CSCI 232:

Data Structures and Algorithms

Graphs (Representation)

Reese Pearsall

Summer 2025

How could we visualize: The US Road Network?

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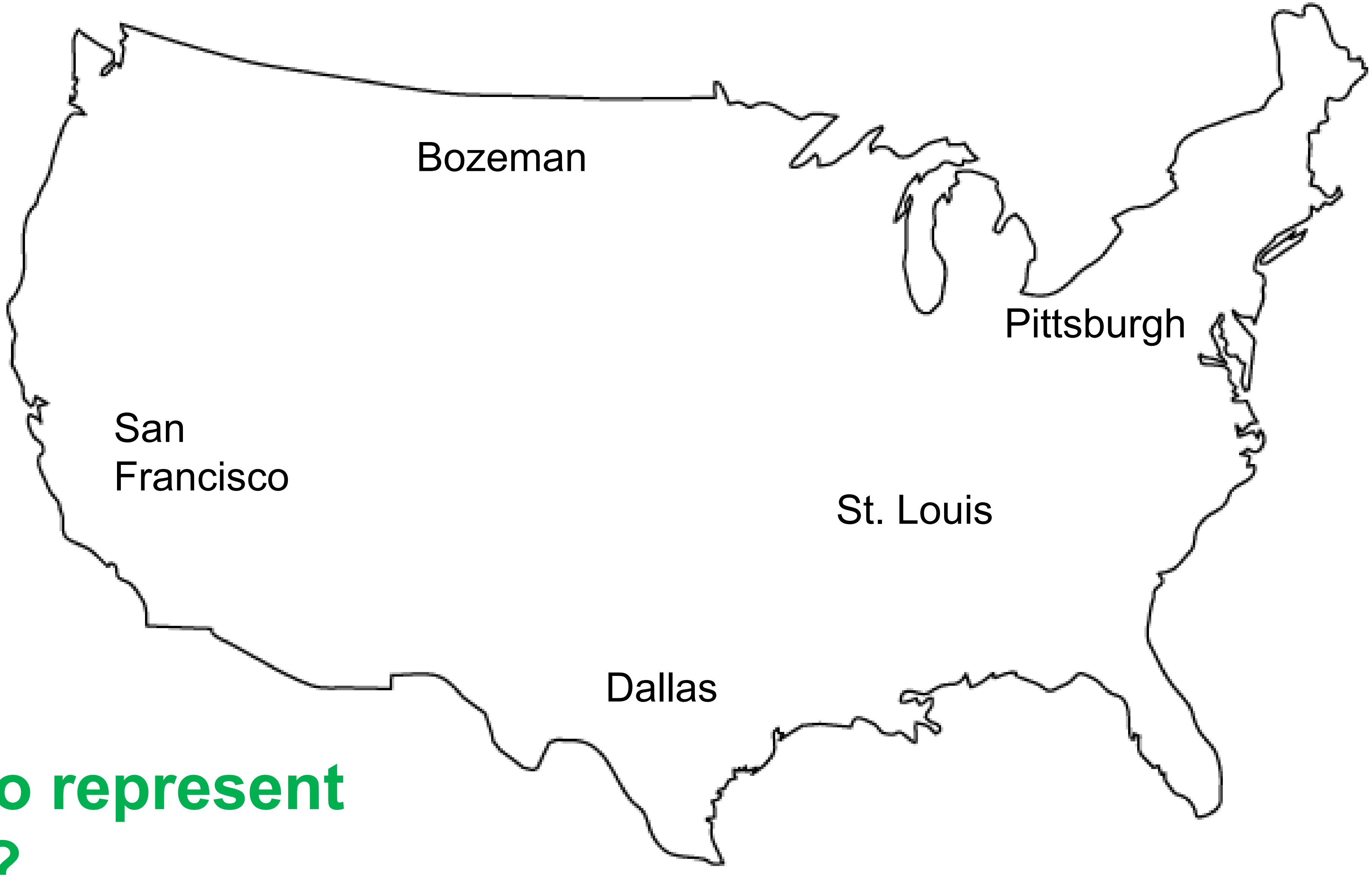

How to represent cities?

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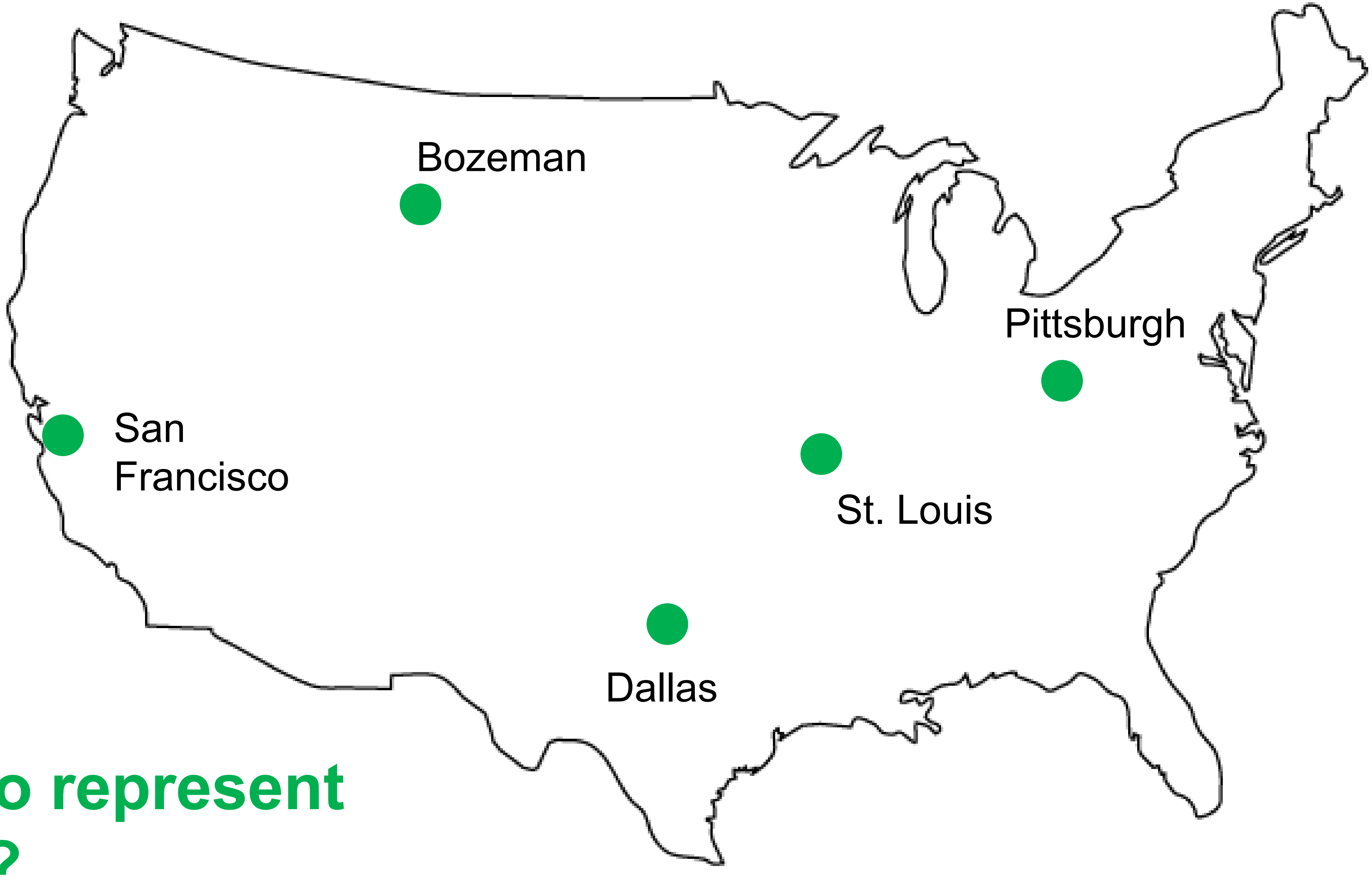

