CSCI 232:
Data Structures and Algorithms

Shortest Path (Part 1)

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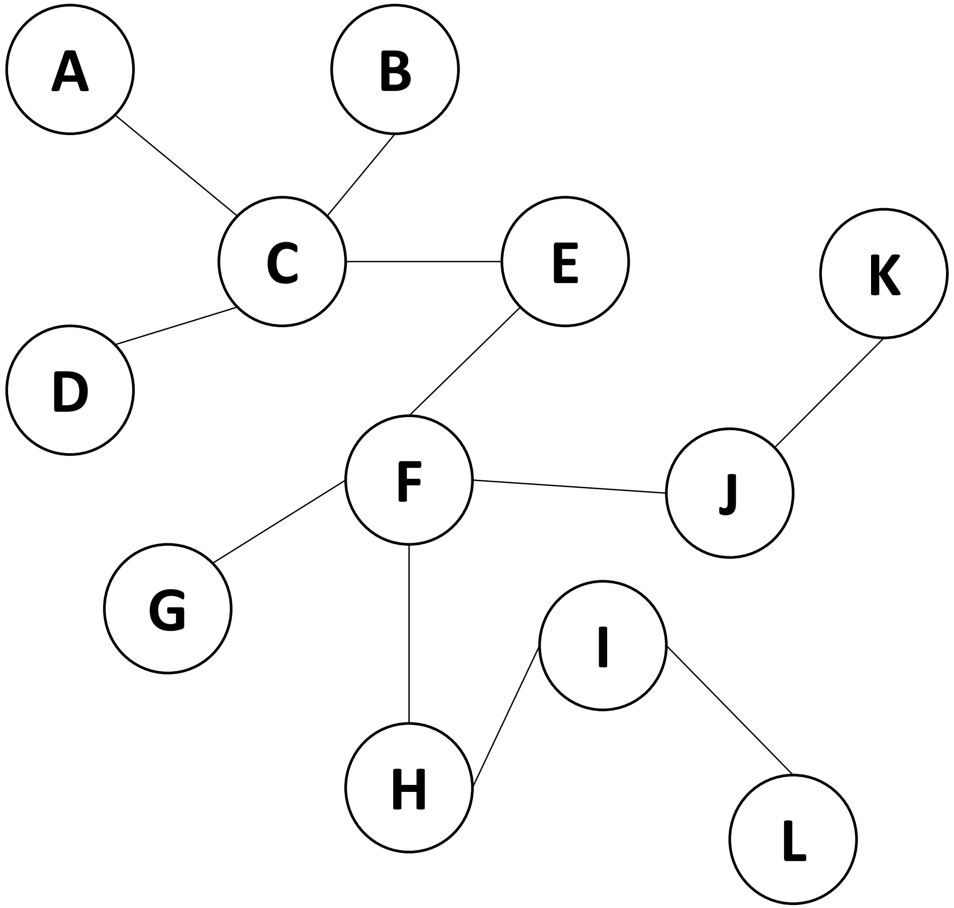
Spring 2025

Announcements

Quiz 2 on Friday
→ Go to your lab section. Must be taken in Roberts 111
→ Same format as quiz 1

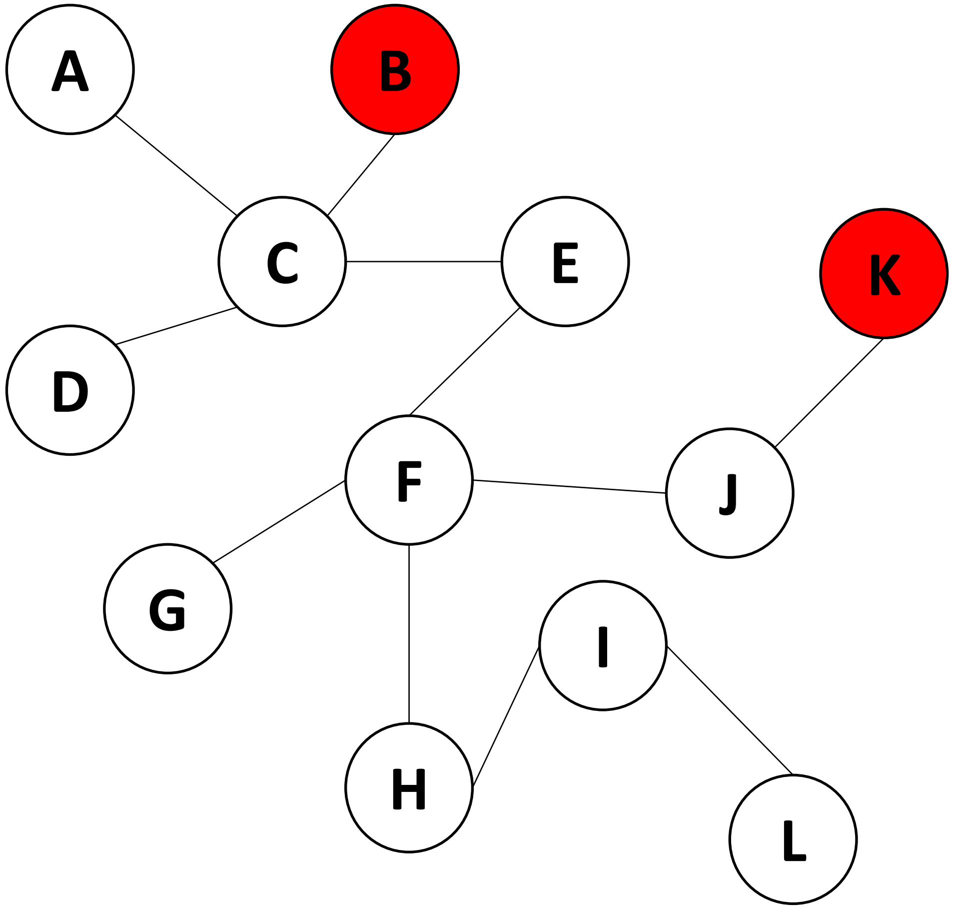
Program 3 posted. Due April 22nd
→ Longer and more challenging than past programs (worth more points)
→ Still very do-able (we have written a lot of the code in class)
→ Get started now



Consider an **Acyclic** graph (a graph with no cycles) (a "tree")

Observation:

Pick any two vertices (V1, V2). There is **only one possible path** that goes from V1 to V2 (and vice versa)



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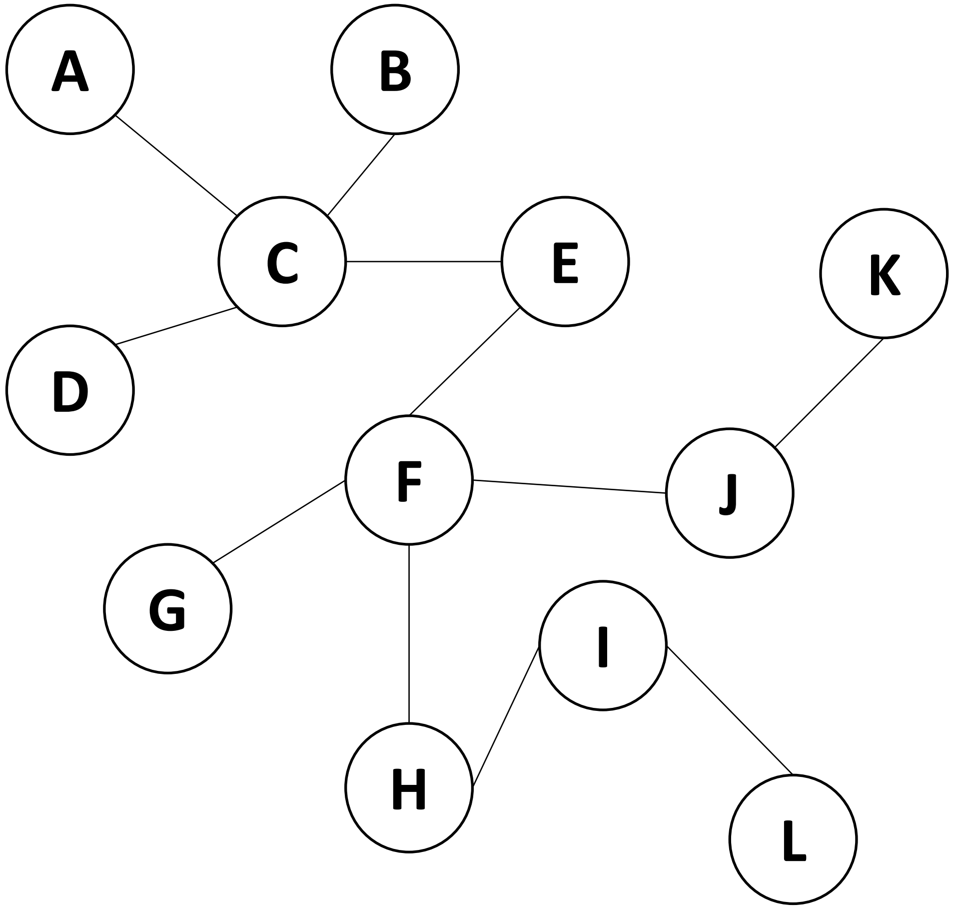
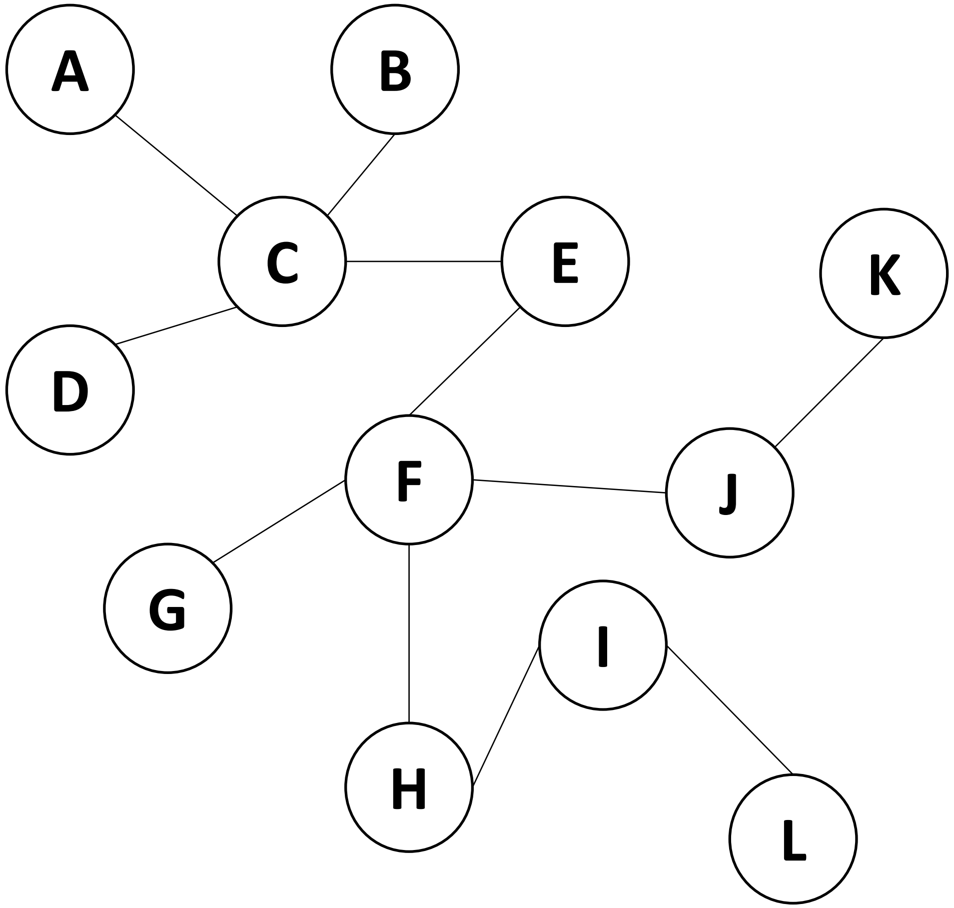


Longest Path ?

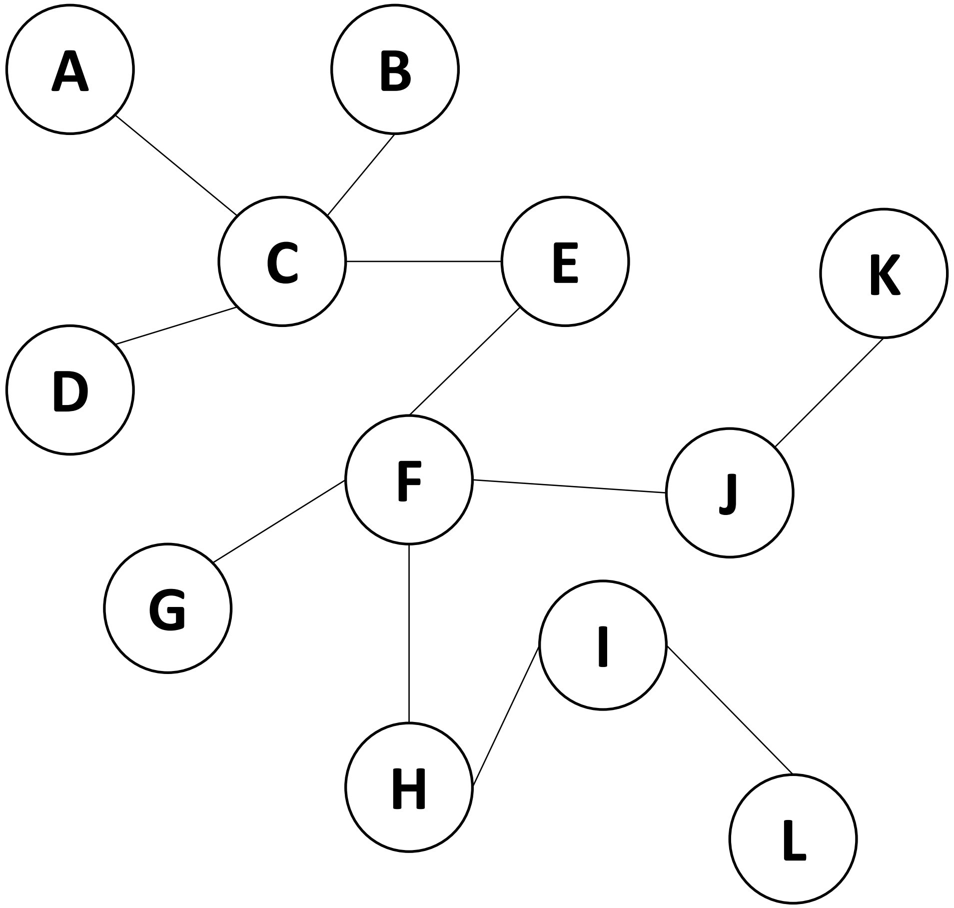


Select any vertex **v1**

```
HashMap<String, LinkedList<String>> adjList
```

```
{ J: [K], A:[C], C:[A,D,B], F: [E, G, H, J], … }
```

HashMaps are unordered, and there is no way to pick a key at an “index”



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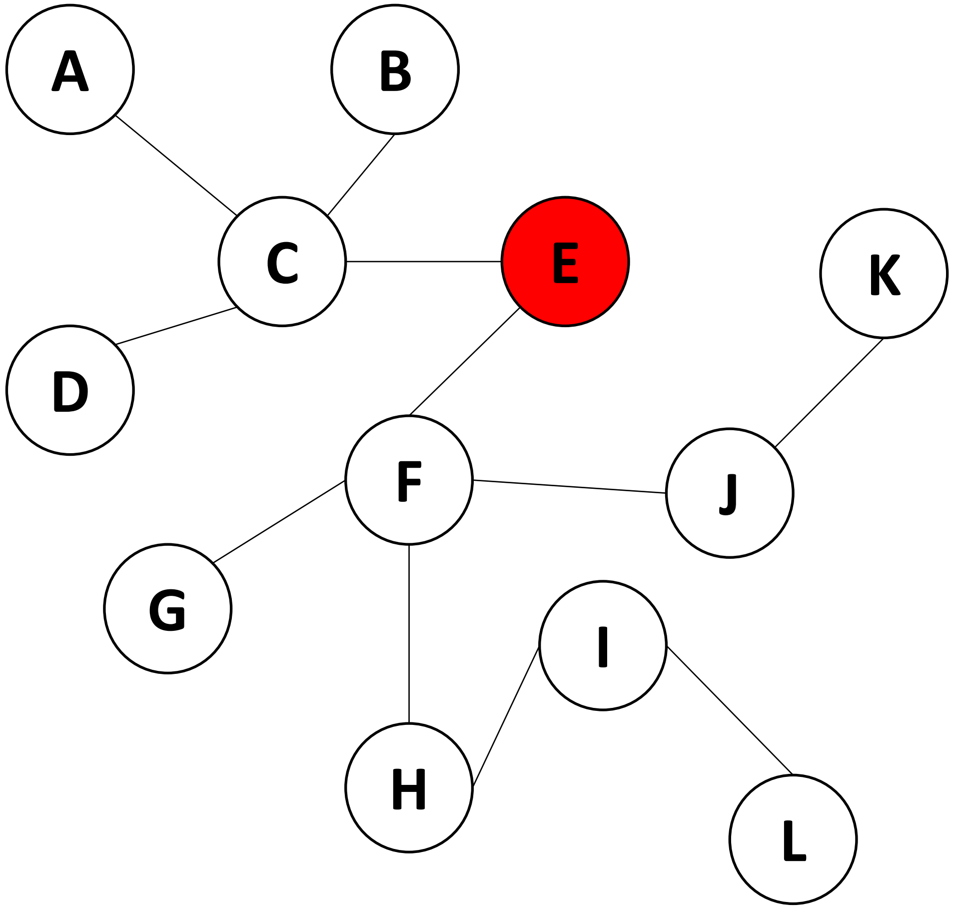
```
HashMap<String, LinkedList<String>> adjList

{ J: [K], A:[C], C:[A,D,B], F: [E, G, H, J], … }
```

HashMaps are unordered, and there is no way to pick a key at an "index"

Just return the first key when iterating over it

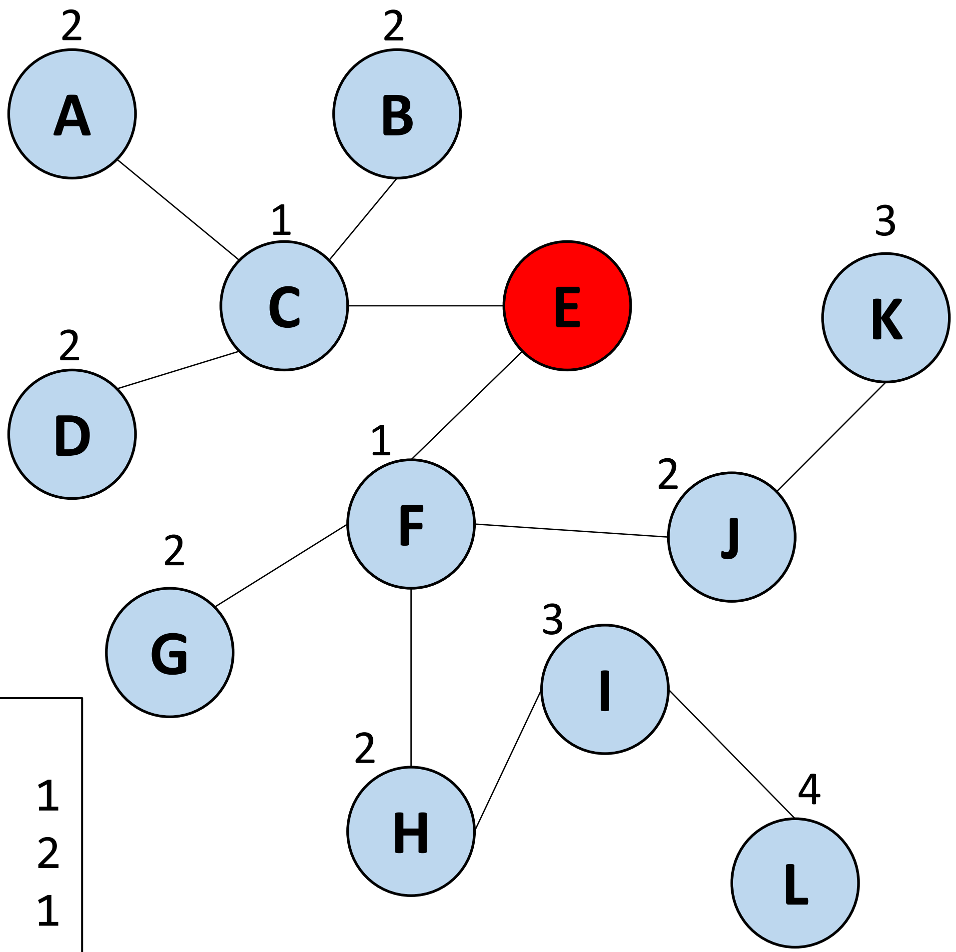
```
for(String key: adjList){
        return key;
}
```



Select any vertex **v1**

Do Breadth First Traversal from vertex **v1** to all other vertices

While doing breadth first, keep track of the distance from vertex **v1**

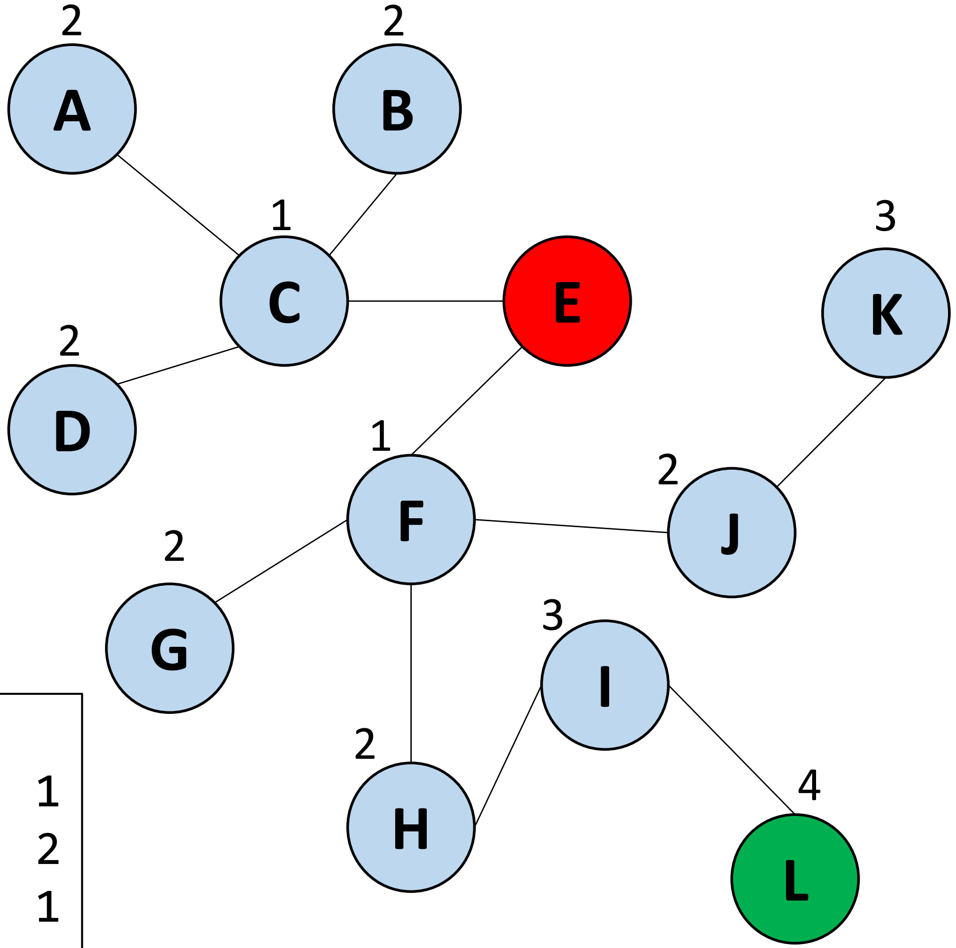



```
{
C: 1
J: 2
F: 1
D: 2
G: 2
L: 4
…

}
```

Select any vertex **v1**

Do Breadth First Traversal from vertex **v1** to all other vertices

While doing breadth first, keep track of the distance from vertex **v1** (store in some kind of data structure)



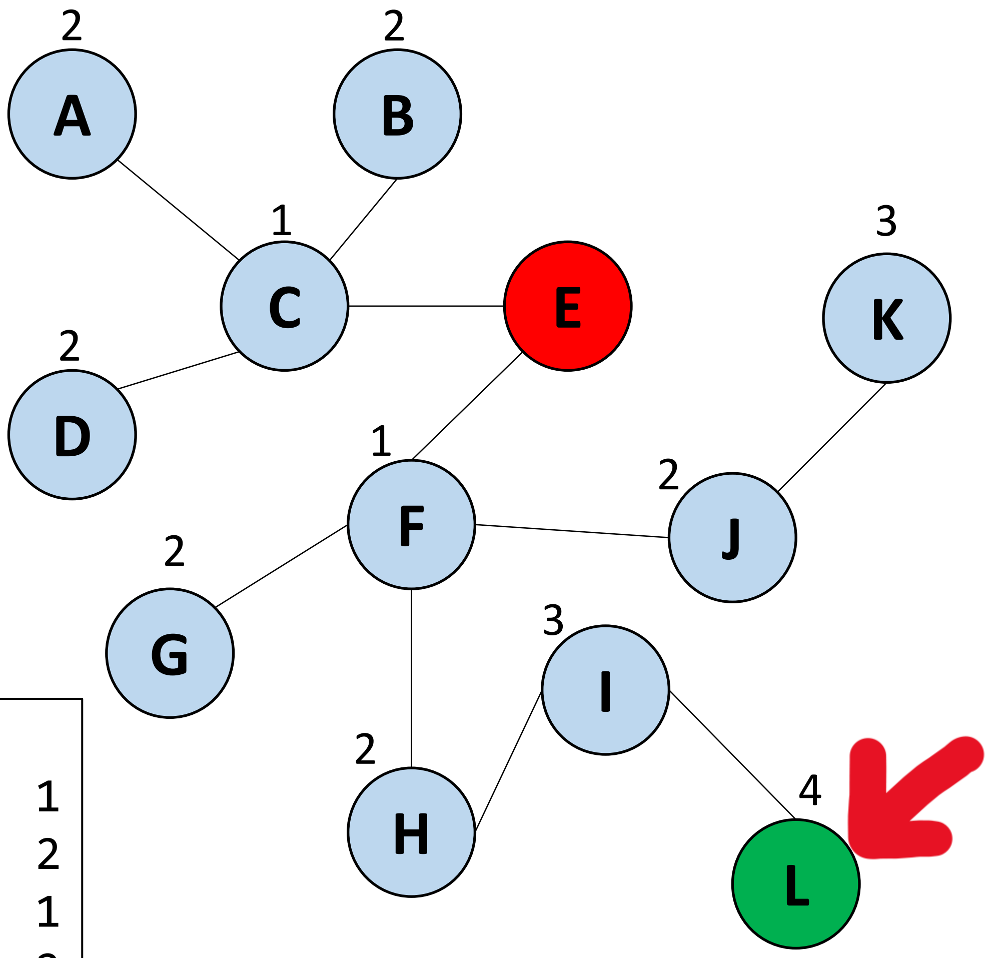

```
{
C: 1
J: 2
F: 1
D: 2
G: 2
L: 4
…

}
```

Select any vertex **v1**

Do Breadth First Traversal from vertex **v1** to all other vertices

While doing breadth first, keep track of the distance from vertex **v1** (store in some kind of data structure)

Select the node that was the furthest away, **v2**




```
{
C:  1
J:  2
F:  1
D:  2
G:  2
L:  4
…

}
```

Select any vertex **v1**

Do Breadth First Traversal from vertex **v1** to all other vertices

While doing breadth first, keep track of the distance from vertex **v1** (store in some kind of data structure)

Select the node that was the furthest away, **v2**

v2 must be an endpoint on the longest path because …

Choose an arbitrary tree node s. Assume $u, v \in V(G)$ are nodes with $d(u,v) = diam(G)$. Assume further that the algorithm finds a node x starting at s first, some node y starting at x next. wlog $d(s,u) \geq d(s,v)$. note that $d(s,x) \geq d(s,y)$ must hold, unless the algorithm's first stage wouldn't end up at x. We will see that $d(x,y) = d(u,v)$.

The most general configuration of all nodes involved can be seen in the following pseudo-graphics (possibly $s = z_{uv}$ or $s = z_{xy}$ or both):

```
(u)                                        (x)
   \                                       /
    \                                     /
     \                                   /
      ( z_uv )---------( s )----------( z_xy )
     /                                   \
    /                                     \
   /                                       \
(v)                                        (y)
```

we know that:

1. $d(z_{uv}, y) \leq d(z_{uv}, v)$. otherwise $d(u,v) < diam(G)$ contradicting the assumption.
2. $d(z_{uv}, x) \leq d(z_{uv}, u)$. otherwise $d(u,v) < diam(G)$ contradicting the assumption.
3. $d(s, z_{xy}) + d(z_{xy}, x) \geq d(s, z_{uv}) + d(z_{uv}, u)$, otherwise stage 1 of the algorithm wouldn't have stopped at x.
4. $d(z_{xy}, y) \geq d(v, z_{uv}) + d(z_{uv}, z_{xy})$, otherwise stage 2 of the algorithm wouldn't have stopped at y.

1) and 2) imply

$$\begin{aligned} d(u,v) &= d(z_{uv}, v) + d(z_{uv}, u) \\ &\geq d(z_{uv}, x) + d(z_{uv}, y) = d(x,y) + 2\, d(z_{uv}, z_{xy}) \\ &\geq d(x,y) \end{aligned}$$

3) and 4) imply

$$\begin{aligned} & d(z_{xy}, y) + d(s, z_{xy}) + d(z_{xy}, x) \\ &\geq d(s, z_{uv}) + d(z_{uv}, u) + d(v, z_{uv}) + d(z_{uv}, z_{xy}) \end{aligned}$$

equivalent to

$$\begin{aligned} d(x,y) &= d(z_{xy}, y) + d(z_{xy}, x) \\ &\geq 2 * d(s, z_{uv}) + d(v, z_{uv}) + d(u, z_{uv}) \\ &\geq d(u,v) \end{aligned}$$

therefore $d(u,v) = d(x,y)$.

analogue proofs hold for the alternative configurations

```
                 (u)                        (x)
                    \                       /
                     \                     /
( s )---------( z_uv )----------( z_xy )
                     /                     \
                    /                       \
                 (v)                        (y)
```

and

```
                 (x)          (u)
                    /            \
                   /              \
( s )---------( z_xy )----------( z_uv )
                   \              /
                    \            /
                 (y)          (v)
```

these are all possible configurations. in particular, $x \notin path(s,u), x \notin path(s,v)$ due to the result of stage 1 of the algorithm and $y \notin path(x,u), y \notin path(x,v)$ due to stage 2.

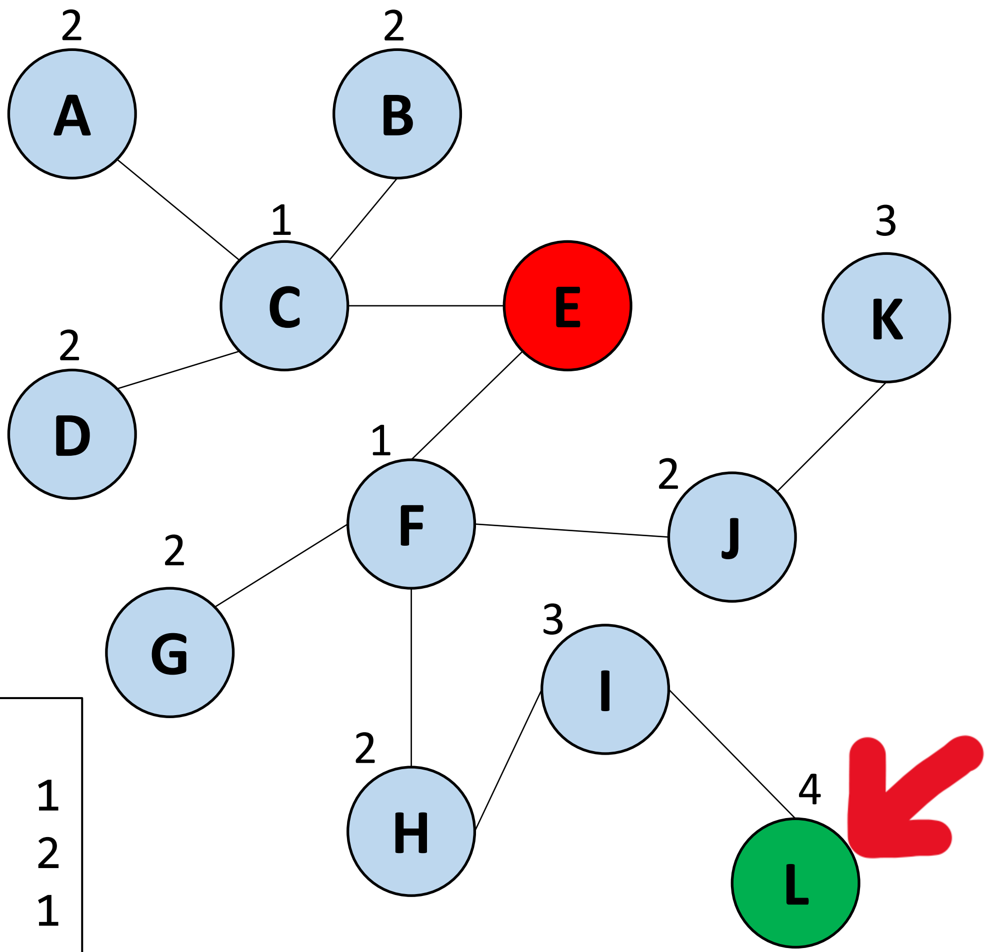
```
{
C: 1
J: 2
F: 1
D: 2
G: 2
L: 4
…

}
```

som

ect t

ay, **v2**

v2 mu … st path because reese told you so



Select any vertex **v1**

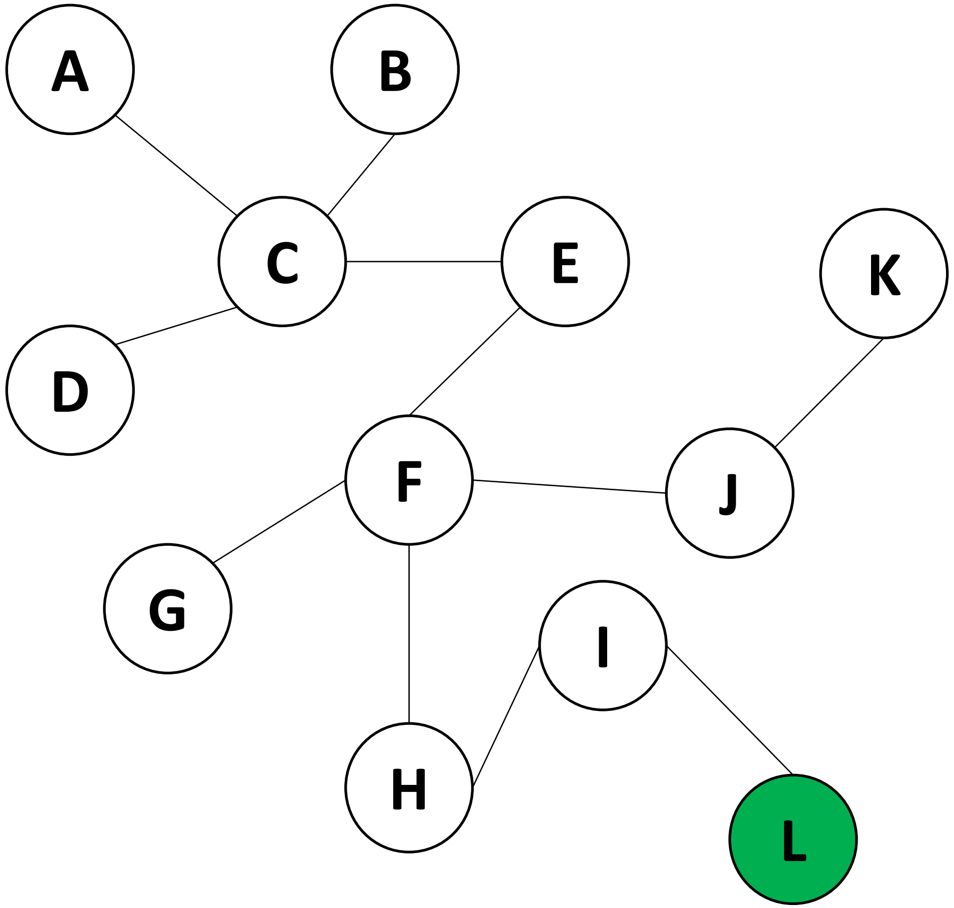
Do Breadth First Traversal from vertex **v1** to all other vertices

