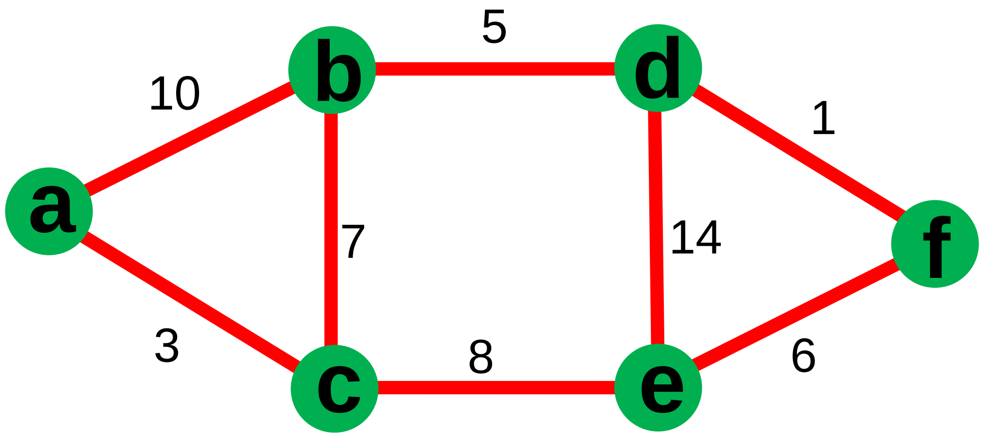
b,d ✓

Graphs



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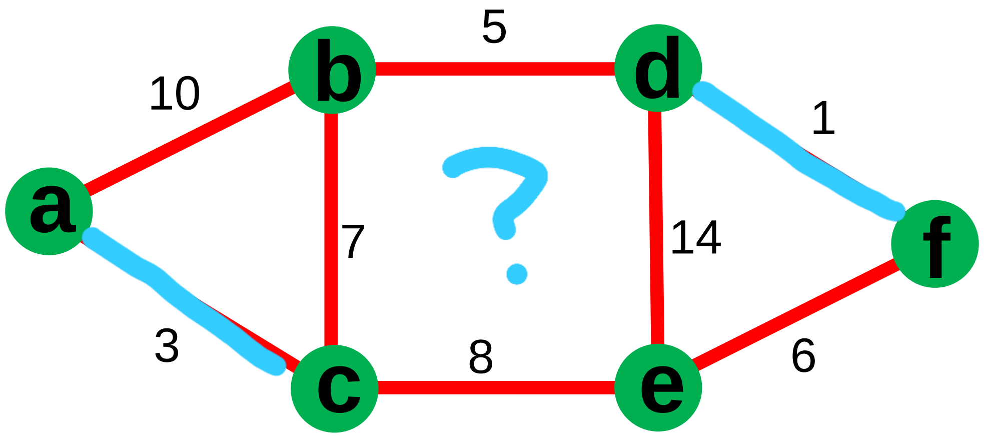

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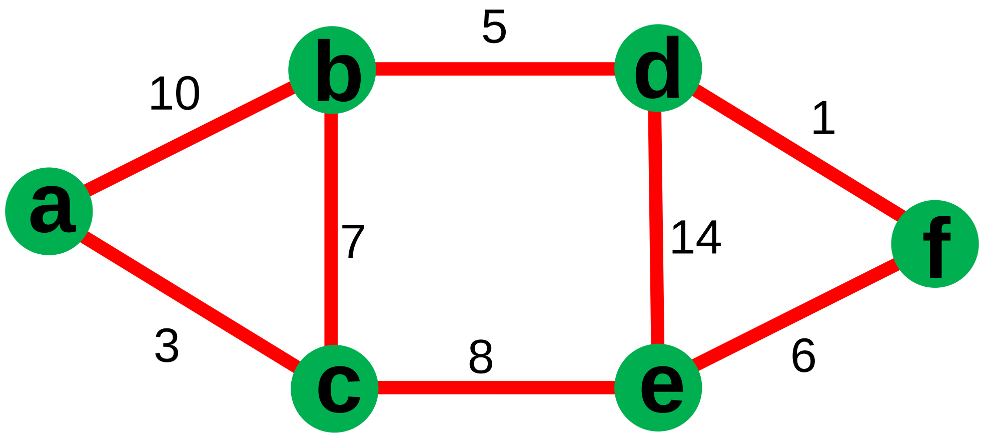
a,c,d,f ✗