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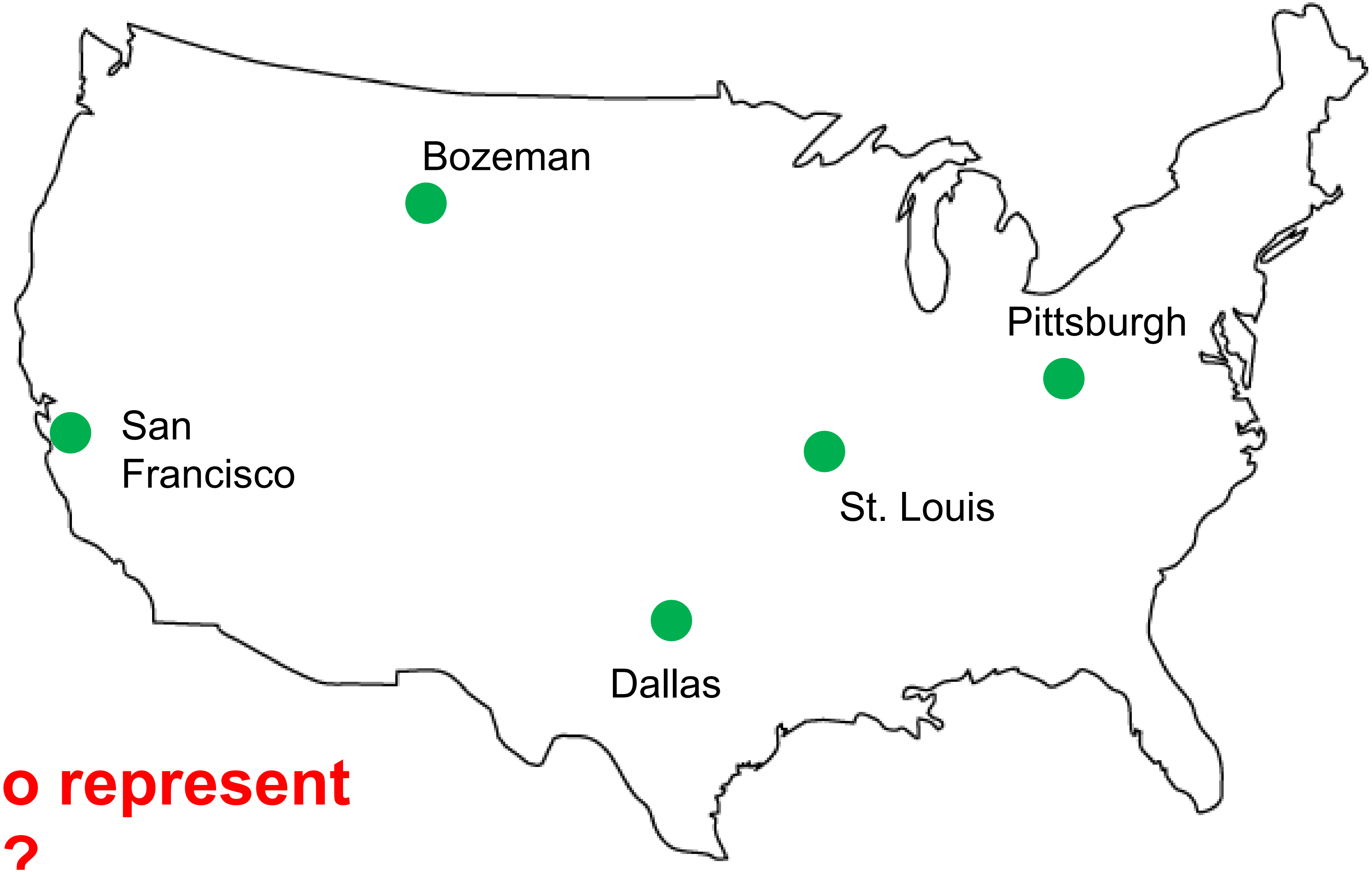

How to represent roads?

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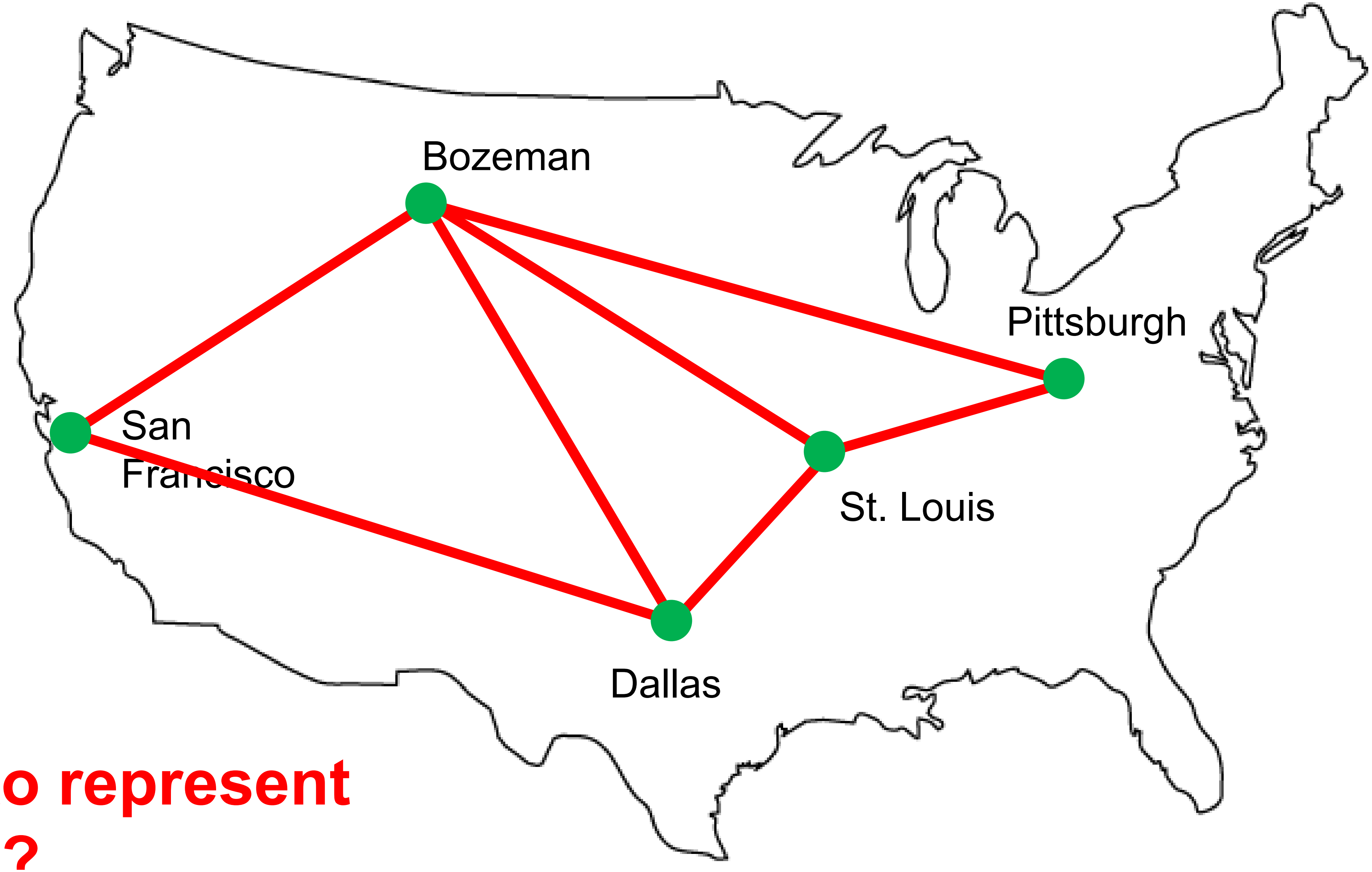

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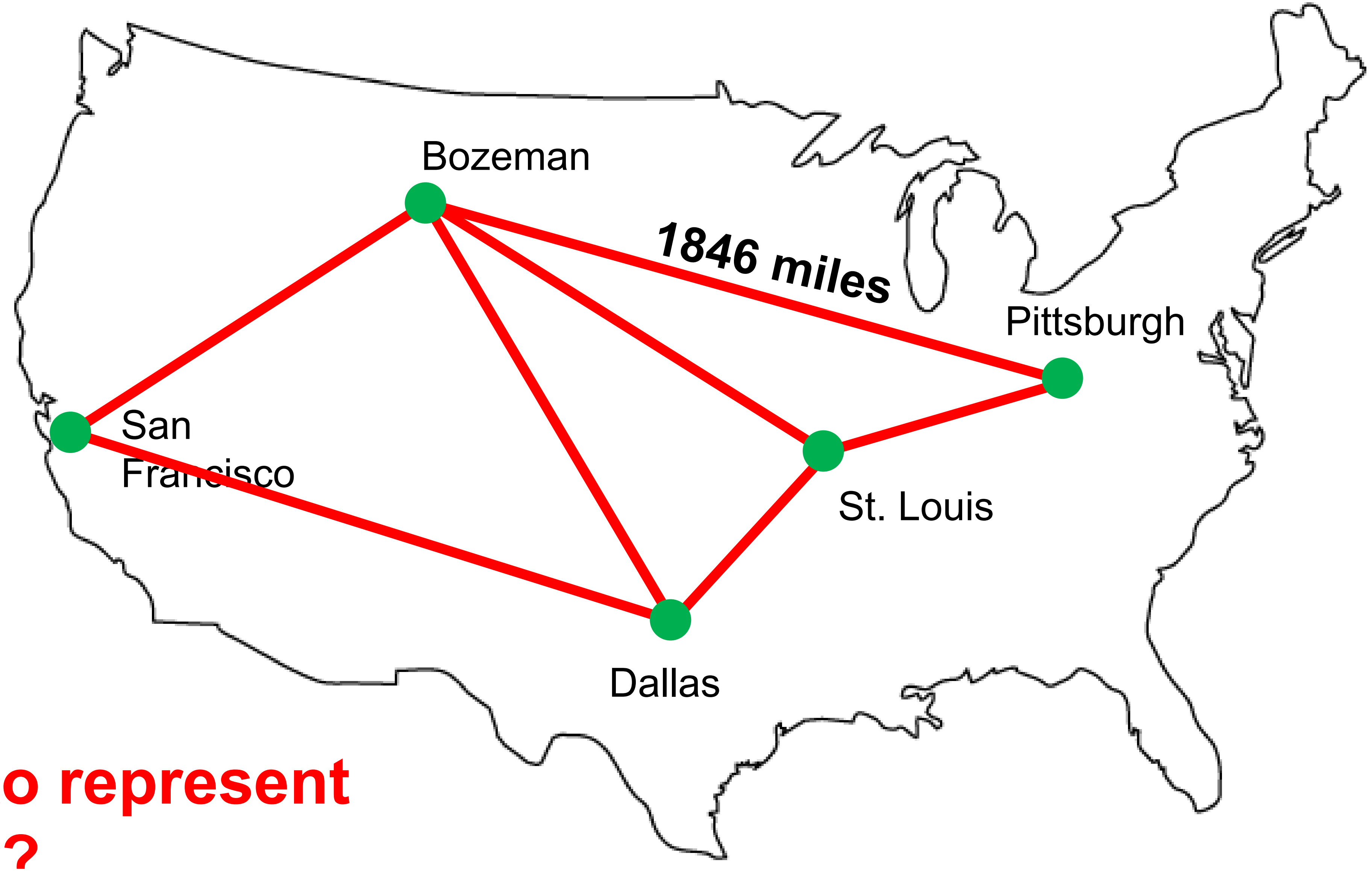