While doing breadth first, keep track of the distance from vertex **v1** (store in some kind of data structure)

Select the node that was the furthest away, **v2**

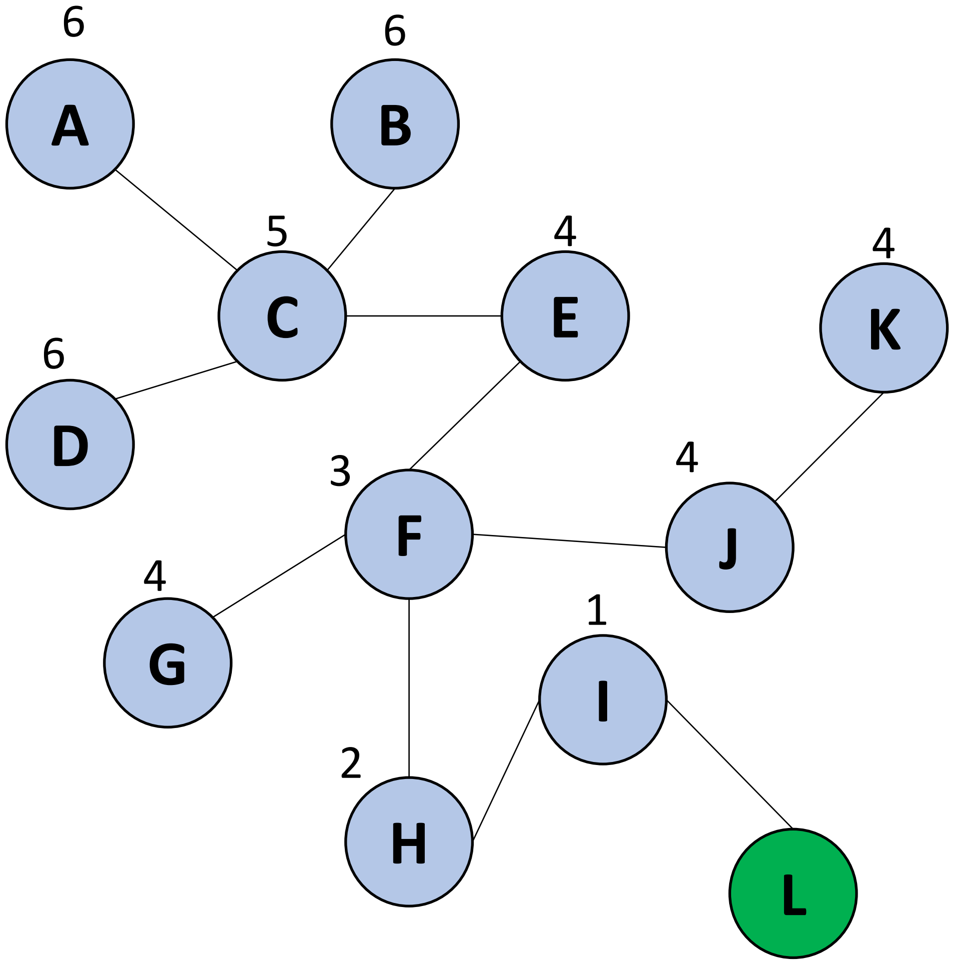
```
{
C: 1
J: 2
F: 1
D: 2
G: 2
L: 4
…

}
```

v2 must be an endpoint on the longest path because otherwise BFS would have found a deeper vertex



Do breadth first search again, but now starting from **v2**

Keep track of distances from **v2**



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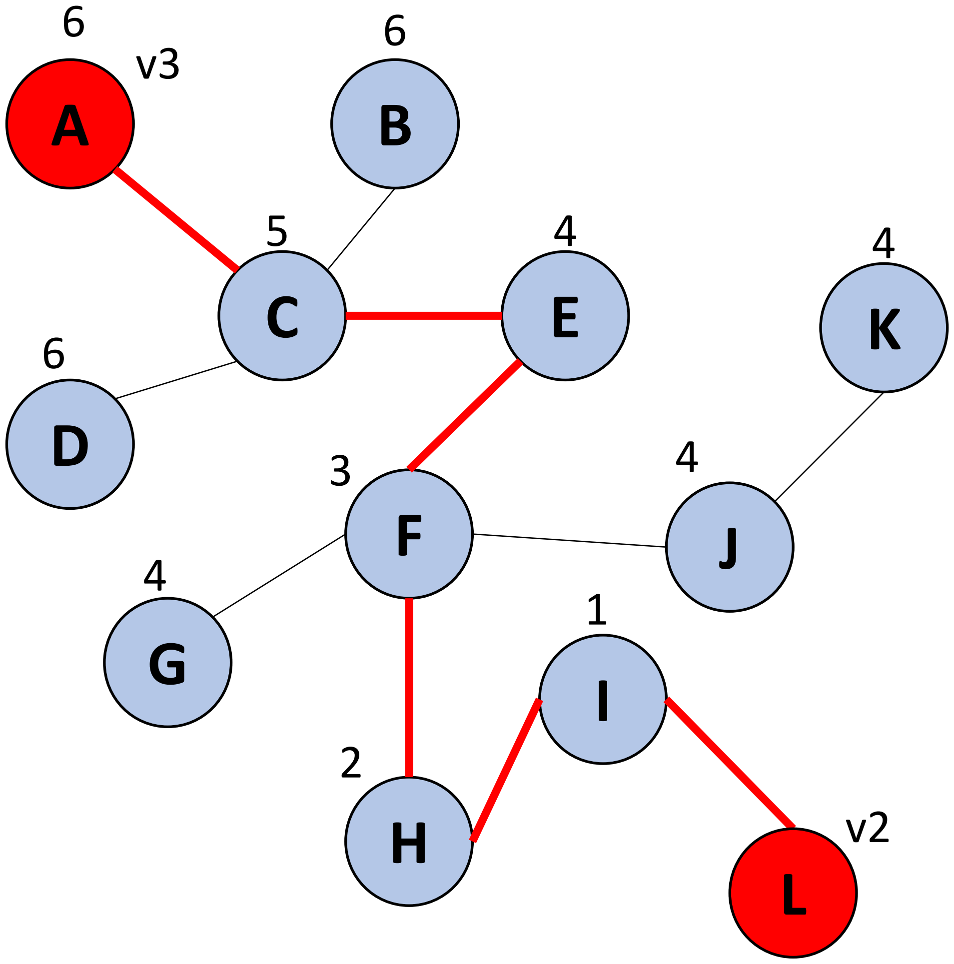
Keep track of distances from **v2**



“Double pass BFS”

Will only work on an acyclic graph

Do breadth first search again, but now starting from **v2**

Keep track of distances from **v2**

Select the vertex with the longest distance, **v3**

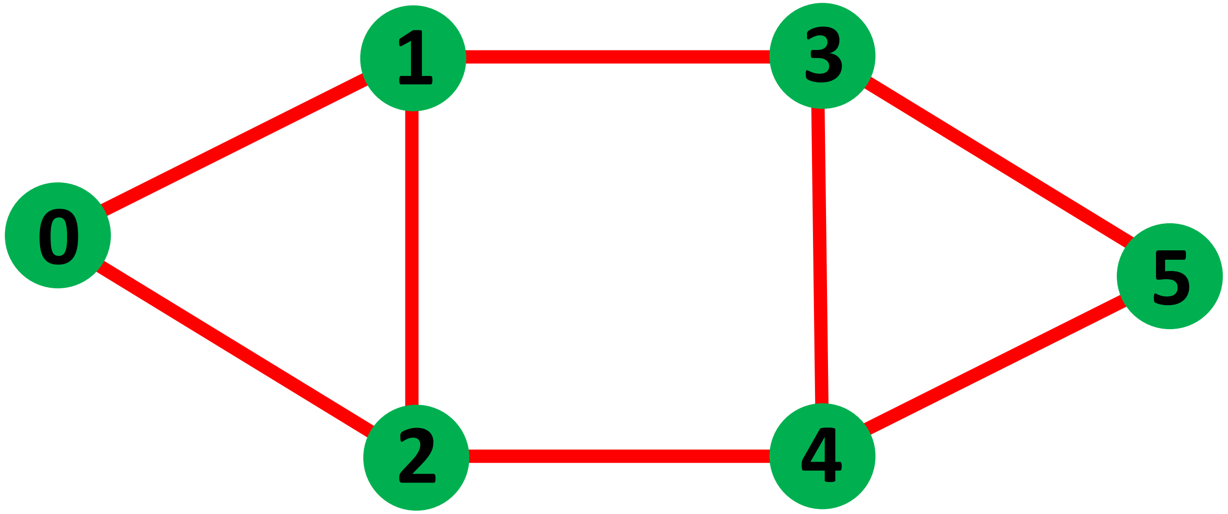
(We have a tie for longest path, so just select one of them)

Breadth First will visit every node, and will always find the farthest away node from some starting point

[V2, V3] is the longest path

Graphs

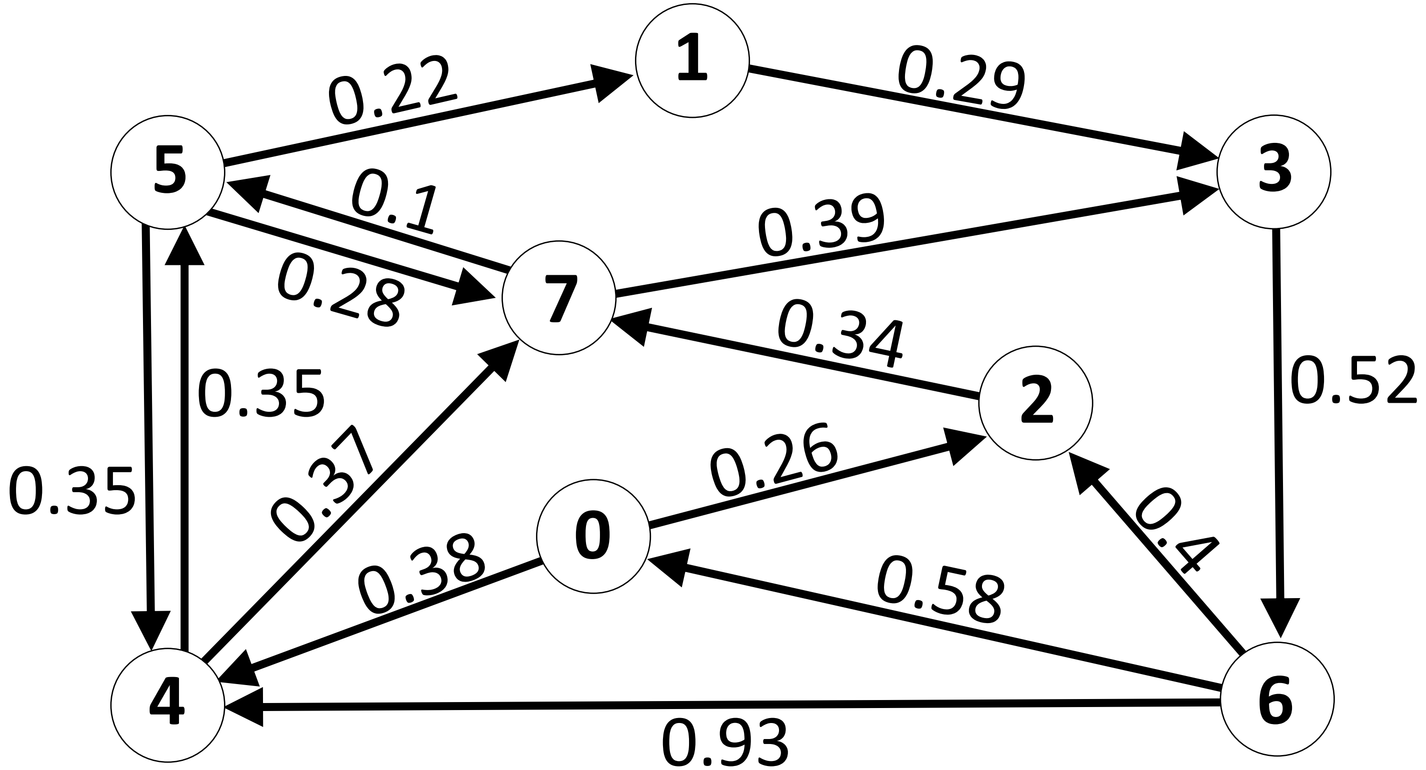
$G = ({\color{green}V}, {\color{red}E})$



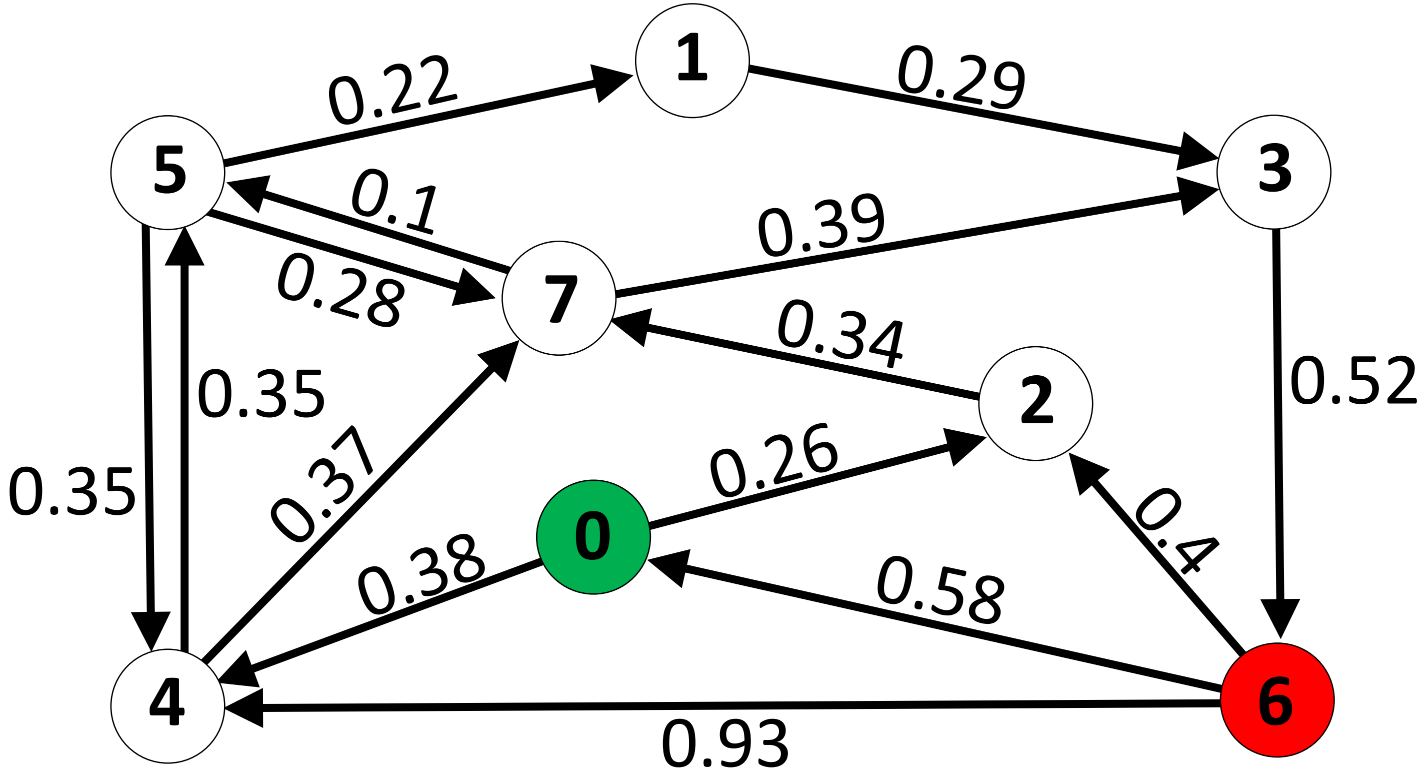
Adjacency List

0	→	{1,2}
1	→	{0,2,3}
2	→	{0,1,4}
3	→	{1,4,5}
4	→	{2,3,5}
5	→	{3,4}

Shortest Path



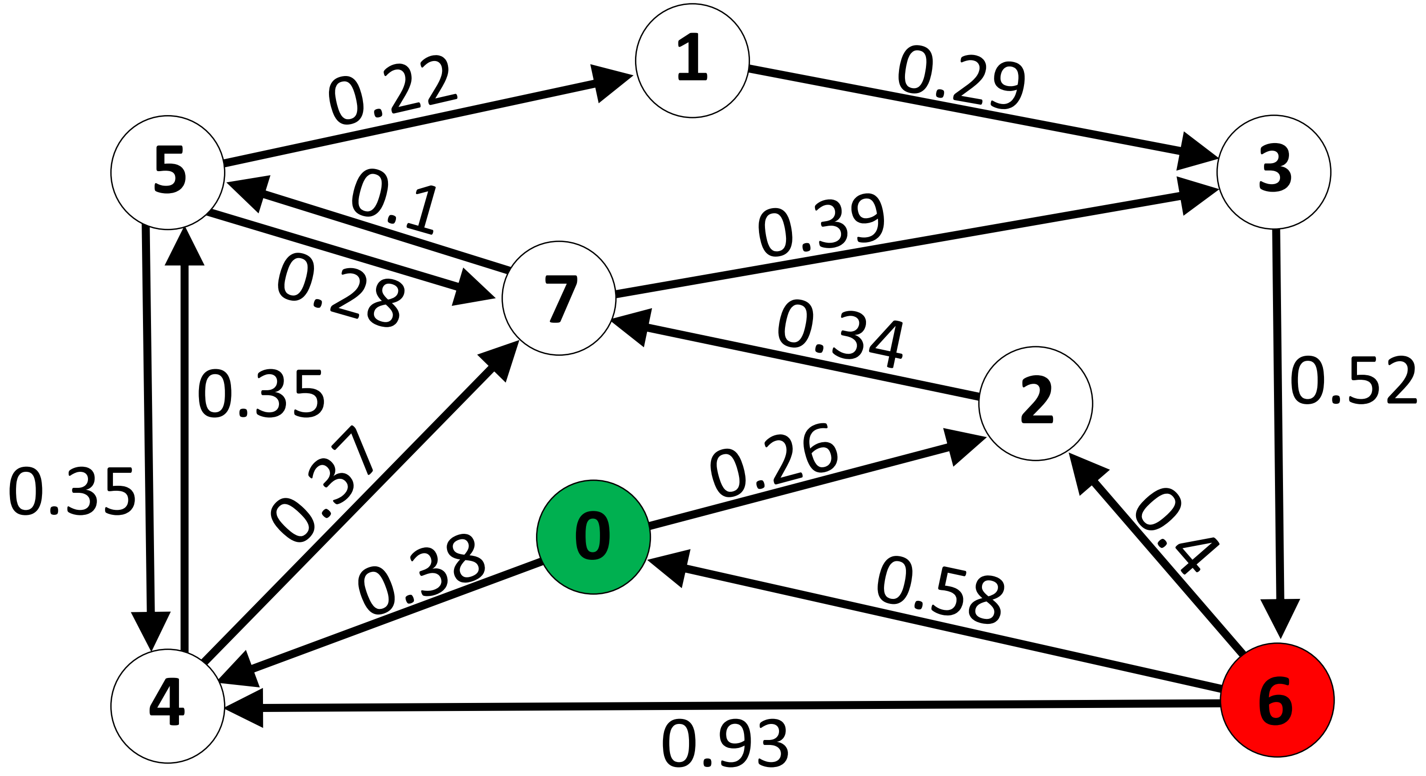
Shortest Path



Path with the smallest sum of edge weights.

What is the shortest path between **vertex 0** and **vertex 6**?

Shortest Path

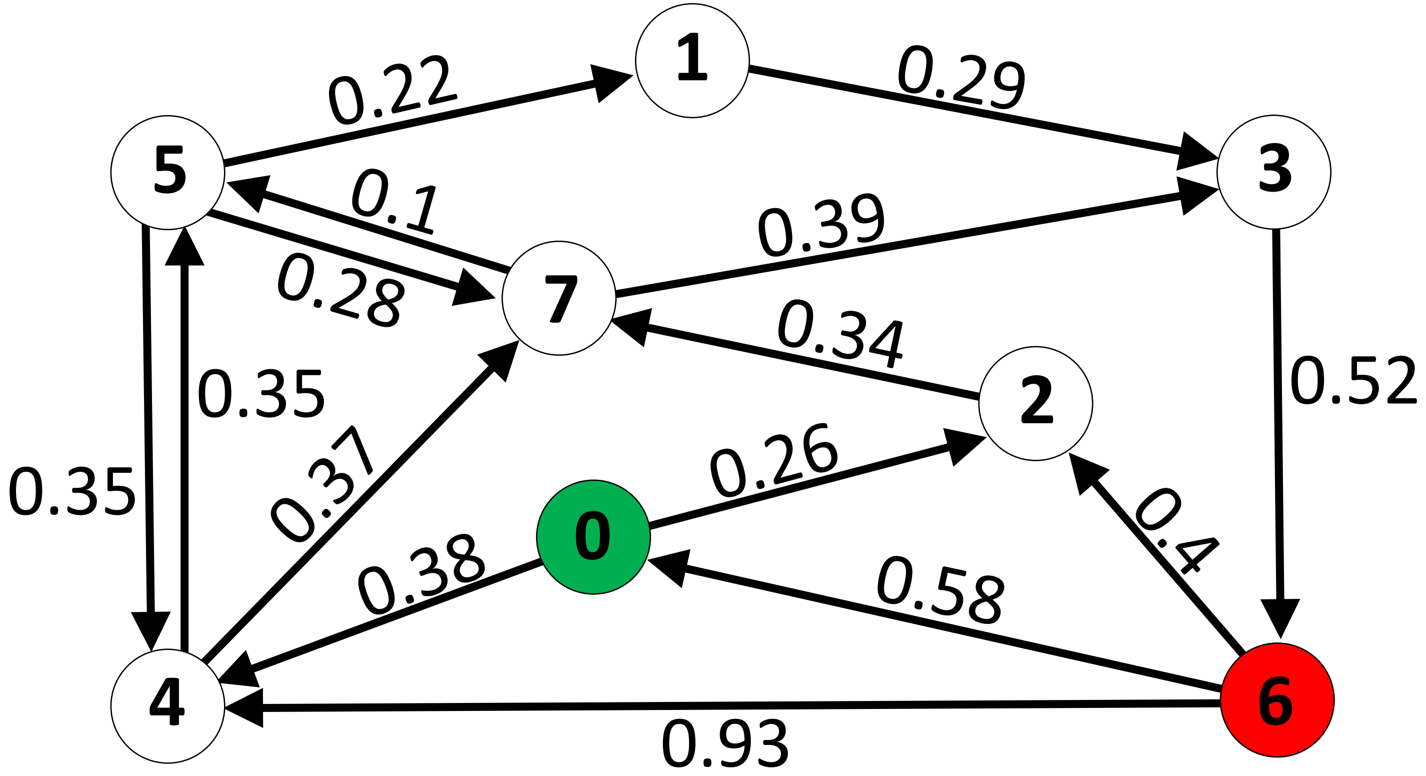


Assumptions:

- Graph is directed.
- Graph is edge-weighted.
- Edge weights are non-negative.
- Graph need not be simple (though our example will be).

What is the shortest path between **vertex 0** and **vertex 6**?

Shortest Path



Assumptions:

- Graph is directed.
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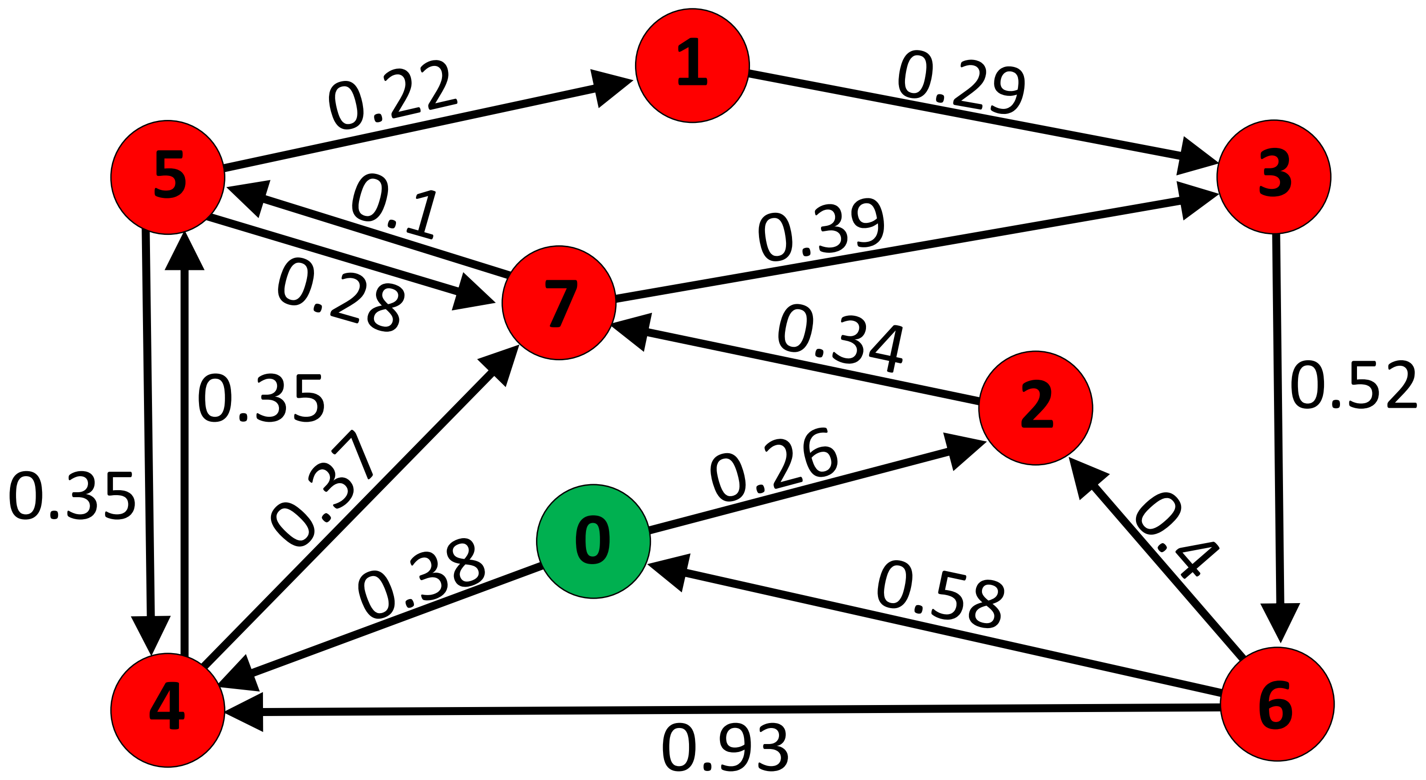
Ideas?

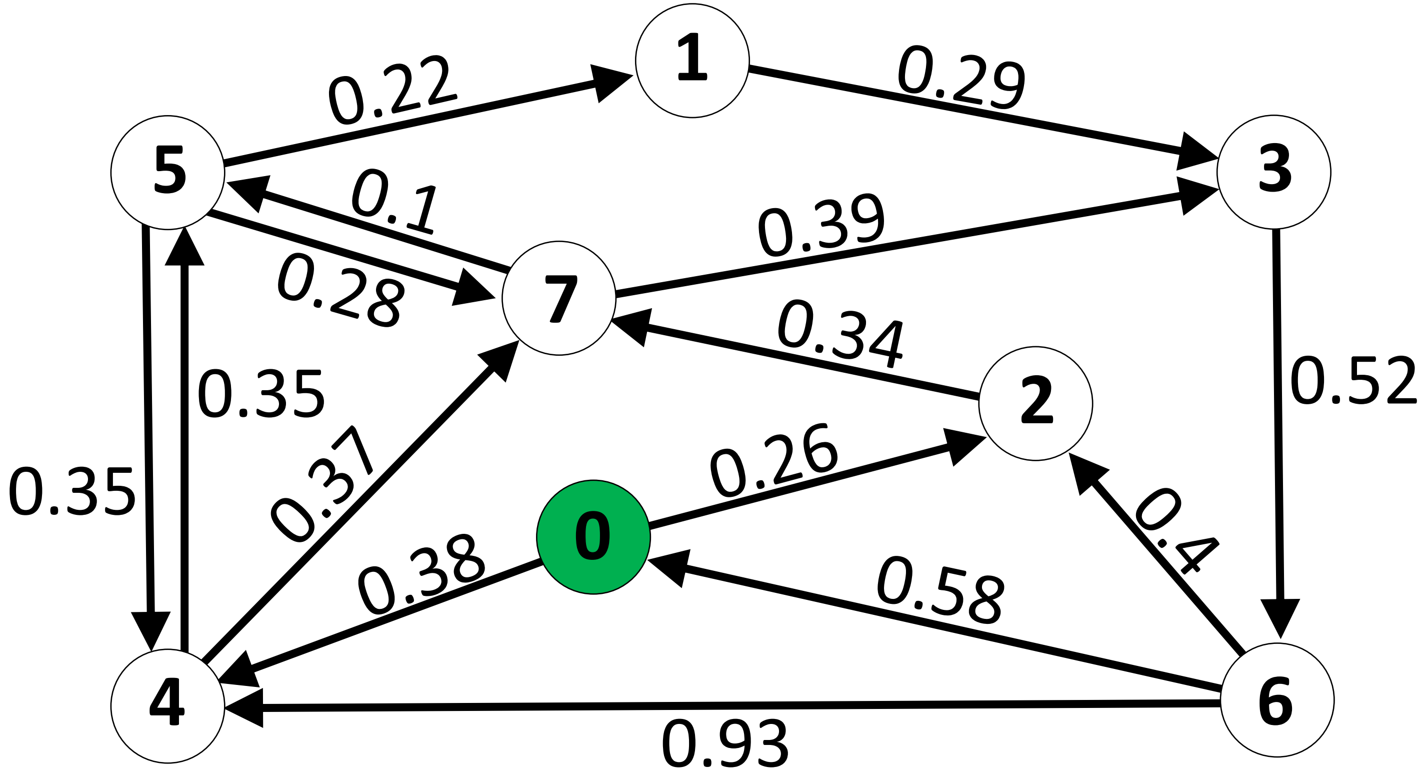
What is the shortest path between **vertex 0** and **vertex 6**?

Shortest Path



We are going to find the shortest path between vertex 0 and every other vertex, flooding out from 0.

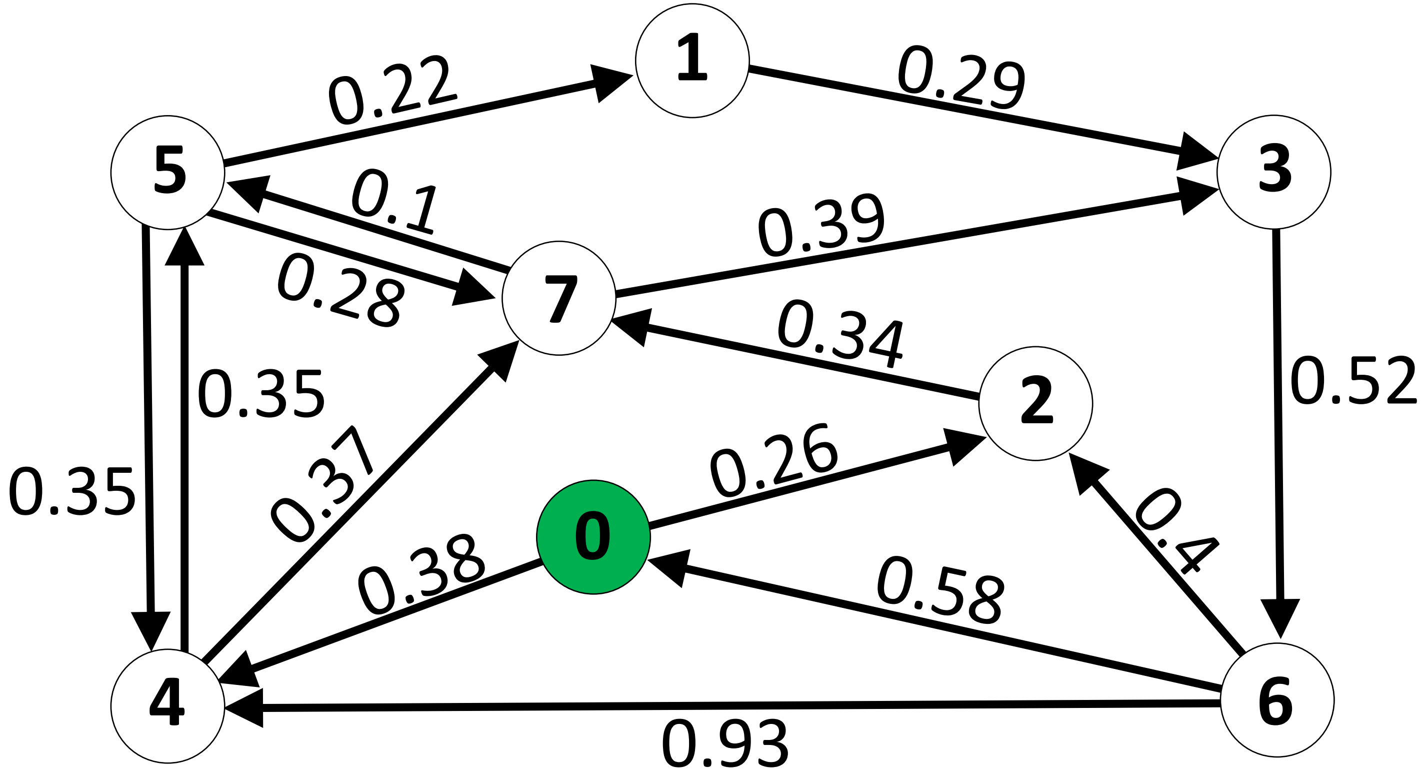
Shortest Path



Distance from 0

0	?
1	?
2	?
3	?
4	?
5	?
6	?
7	?

Shortest Path



Distance from 0

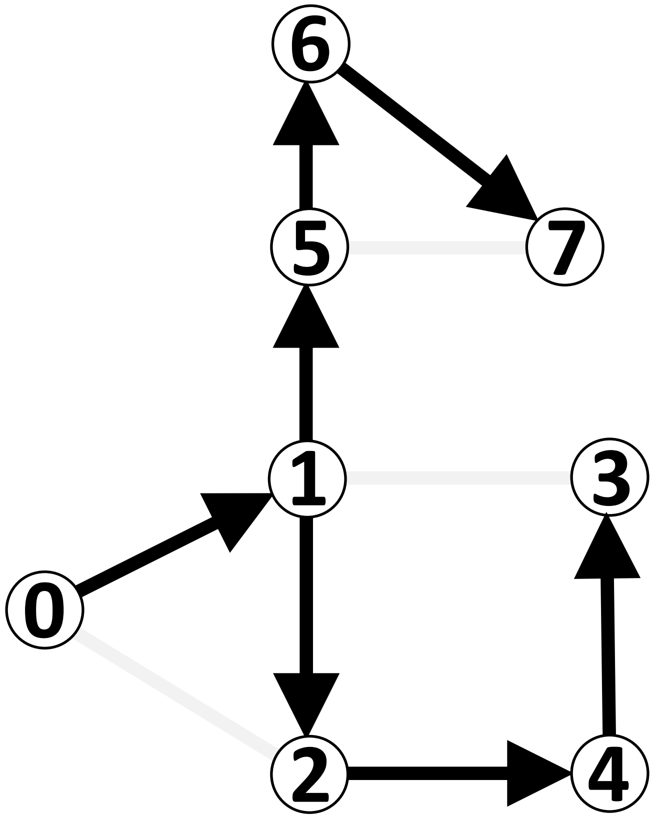
0	0
1	∞
2	∞
3	∞
4	∞
5	∞
6	∞
7	∞

How can we keep track of routes?