Graphs



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c,e,d,f,e

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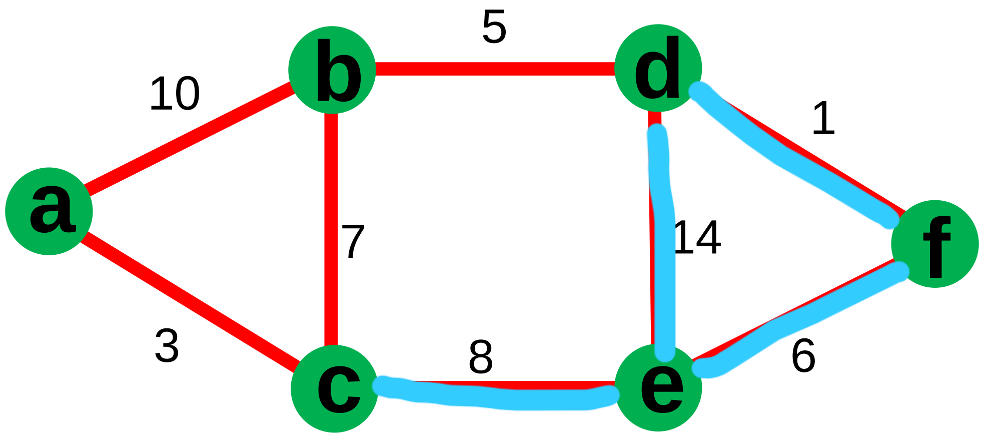

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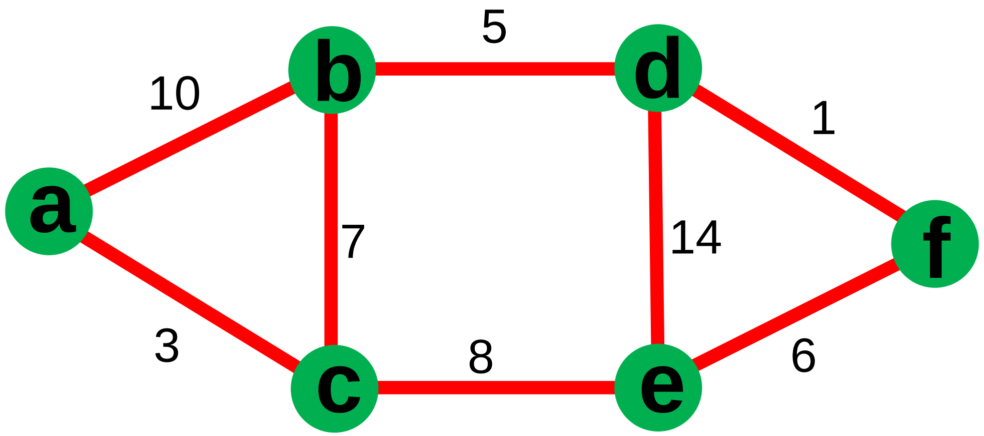

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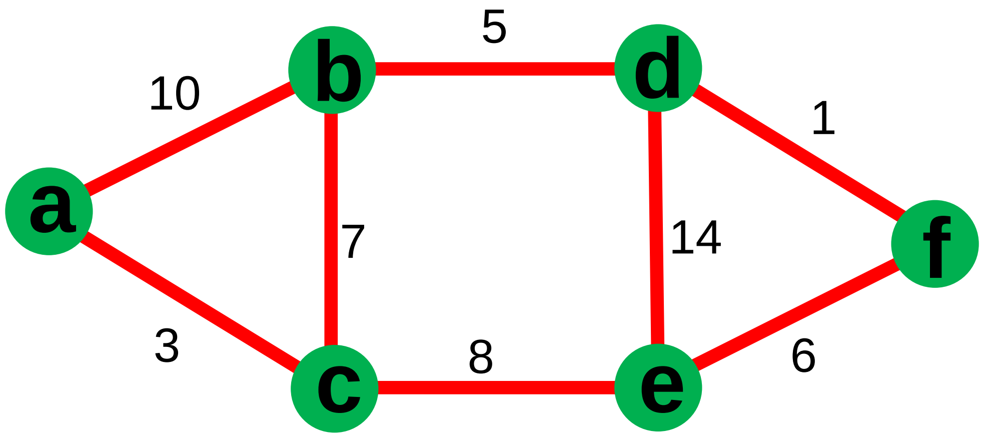
- Cycle = Sequence of vertices connected by edge with loop(s).