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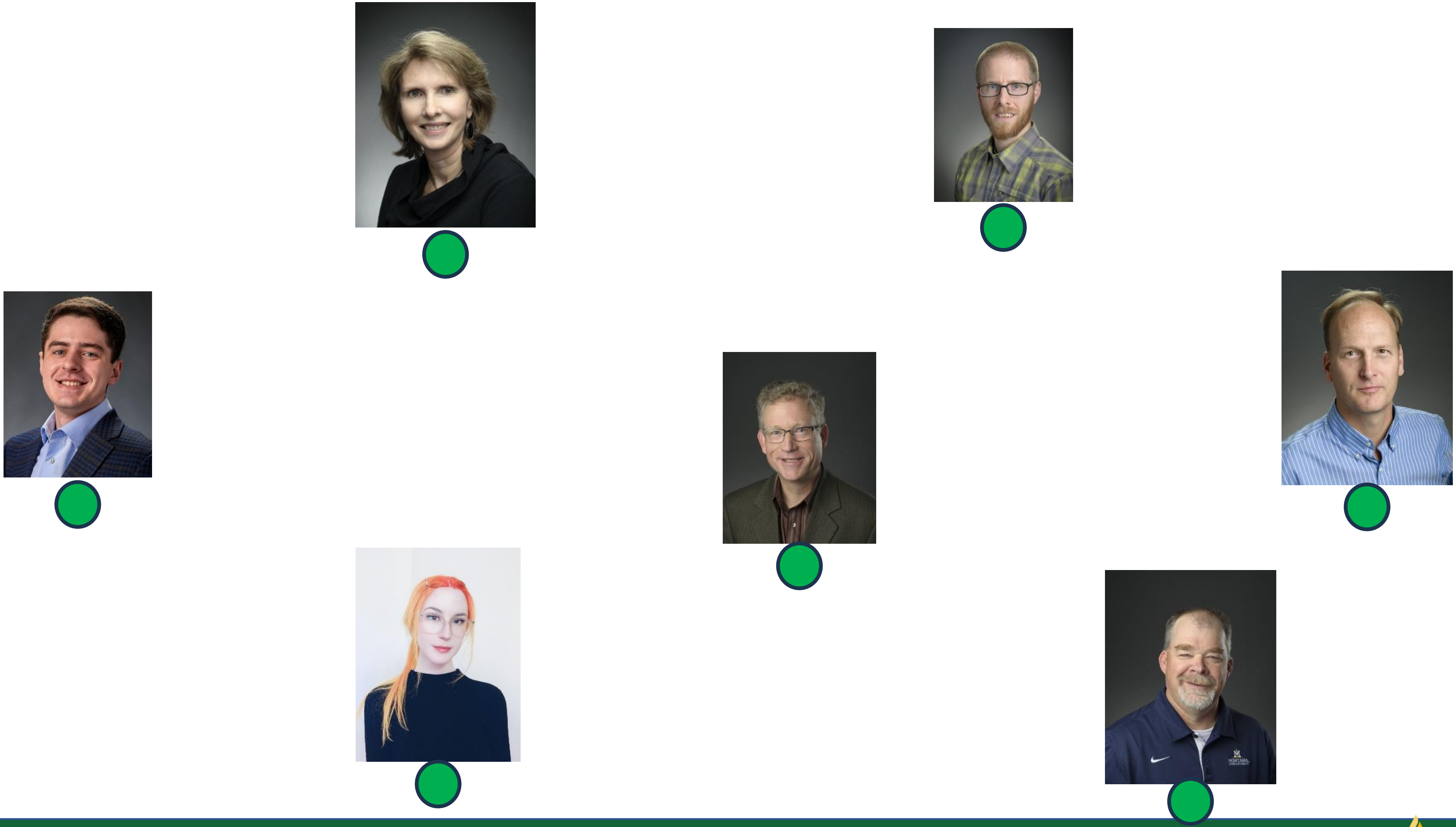

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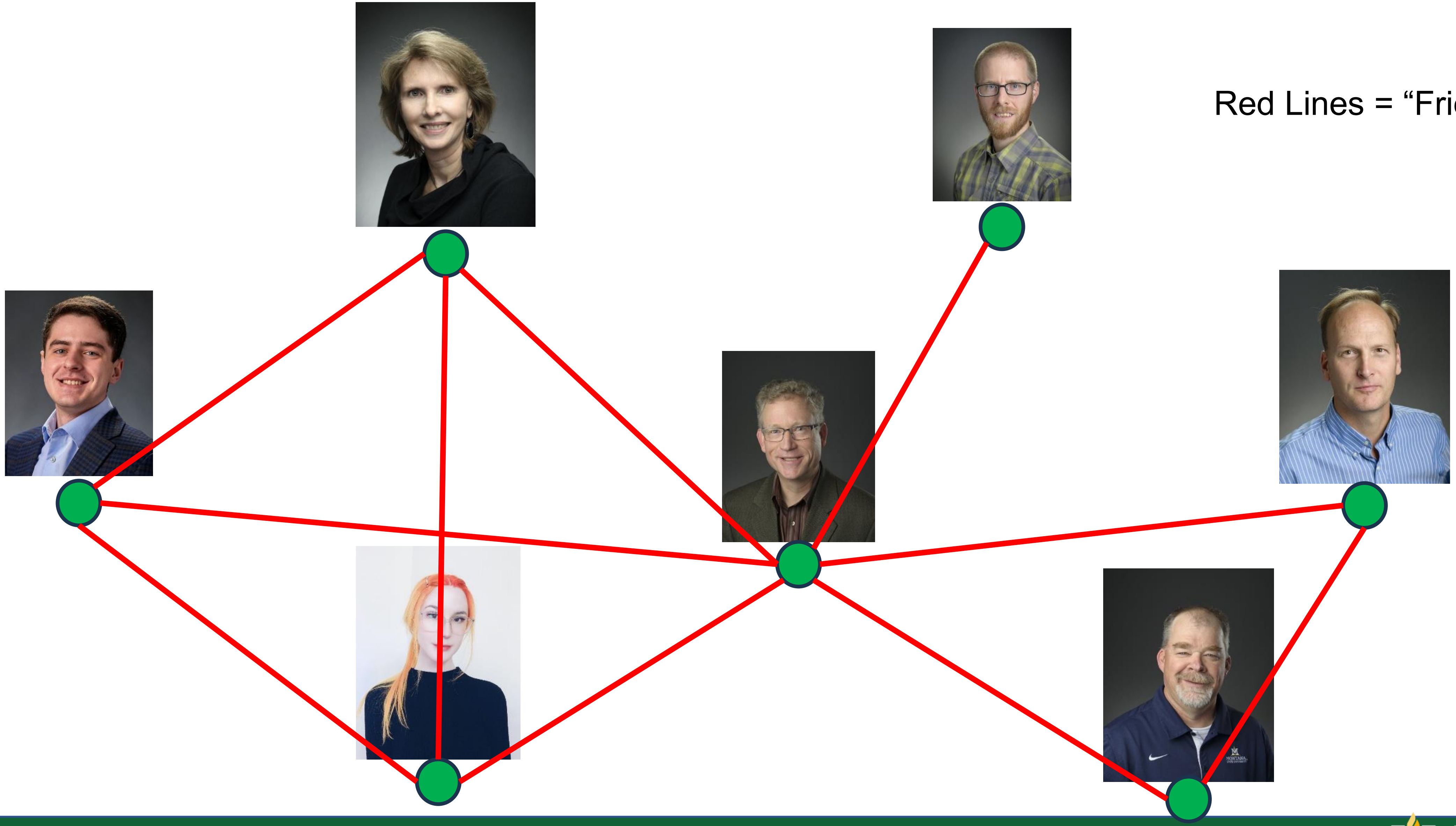
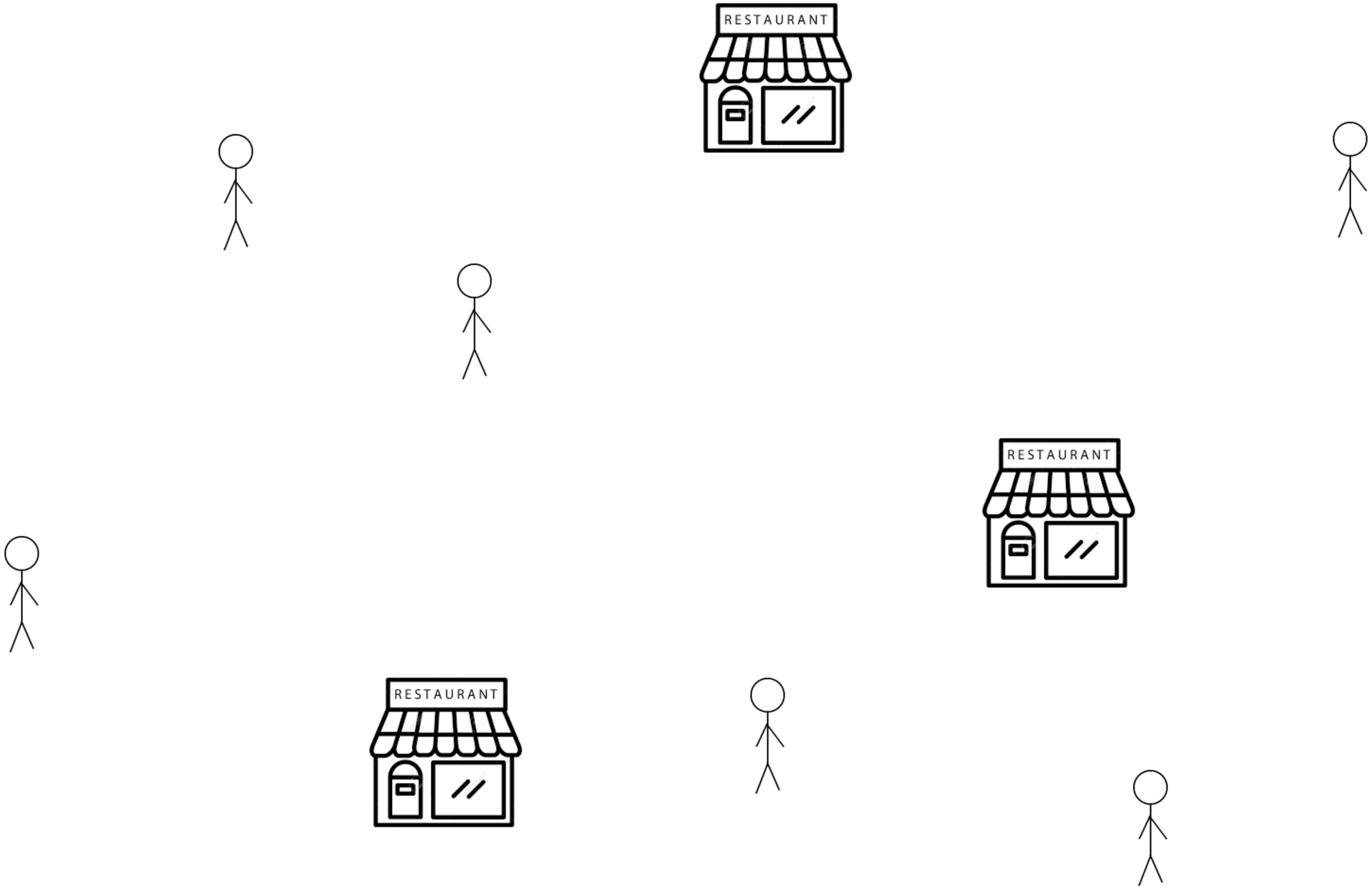