Graphs - Paths

`int[] previousVertex`

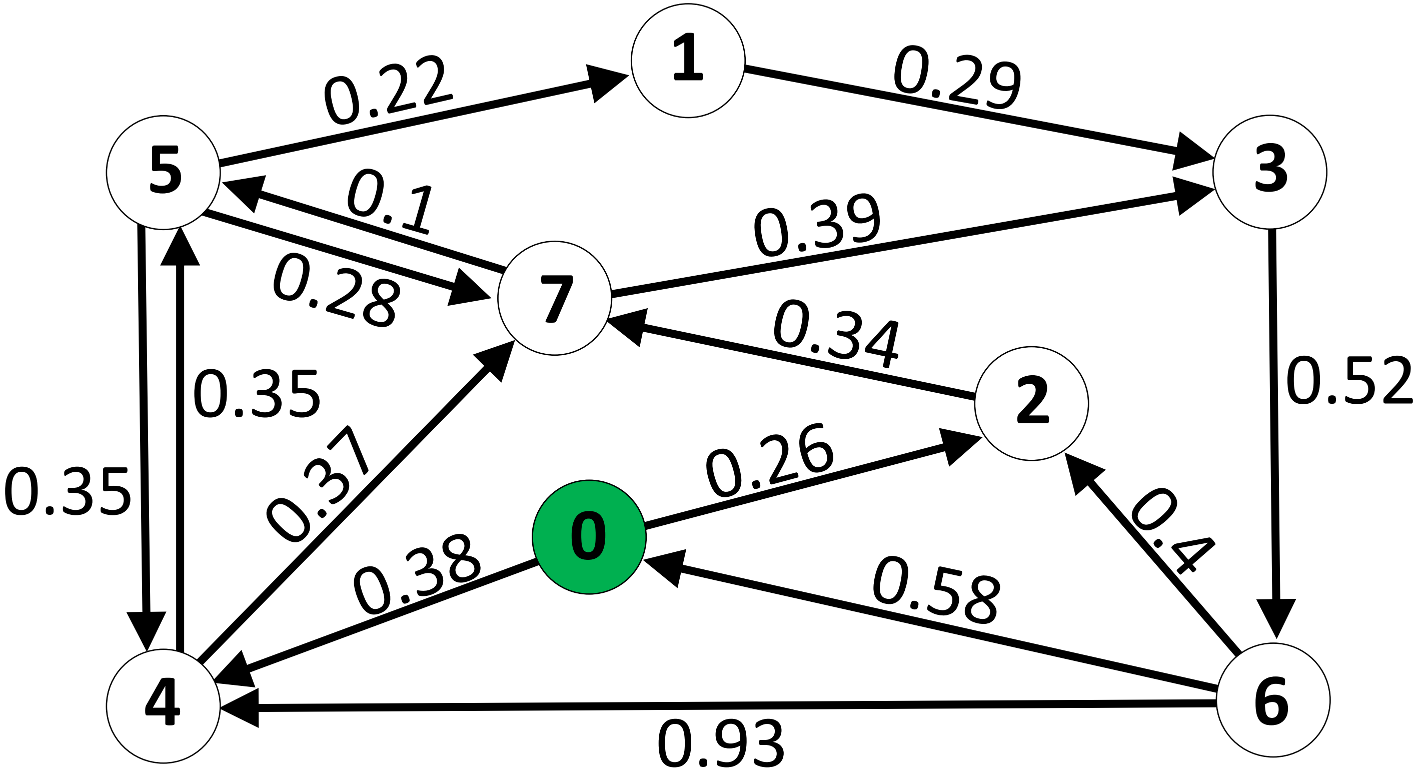
0	-
1	0
2	1
3	4
4	2
5	1
6	5
7	6



How do we determine the path from 0 to 6?

Start at vertex 6. Find its previous vertex. Find its previous vertex... until we get back to the start (0).

Shortest Path

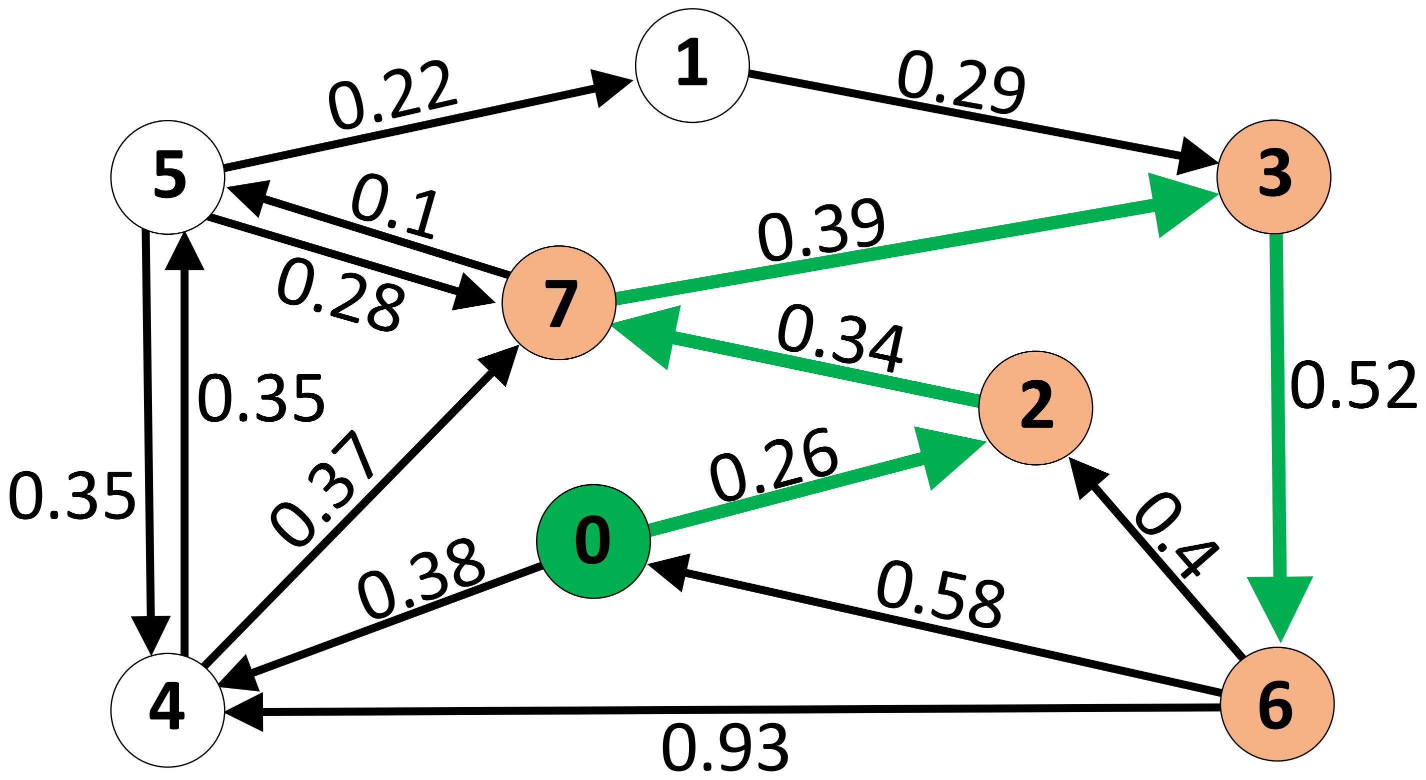


	Distance from 0
0	0
1	∞
2	∞
3	∞
4	∞
5	∞
6	∞
7	∞

	Previous vertex
0	-
1	
2	
3	
4	
5	
6	
7	

How can we keep track of routes?

Shortest Path

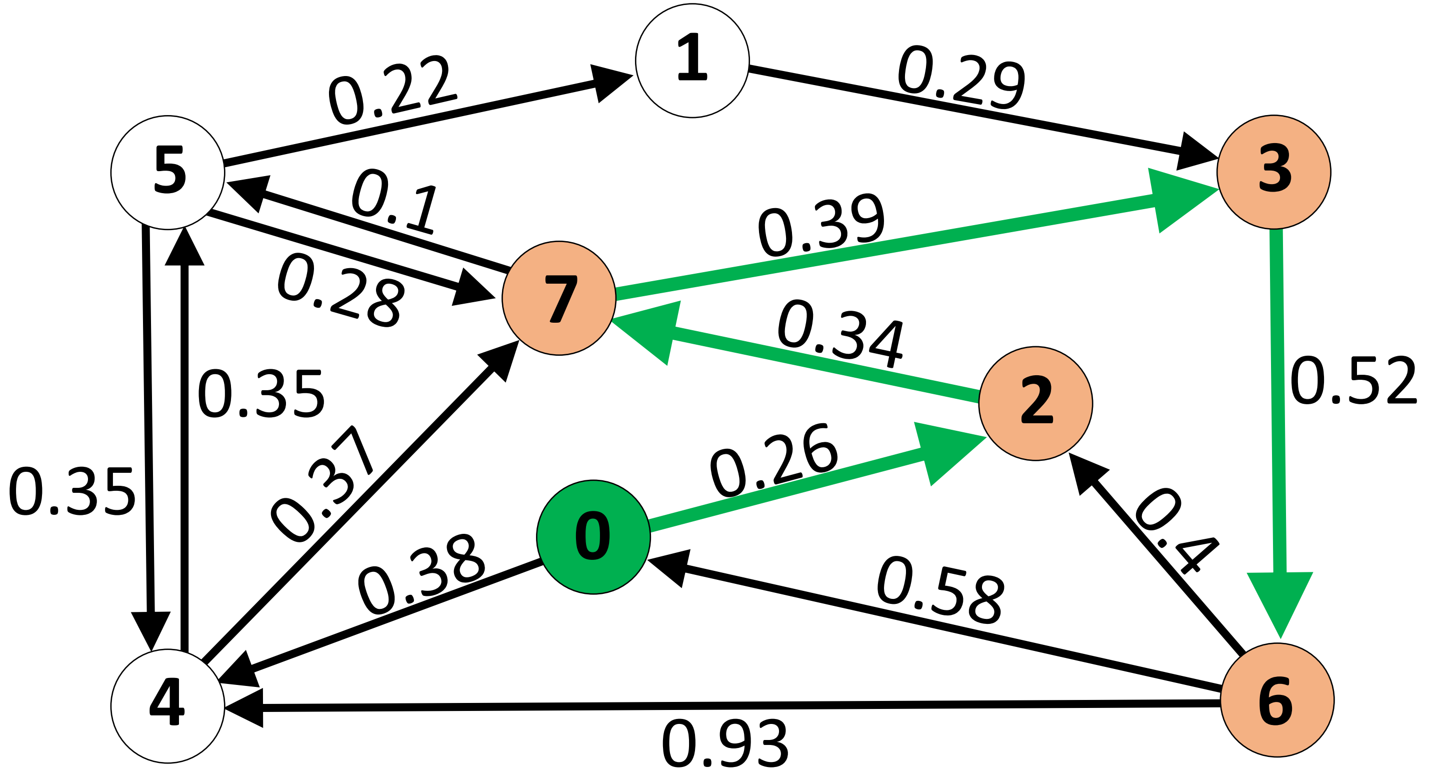


	Distance from 0
0	0
1	∞
2	0.26
3	0.99
4	∞
5	∞
6	1.51
7	0.60

	Previous vertex
0	-
1	
2	0
3	7
4	
5	
6	3
7	2

How can we keep track of routes?

Shortest Path



Distance from 0

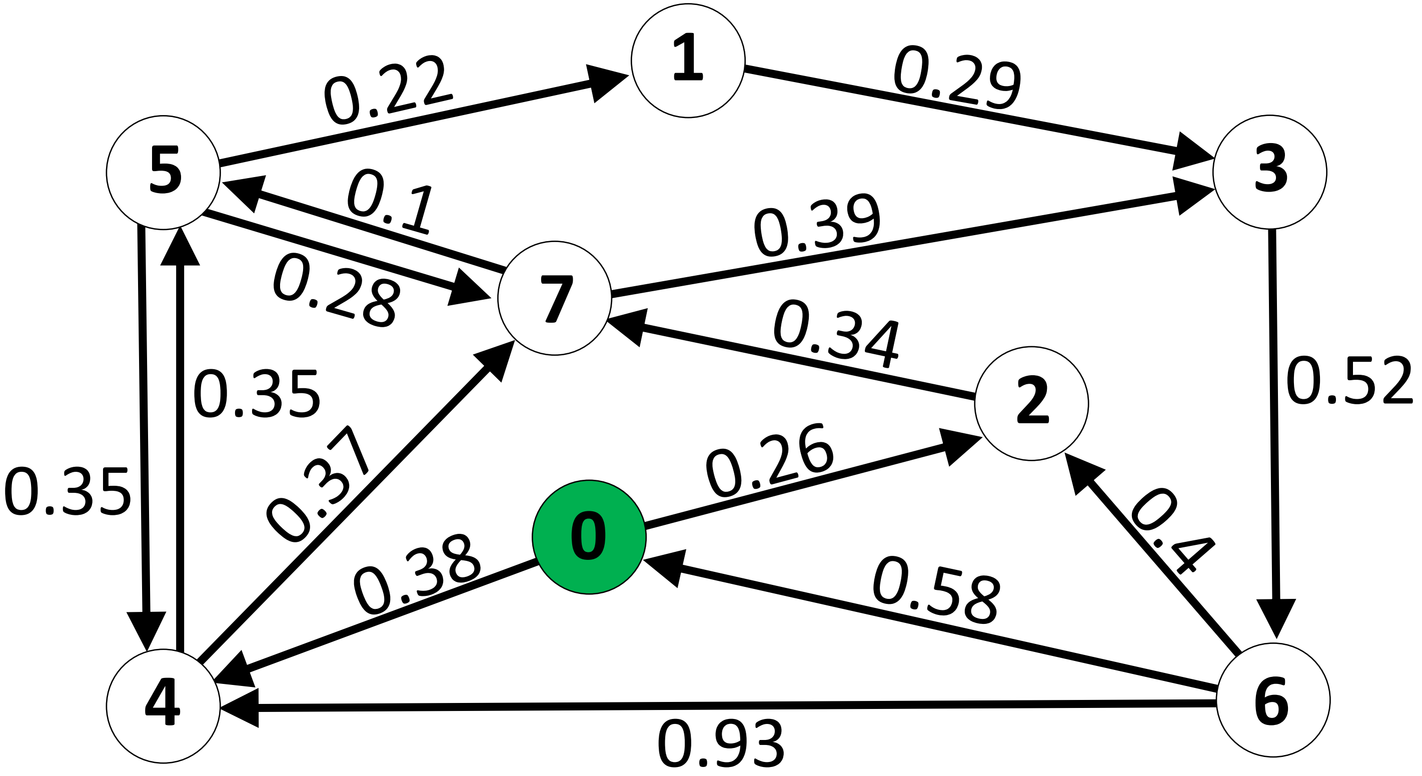
0	0
1	∞
2	∞
3	∞
4	∞
5	∞
6	∞
7	∞

Previous vertex

0	-
1	
2	0
3	7
4	
5	
6	3
7	2

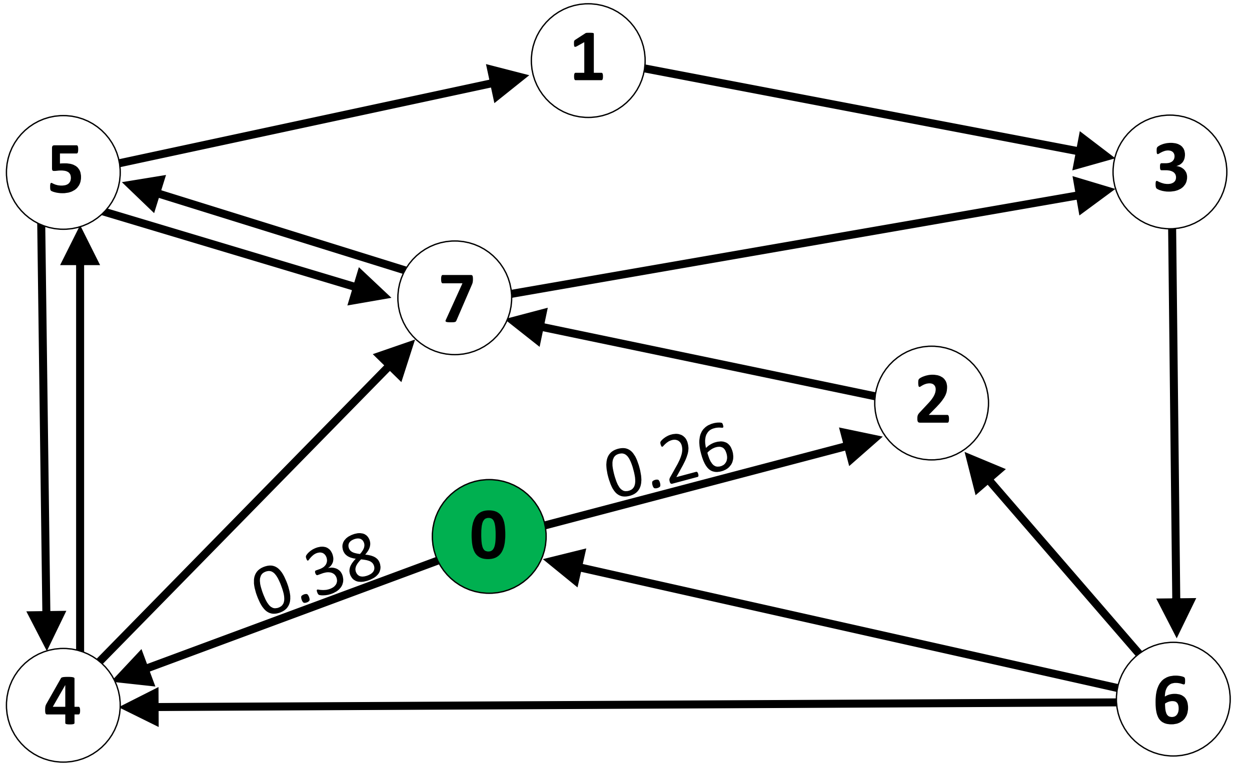
If this is the shortest path from 0 to 6, what can we say about the shortest path from 0 to 3?

Shortest Path



Vertex	Distance from 0	Previous vertex
0	0	-
1	∞	
2	∞	
3	∞	
4	∞	
5	∞	
6	∞	
7	∞	

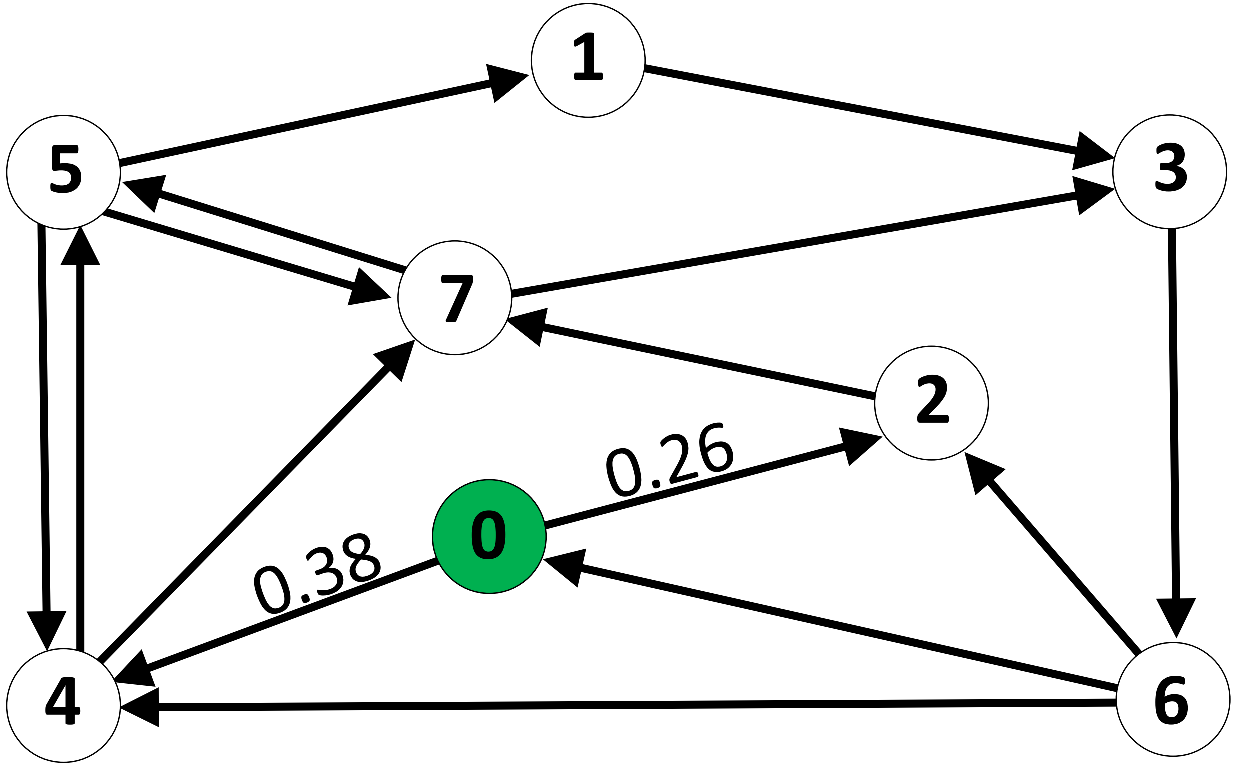
Shortest Path



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	∞	
3	∞	
4	∞	
5	∞	
6	∞	
7	∞	

Claim: There cannot possibly be a shorter path from 0 to 2 than the edge from 0 to 2 because…?

Shortest Path



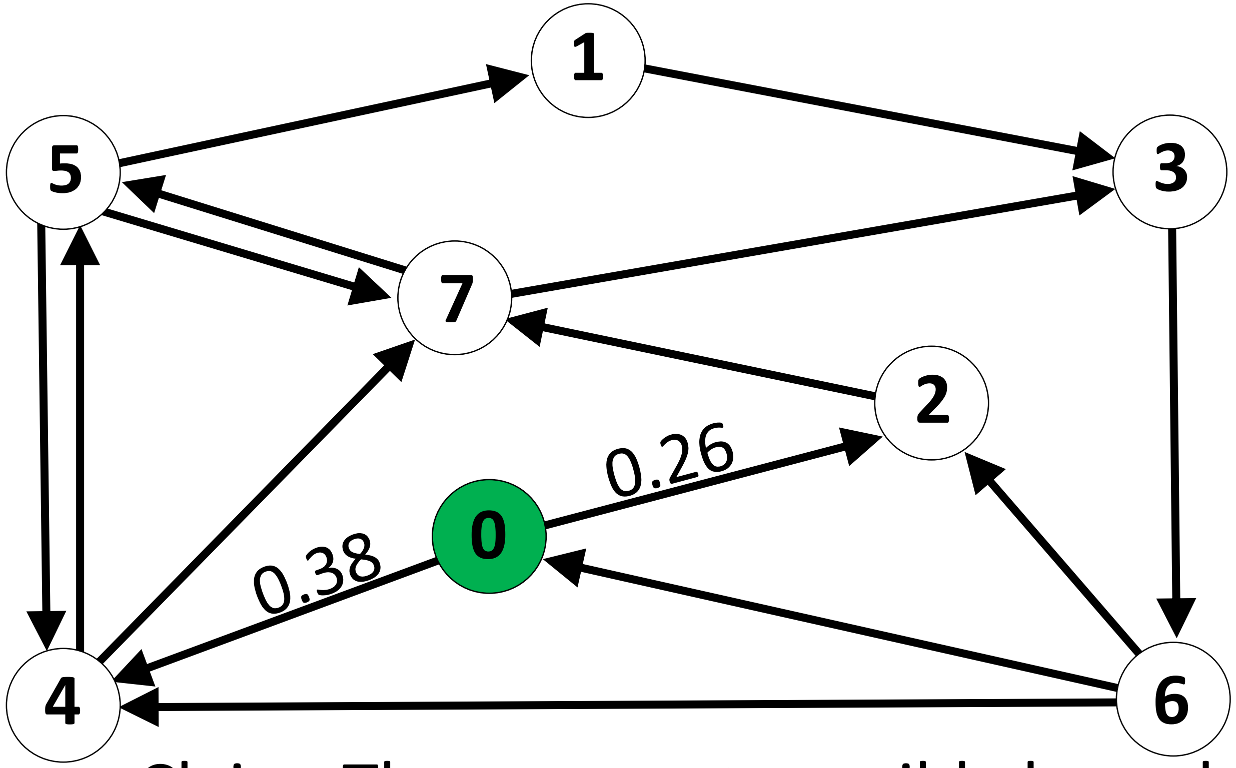
	Distance from 0	Previous vertex
0	0	-
1	∞	
2	∞	
3	∞	
4	∞	
5	∞	
6	∞	
7	∞	

Claim: There cannot possibly be a shorter path from 0 to 2 than the edge from 0 to 2 because non-negative edge weights mean every other path is at least 0.38 or 0.26.

Shortest Path

	Distance from 0
0	0
1	∞
7	∞

	Previous vertex
0	-
1	
7	

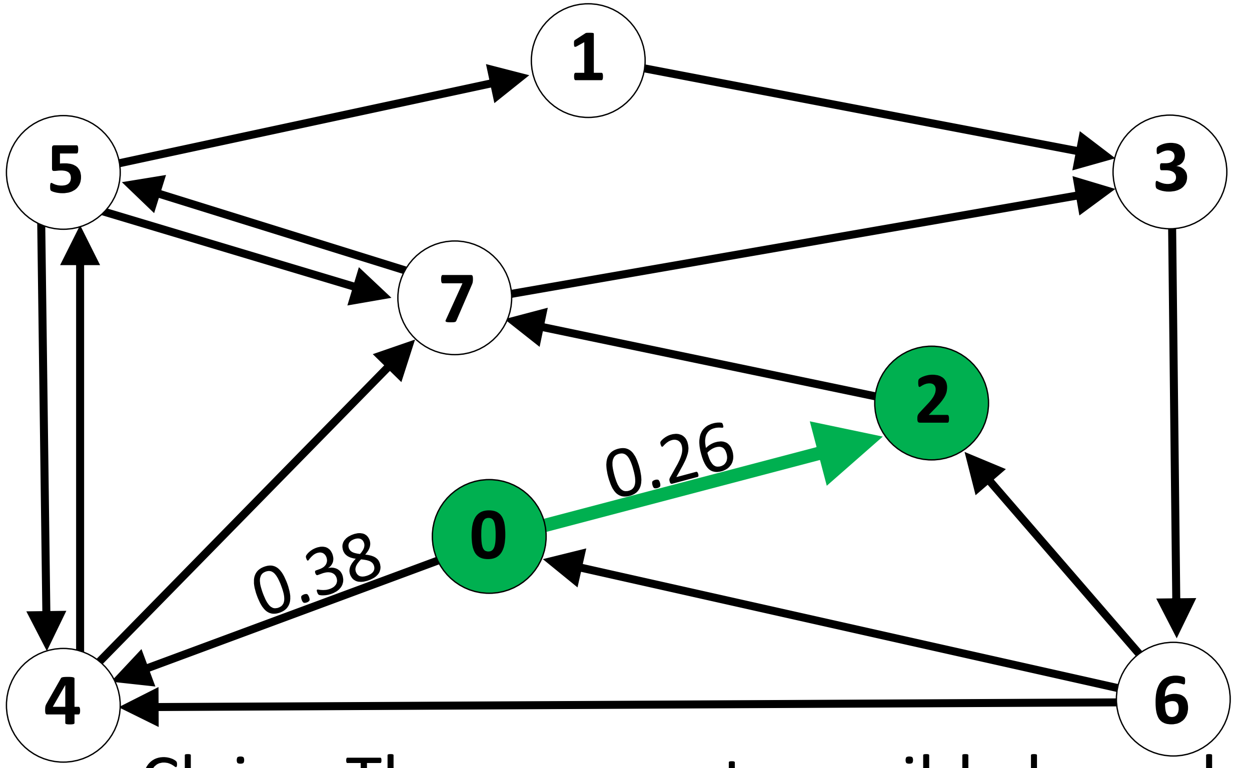


Can we say the same thing about the edge from 0 to 4?

I.e., Could there be a shorter path from 0 to 4 other than the edge from 0 to 4?

Claim: There cannot possibly be a shorter path from 0 to 2 than the edge from 0 to 2 because non-negative edge weights mean every other path is at least 0.38 or 0.26.

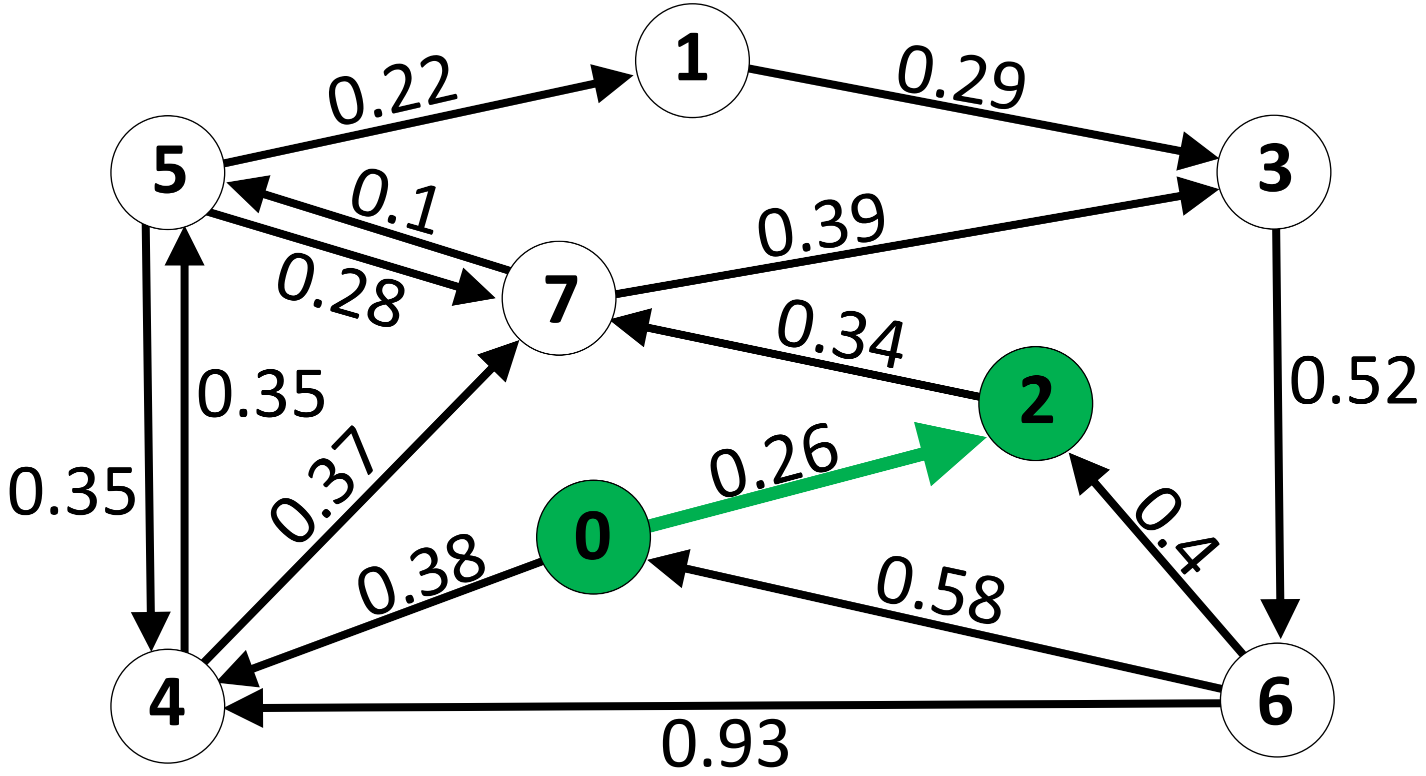
Shortest Path



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	∞	
5	∞	
6	∞	
7	∞	

Claim: There cannot possibly be a shorter path from 0 to 2 than the edge from 0 to 2 because non-negative edge weights mean every other path is at least 0.38 or 0.26.

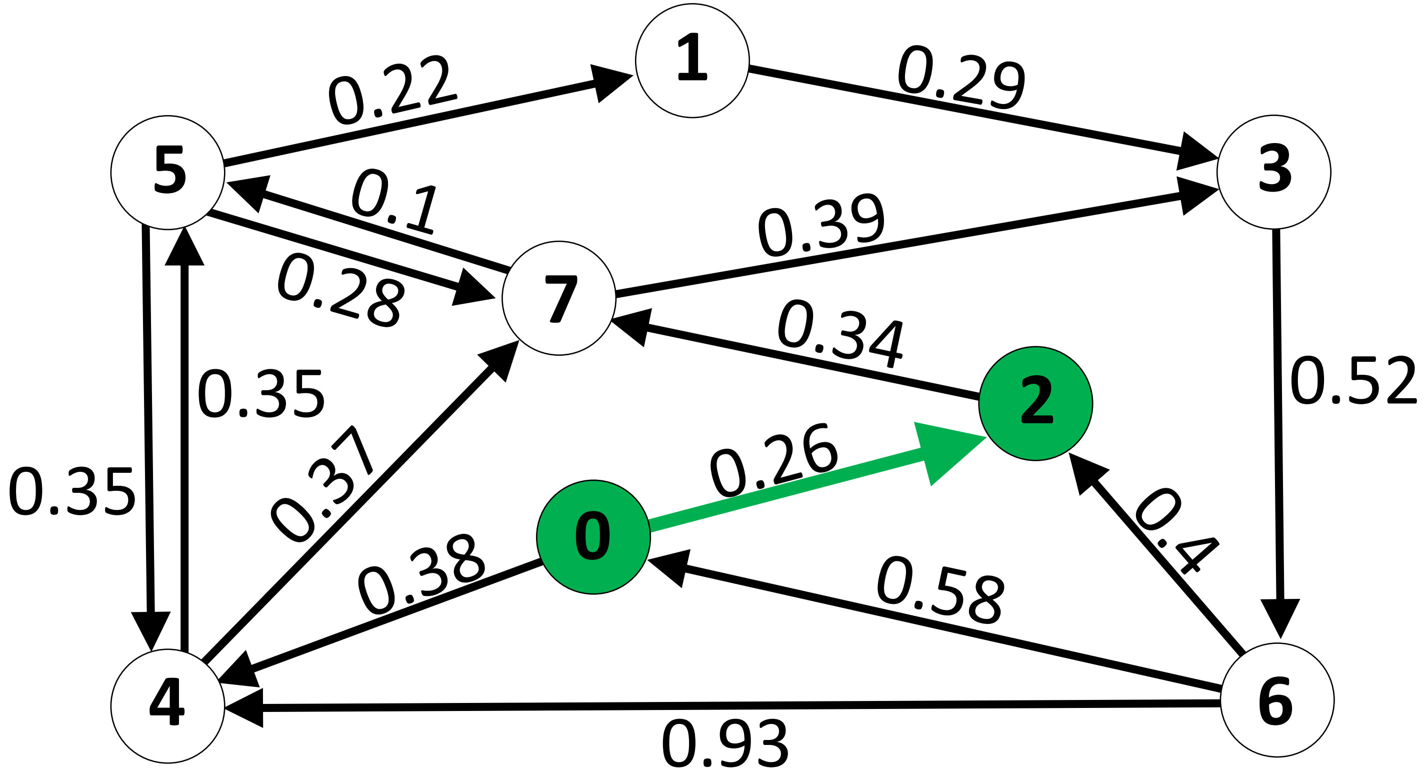
Shortest Path



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	∞	
5	∞	
6	∞	
7	∞	

We need some process for progressing through the graph.

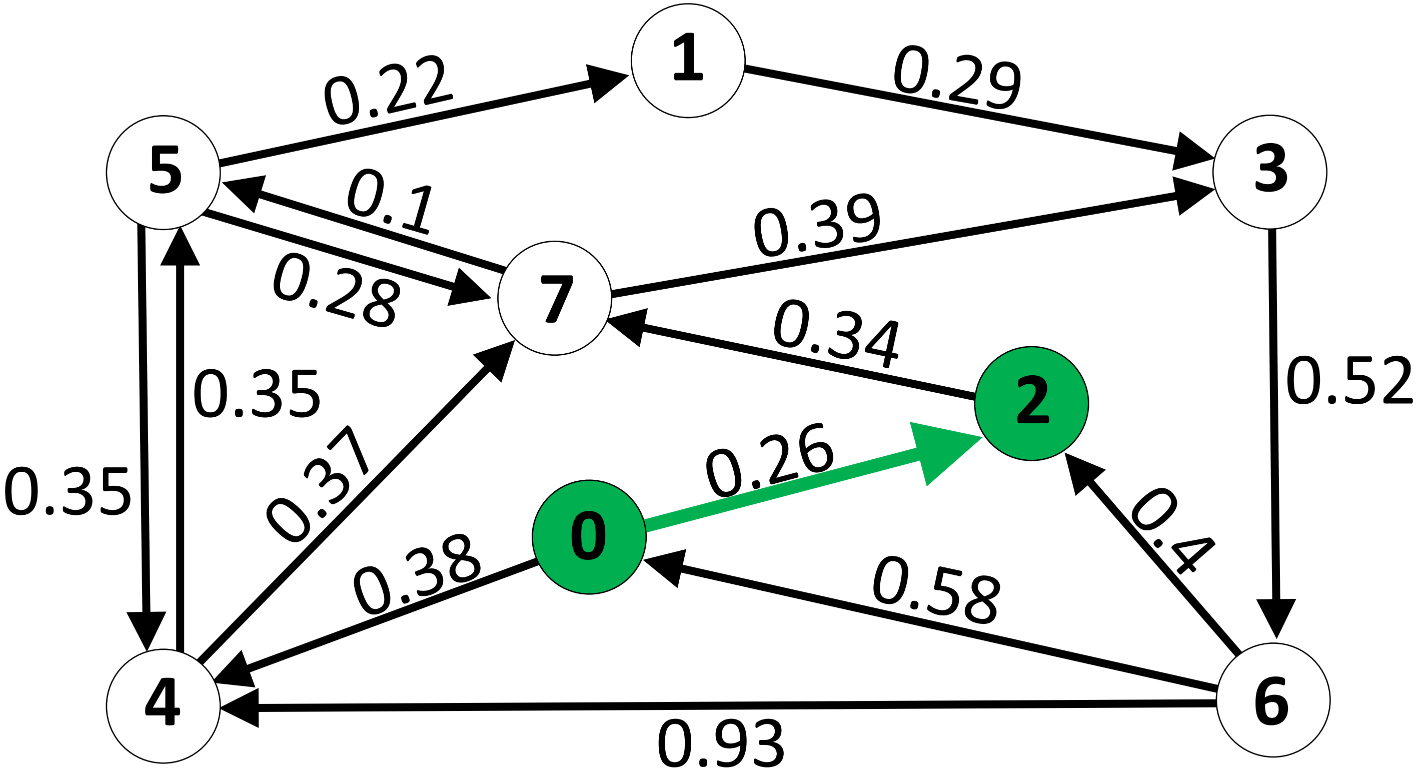
Shortest Path



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	∞	
5	∞	
6	∞	
7	∞	

We need some process for progressing through the graph.
What if we prioritized neighbors based on path (not edge) distance?