Graphs



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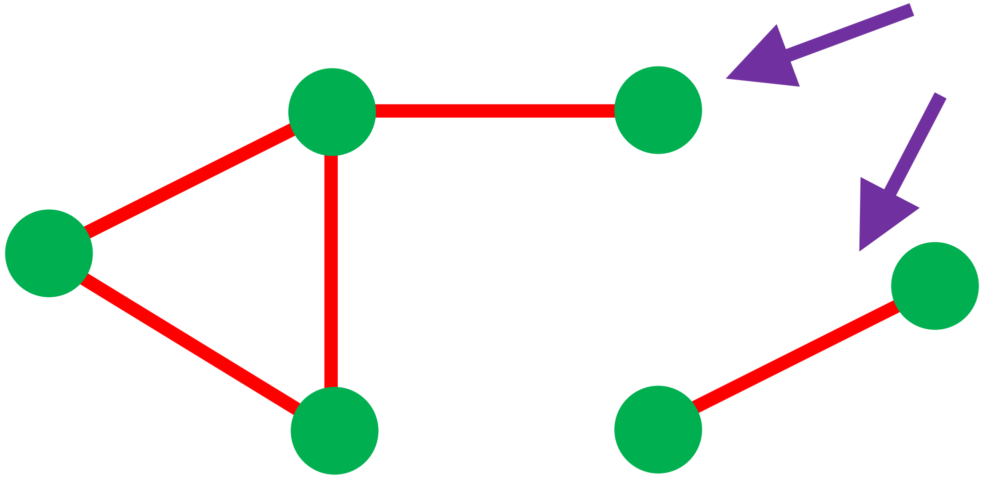
Graphs



Two Connected Components

Vertices (or Nodes)
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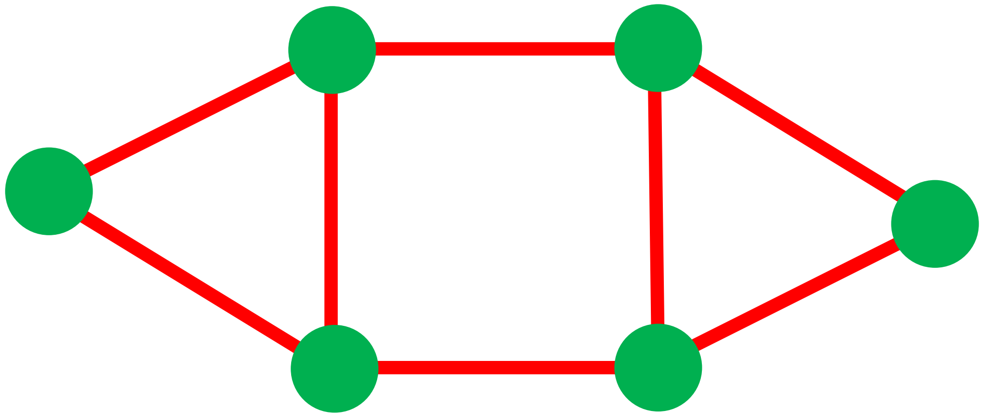