Red Lines = “Friends with”

How could we visualize: Restaurants and Potential Customers?



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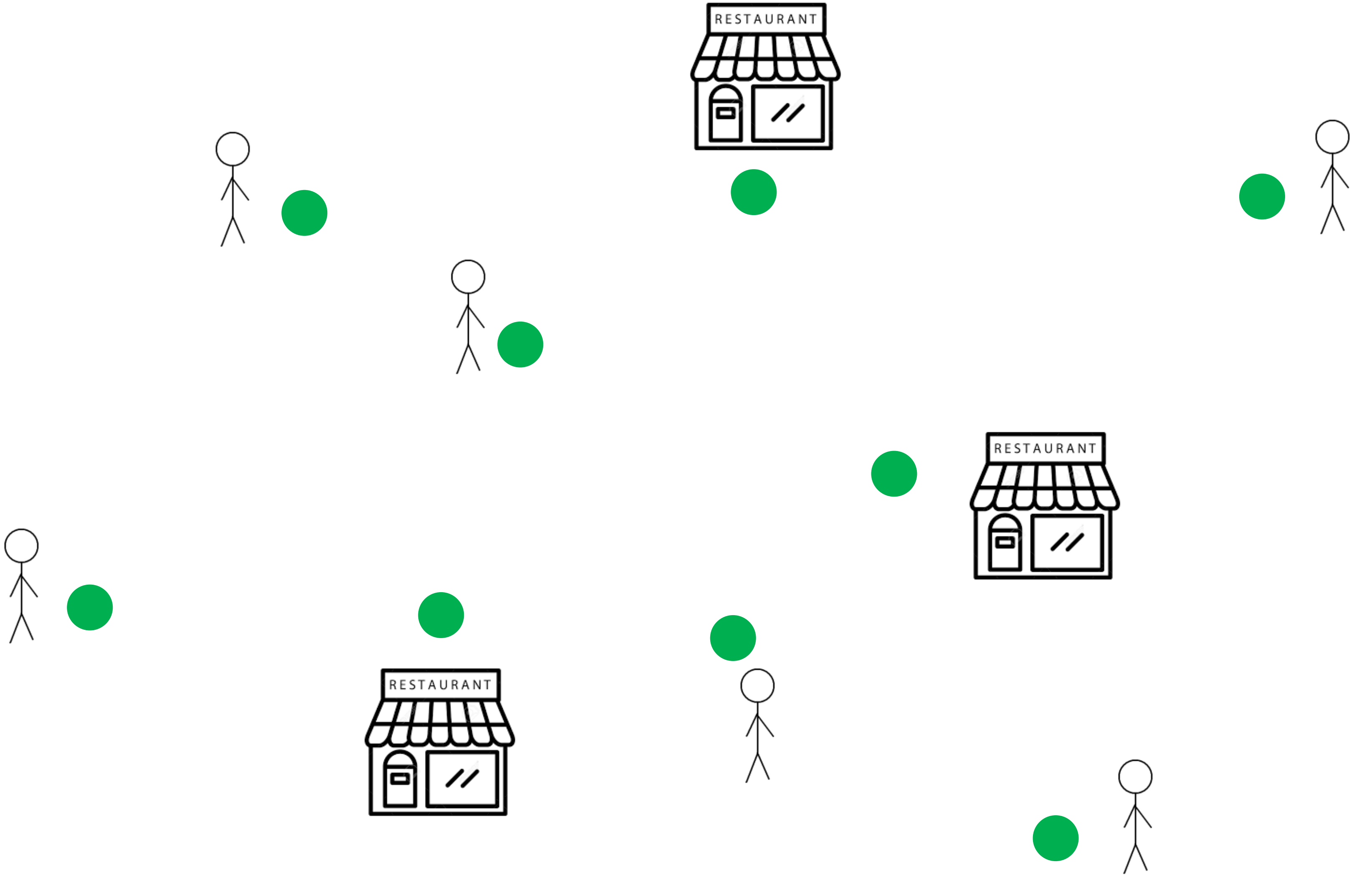

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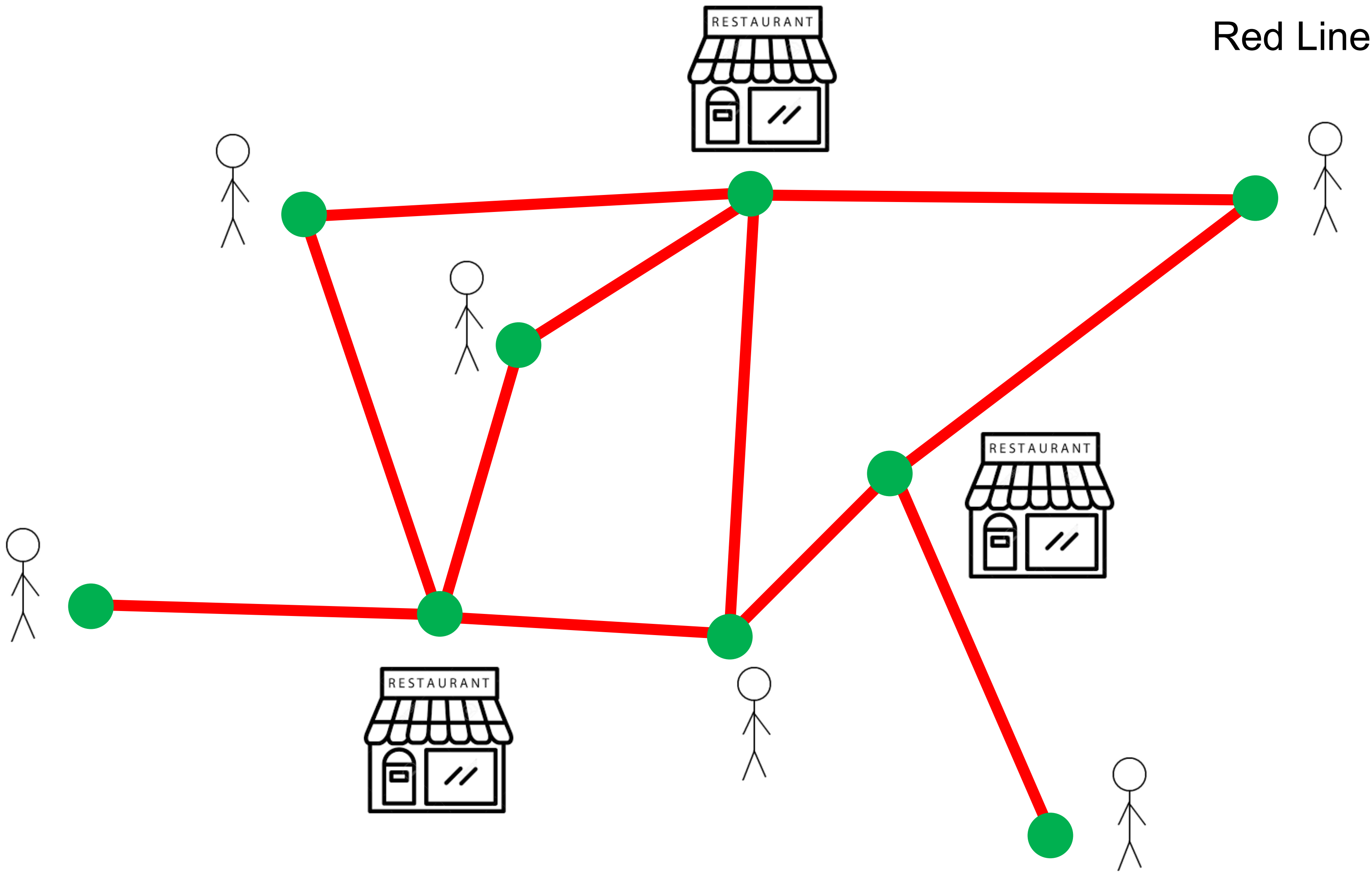