Shortest Path



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	∞	
5	∞	
6	∞	
7	∞	

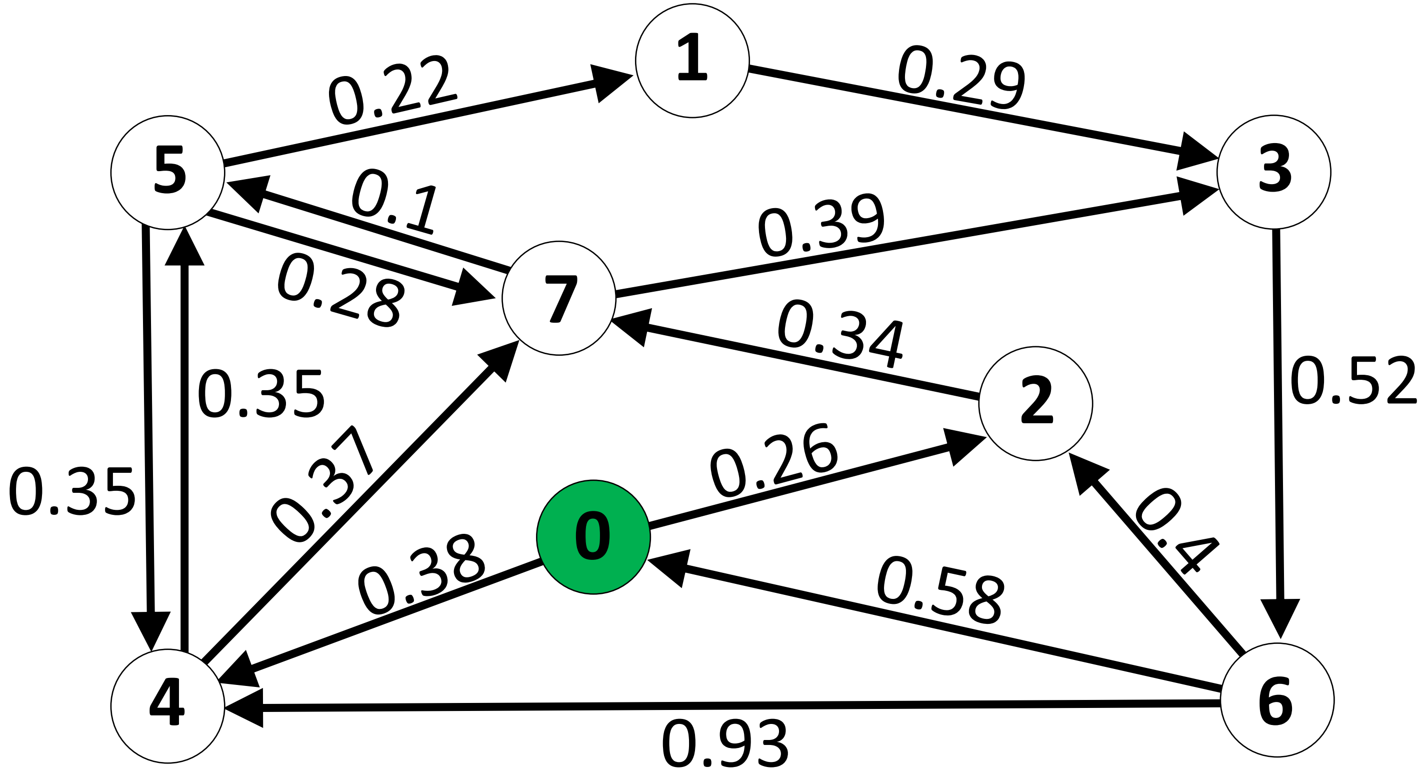
Priority queue

We need some process for progressing through the graph.

What if we prioritized neighbors based on path (not edge) distance?

vertex (distance)

Shortest Path



Distance from 0

0	0
1	∞
2	∞
3	∞
4	∞
5	∞
6	∞
7	∞

Previous vertex

0	-
1	
2	
3	
4	
5	
6	
7	

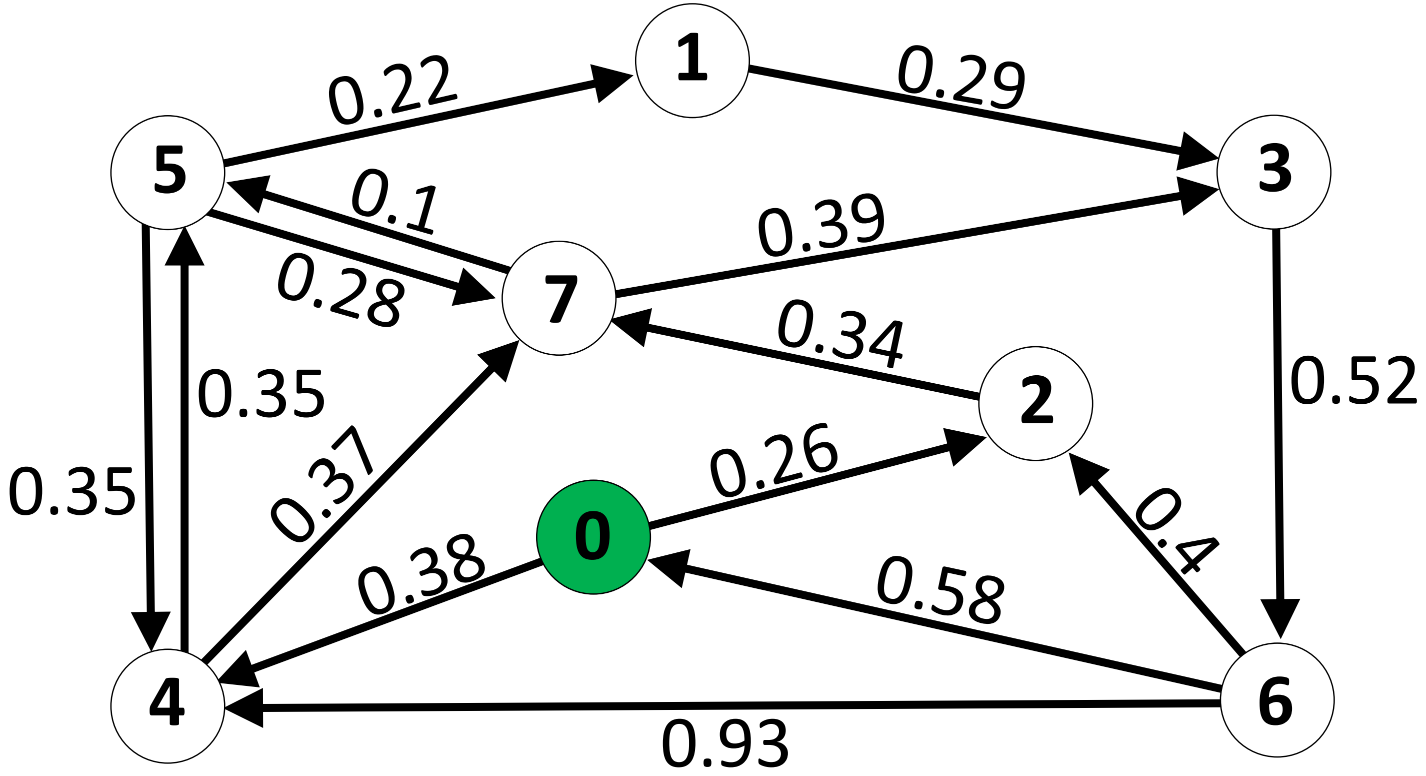
Priority queue

We need some process for progressing through the graph.

What if we prioritized neighbors based on path (not edge) distance?

vertex (distance)

Shortest Path

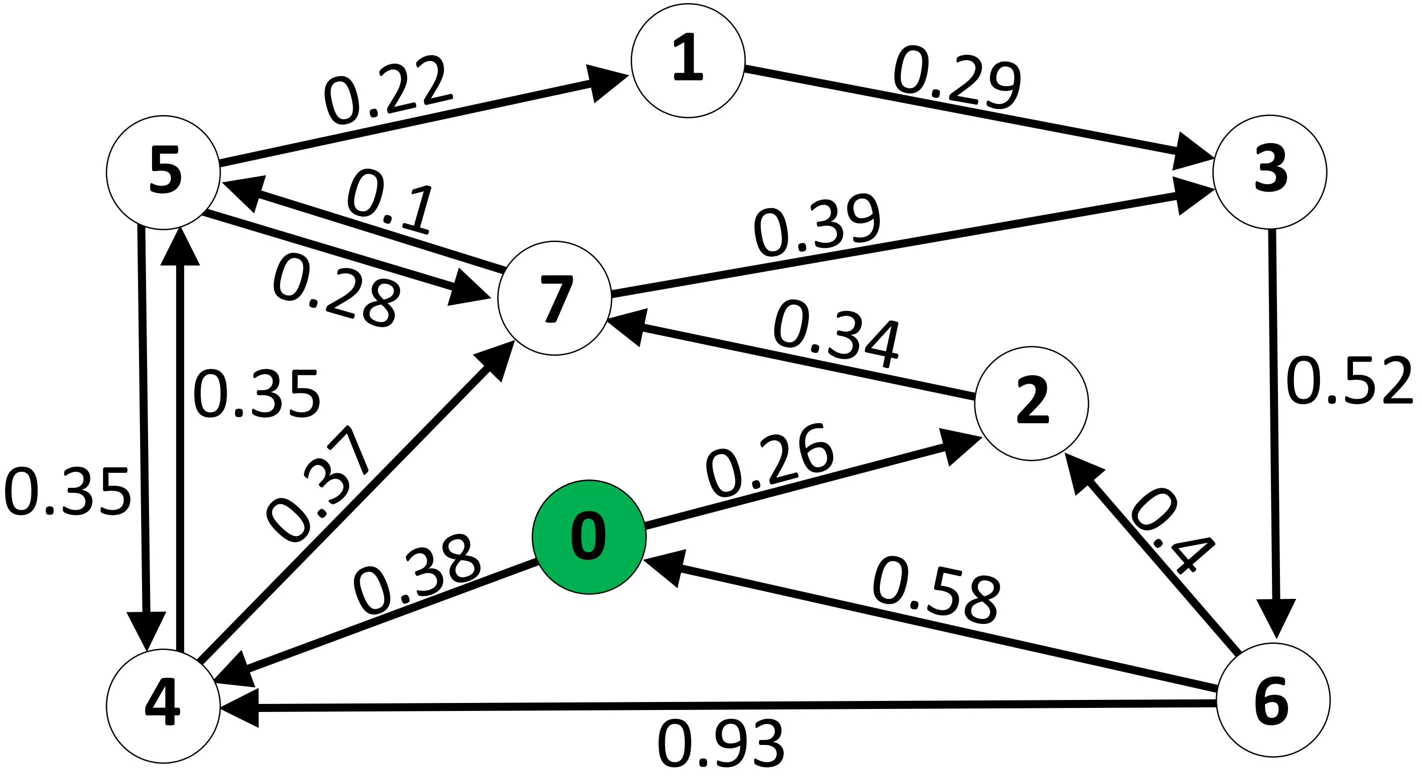


	Distance from 0	Previous vertex	Priority queue
0	0	-	
1	∞		
2	∞		
3	∞		
4	∞		
5	∞		
6	∞		
7	∞		

vertex (distance)

What can we reach from connected vertices and at what distance (from 0)?

Shortest Path

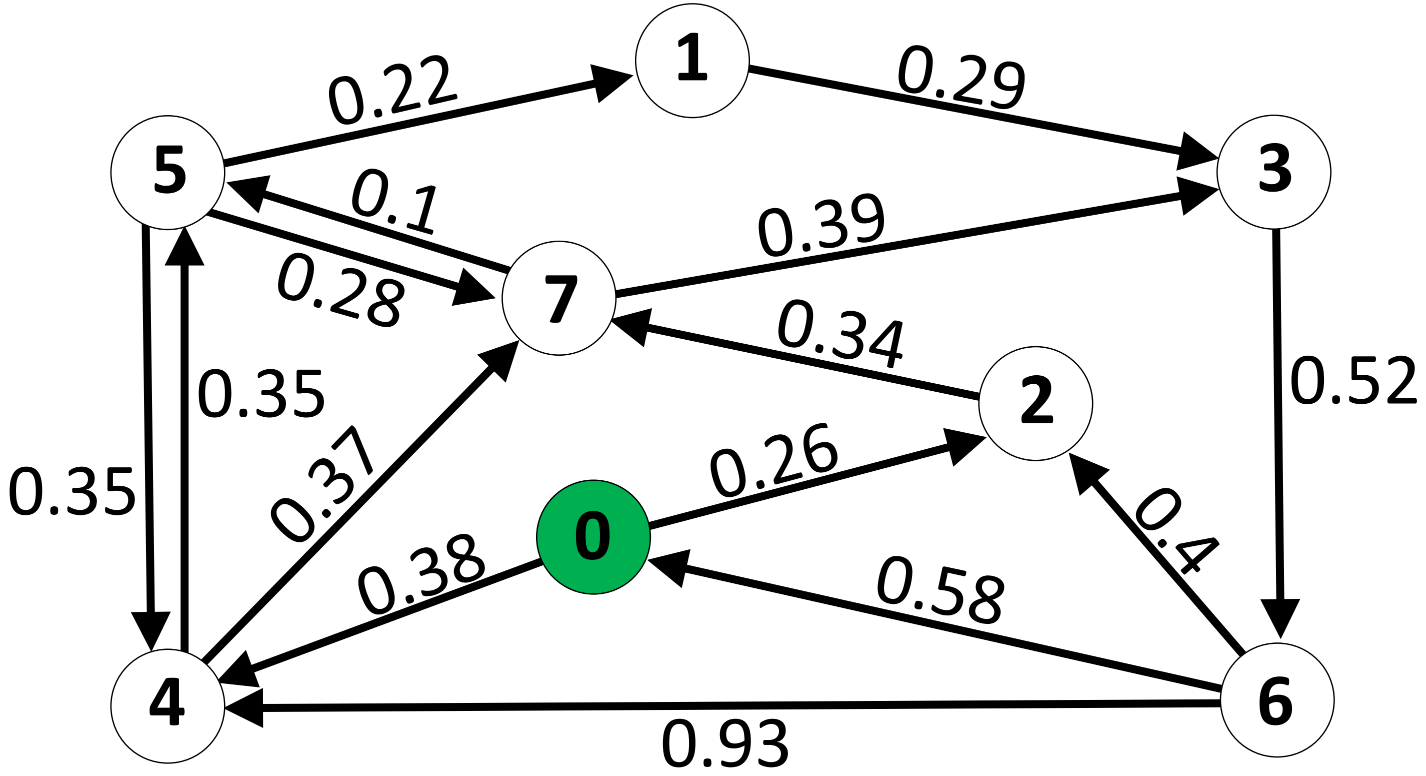


Vertex	Distance from 0	Previous vertex	Priority queue
0	0	-	2 (0.26)
1	∞		4 (0.38)
2	∞		
3	∞		
4	∞		
5	∞		
6	∞		
7	∞		

vertex (distance)

What can we reach from connected vertices and at what distance (from 0)?

Shortest Path



Distance from 0

0	0
1	∞
2	∞
3	∞
4	∞
5	∞
6	∞
7	∞

Previous vertex

0	-
1	
2	0
3	
4	0
5	
6	
7	

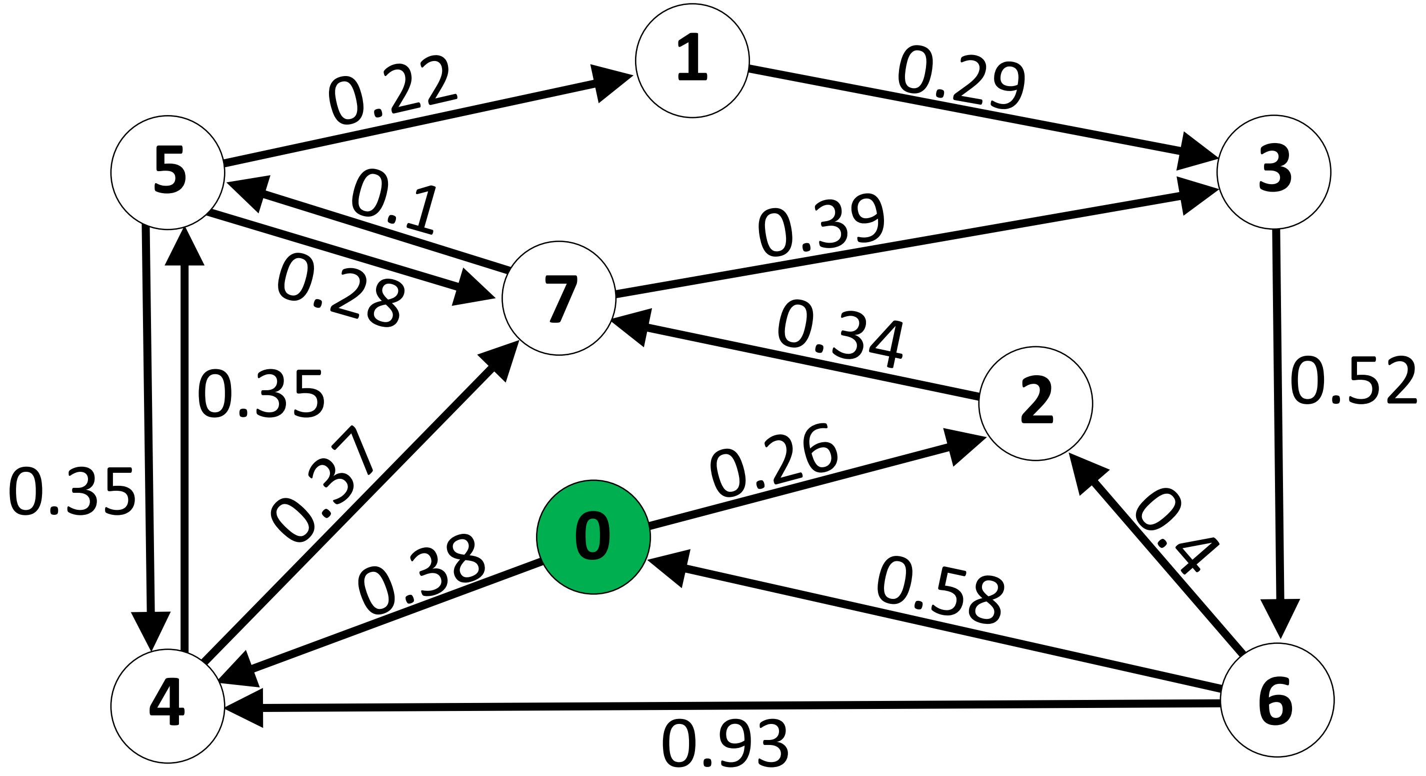
Priority queue

2 (0.26)
4 (0.38)

vertex (distance)

What can we reach from connected vertices and at what distance (from 0)?

Shortest Path



	Distance from 0
0	0
1	∞
2	0.26
3	∞
4	0.38
5	∞
6	∞
7	∞

	Previous vertex
0	-
1	
2	0
3	
4	0
5	
6	
7	

Priority queue

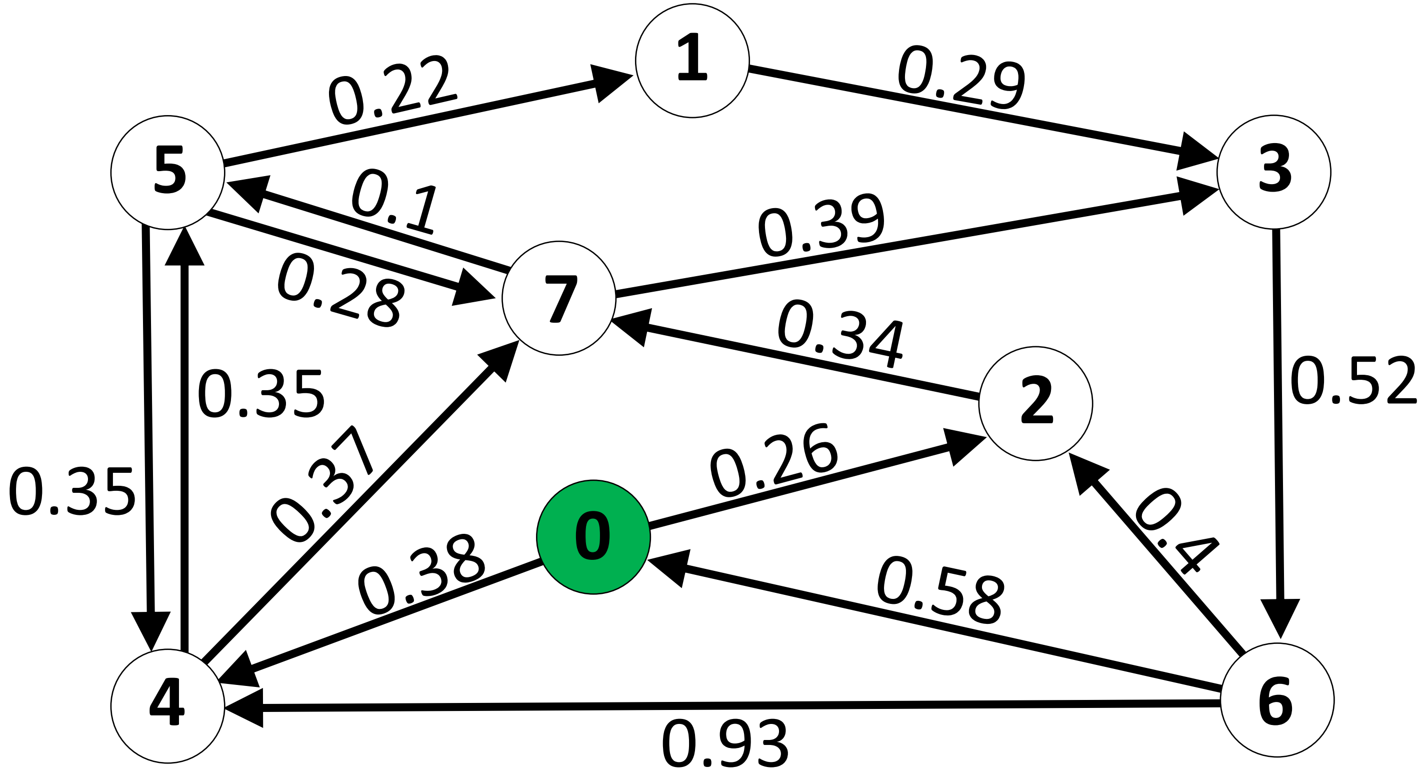
2 (0.26)
4 (0.38)

vertex (distance)

What can we reach from connected vertices and at what distance (from 0)?

Shortest Path

queue top = 2 (0.26)



	Distance from 0
0	0
1	∞
2	0.26
3	∞
4	0.38
5	∞
6	∞
7	∞

	Previous vertex
0	-
1	
2	0
3	
4	0
5	
6	
7	

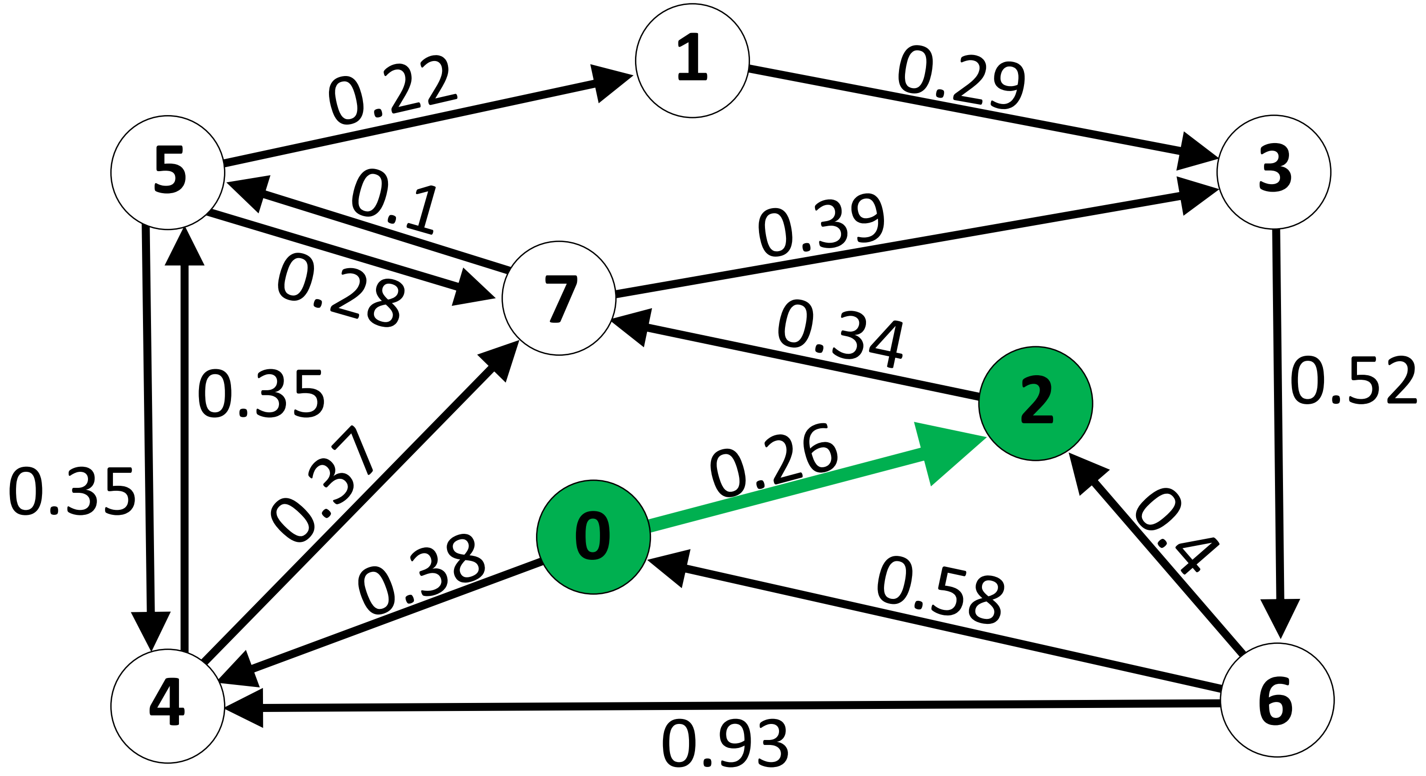
Priority queue
4 (0.38)

vertex (distance)

What can we reach from connected vertices and at what distance (from 0)?

Shortest Path

queue top = 2 (0.26)



Vertex	Distance from 0
0	0
1	∞
2	0.26
3	∞
4	0.38
5	∞
6	∞
7	∞

Vertex	Previous vertex
0	-
1	
2	0
3	
4	0
5	
6	
7	

Priority queue

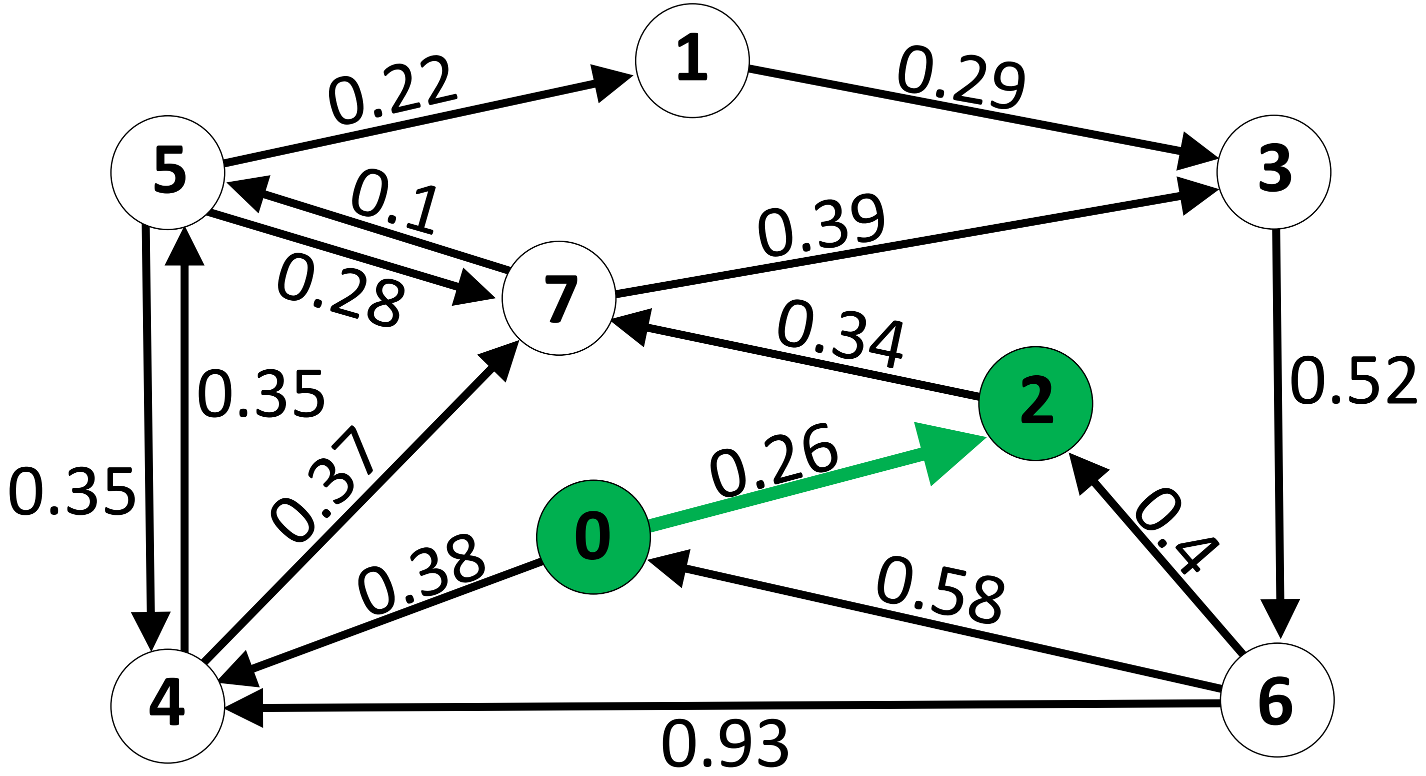
4 (0.38)

vertex (distance)

What can we reach from connected vertices and at what distance (from 0)?

Shortest Path

queue top = 2 (0.26)



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	0.38	0
5	∞	
6	∞	
7	0.60	2

Priority queue

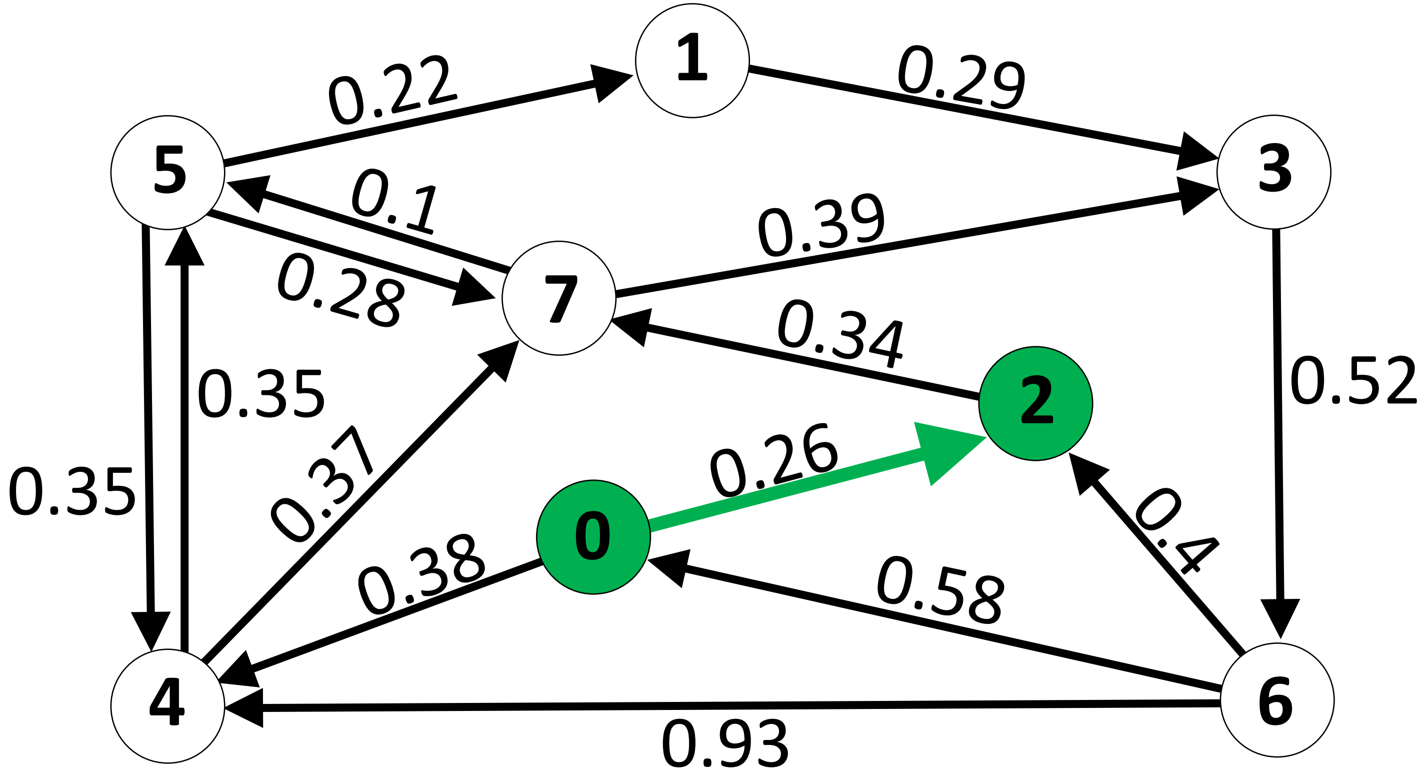
4 (0.38)
7 (0.60)

vertex (distance)

What can we reach from connected vertices and at what distance (from 0)?