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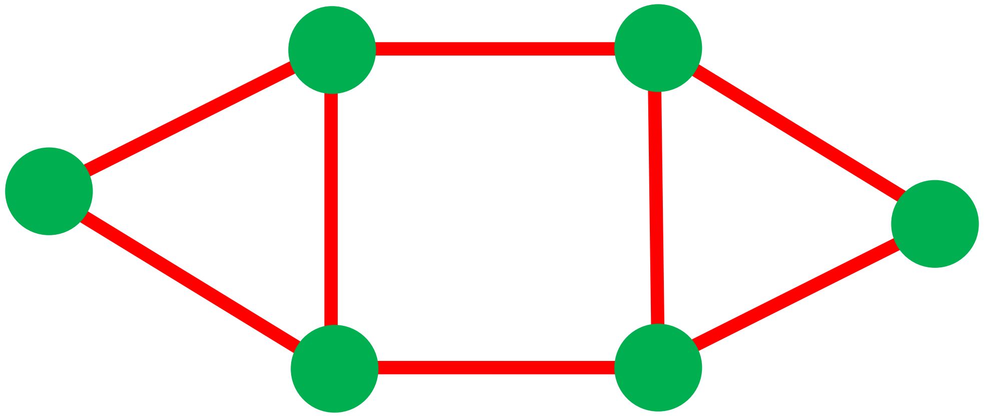
- Degree of a vertex = $\deg(v)$ = # of edges touching it (undirected).

Graphs



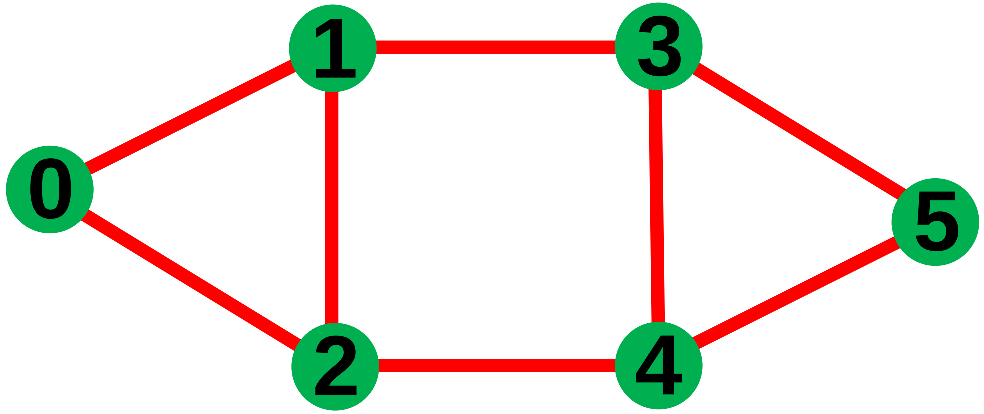
Vertices (or Nodes)
Edges

$$G = (V, E)$$

What are some operations we may want to perform on a graph?

- Add vertices/edges.
- Find path between vertex pair.
- Is graph connected?
- Find degree of vertex.
- Is the graph simple?
- Get number of vertices/edges.
- Get neighbors of vertex.
- Is there a cycle?
- Find max degree of graph.

How can we represent a graph in a computer?



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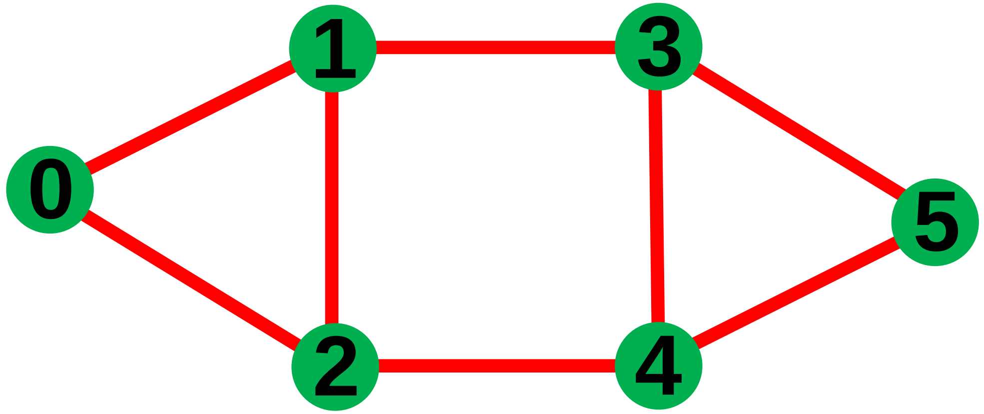