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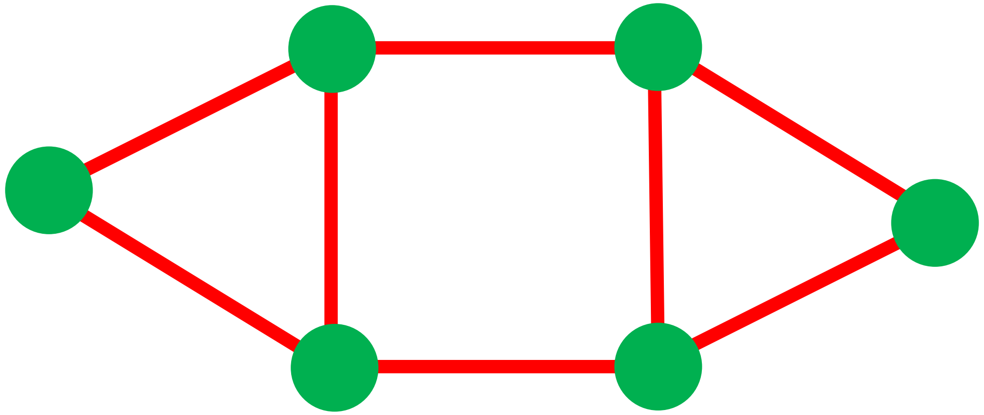
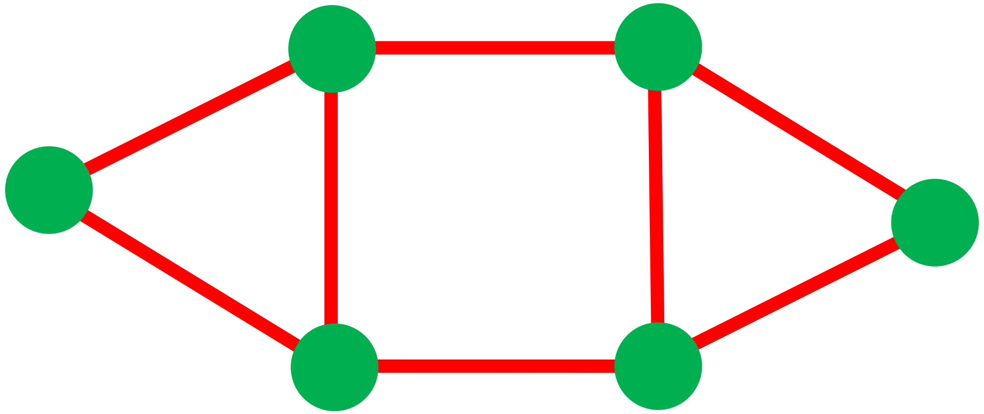
Graphs



Vertices (or Nodes)

Graphs



Vertices (or Nodes)

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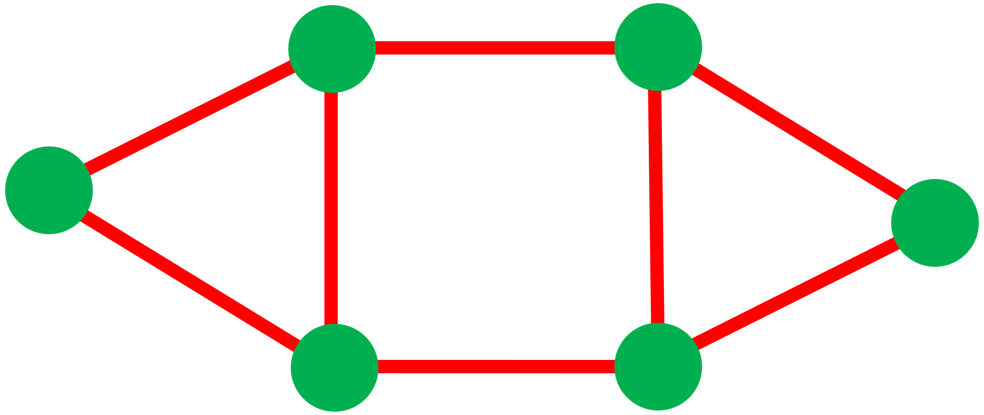

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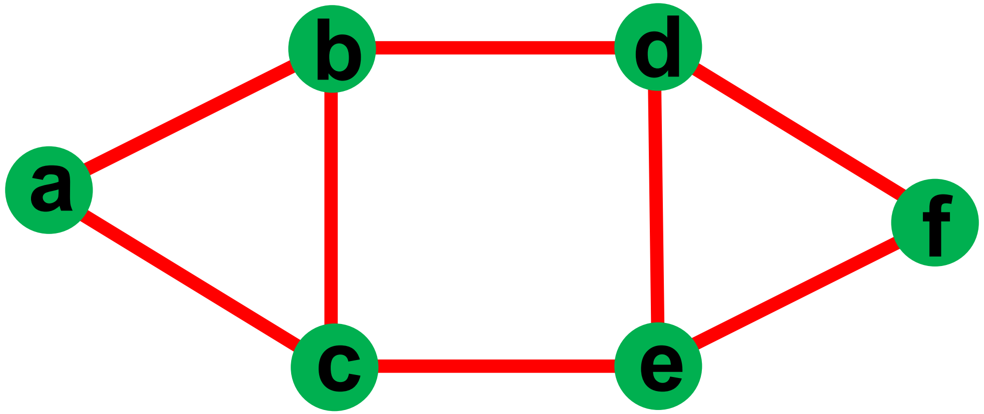

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V = {a, b, c, d, e, f}

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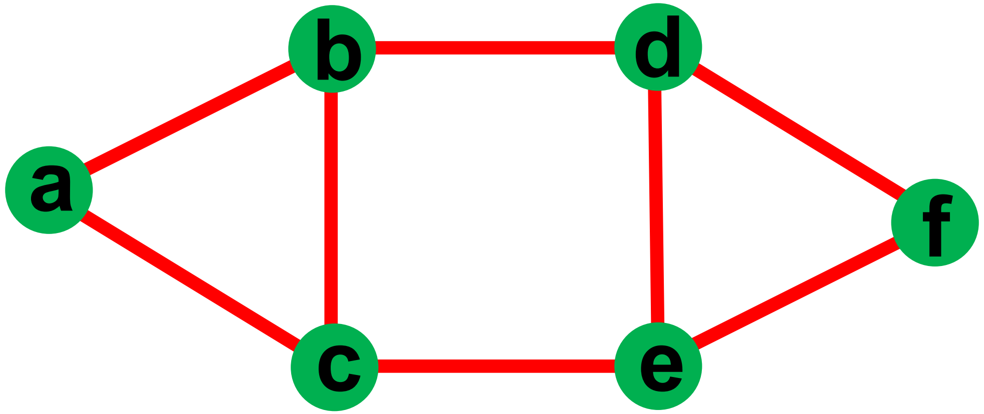
Graphs



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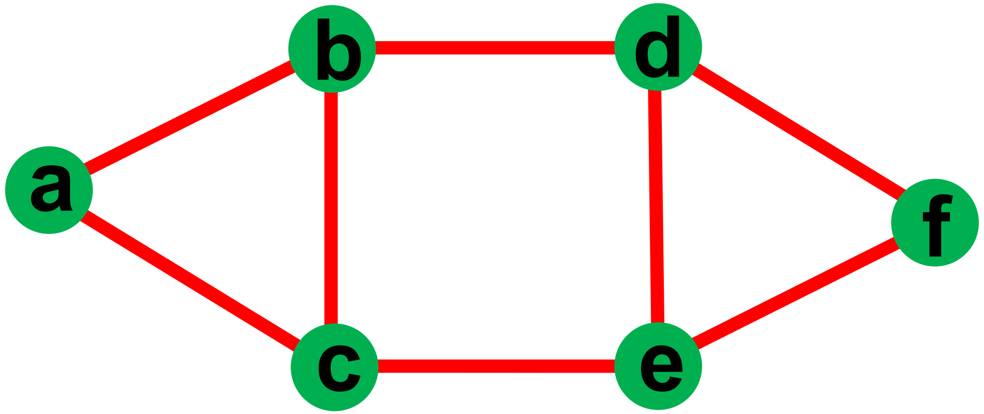