Shortest Path

queue top = 4 (0.38)



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	0.38	0
5	∞	
6	∞	
7	0.60	2

Priority queue

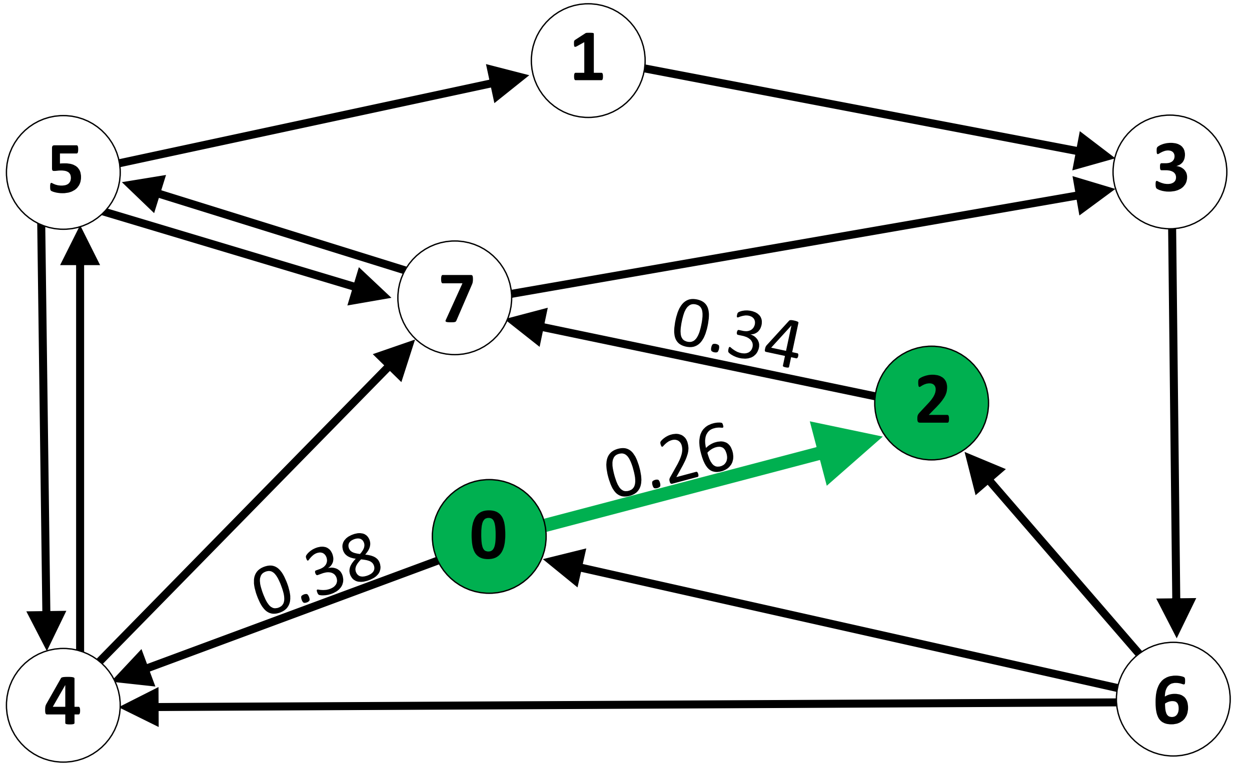
7 (0.60)

vertex (distance)

Repeat.

Shortest Path

queue top = 4 (0.38)



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	0.38	0
5	∞	
6	∞	
7	0.60	2

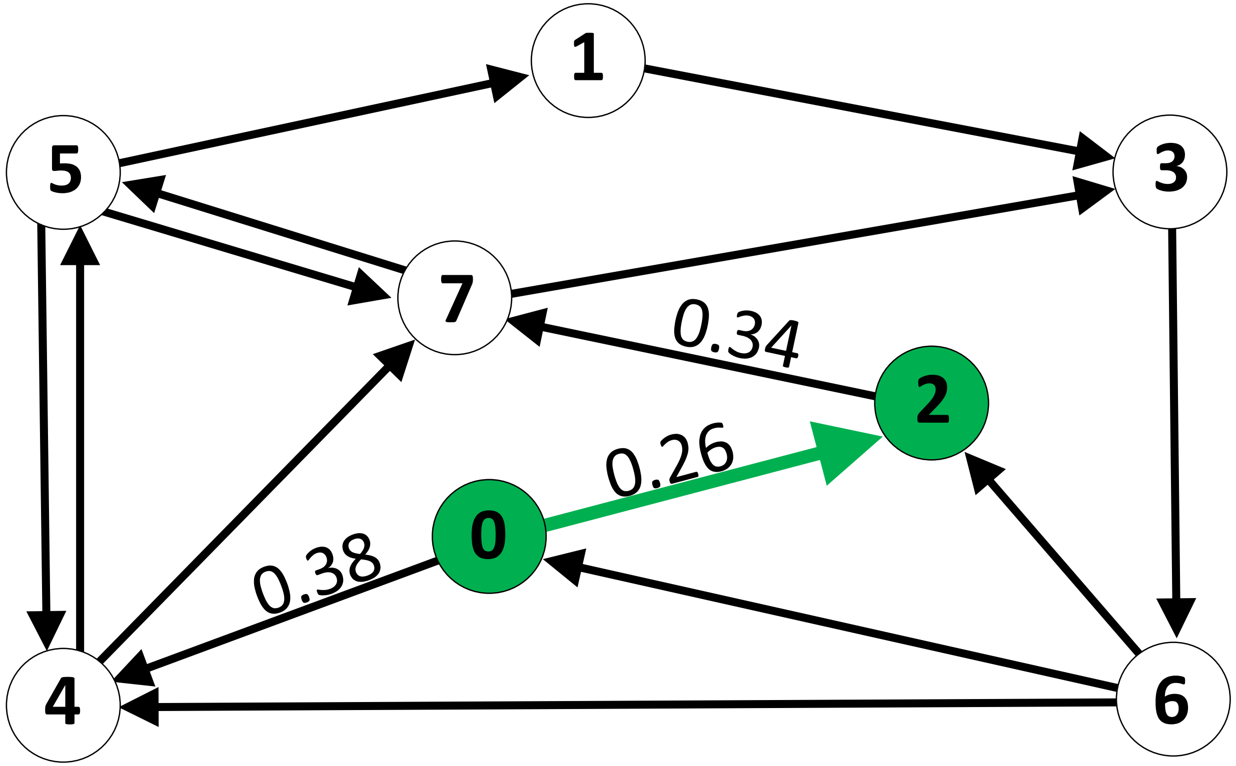
Priority queue
7 (0.60)

vertex (distance)

What can we say about the shortest path from 0 to 4?

Shortest Path

queue top = 4 (0.38)



Vertex	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	0.38	0
5	∞	
6	∞	
7	0.60	2

Priority queue

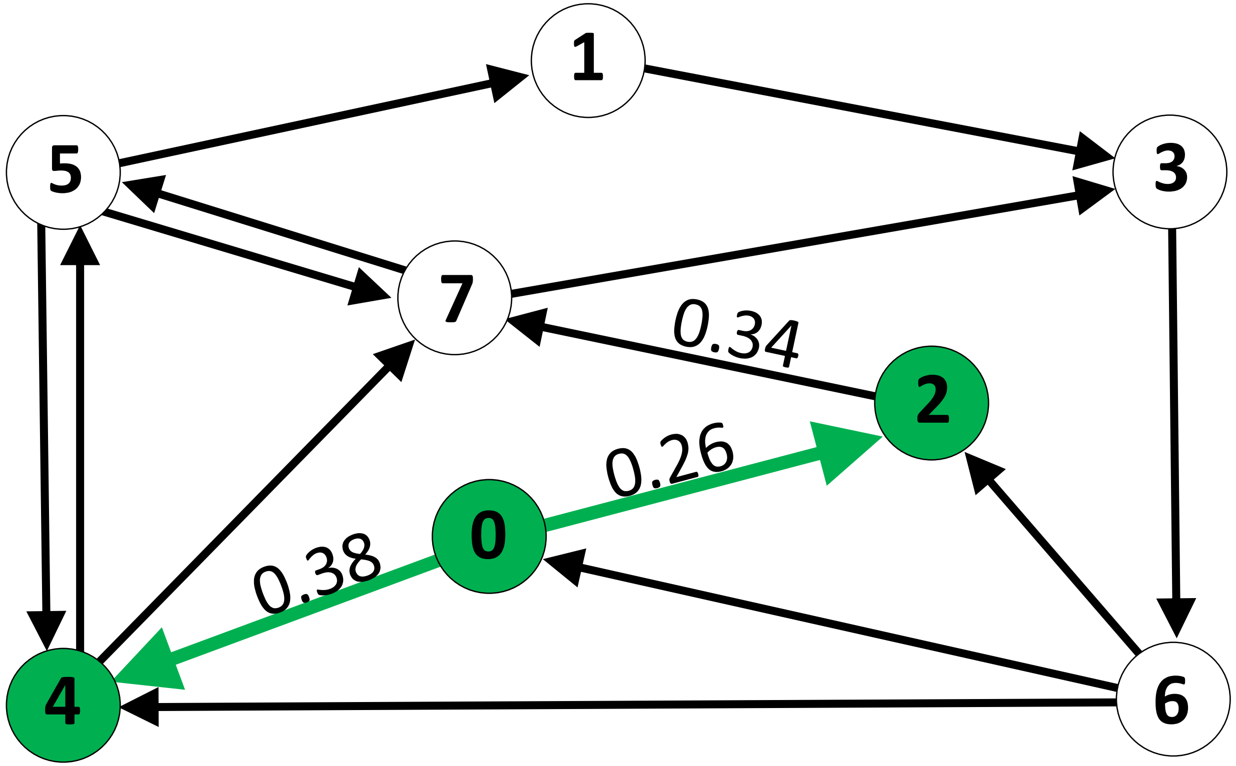
7 (0.60)

vertex (distance)

The 0 to 4 edge has to be the shortest path between 0 and 4, since any other path would go from 0 -> 2 -> 7 -> ? at cost at least 0.26 + 0.34 = 0.6 > 0.38

Shortest Path

queue top = 4 (0.38)



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	0.38	0
5	∞	
6	∞	
7	0.60	2

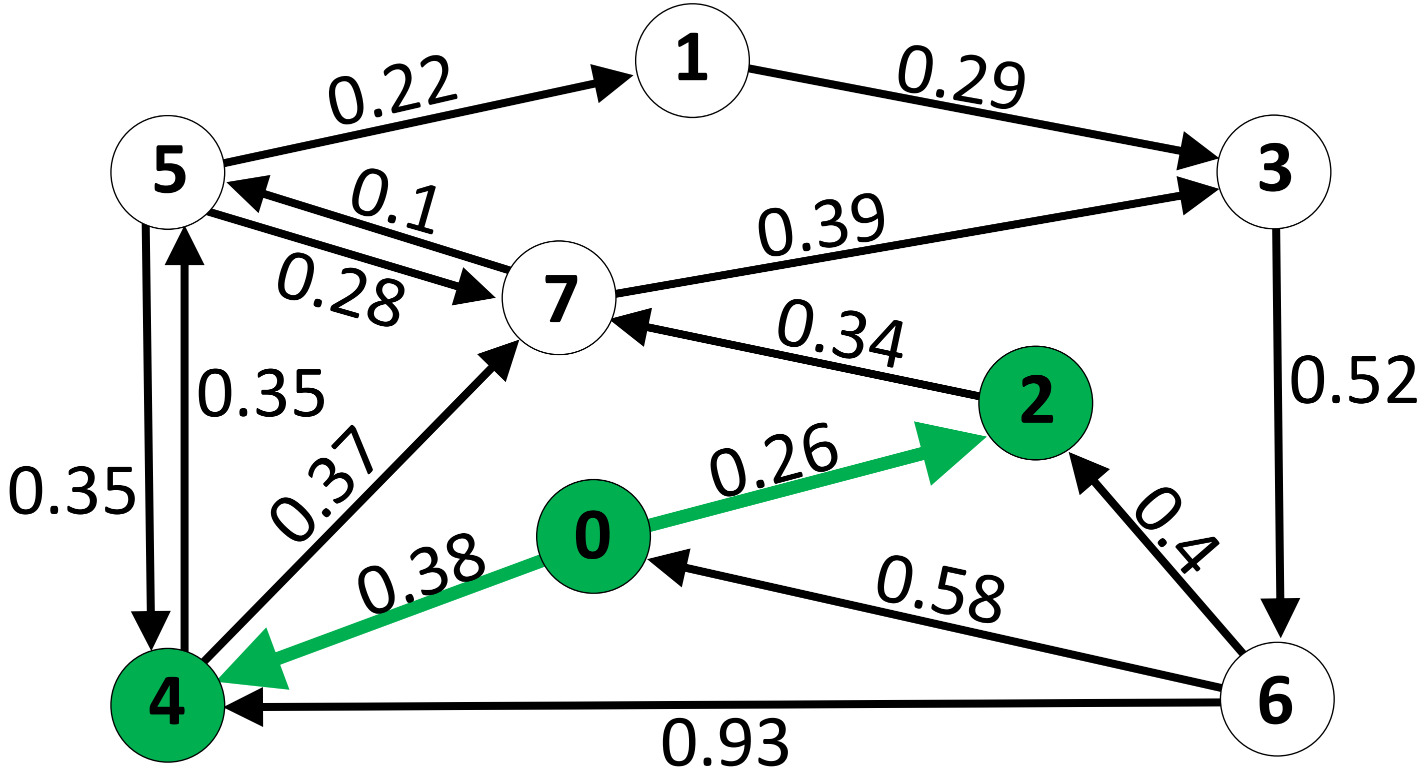
Priority queue
7 (0.60)

vertex (distance)

The 0 to 4 edge has to be the shortest path between 0 and 4, since any other path would go from 0 -> 2 -> 7 -> ? at cost at least 0.26 + 0.34 = 0.6 > 0.38

Shortest Path

queue top	= 4 (0.38)



Add neighbors to queue/previous.

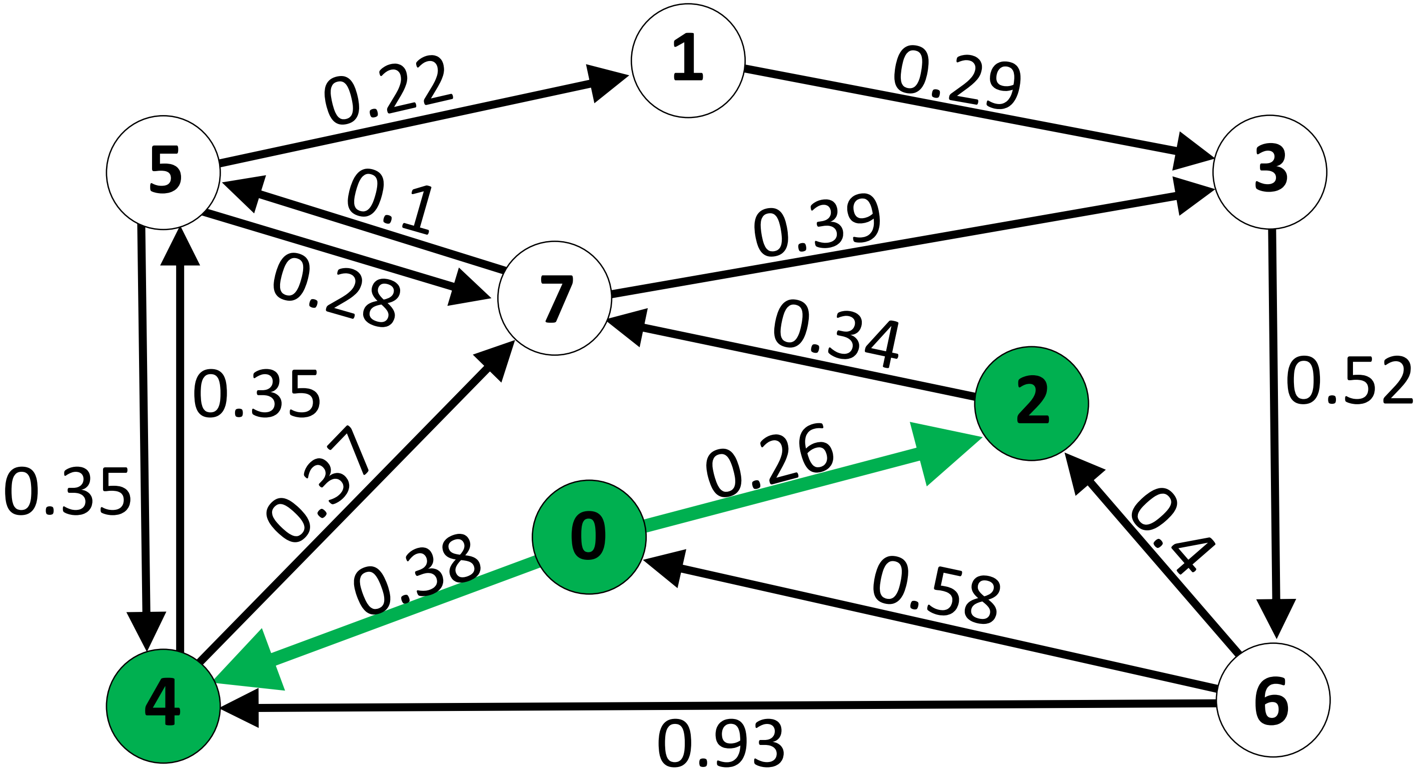
	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	0.38	0
5	∞	
6	∞	
7	0.60	2

Priority queue
7 (0.60)

vertex (distance)

Shortest Path

queue top = 4 (0.38)



Add neighbors to queue/previous.

	Distance from 0
0	0
1	∞
2	0.26
3	∞
4	0.38
5	0.73
6	∞
7	0.60

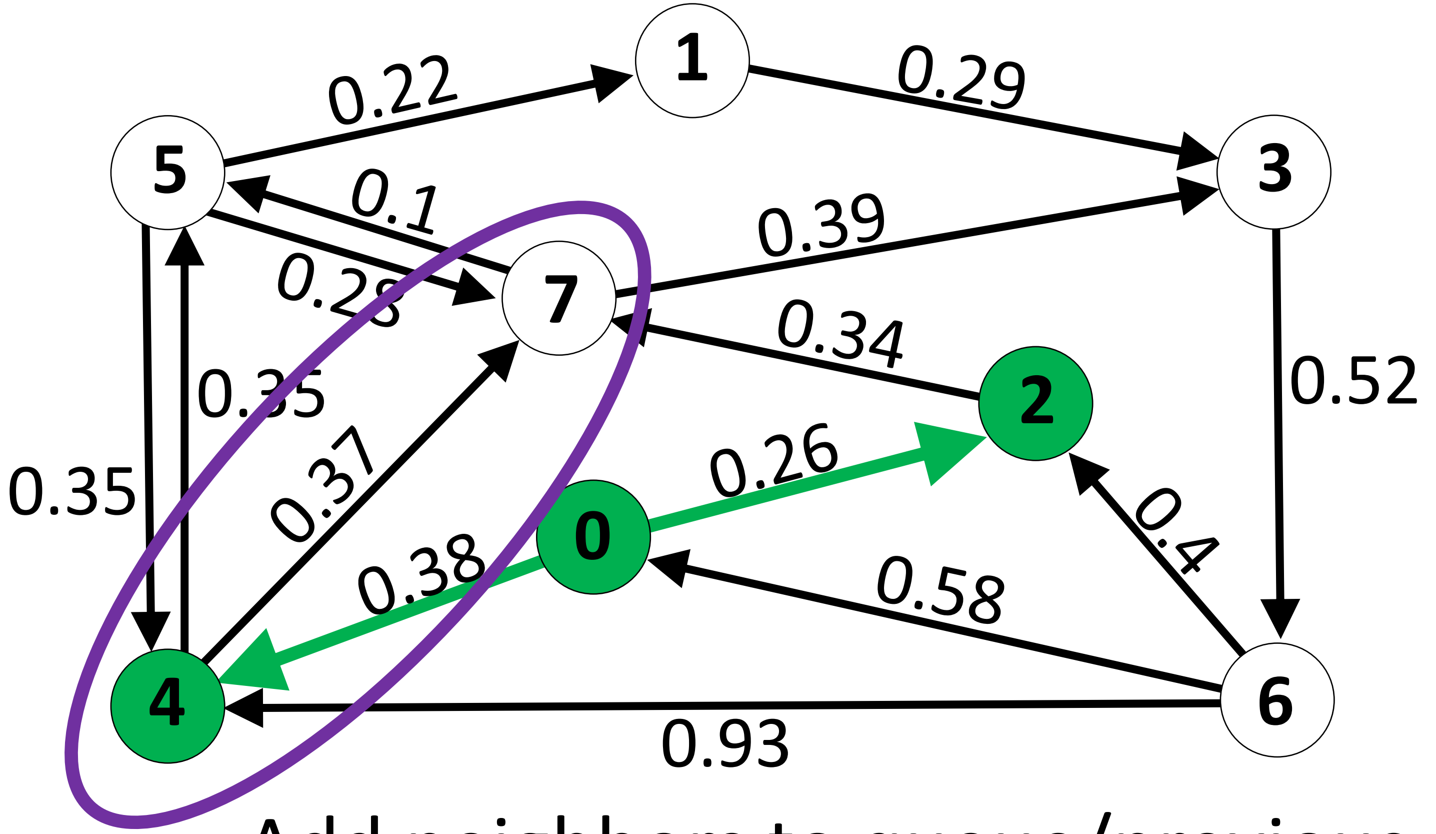
	Previous vertex
0	-
1	
2	0
3	
4	0
5	4
6	
7	2

Priority queue
7 (0.60)
5 (0.73)

vertex (distance)

Shortest Path

queue top = 4 (0.38)



Add neighbors to queue/previous.

We have another route to 7!

	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	0.38	0
5	0.73	4
6	∞	
7	0.60	2

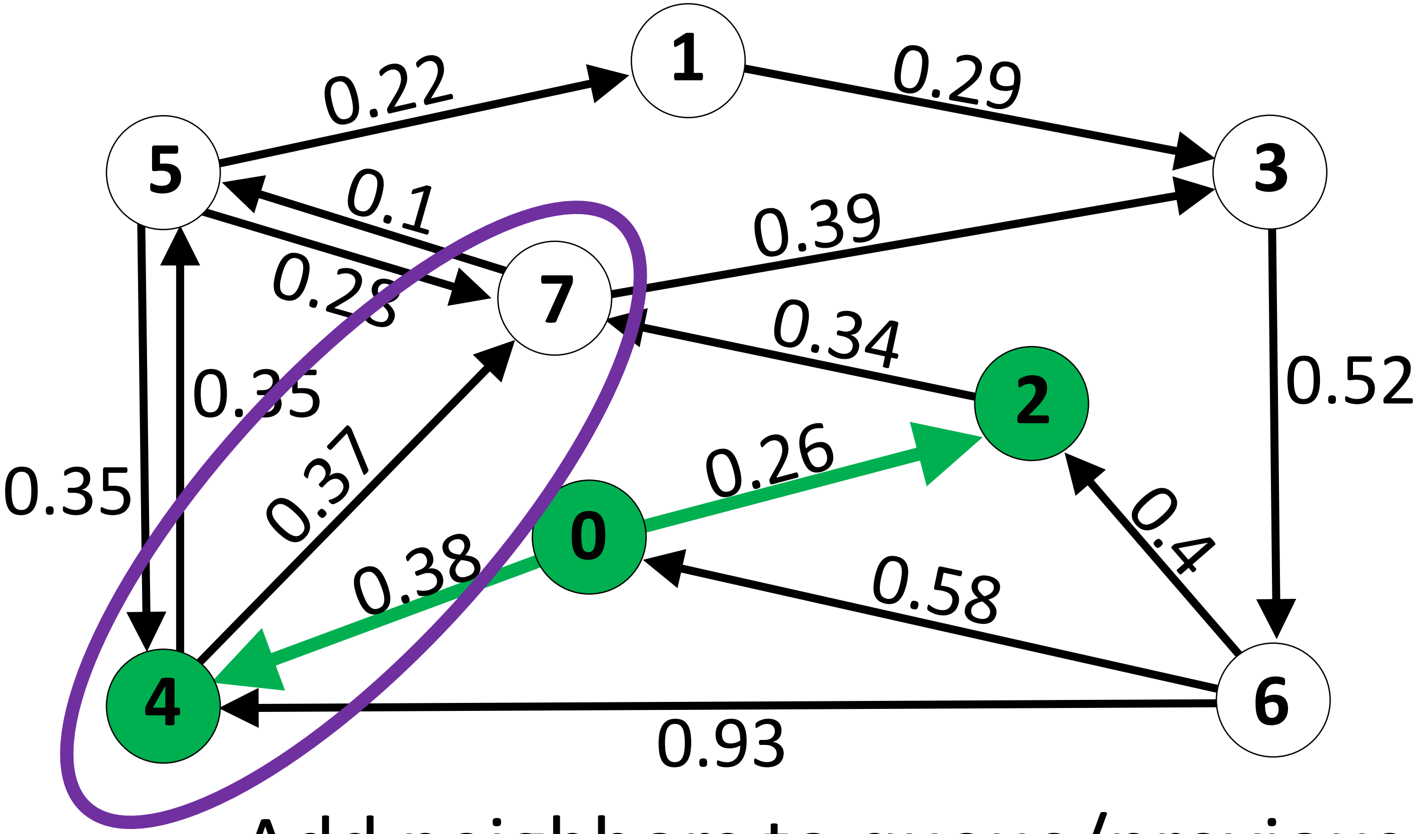
Priority queue

7 (0.60)
5 (0.73)

vertex (distance)

Shortest Path

queue top = 4 (0.38)



Add neighbors to queue/previous.

We have another route to 7! Check to see if it is shorter!

	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	0.38	0
5	0.73	4
6	∞	
7	0.60	2

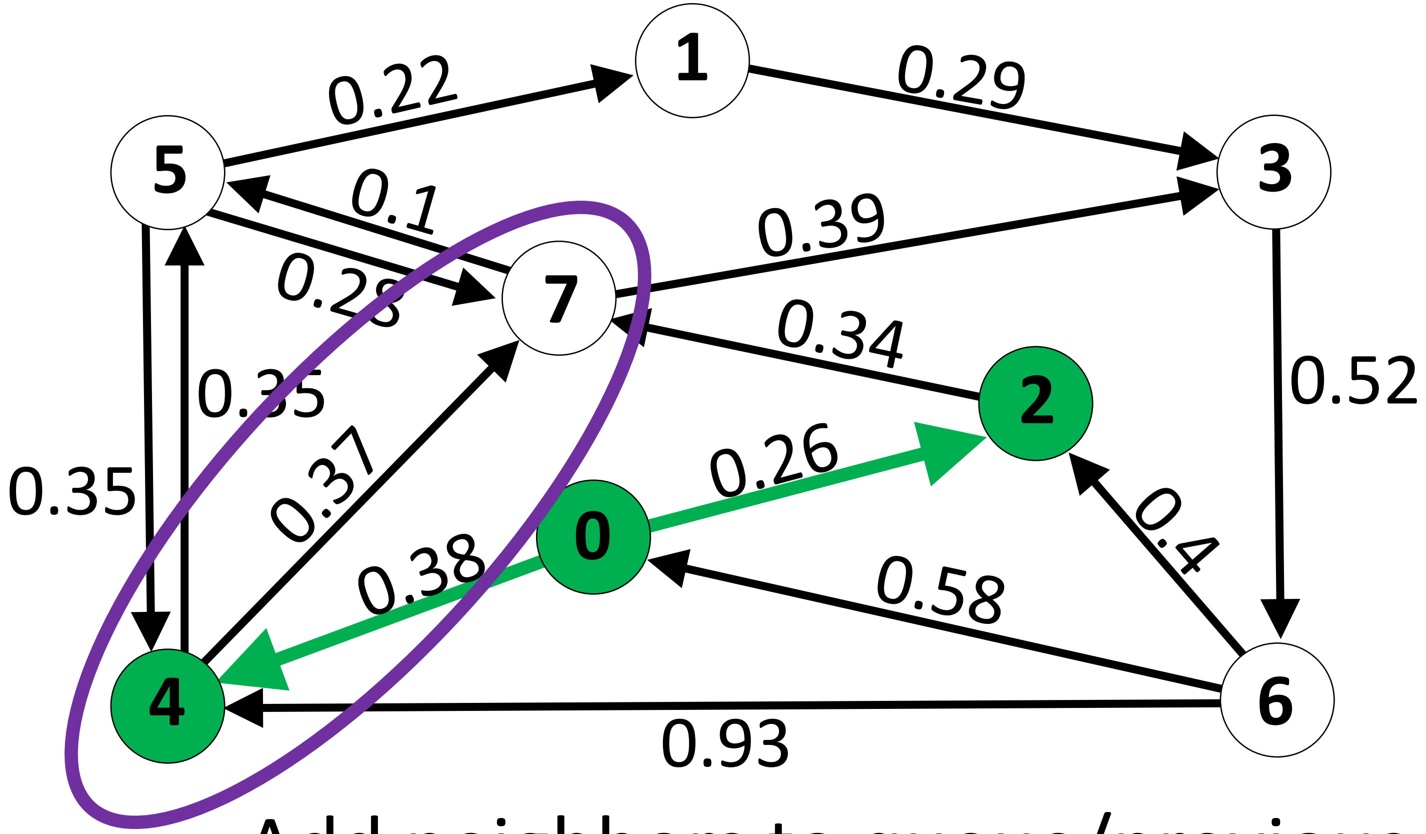
Priority queue

7 (0.60)
5 (0.73)

vertex (distance)

Shortest Path

queue top = 4 (0.38)



	Distance from 0
0	0
1	∞
2	0.26
3	∞
4	0.38
5	0.73
6	∞
7	0.60

	Previous vertex
0	-
1	
2	0
3	
4	0
5	4
6	
7	2

Priority queue
7 (0.60)
5 (0.73)

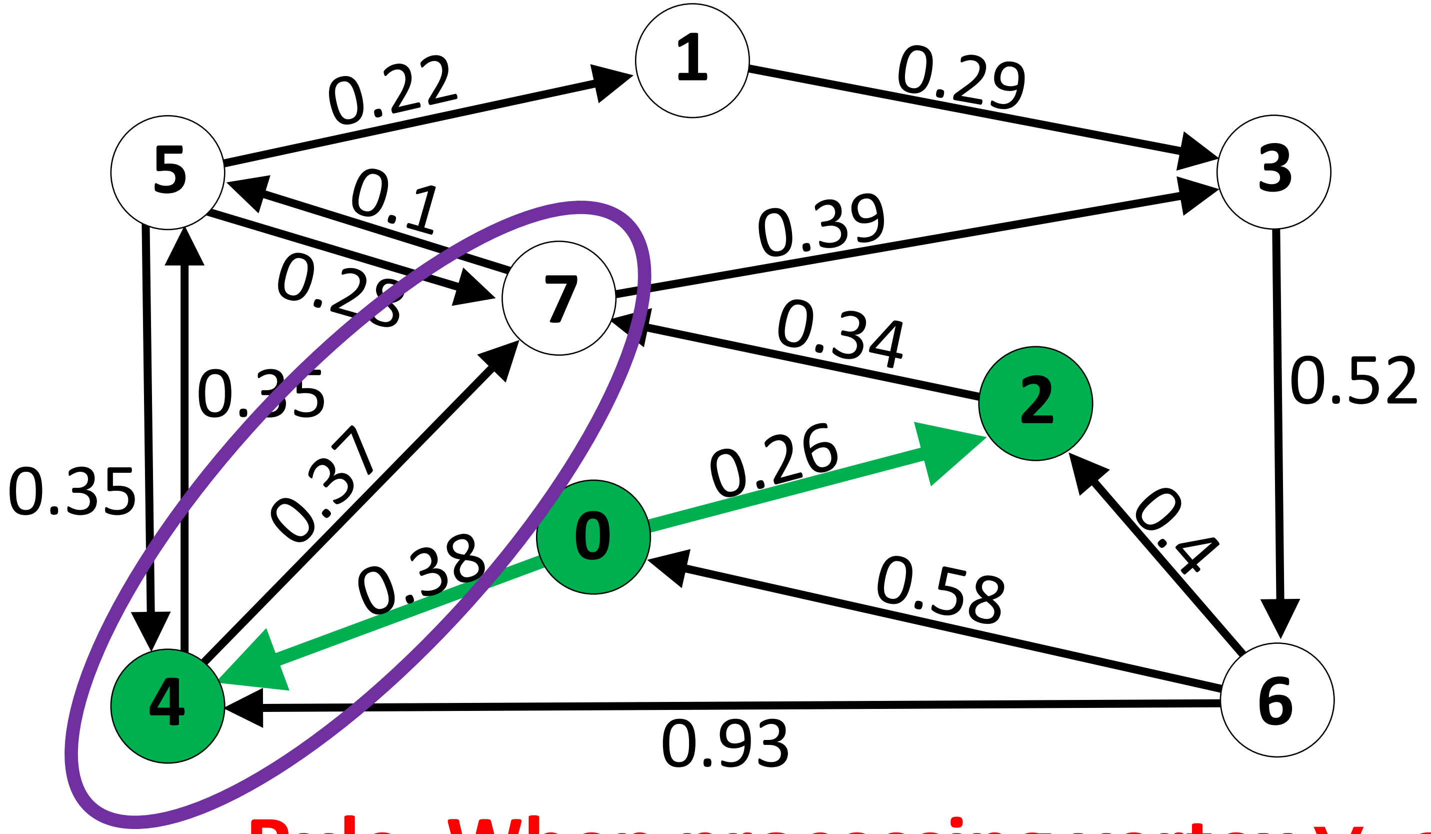
vertex (distance)

Add neighbors to queue/previous.

We have another route to 7! Check to see if it is shorter! It's not (0.38 + 0.37 = 0.75 > 0.60).

Shortest Path

queue top = 4 (0.38)



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	∞	
4	0.38	0
5	0.73	4
6	∞	
7	0.60	2

Priority queue

7 (0.60)
5 (0.73)

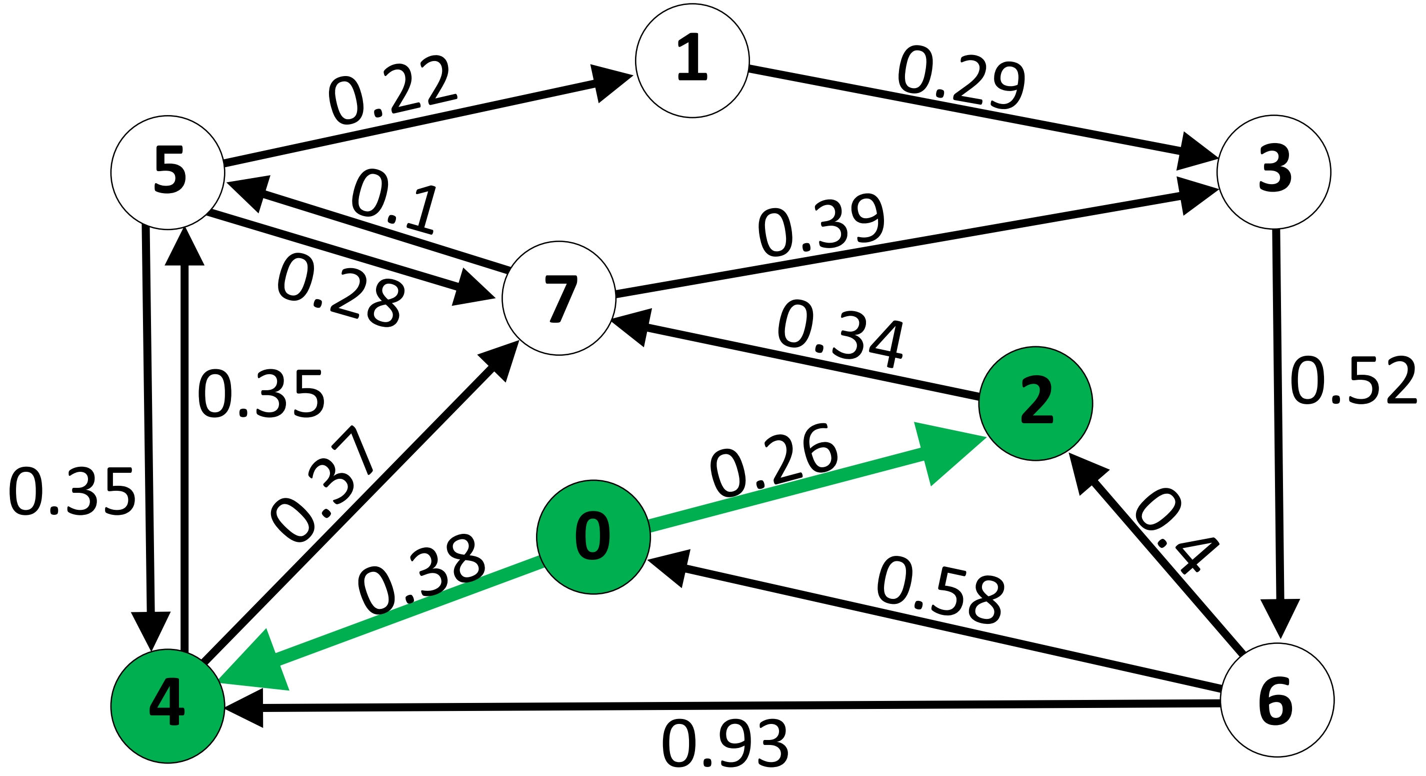
vertex (distance)

Rule: **When processing vertex v, only add/modify queue for neighbor u if and only if:**

`distance[v] + weight(v, u) < distance[u]`

Shortest Path

queue top =



Repeat.