1. Adjacency List

0	→	{1,2}
1	→	{0,2,3}
2	→	{0,1,4}
3	→	{1,4,5}
4	→	{2,3,5}
5	→	{3,4}

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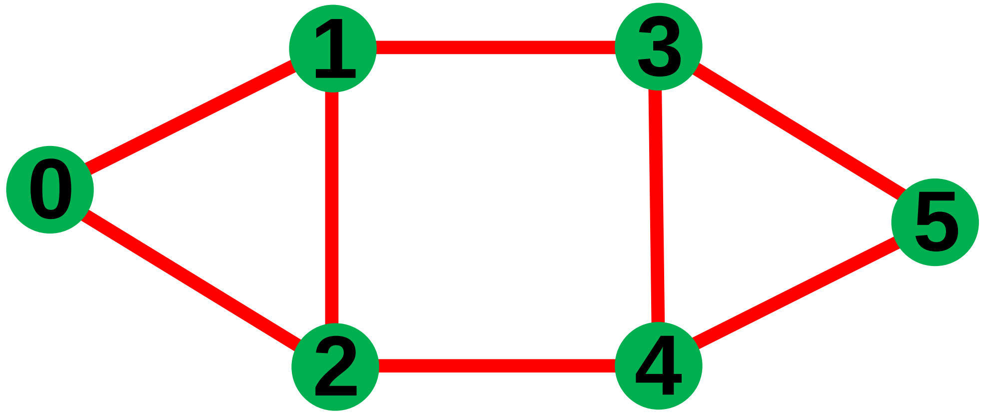

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2. Adjacency Matrix

	0	1	2	3	4	5
0	F	T	T	F	F	F
1	T	F	T	T	F	F
2	T	T	F	F	T	F
3	F	T	F	F	T	T
4	F	F	T	T	F	T
5	F	F	F	T	T	F

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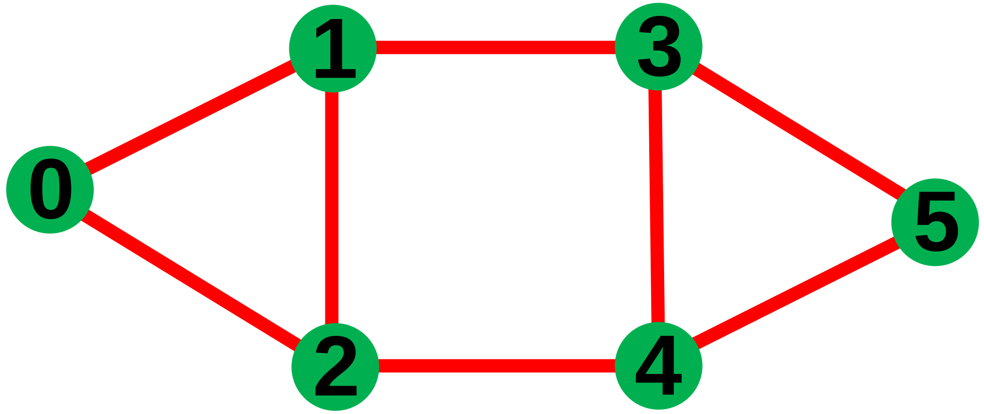

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3	F	T	F	F	T	T
4	F	F	T	T	F	T
5	F	F	F	T	T	F

3. Objects

```
public class Node {
    private Set<Node> neighbors;

    …
}
```

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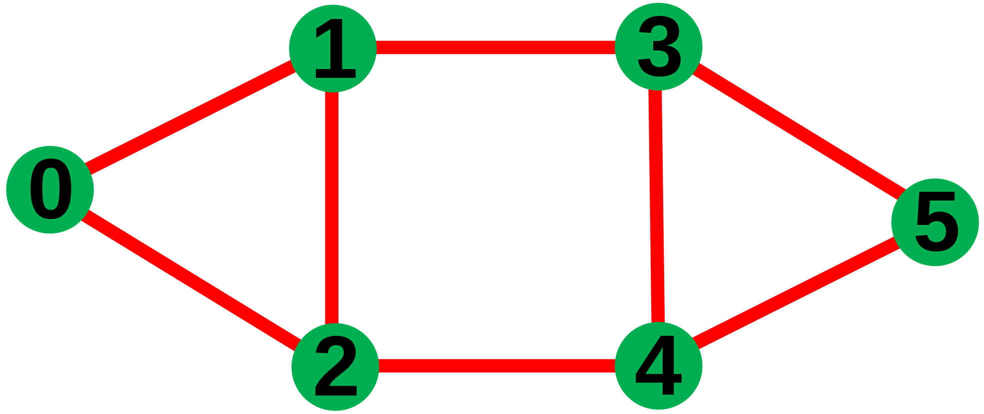