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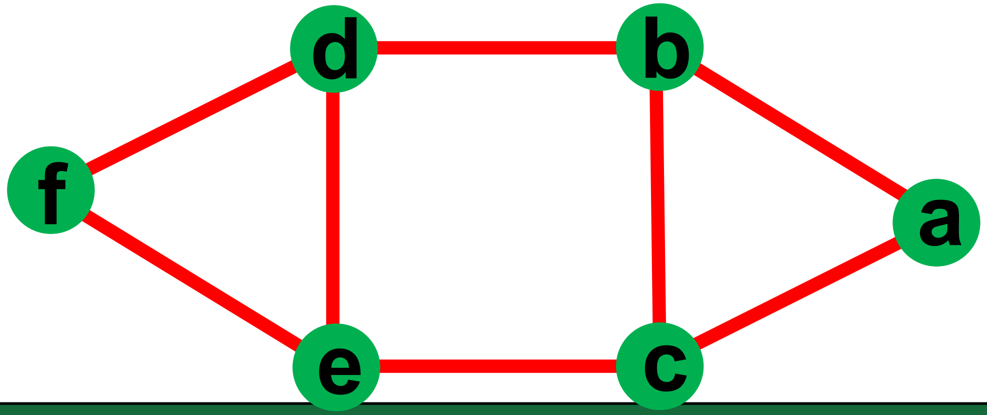
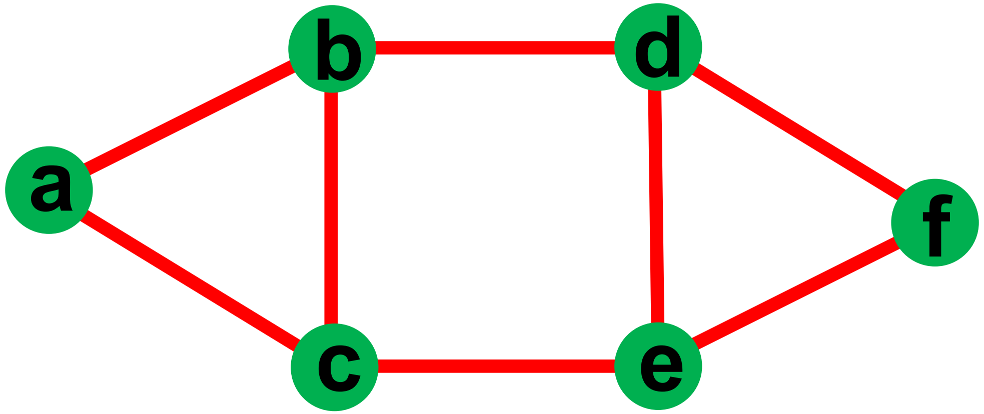


Same graph? ✓

Graphs



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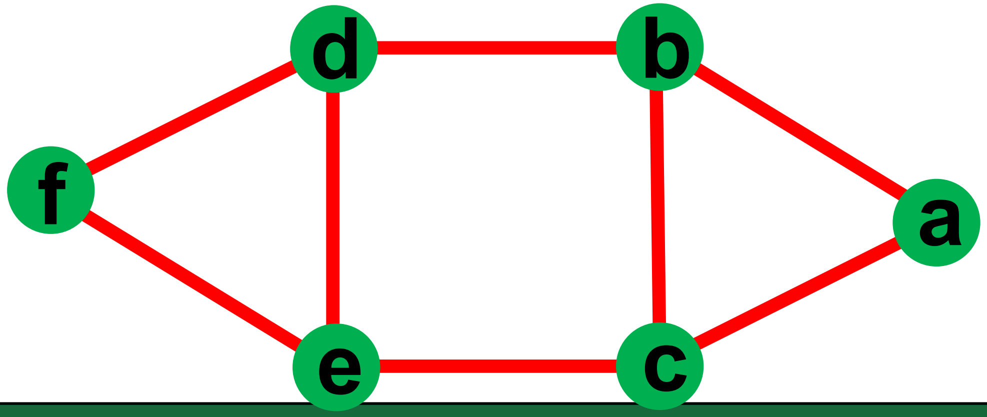

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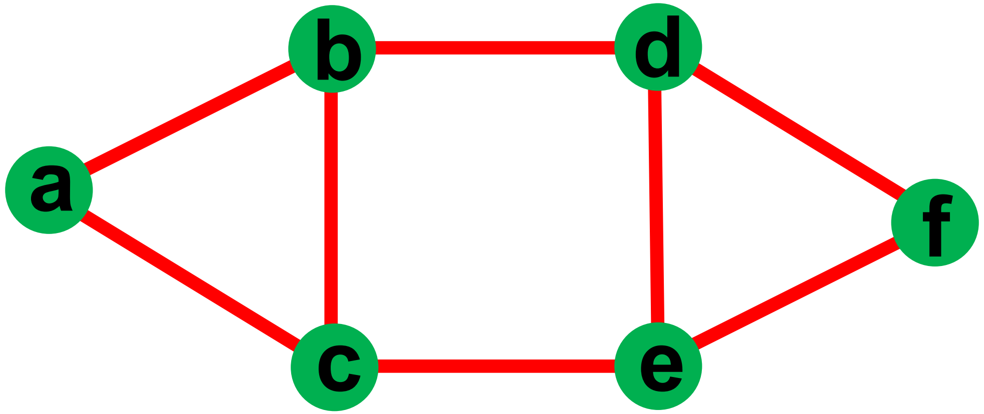

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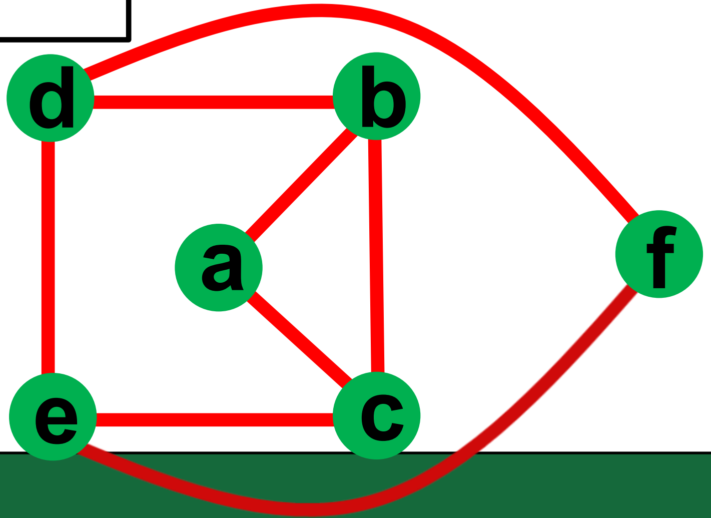

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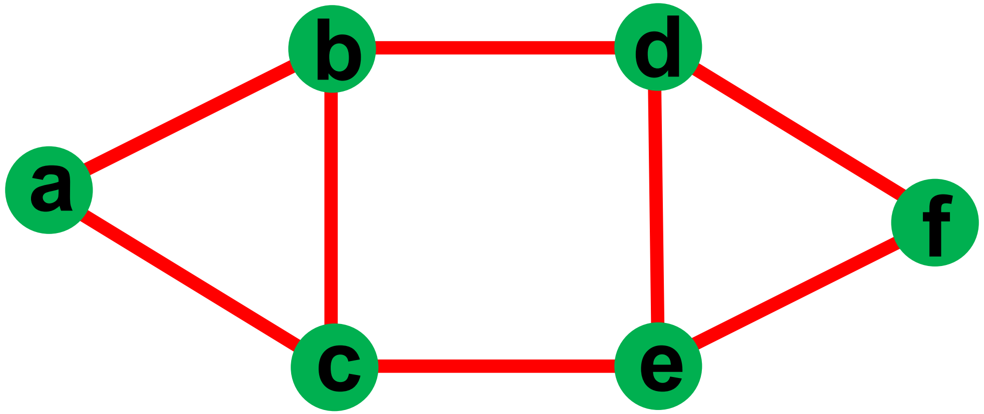

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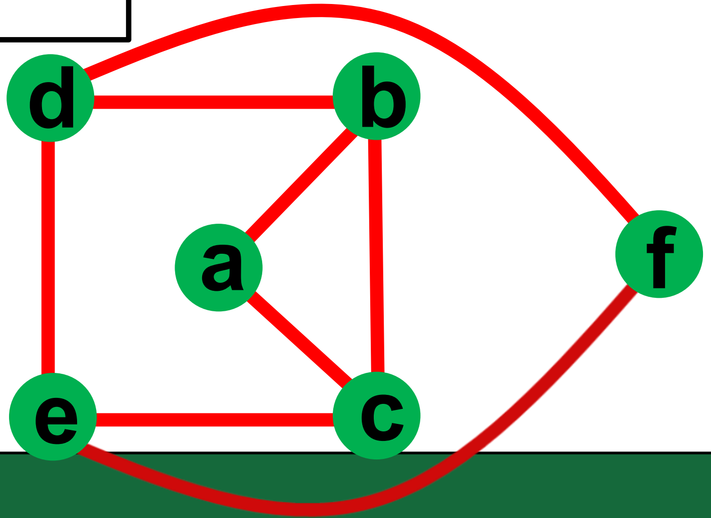

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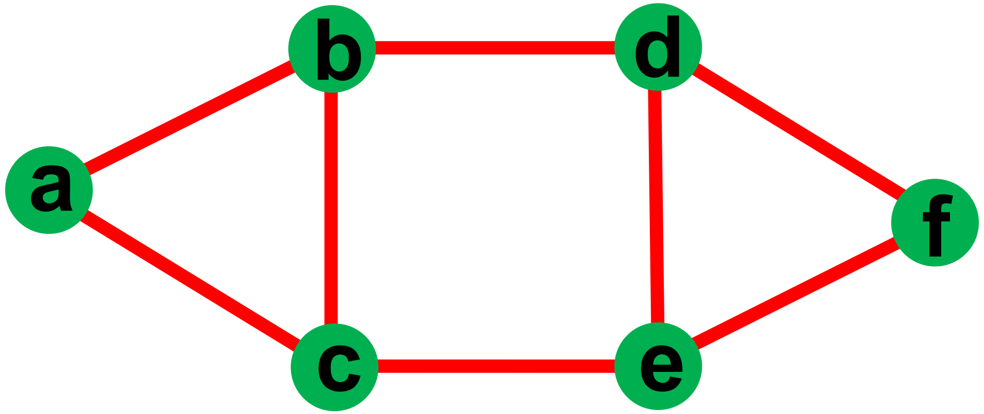