Distance from 0

0	0
1	∞
2	0.26
3	∞
4	0.38
5	0.73
6	∞
7	0.60

Previous vertex

0	-
1	
2	0
3	
4	0
5	4
6	
7	2

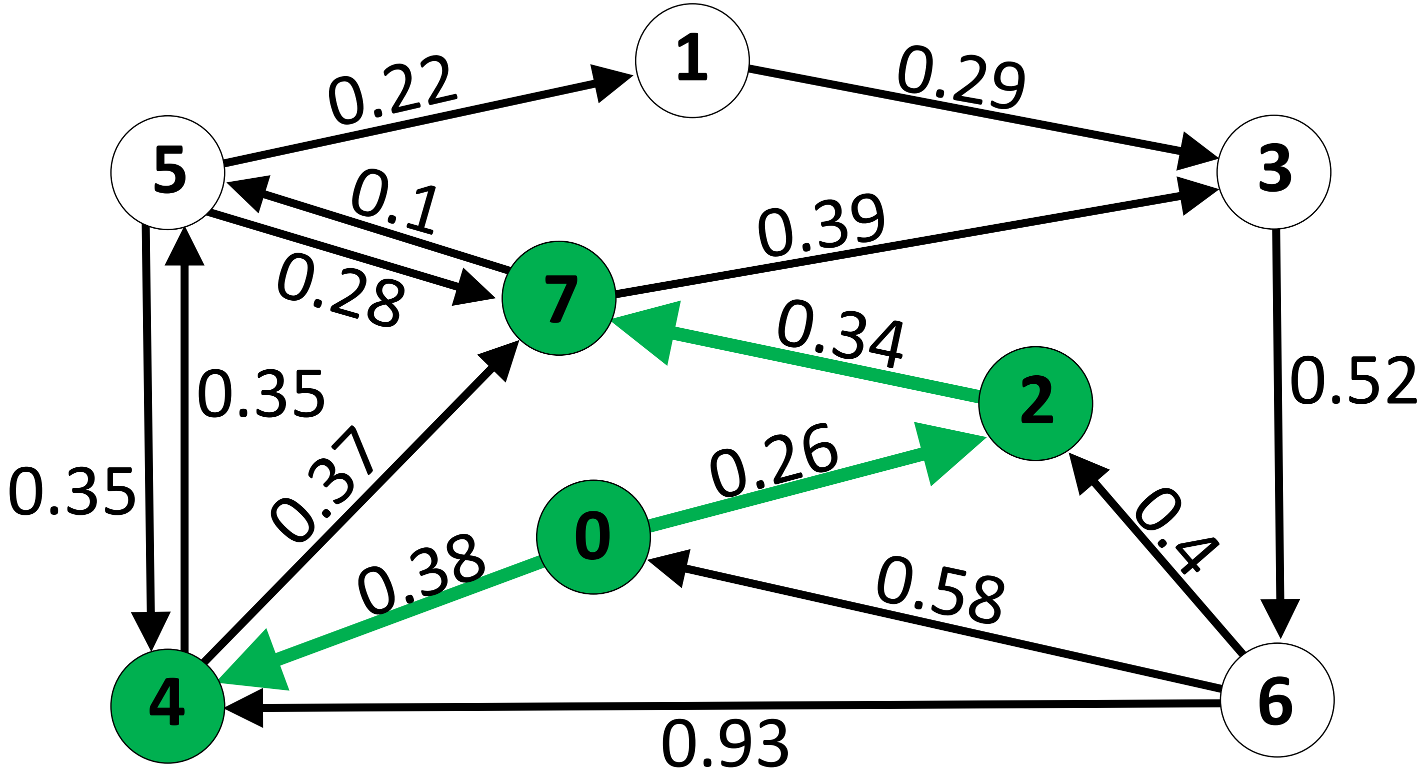
Priority queue

7 (0.60)
5 (0.73)

vertex (distance)

Shortest Path

queue top = 7 (0.60)



Repeat.

	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	0.99	7
4	0.38	0
5	0.73	4
6	∞	
7	0.60	2

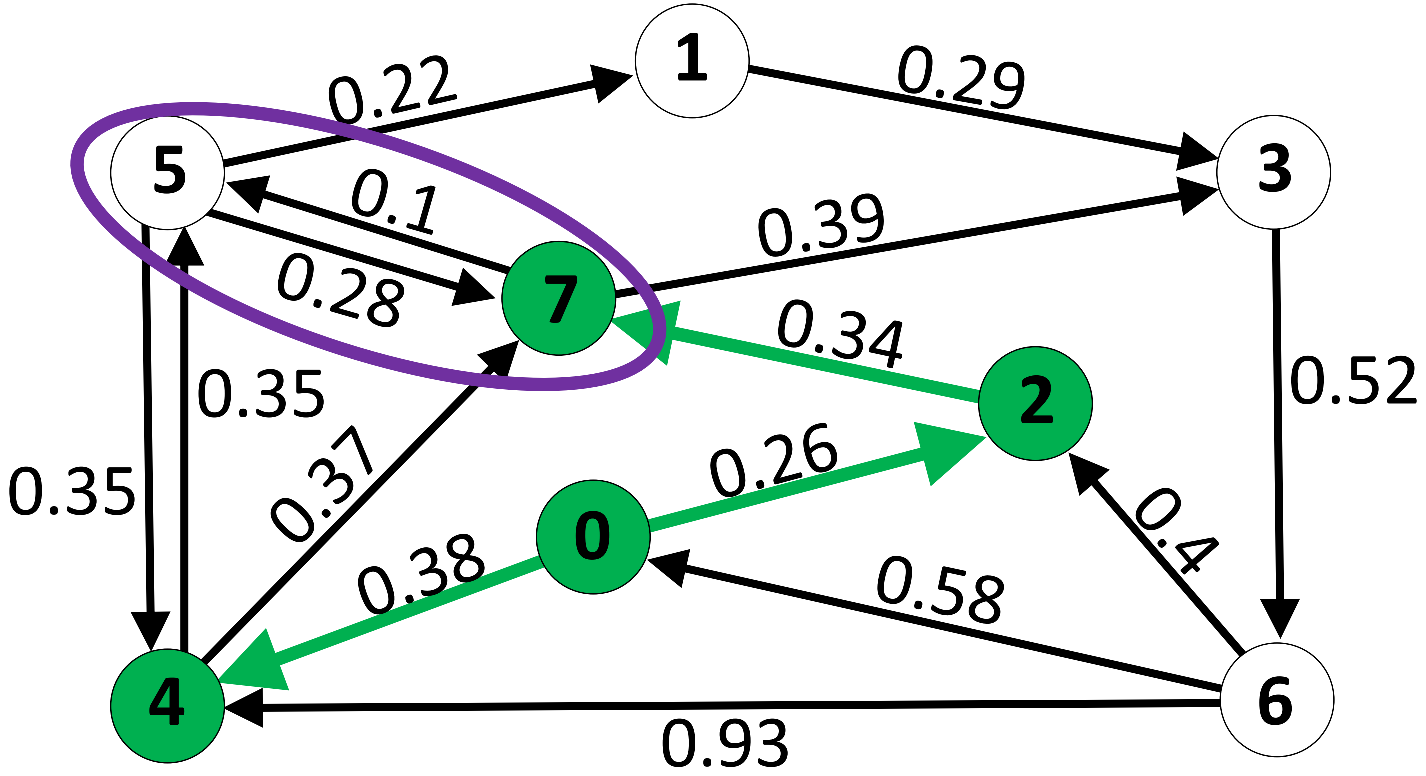
Priority queue

5 (0.73)
3 (0.99)

vertex (distance)

Shortest Path

queue top = 7 (0.60)



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	0.99	7
4	0.38	0
5	0.73	4
6	∞	
7	0.60	2

Priority queue

5 (0.73)
3 (0.99)

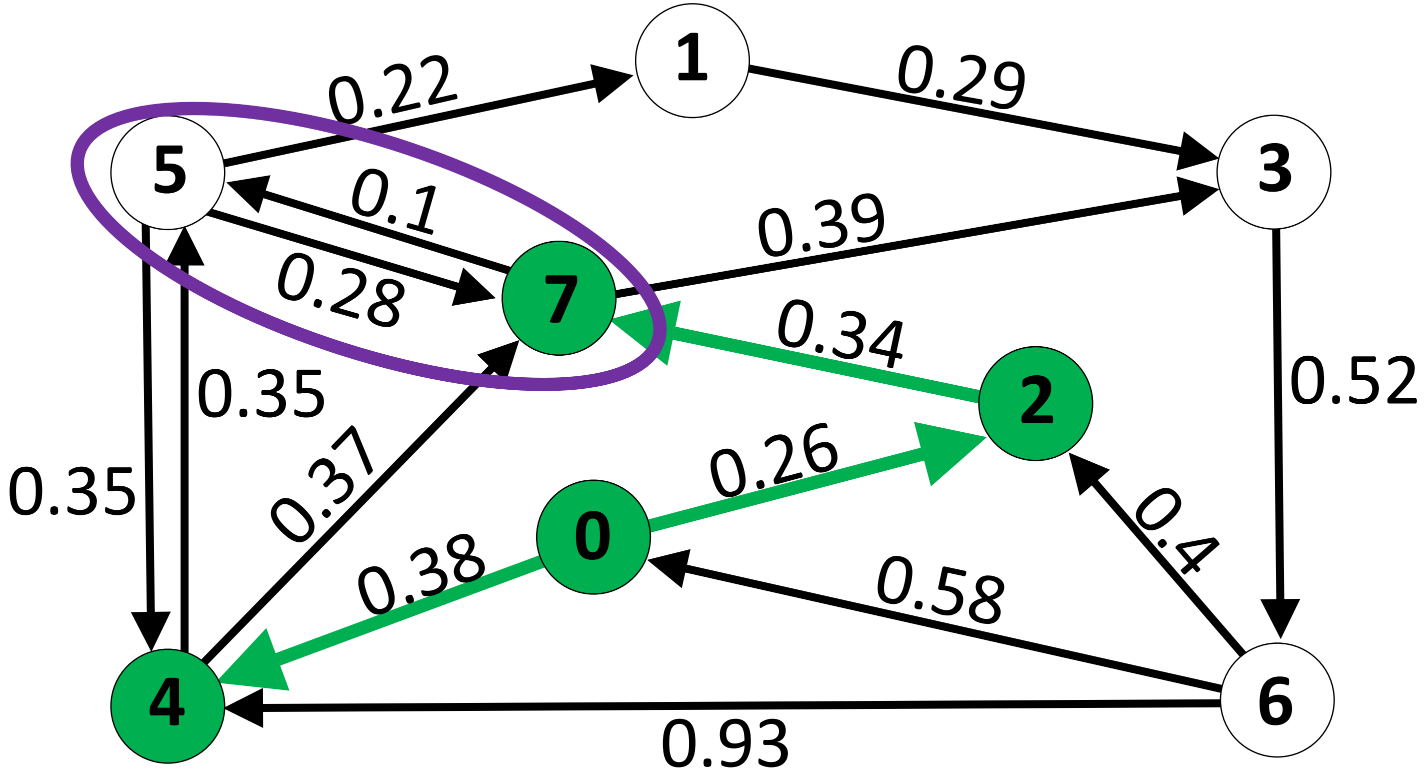
vertex (distance)

Repeat.

We have another route to 5, and at cost 0.7 < 0.73.

Shortest Path

queue top = 7 (0.60)



	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	0.99	7
4	0.38	0
5	0.73	4
6	∞	
7	0.60	2

Priority queue

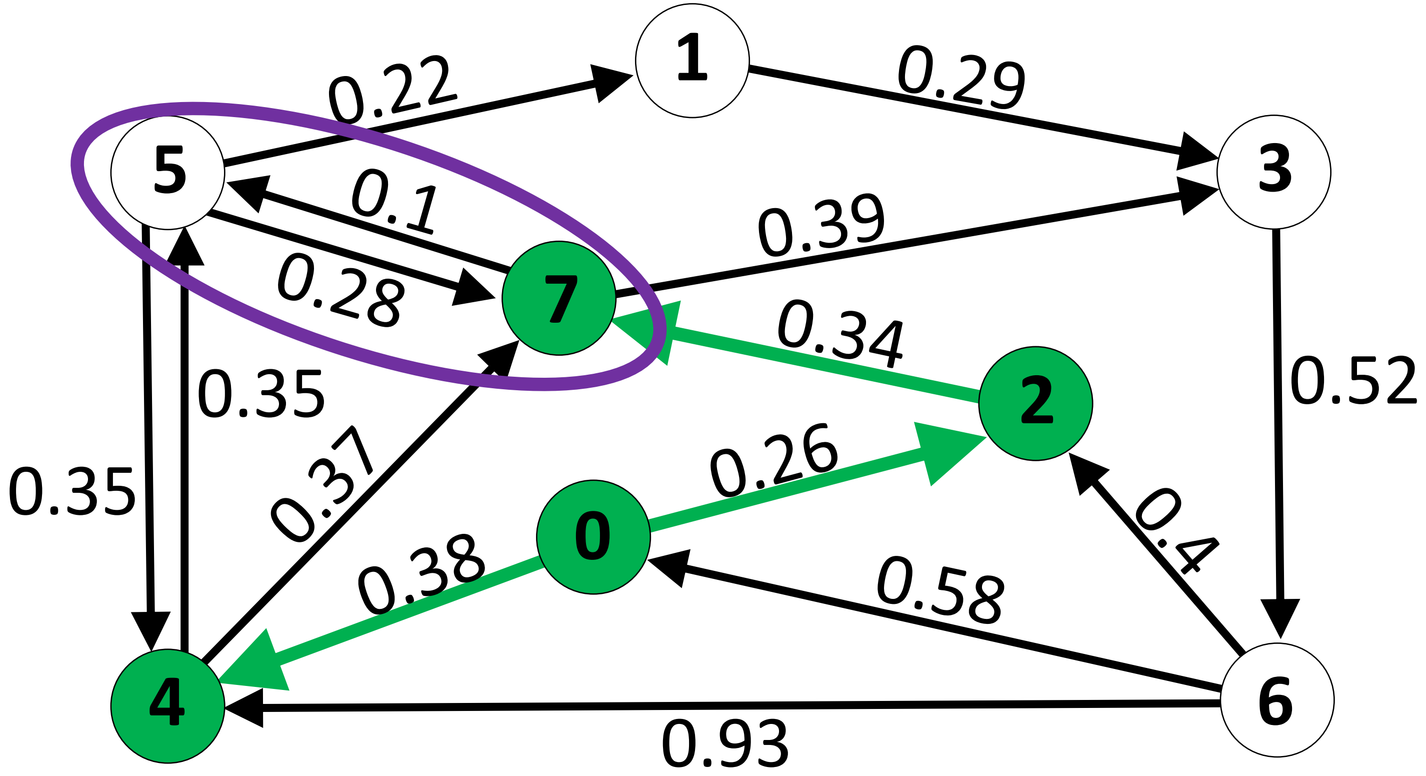
5 (0.73)
3 (0.99)

vertex (distance)

Repeat. **We have another route to 5, and at cost 0.7 < 0.73.**
i.e., `distance[v] + weight(v, u) < distance[u]`

Shortest Path

queue top = 7 (0.60)



Vertex	Distance from 0
0	0
1	∞
2	0.26
3	0.99
4	0.38
5	~~0.73~~ **0.70**
6	∞
7	0.60

Vertex	Previous vertex
0	-
1	
2	0
3	7
4	0
5	~~4~~ **7**
6	
7	2

Priority queue

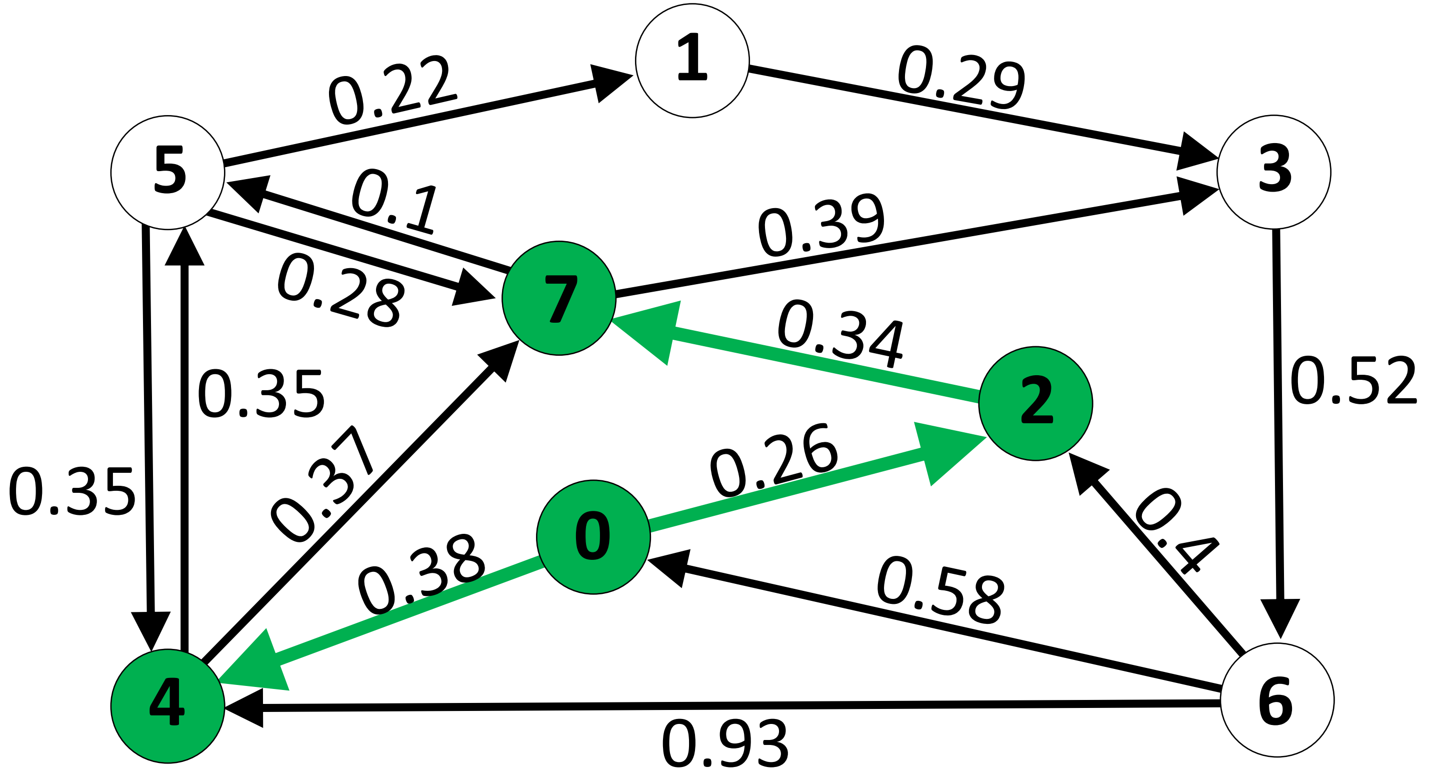
5 (~~0.73~~ **0.70**)
3 (0.99)

vertex (distance)

Repeat. **We have another route to 5, and at cost 0.7 < 0.73. So updated queue/previous/distance.**

Shortest Path

queue top	= 7 (0.60)



Distance from 0

0	0
1	∞
2	0.26
3	0.99
4	0.38
5	0.70
6	∞
7	0.60

Previous vertex

0	-
1	
2	0
3	7
4	0
5	7
6	
7	2

Priority queue

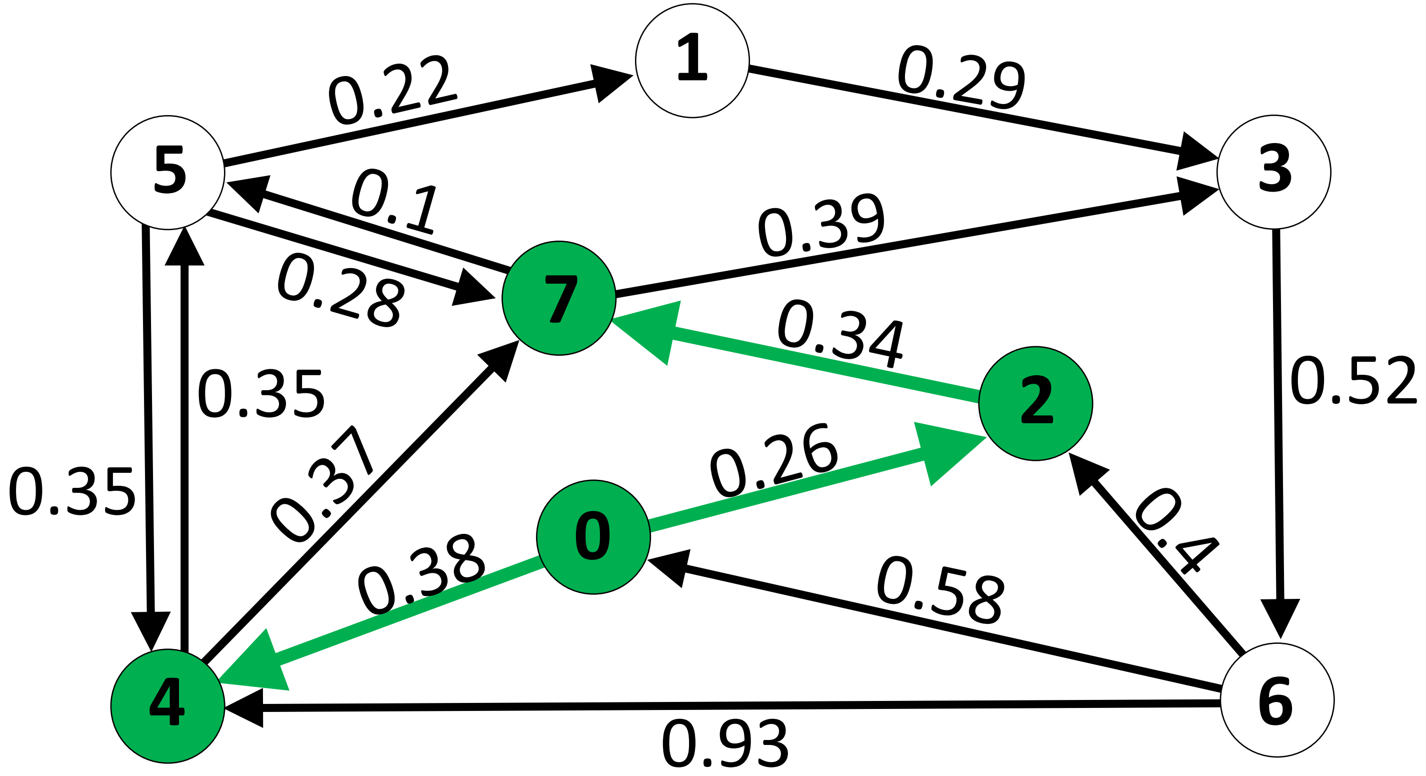
5 (0.70)
3 (0.99)

vertex (distance)

Repeat. **We have another route to 5, and at cost 0.7 < 0.73. So updated queue/previous/distance.**

Shortest Path

queue top = 7 (0.60)



Repeat.

	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	0.99	7
4	0.38	0
5	0.70	7
6	∞	
7	0.60	2

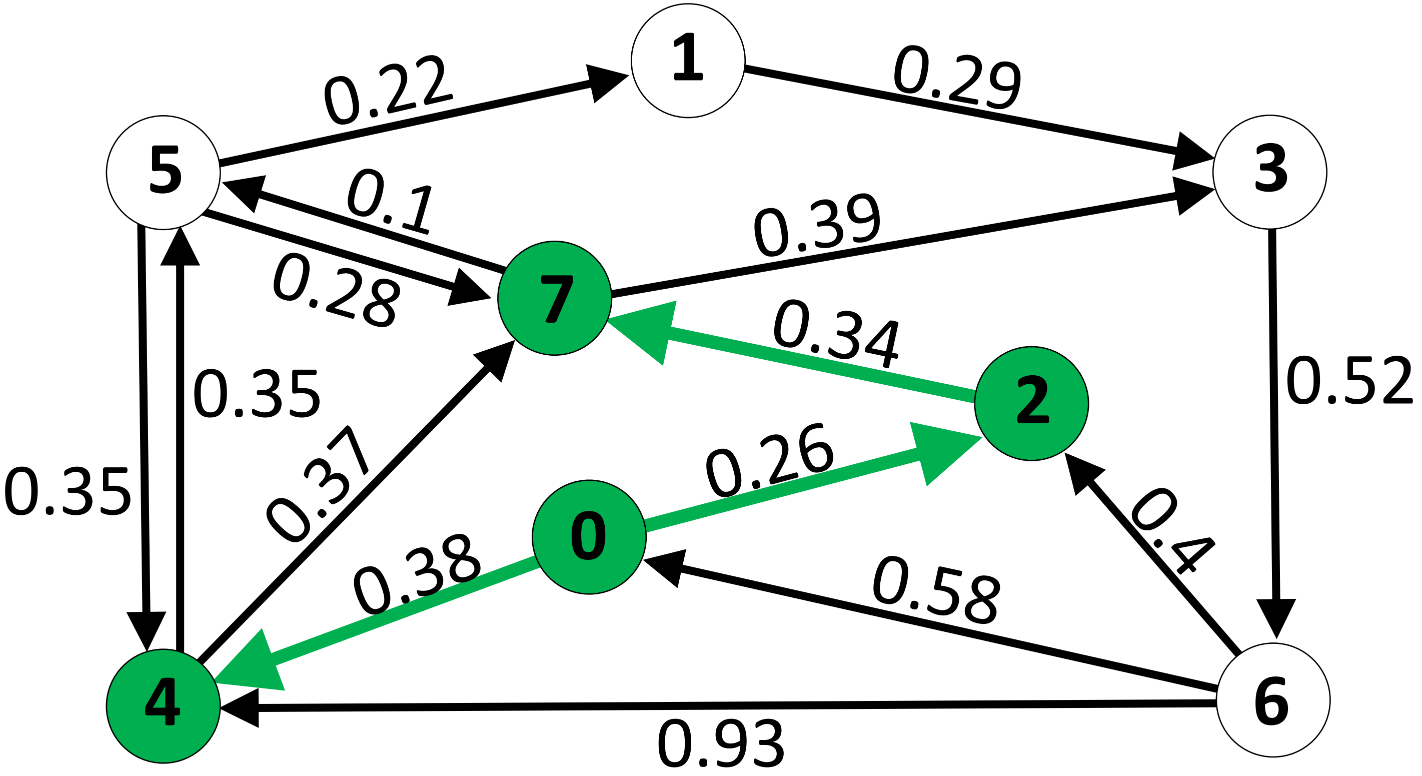
Priority queue

5 (0.70)
3 (0.99)

vertex (distance)

Shortest Path

queue top = 5 (0.70)



Repeat.

	Distance from 0	Previous vertex
0	0	-
1	∞	
2	0.26	0
3	0.99	7
4	0.38	0
5	0.70	7
6	∞	
7	0.60	2

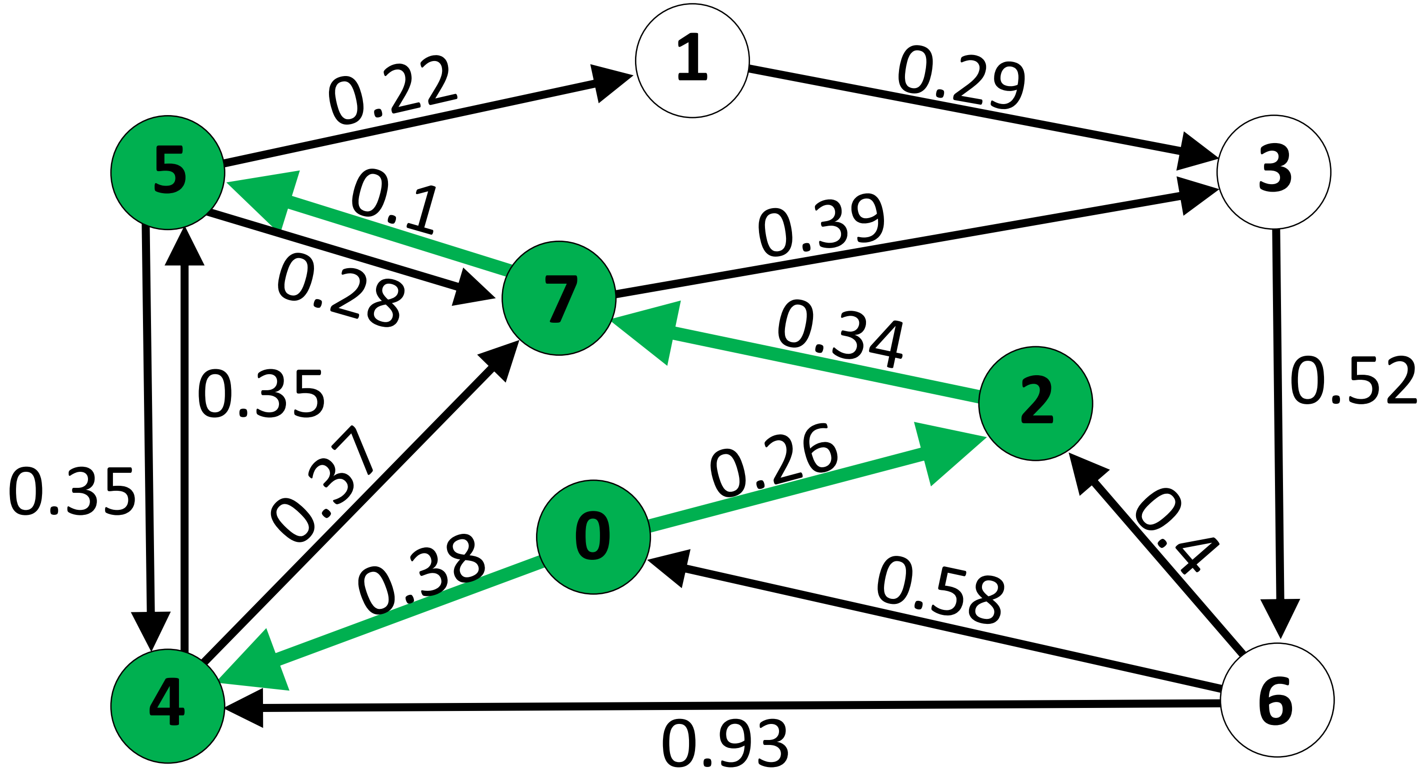
Priority queue

3 (0.99)

vertex (distance)

Shortest Path

queue top	= 5 (0.70)



Repeat.

	Distance from 0
0	0
1	∞
2	0.26
3	0.99
4	0.38
5	0.70
6	∞
7	0.60

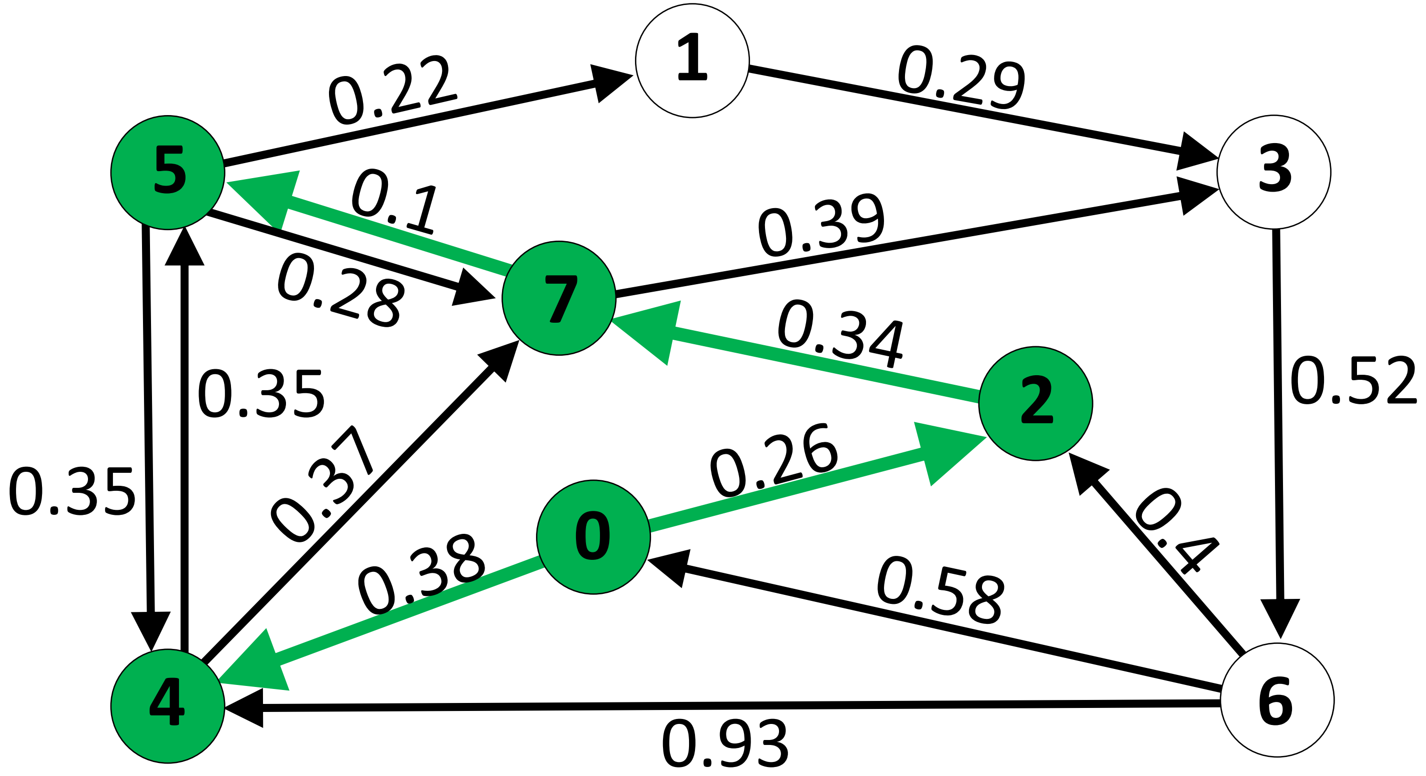
	Previous vertex
0	-
1	
2	0
3	7
4	0
5	7
6	
7	2

Priority queue
3 (0.99)

vertex (distance)

Shortest Path

queue top = 5 (0.70)



Repeat.

	Distance from 0
0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	∞
7	0.60

	Previous vertex
0	-
1	5
2	0
3	7
4	0
5	7
6	
7	2

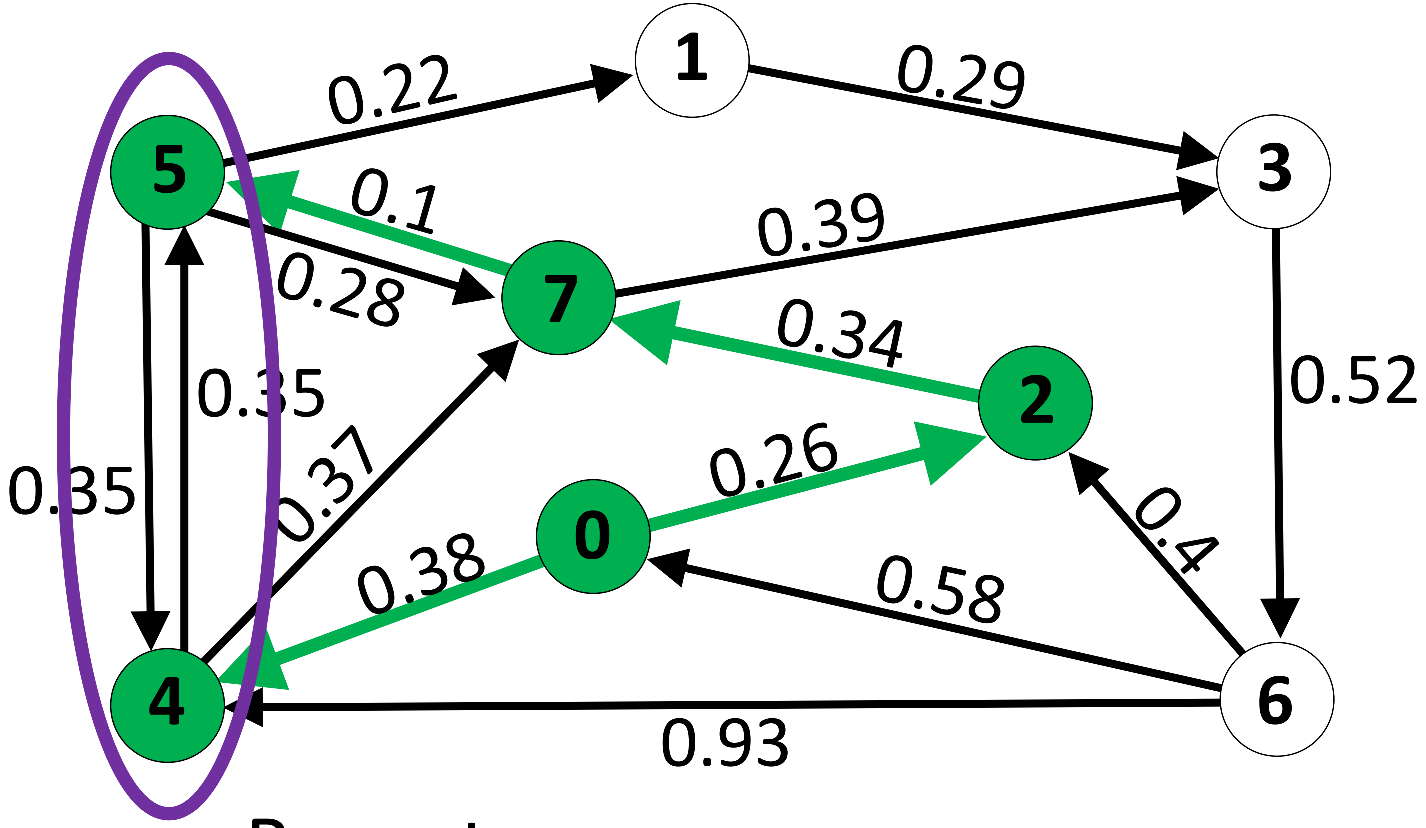
Priority queue

1 (0.92)
3 (0.99)

vertex (distance)

Shortest Path

queue top = 5 (0.70)



Repeat.