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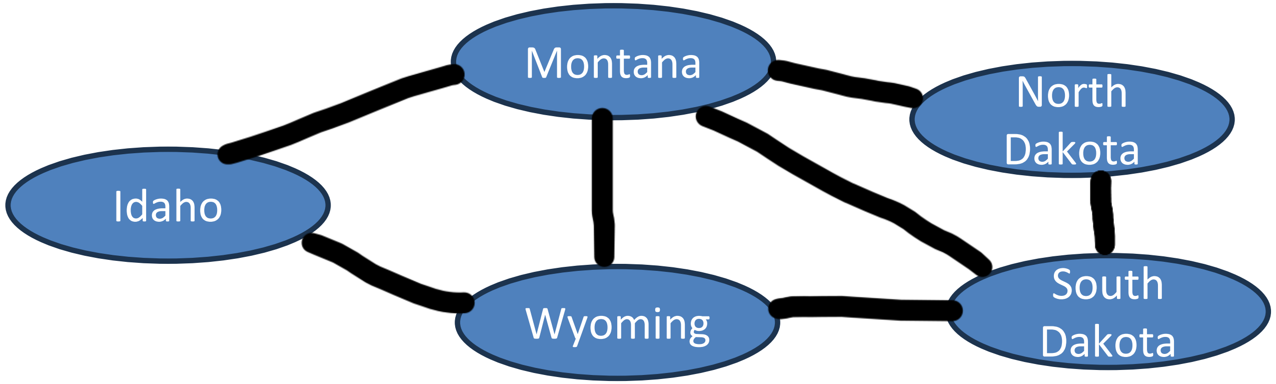
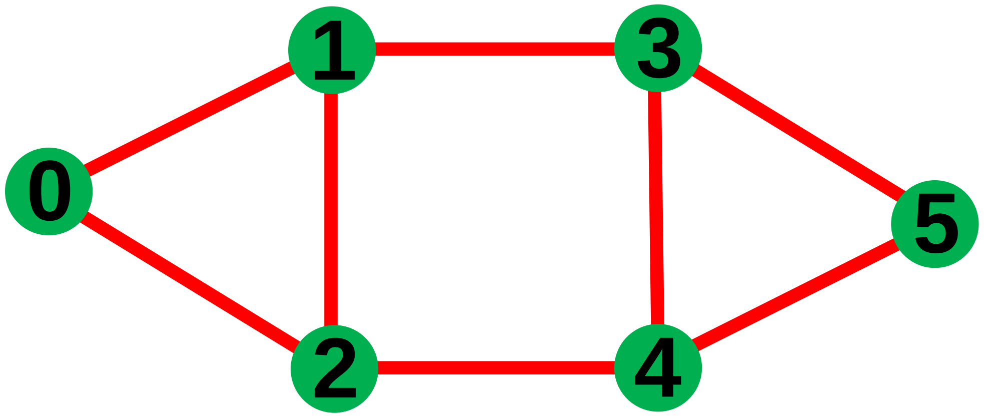
1. Adjacency Lists