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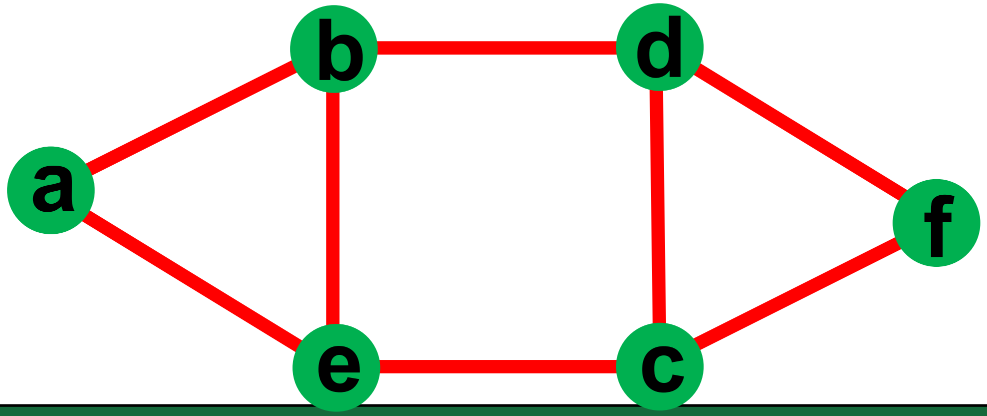

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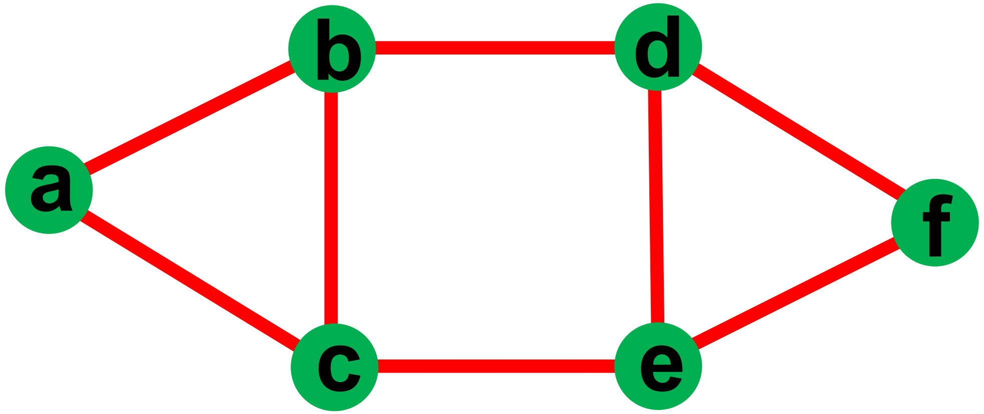

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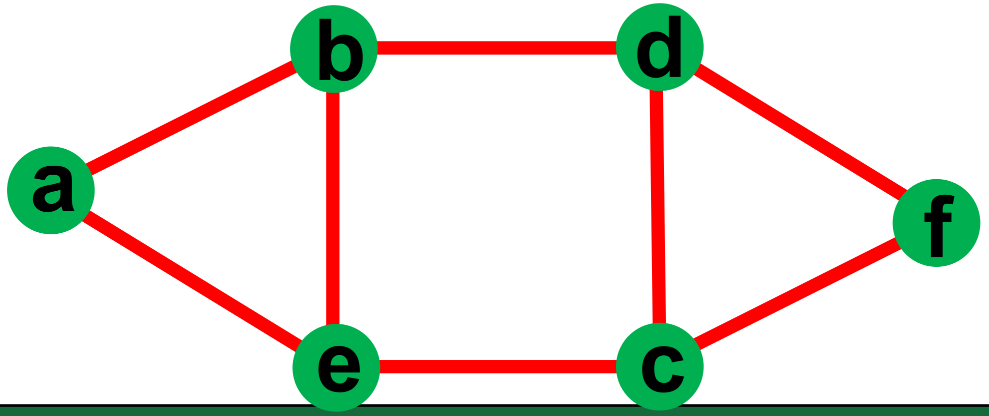

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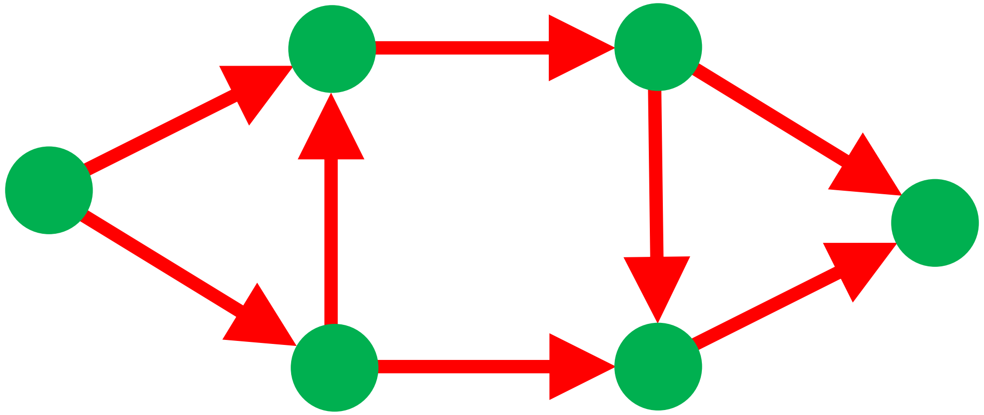

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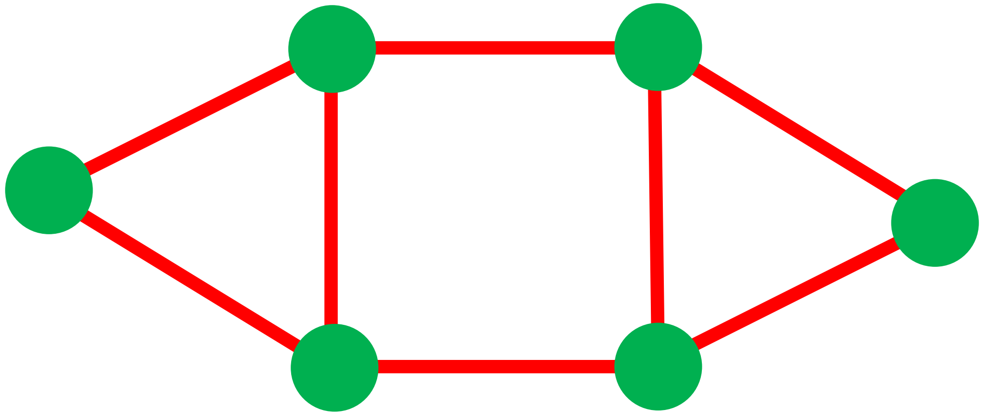
- Edges can be directed…

Graphs



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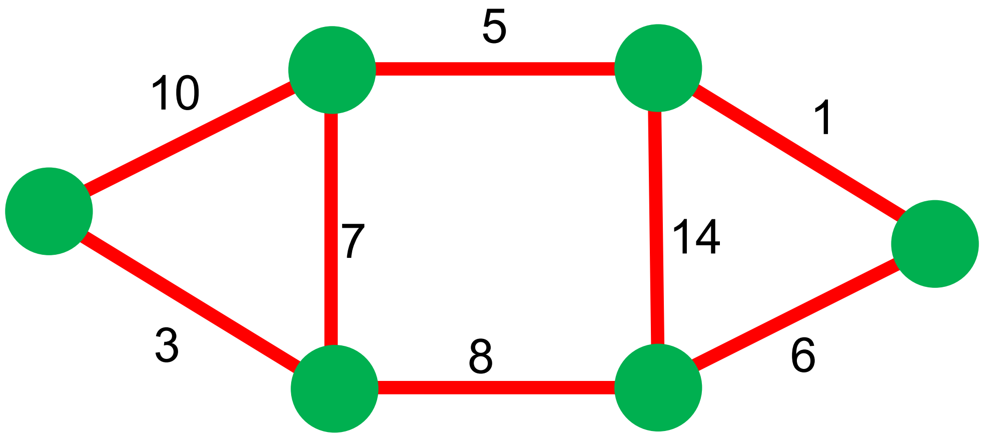

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- Edges can be directed or undirected.
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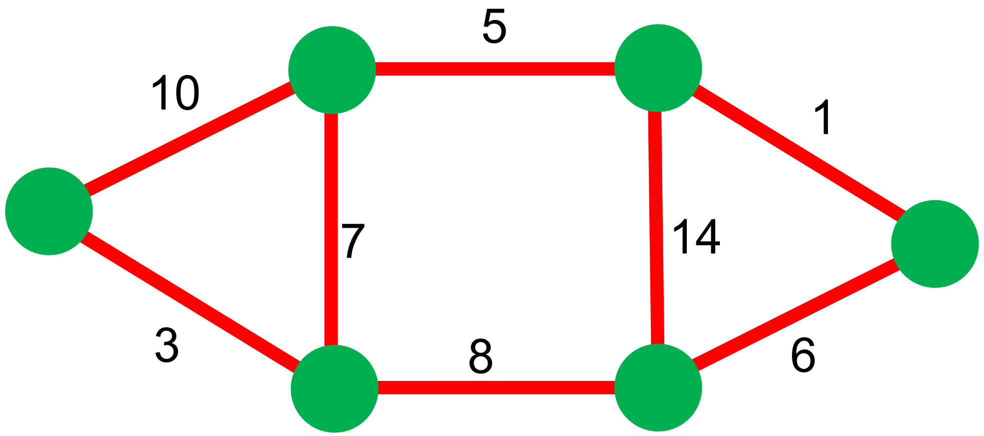