What about neighbor 4?

	Distance from 0
0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	∞
7	0.60

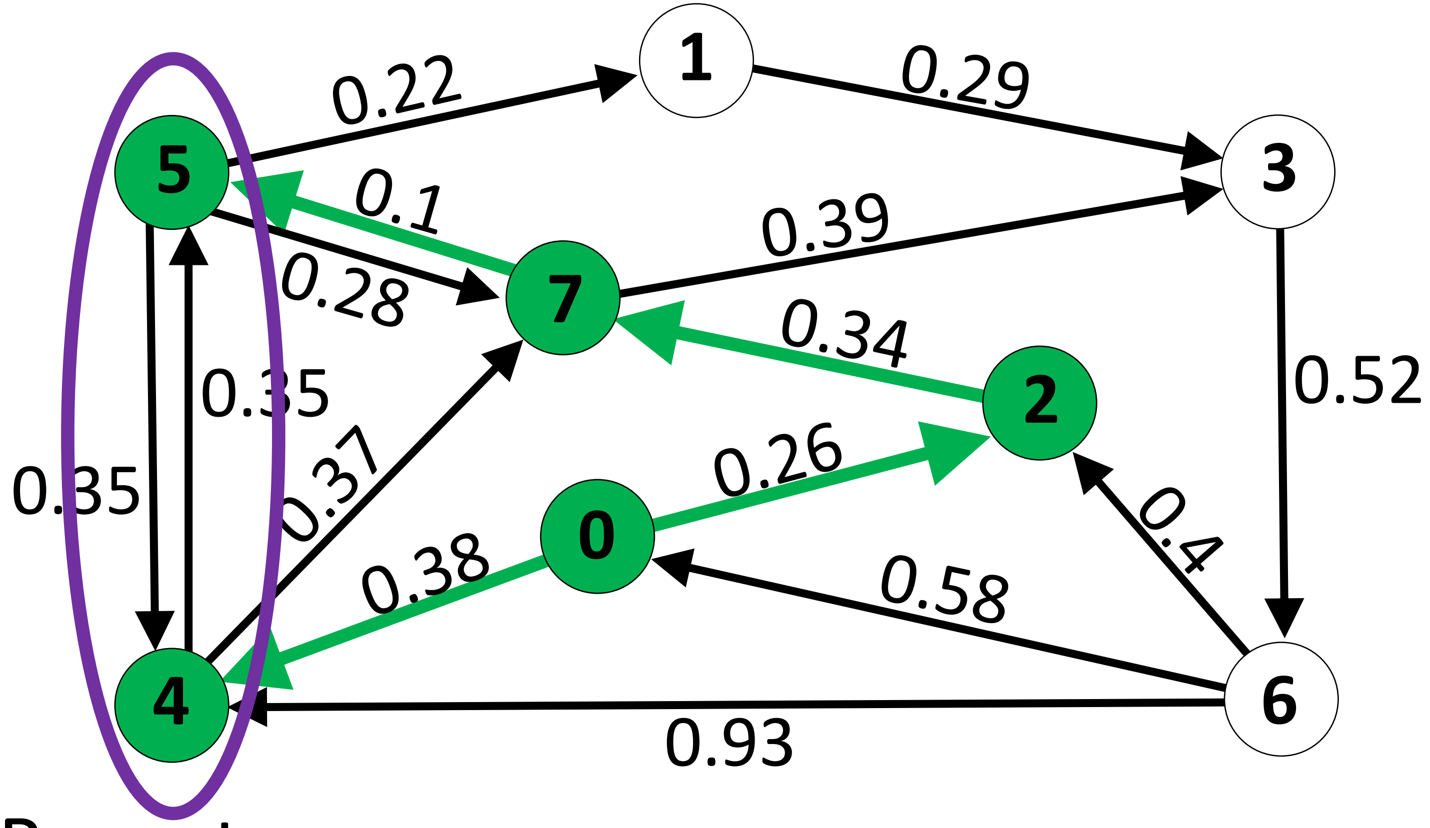
	Previous vertex
0	-
1	5
2	0
3	7
4	0
5	7
6	
7	2

Priority queue
1 (0.92)
3 (0.99)

vertex (distance)

Shortest Path

queue top	= 5 (0.70)



Distance from 0

0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	∞
7	0.60

Previous vertex

0	-
1	5
2	0
3	7
4	0
5	7
6	
7	2

Priority queue

1 (0.92)
3 (0.99)

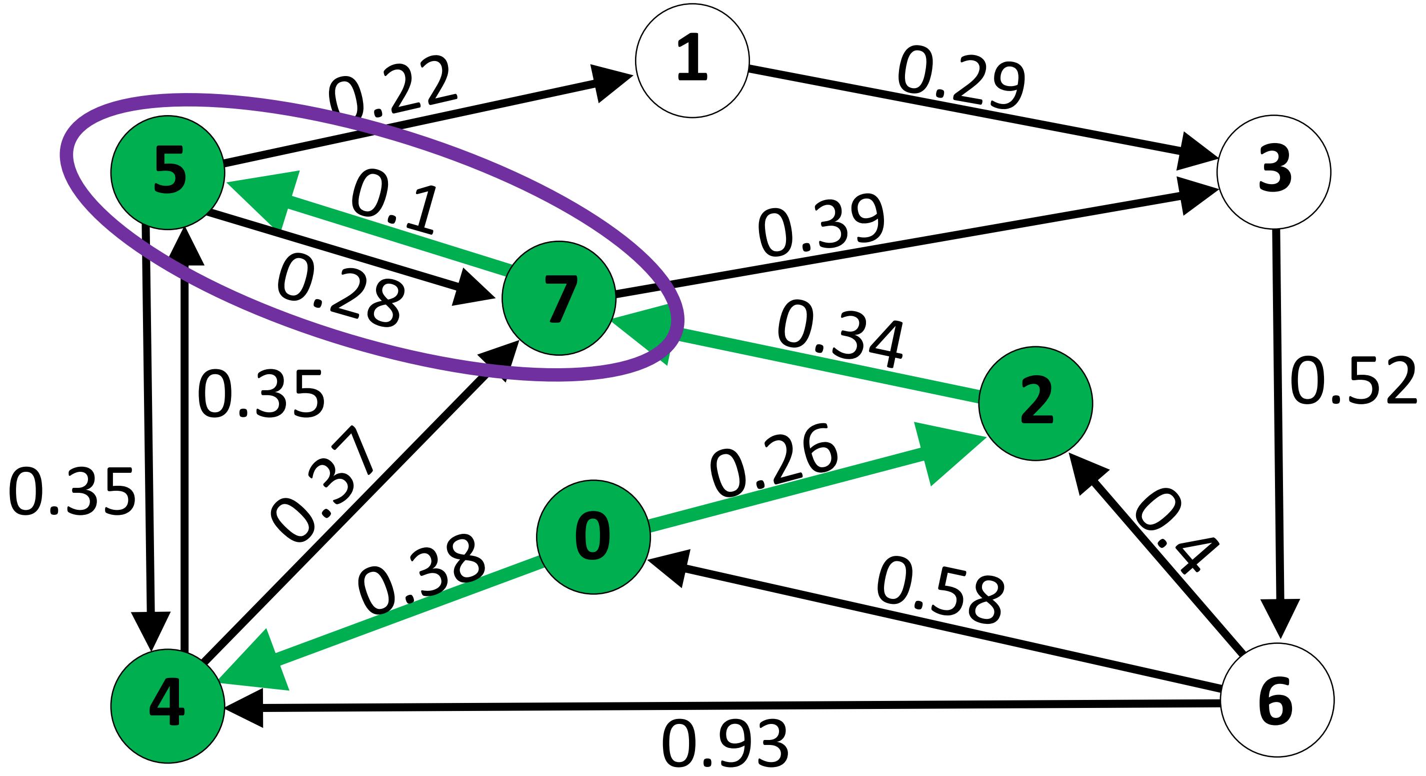
vertex (distance)

Repeat.

What about neighbor 4? distance[5] + weight(5, 4) = 0.70 + 0.35 = 1.05 $\not<$ 0.38 = distance[4]

Shortest Path

queue top = 5 (0.70)



Vertex	Distance from 0	Previous vertex	Priority queue
0	0	-	1 (0.92)
1	0.92	5	3 (0.99)
2	0.26	0	
3	0.99	7	
4	0.38	0	
5	0.70	7	
6	∞		
7	0.60	2	

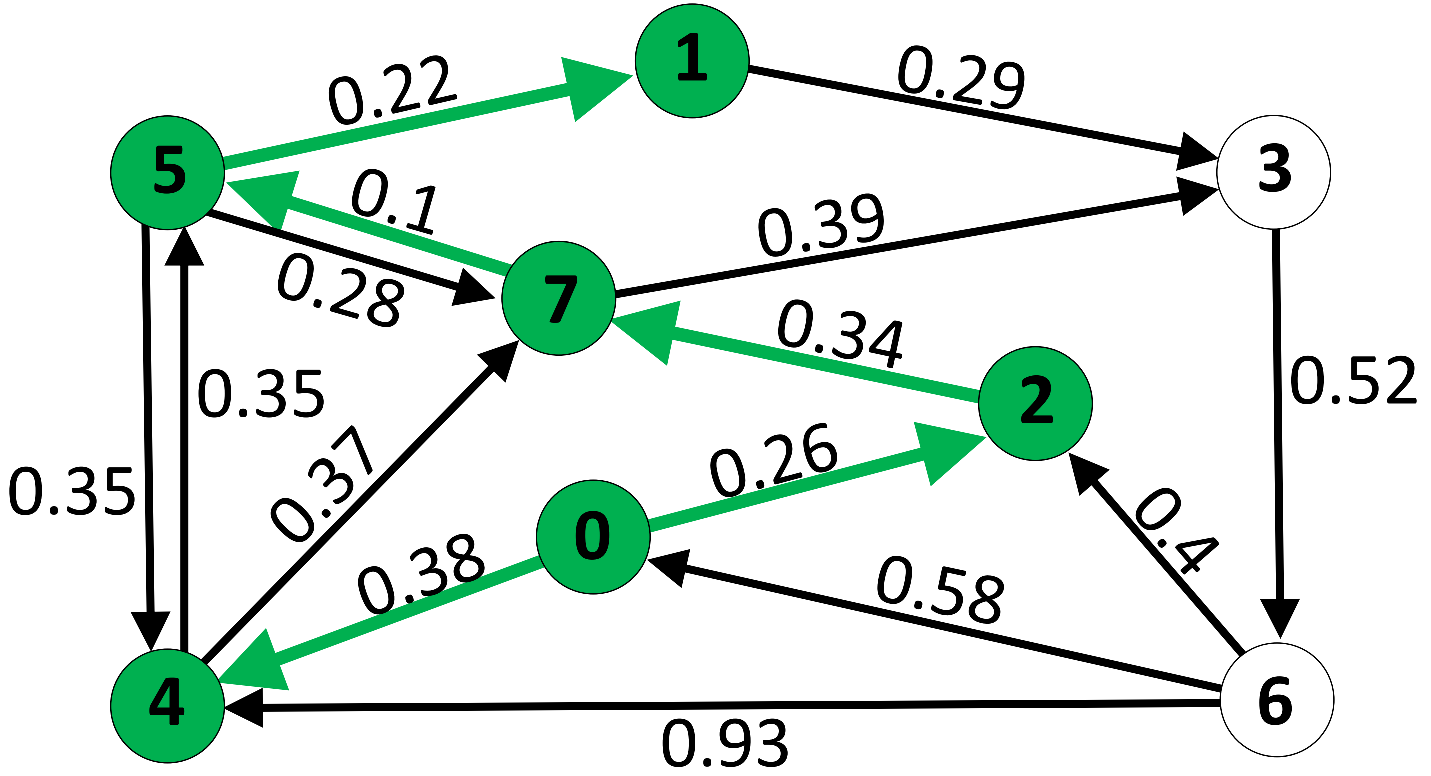
vertex (distance)

Repeat. **What about neighbor 7?**

distance[5] + weight(5, 7) = 0.70 + 0.28 = 0.98 $\not<$ 0.60 = distance[7]

Shortest Path

queue top = 1 (0.92)



Repeat.

Distance from 0

0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	∞
7	0.60

Previous vertex

0	-
1	5
2	0
3	7
4	0
5	7
6	
7	2

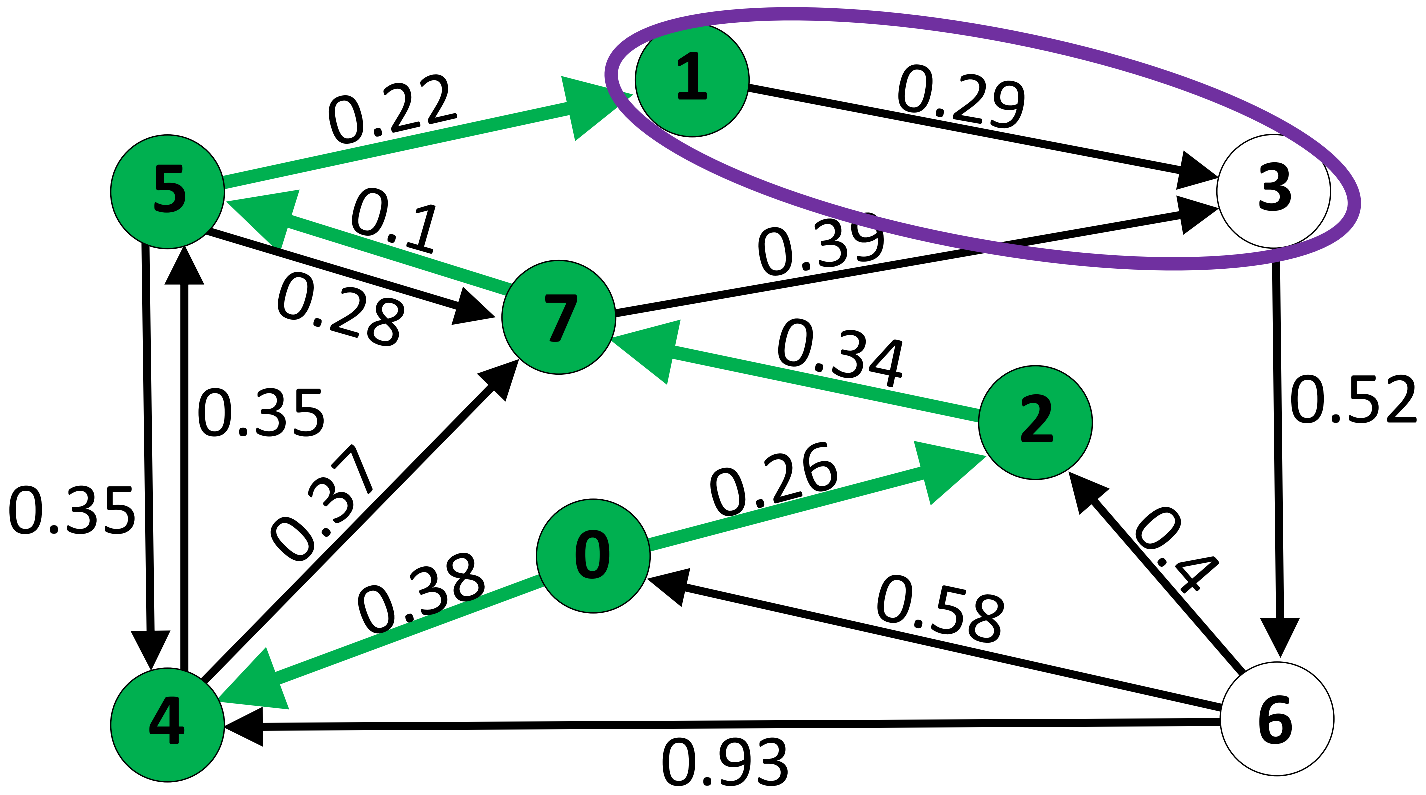
Priority queue

3 (0.99)

vertex (distance)

Shortest Path

queue top	= 1 (0.92)



	Distance from 0	Previous vertex
0	0	-
1	0.92	5
2	0.26	0
3	0.99	7
4	0.38	0
5	0.70	7
6	∞	
7	0.60	2

Priority queue

3 (0.99)

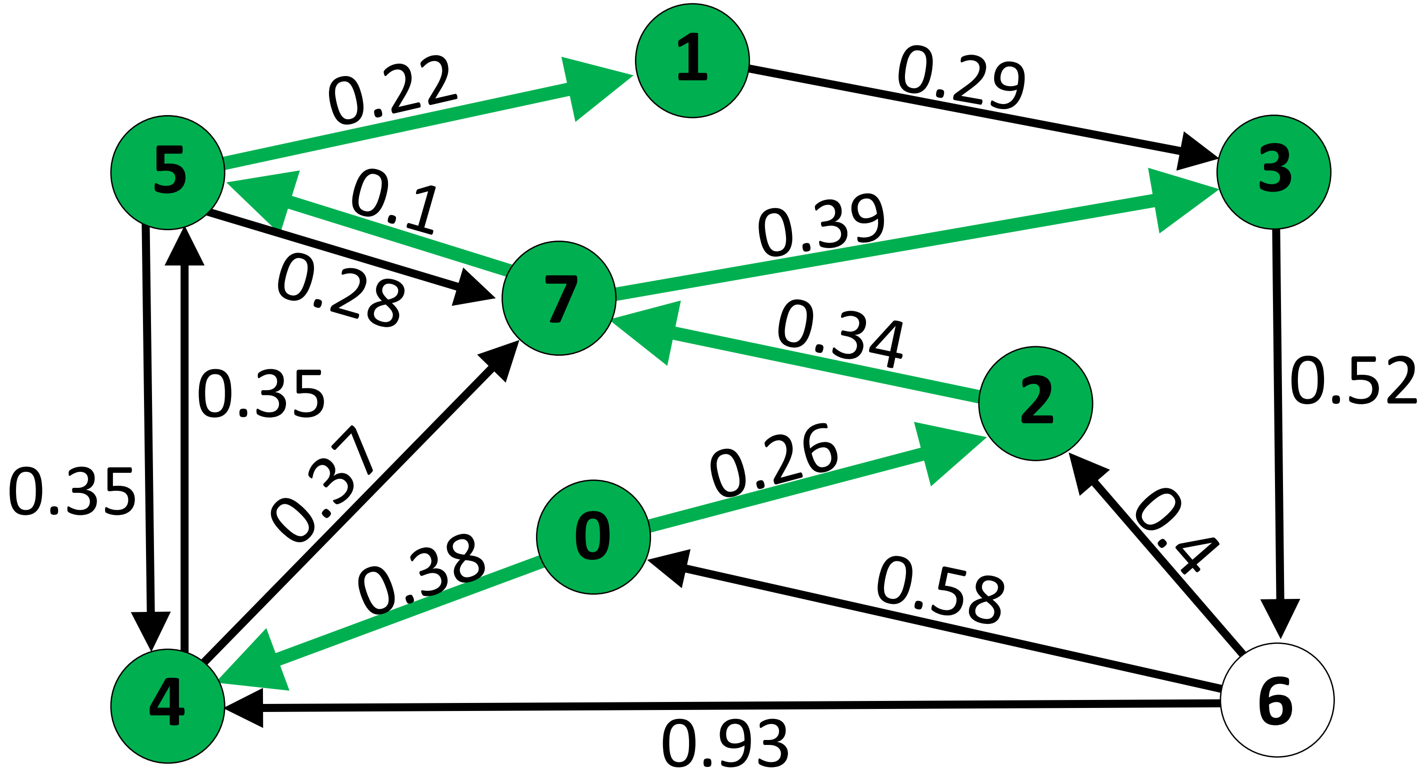
vertex (distance)

Repeat.

What about neighbor 3? 0.92 + 0.29 = 1.21 > 0.99

Shortest Path

queue top = 3 (0.99)



Repeat.

	Distance from 0
0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	∞
7	0.60

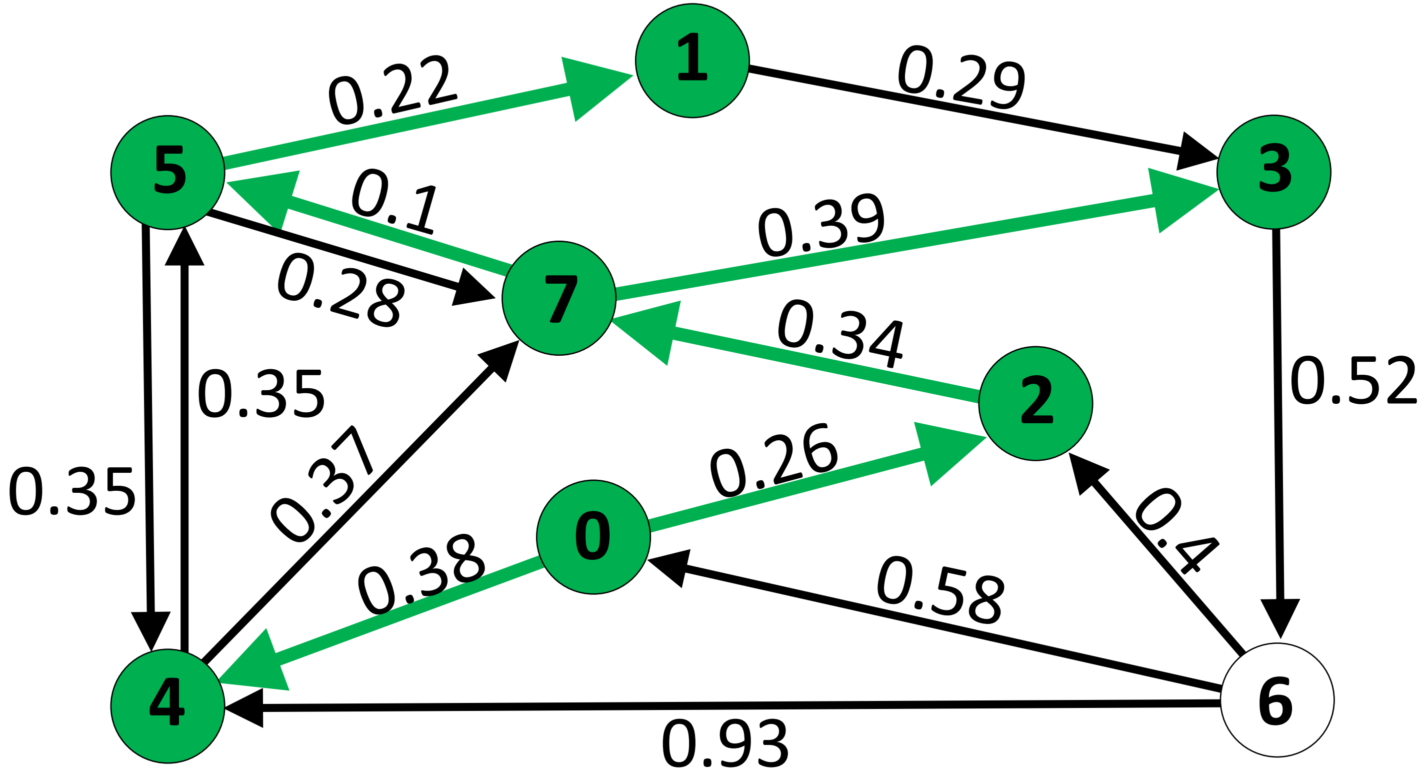
	Previous vertex
0	-
1	5
2	0
3	7
4	0
5	7
6	
7	2

Priority queue

vertex (distance)

Shortest Path

queue top	= 3 (0.99)



Repeat.

	Distance from 0
0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	1.51
7	0.60

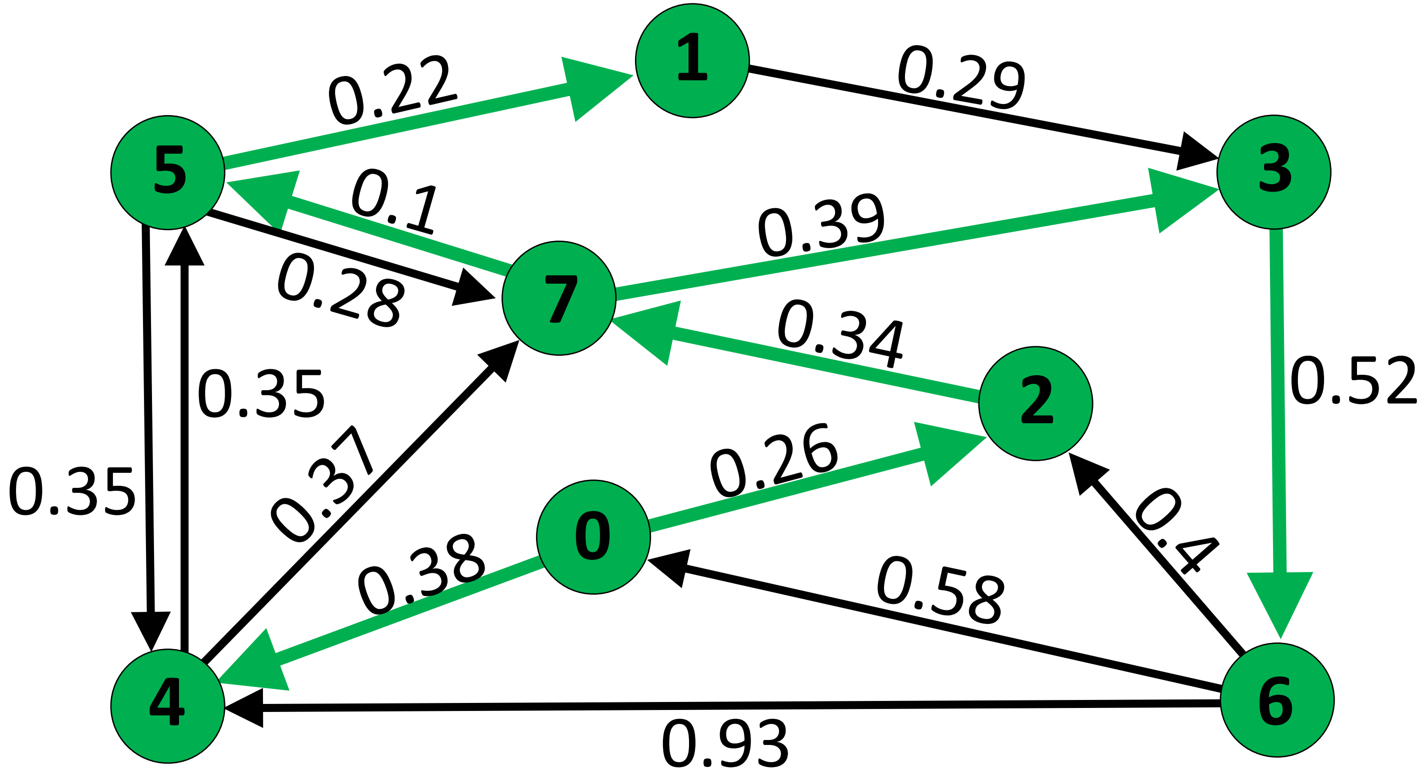
	Previous vertex
0	-
1	5
2	0
3	7
4	0
5	7
6	3
7	2

Priority queue
6 (1.51)

vertex (distance)

Shortest Path

queue top = 6 (1.51)



Distance from 0	
0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	1.51
7	0.60

Previous vertex	
0	-
1	5
2	0
3	7
4	0
5	7
6	3
7	2

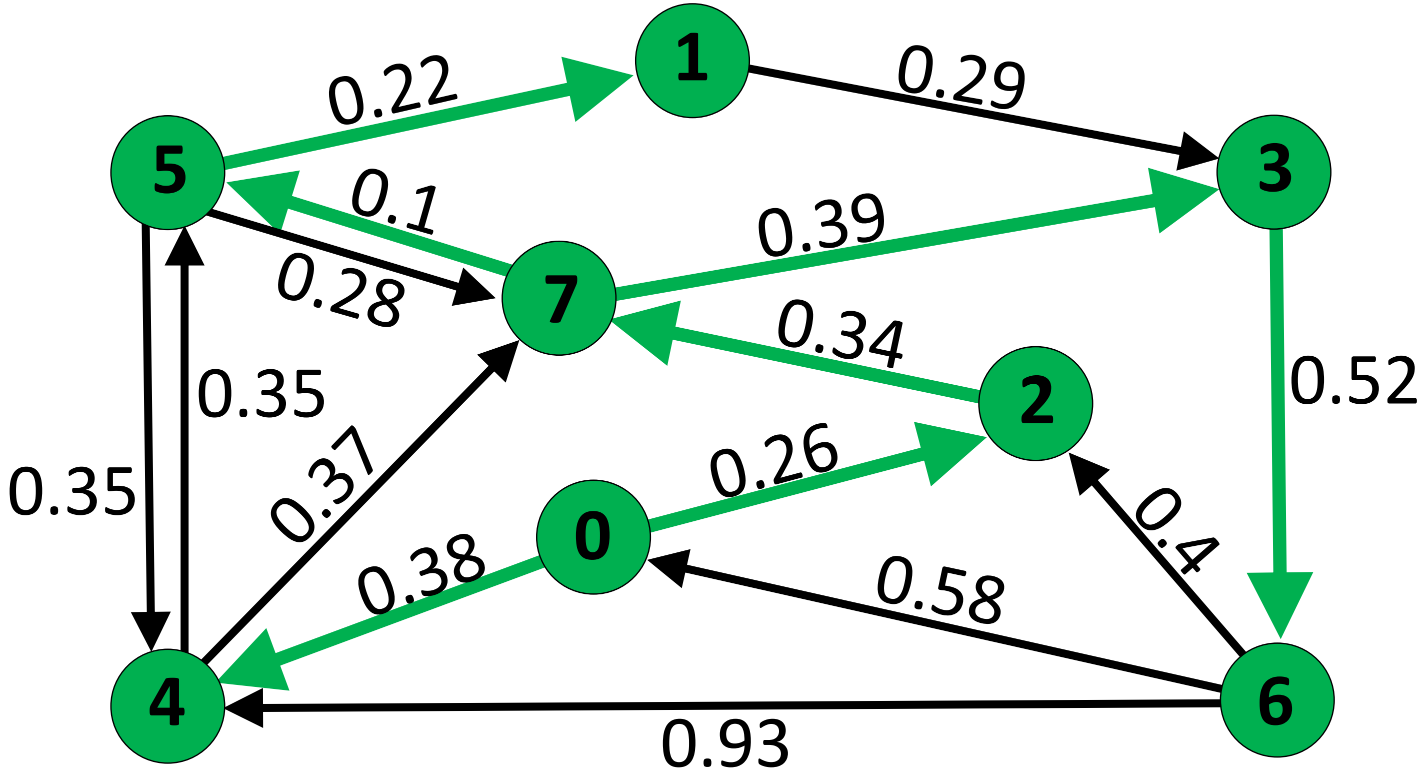
Priority queue

vertex (distance)

Repeat.

Shortest Path

queue top = 6 (1.51)



Repeat?

	Distance from 0
0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	1.51
7	0.60

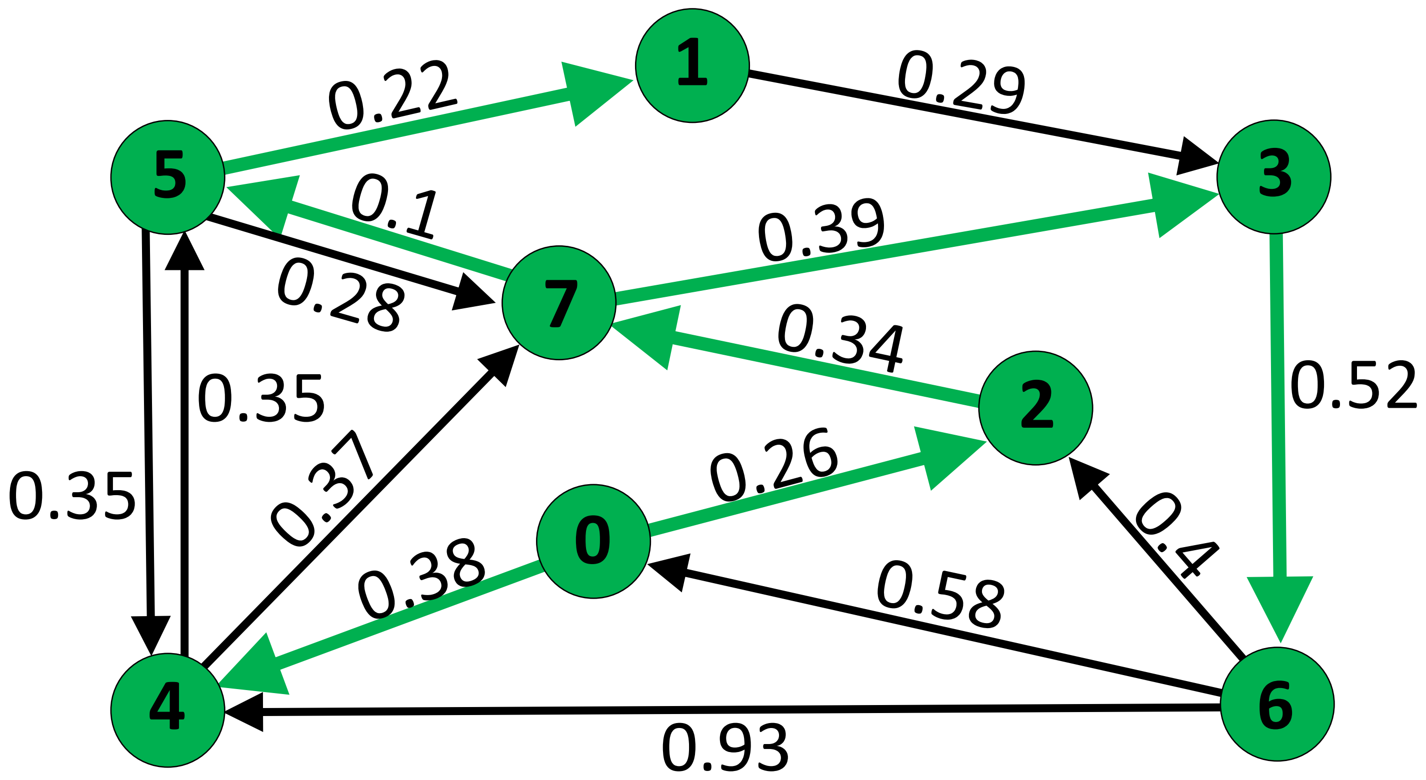
	Previous vertex
0	-
1	5
2	0
3	7
4	0
5	7
6	3
7	2

Priority queue

vertex (distance)

Shortest Path

queue top = 6 (1.51)



Repeat?