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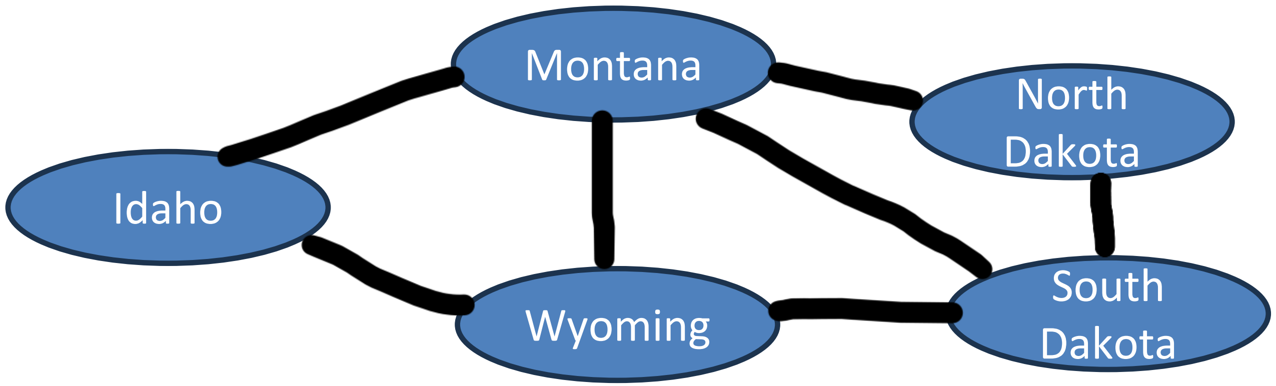
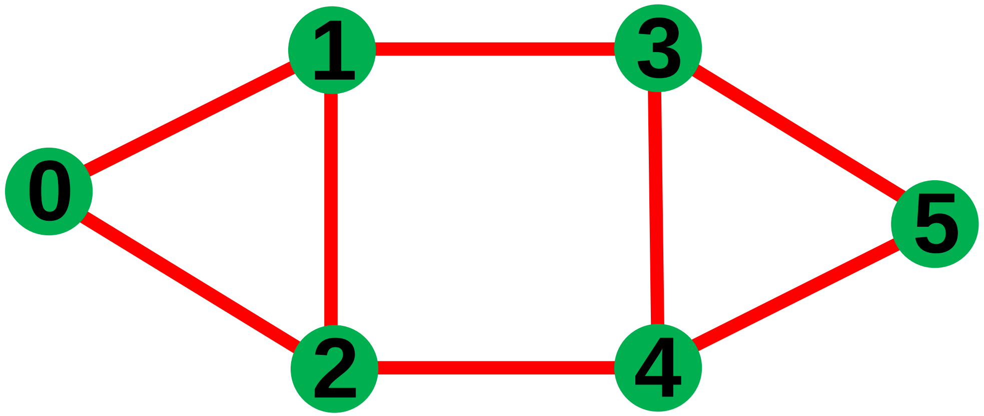
Montana: [North Dakota, South Dakota, Wyoming, Idaho]
Idaho: [Montana, Wyoming]
Wyoming: [Idaho, Montana, South Dakota]
North Dakota: [Montana, South Dakota]
South Dakota: [Wyoming, Montana, North Dakota]

?

1. Adjacency Lists





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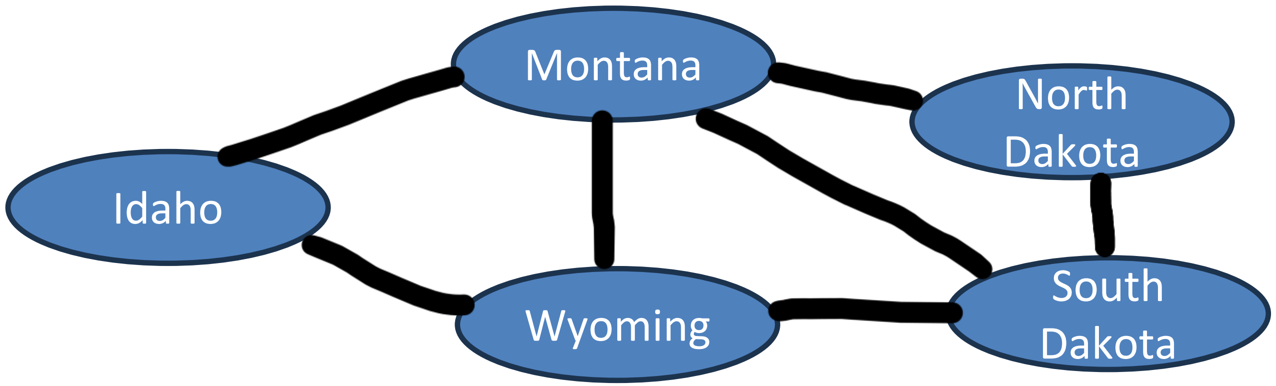

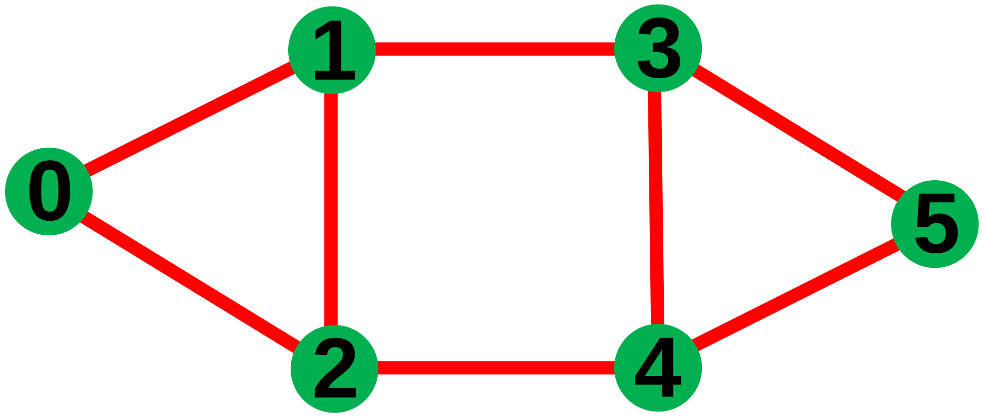

0	→ {1,2} ?
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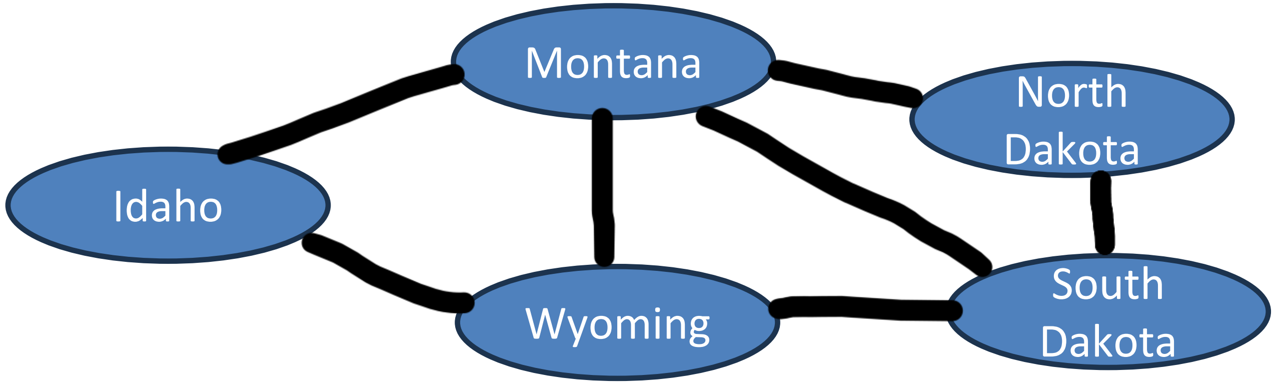

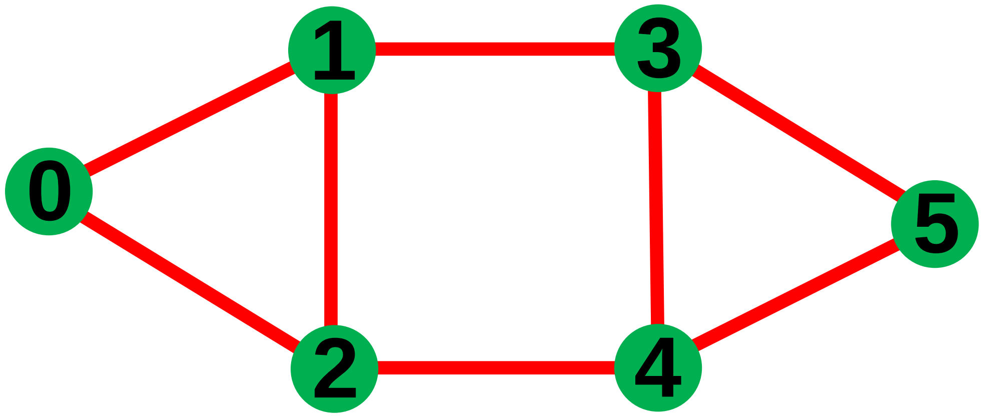

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Array of Linked Lists