Neighbor 4? 1.51 + 0.93 > 0.38

Distance from 0

0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	1.51
7	0.60

Previous vertex

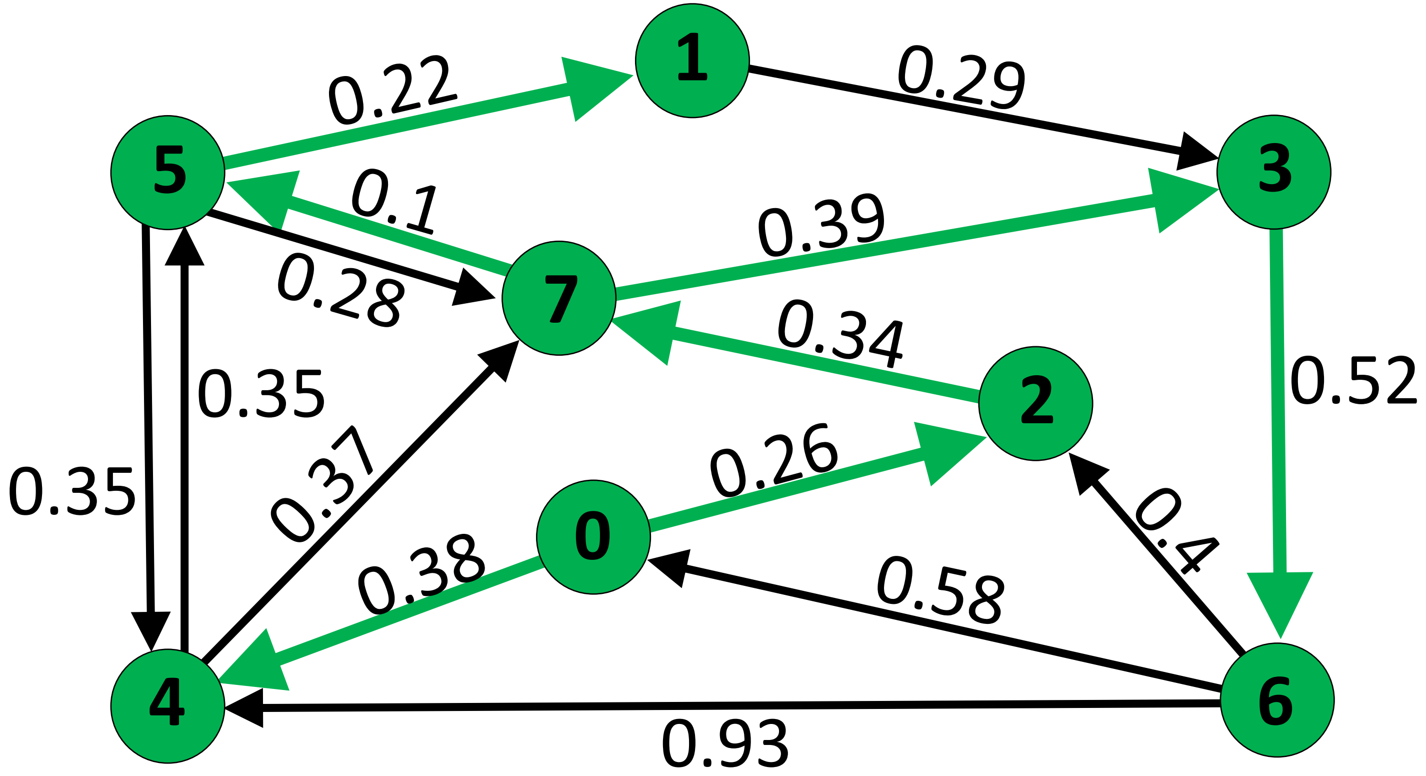
0	-
1	5
2	0
3	7
4	0
5	7
6	3
7	2

Priority queue

vertex (distance)

Shortest Path

queue top = 6 (1.51)



	Distance from 0	Previous vertex
0	0	-
1	0.92	5
2	0.26	0
3	0.99	7
4	0.38	0
5	0.70	7
6	1.51	3
7	0.60	2

Priority queue

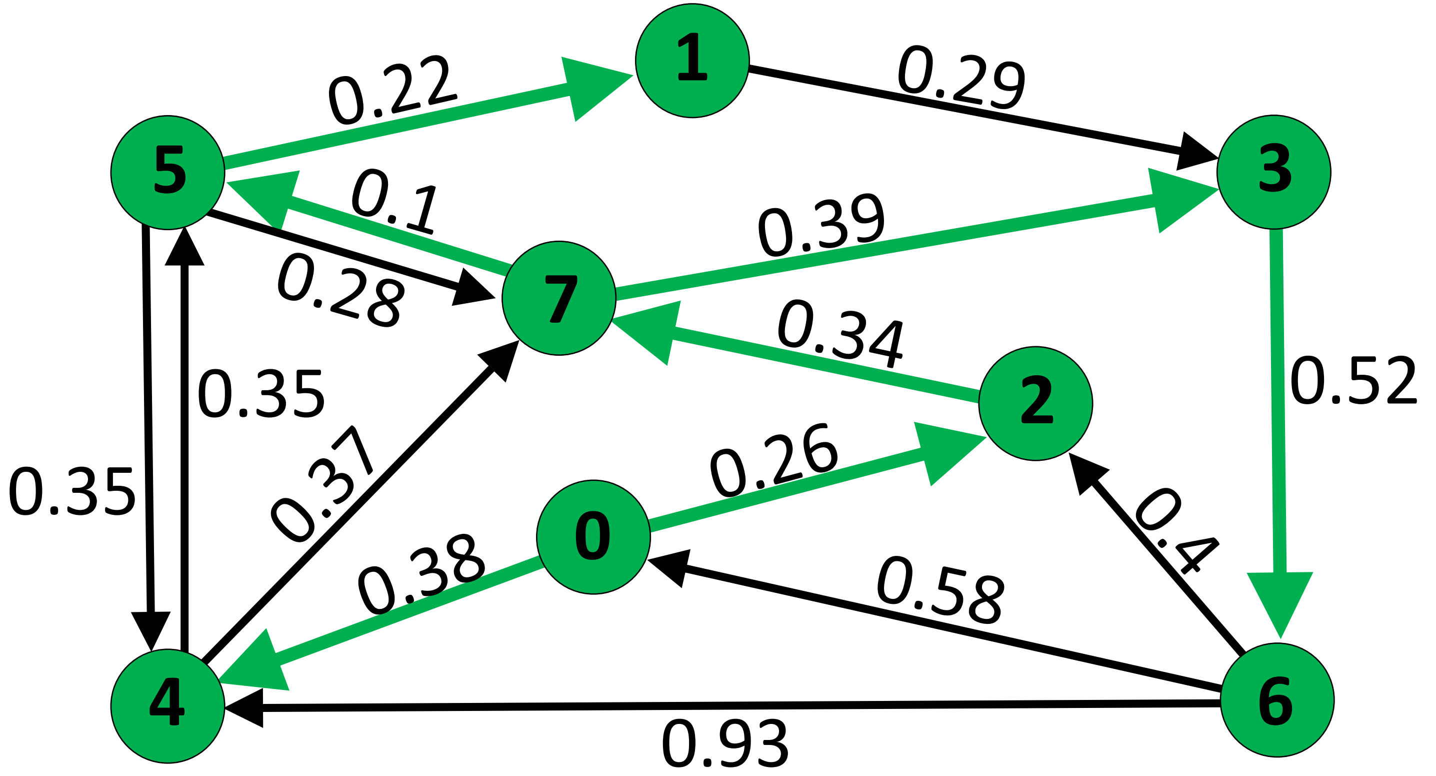
vertex (distance)

Repeat?

Neighbor 0? 1.51 + 0.58 > 0

Shortest Path

queue top = 6 (1.51)



	Distance from 0
0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	1.51
7	0.60

	Previous vertex
0	-
1	5
2	0
3	7
4	0
5	7
6	3
7	2

Priority queue

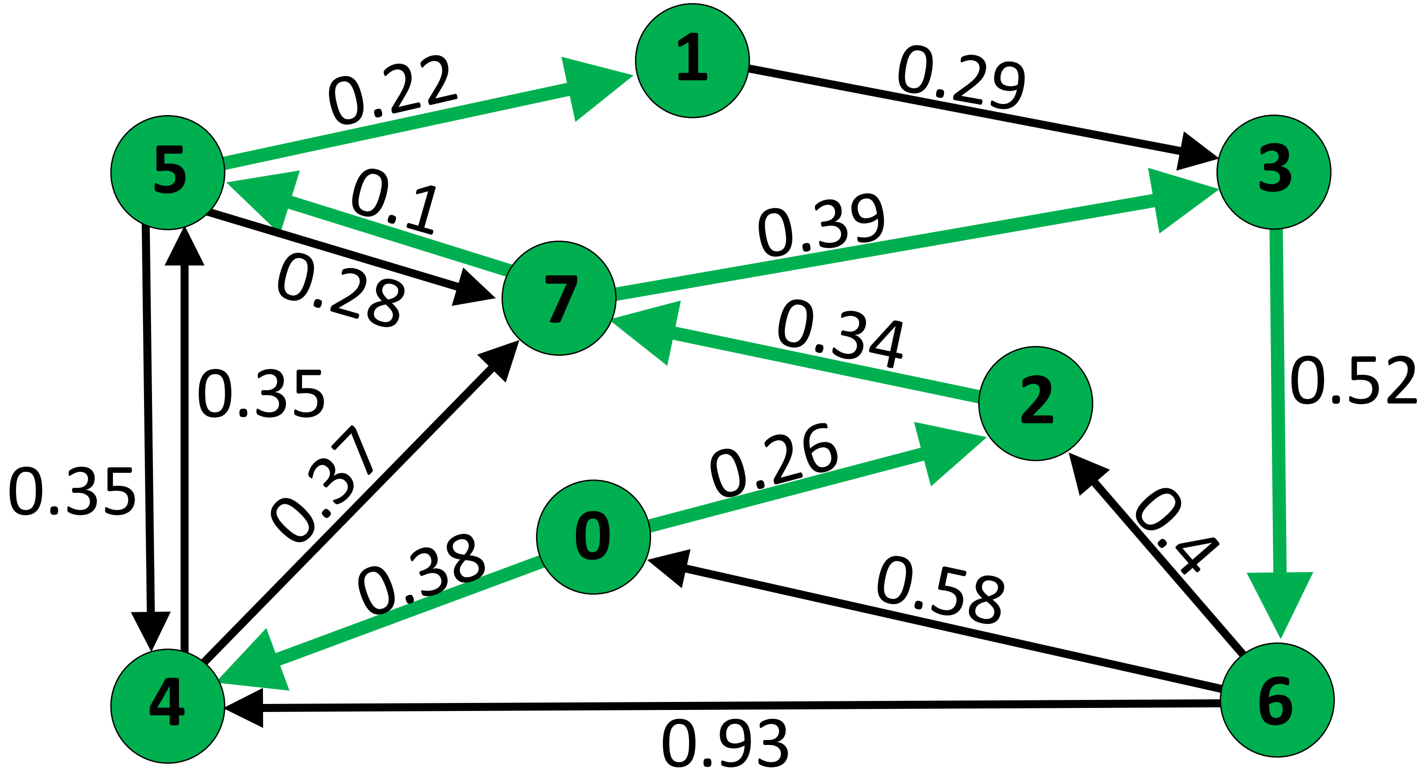
vertex (distance)

Repeat?

Neighbor 2? 1.51 + 0.4 > 0.26

Shortest Path

queue top = 6 (1.51)



When are we done?

	Distance from 0
0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	1.51
7	0.60

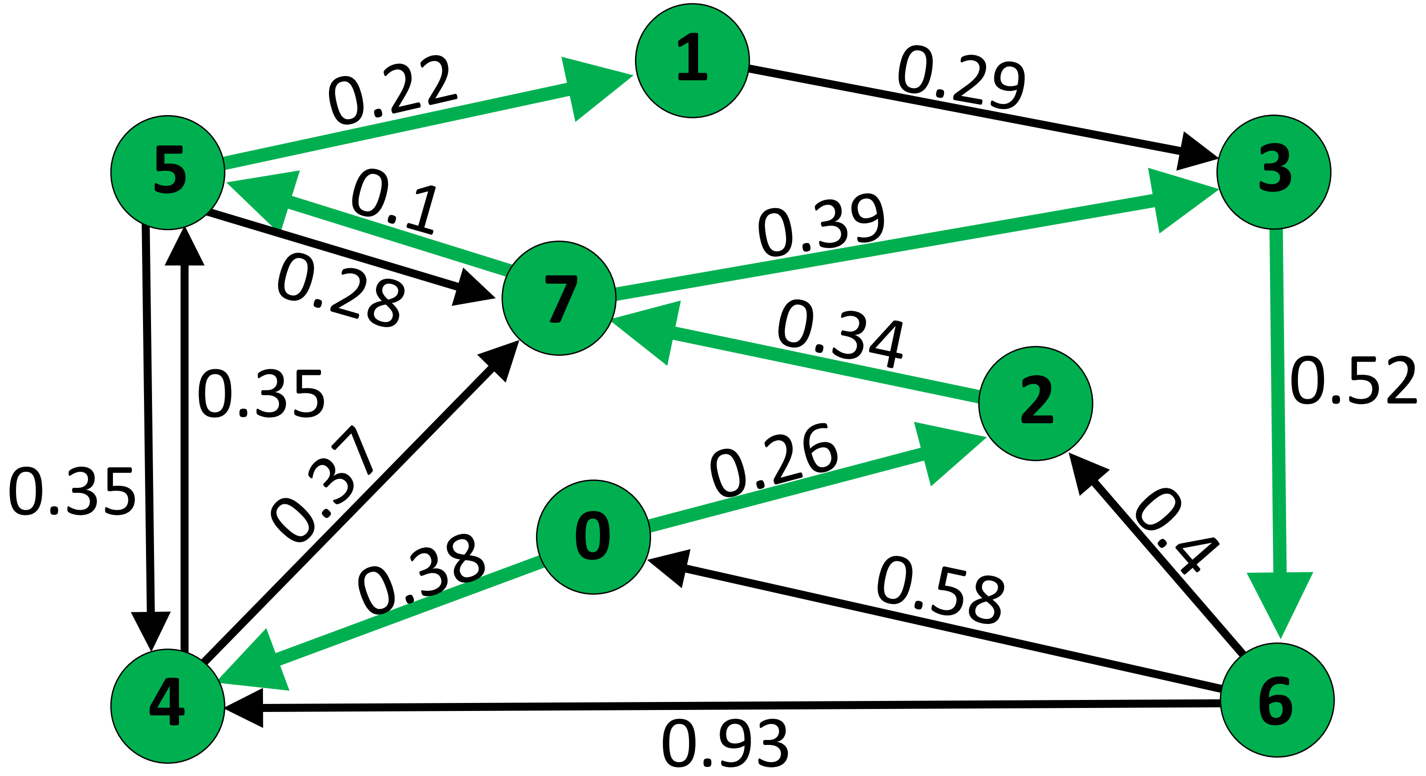
	Previous vertex
0	-
1	5
2	0
3	7
4	0
5	7
6	3
7	2

Priority queue

vertex (distance)

Shortest Path

queue top = 6 (1.51)



Distance from 0	
0	0
1	0.92
2	0.26
3	0.99
4	0.38
5	0.70
6	1.51
7	0.60

Previous vertex	
0	-
1	5
2	0
3	7
4	0
5	7
6	3
7	2

Priority queue

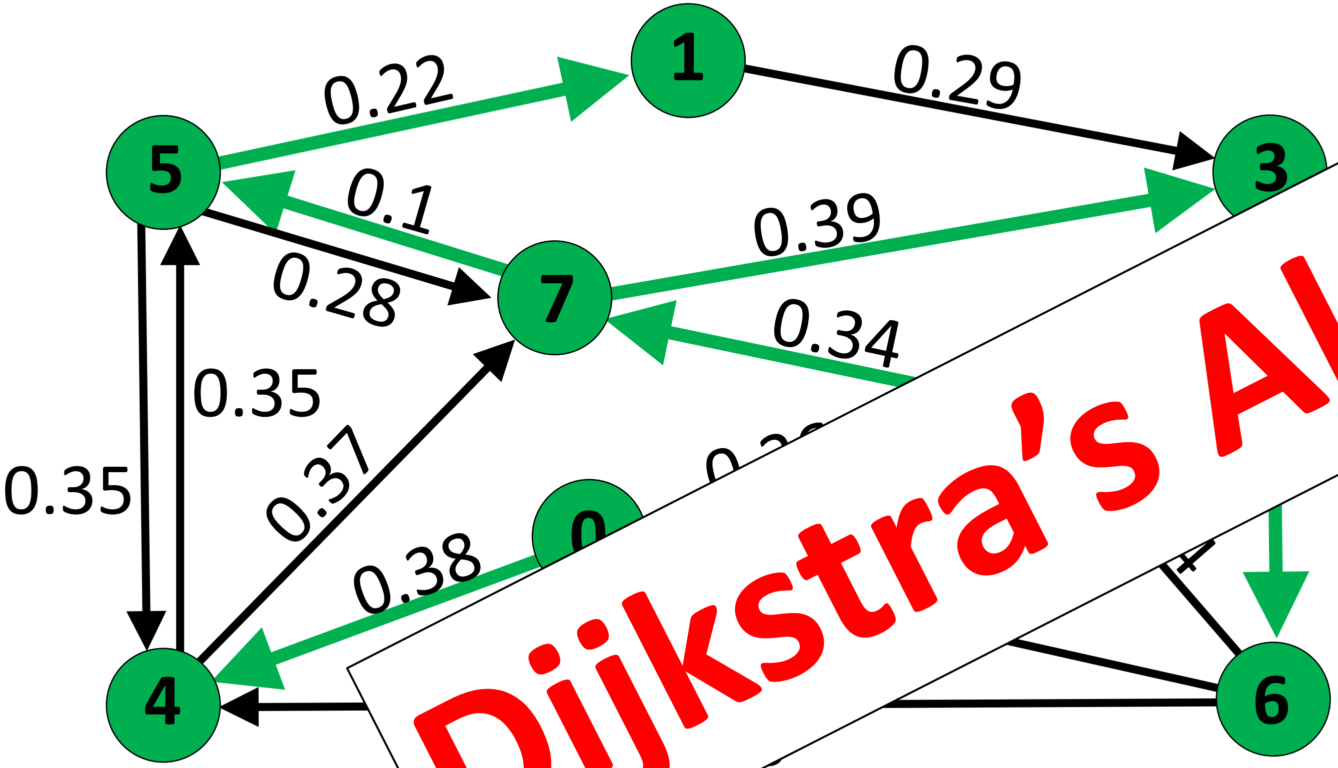
vertex (distance)

When are we done?

When the queue is empty!

Shortest Path

queue top = 6 (1.51)



	Distance from 0	Previous vertex
0	0	-
1	[illegible]	[illegible]
2	[illegible]	0
3	[illegible]	7
4	0.38	0
5	0.70	7
6	1.51	3
7	0.60	2

Priority queue

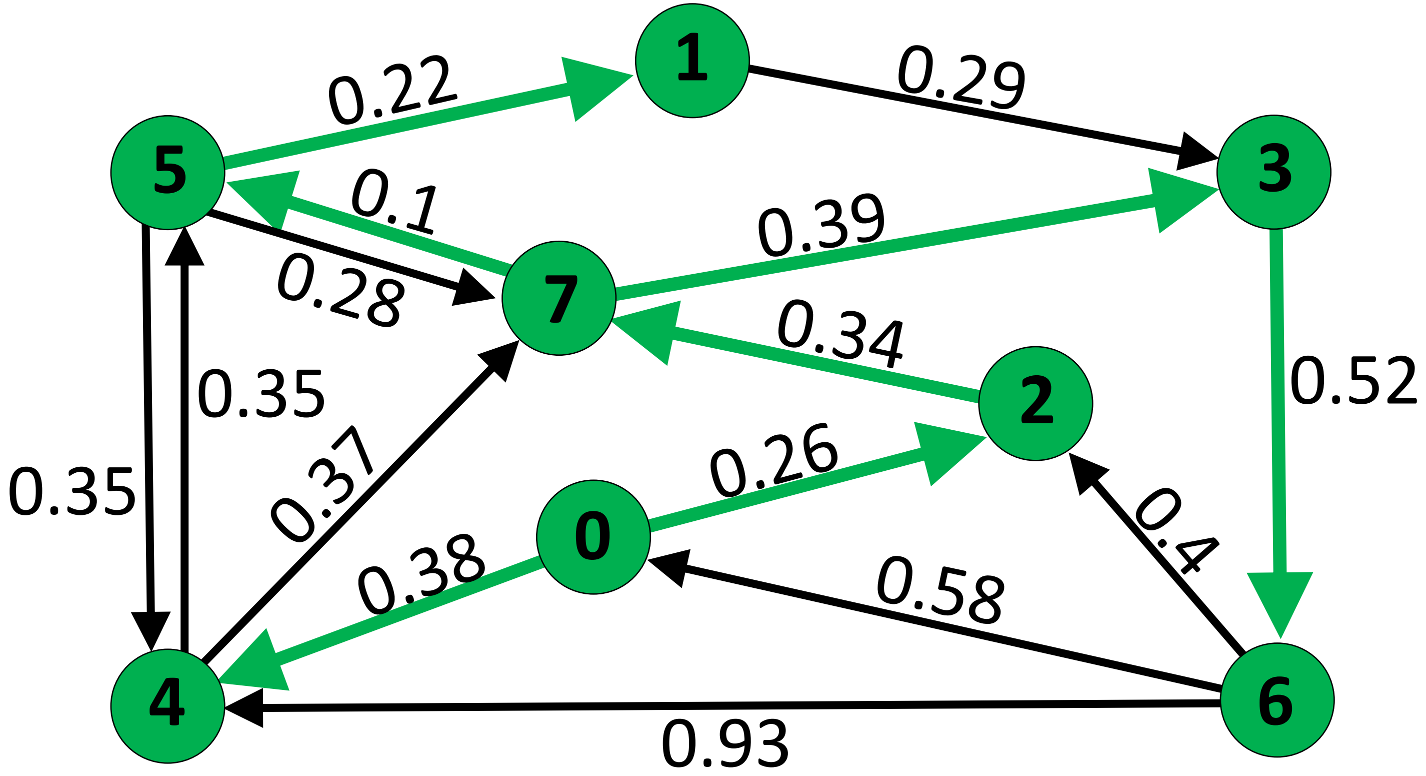
vertex (distance)

Dijkstra's Algorithm

When … we done?

When the queue is empty!

Shortest Path

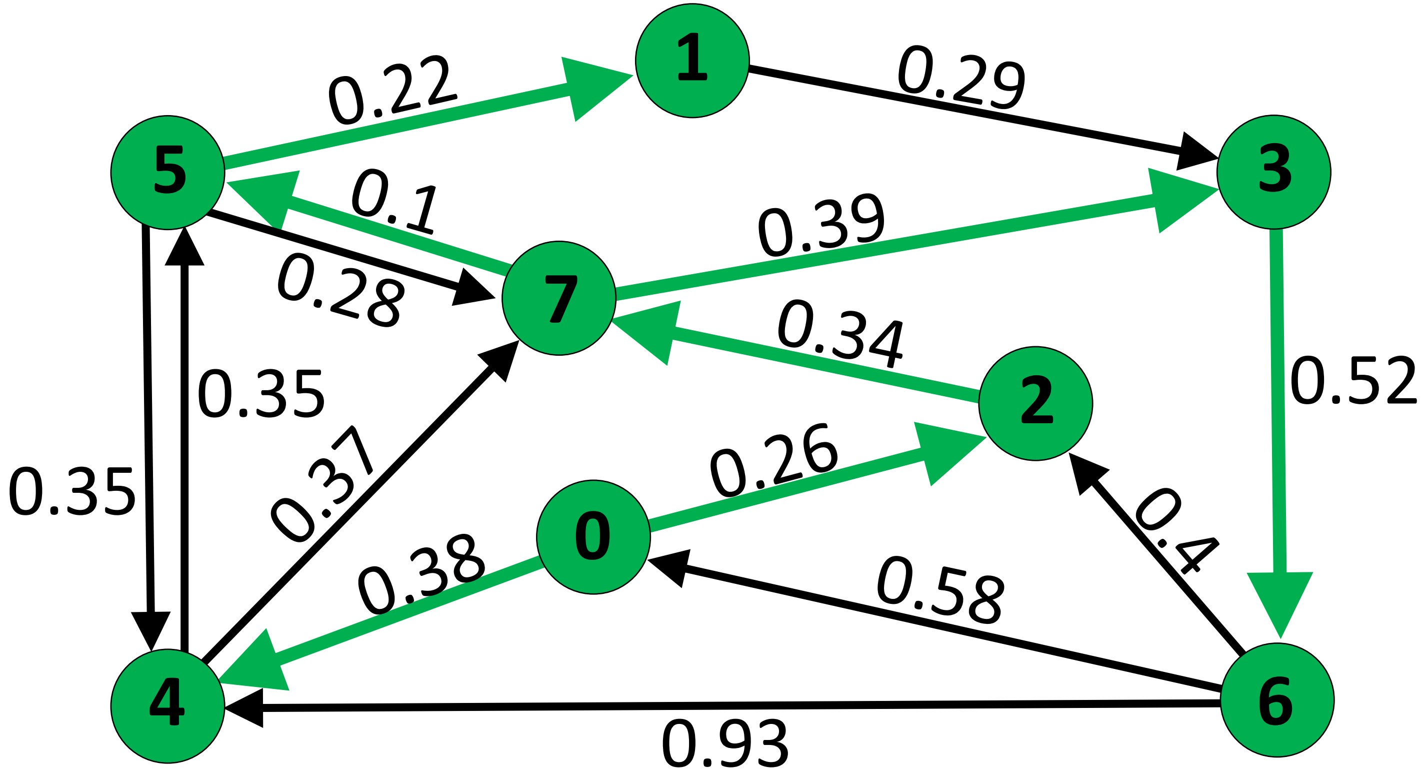


Assumptions:

- Graph is directed.
- Graph is edge-weighted.
- Edge weights are non-negative.
- Graph need not be simple (though our example will be).

What happens if there are self-loops?

Shortest Path



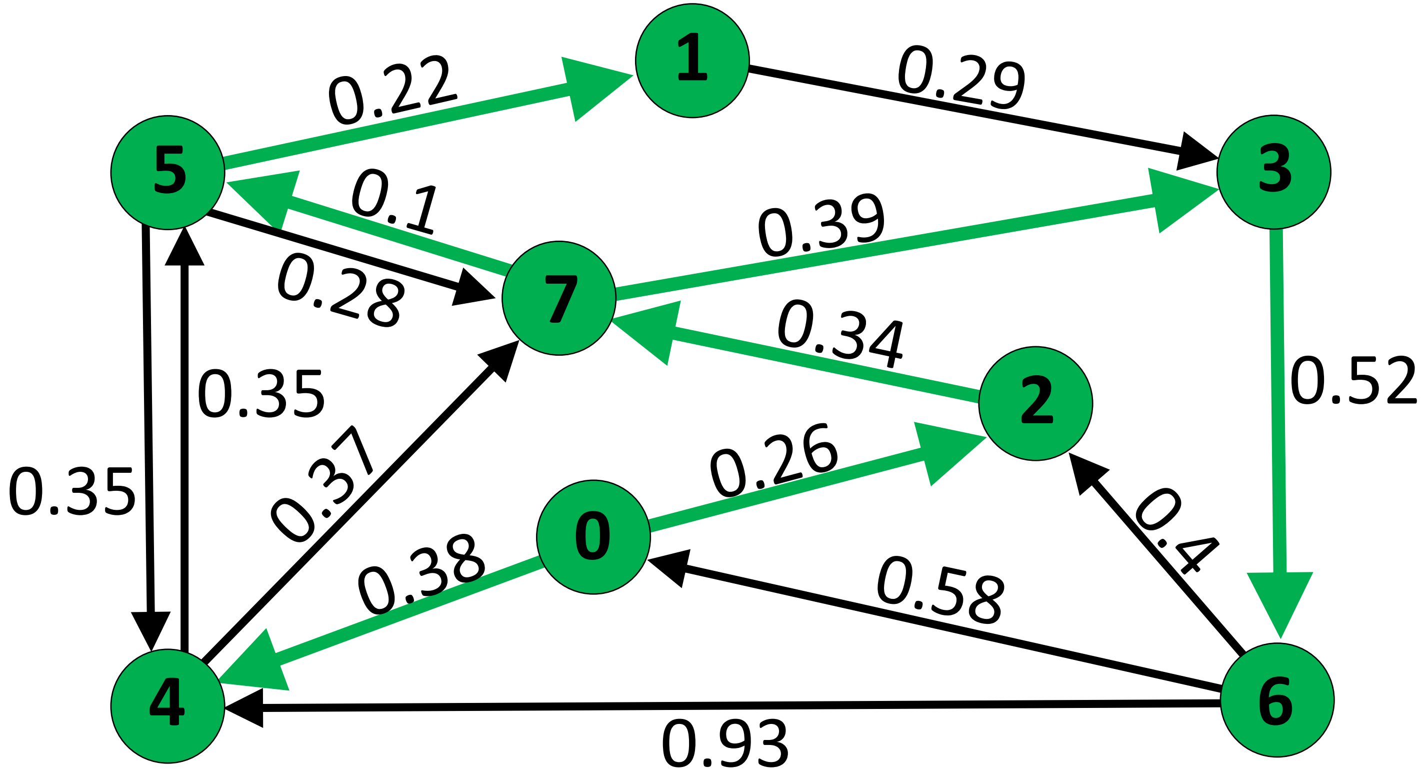
Assumptions:

- Graph is directed.
- Graph is edge-weighted.
- Edge weights are non-negative.
- Graph need not be simple (though our example will be).

What happens if there are self-loops?

They are never taken, since they will never lower the cost of a path.

Shortest Path

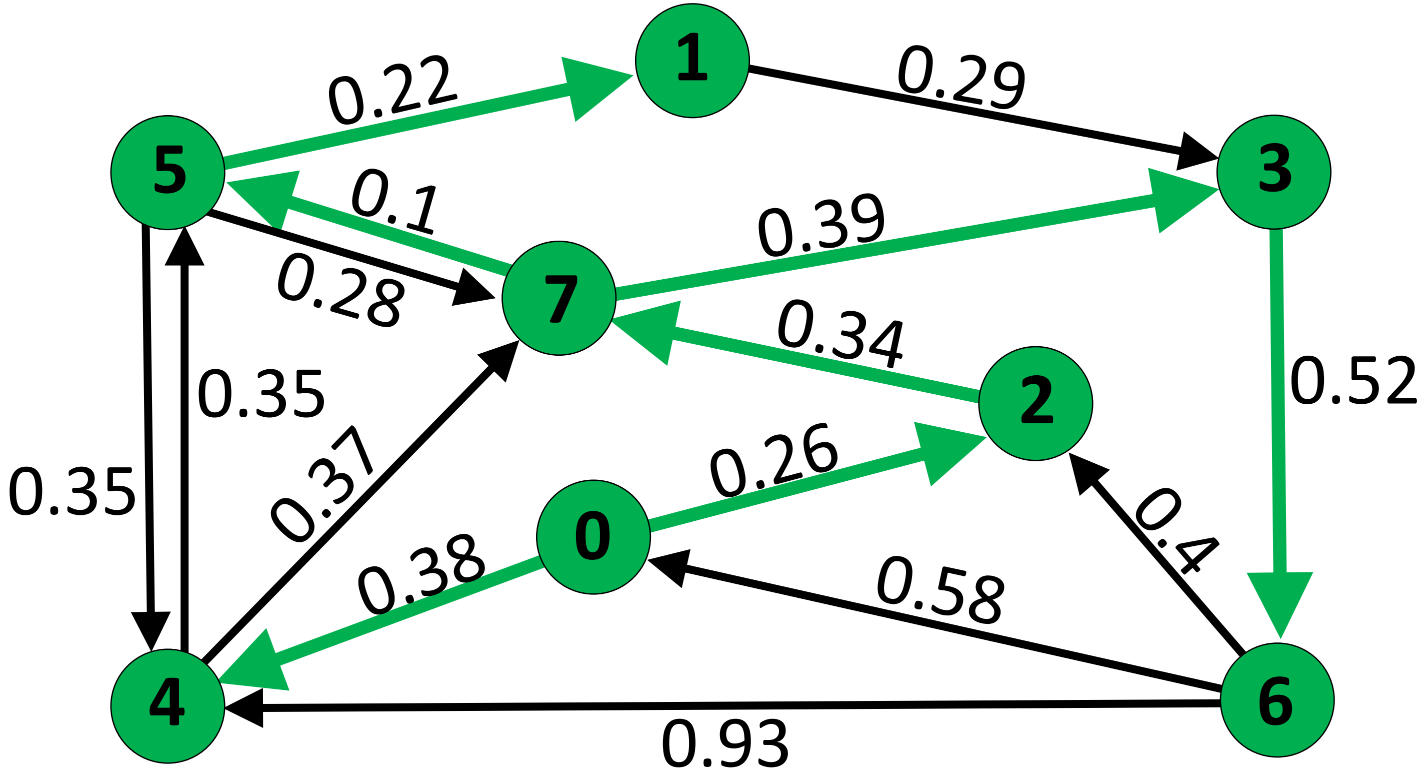


Assumptions:

- Graph is directed.
- Graph is edge-weighted.
- Edge weights are non-negative.
- Graph need not be simple (though our example will be).

What happens if there are parallel edges?

Shortest Path



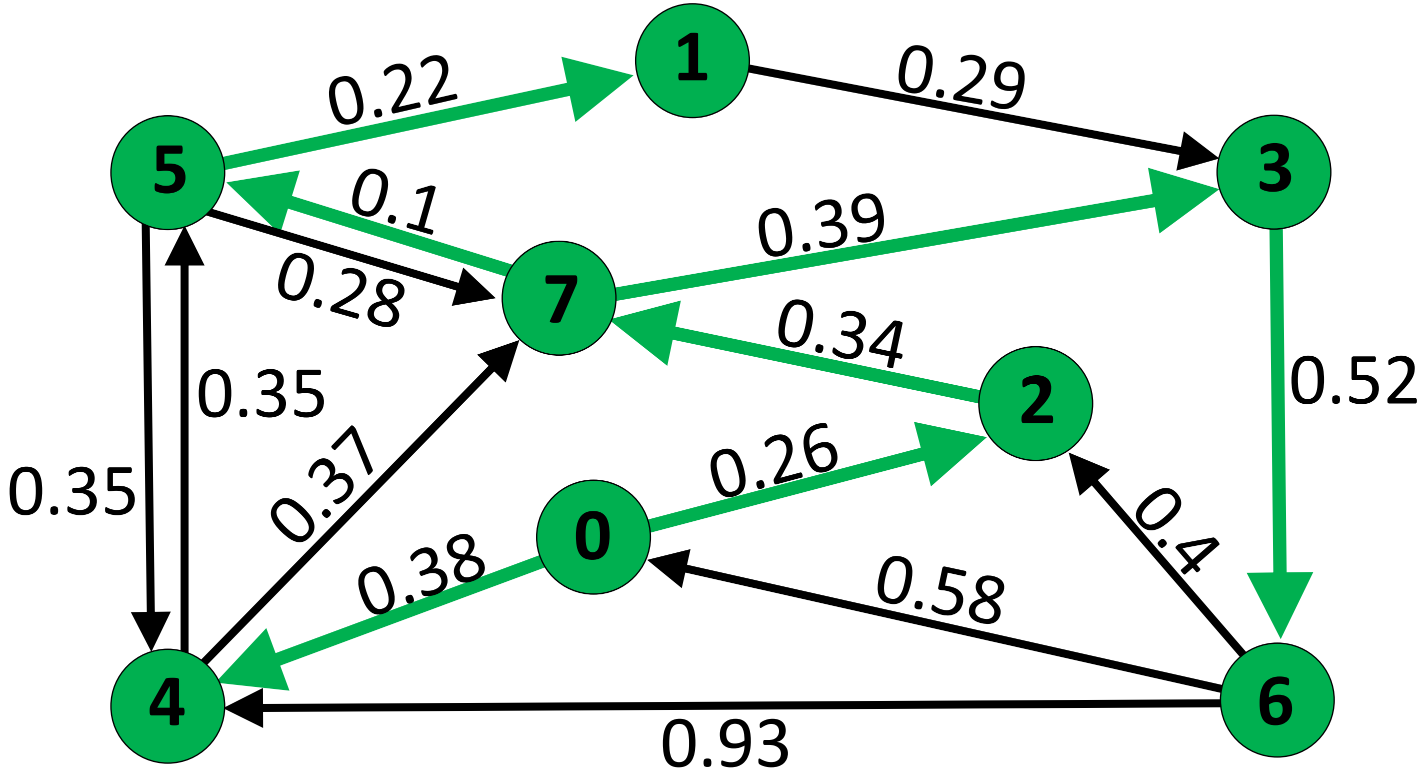
Assumptions:

- Graph is directed.
- Graph is edge-weighted.
- Edge weights are non-negative.
- Graph need not be simple (though our example will be).

What happens if there are parallel edges?

The cheapest one is taken and all others are ignored.

Shortest Path



Assumptions:

- Graph is directed.
- Graph is edge-weighted.
- Edge weights are non-negative.
- Graph need not be simple (though our example will be).

What happens if there are negative weights?

```java
public class Edge implements Comparable<Edge>{

	private int sourceVertex;
	private int destVertex;
	private double weight;

	public Edge(int vertex1, int vertex2, double weight) {
		this.sourceVertex = vertex1;
		this.destVertex = vertex2;
		this.weight = weight;
	}
```