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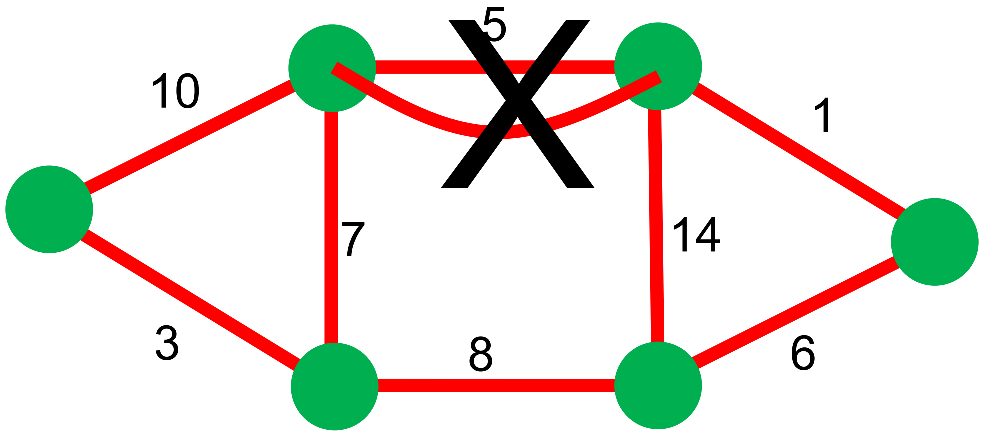

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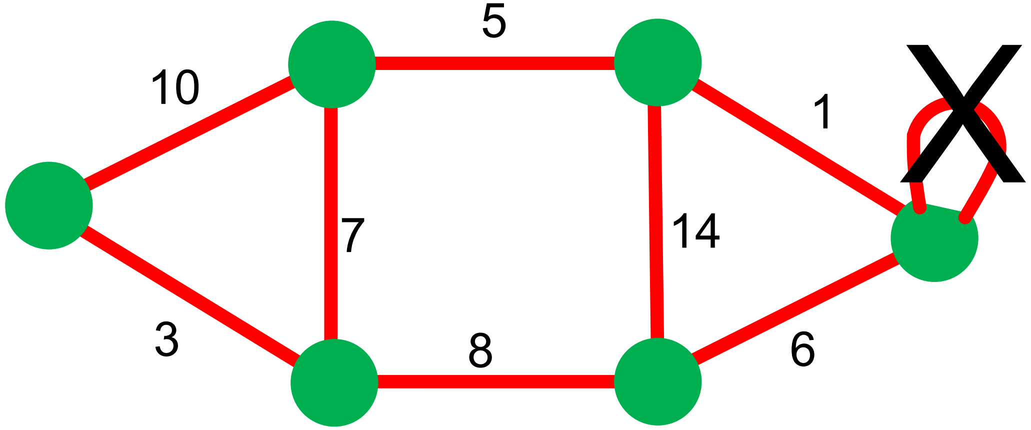

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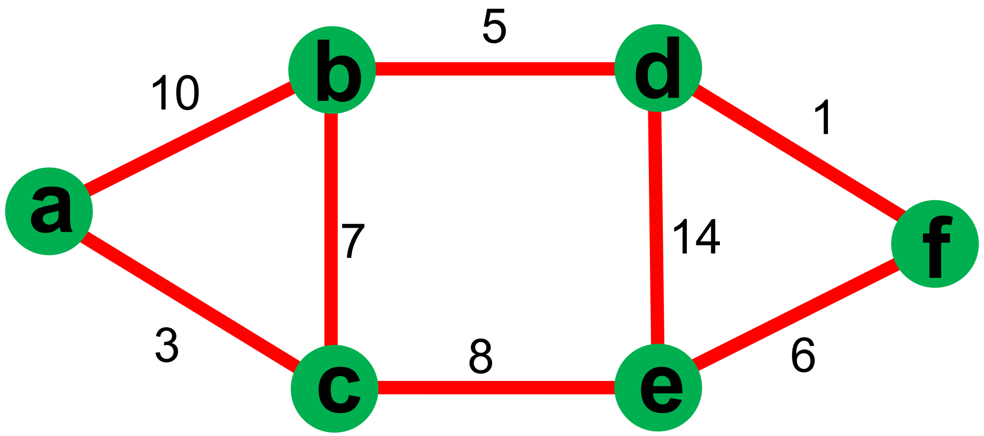

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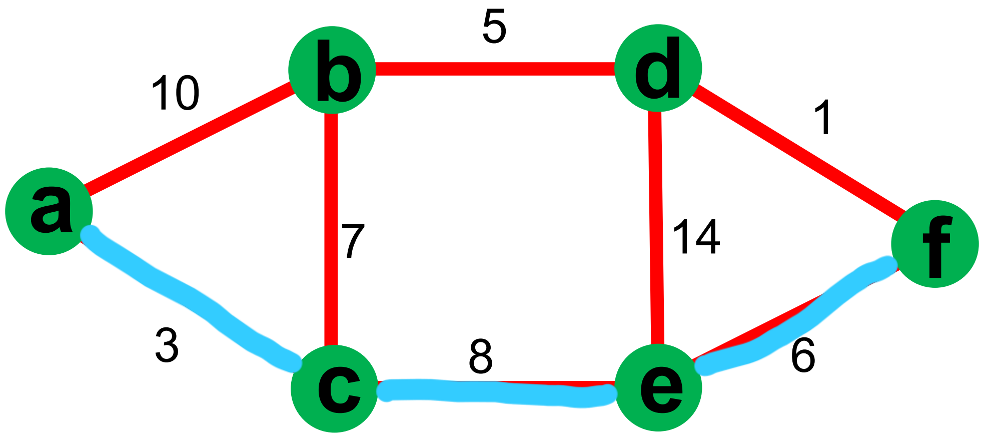

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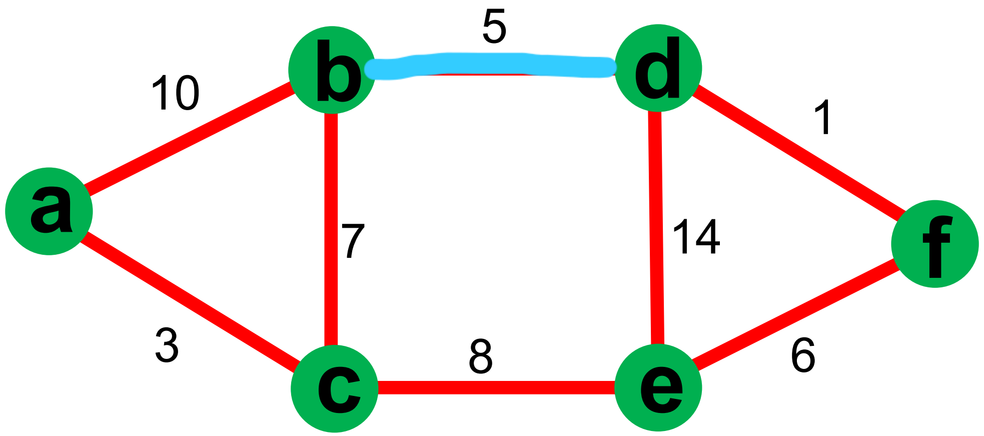
a,c,e,f ✓ "cost" of path = 17

Graphs



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b,d ✓

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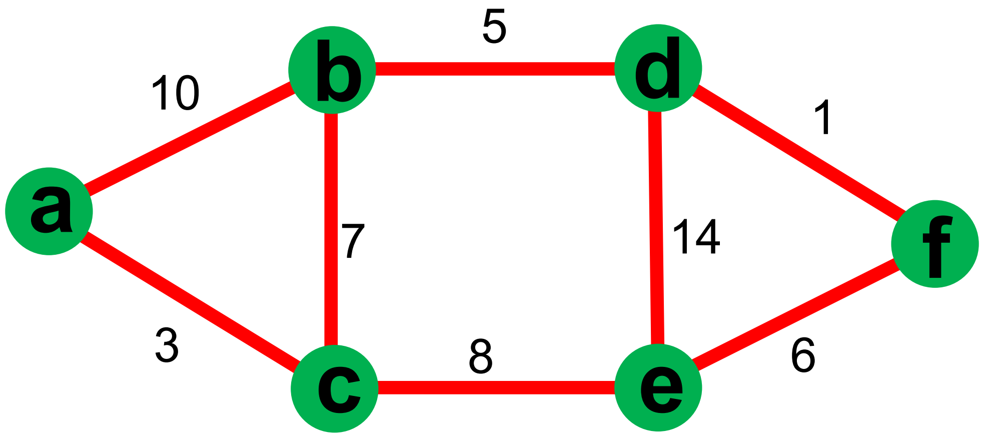

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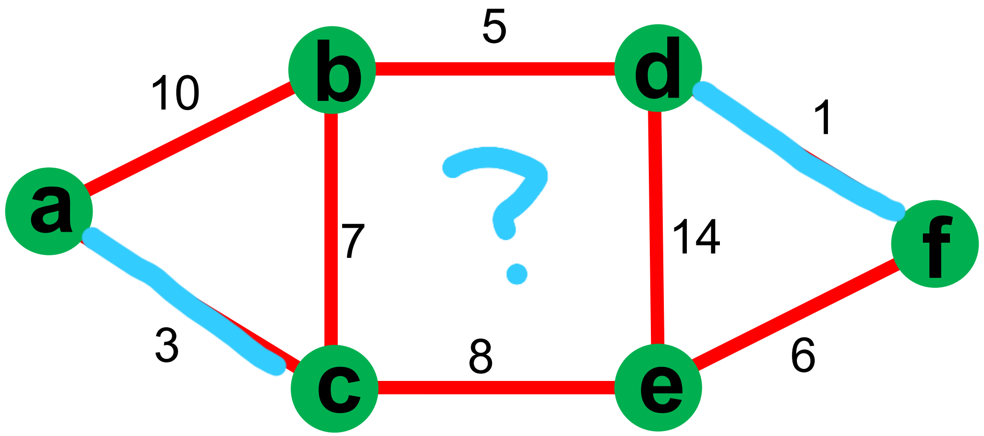

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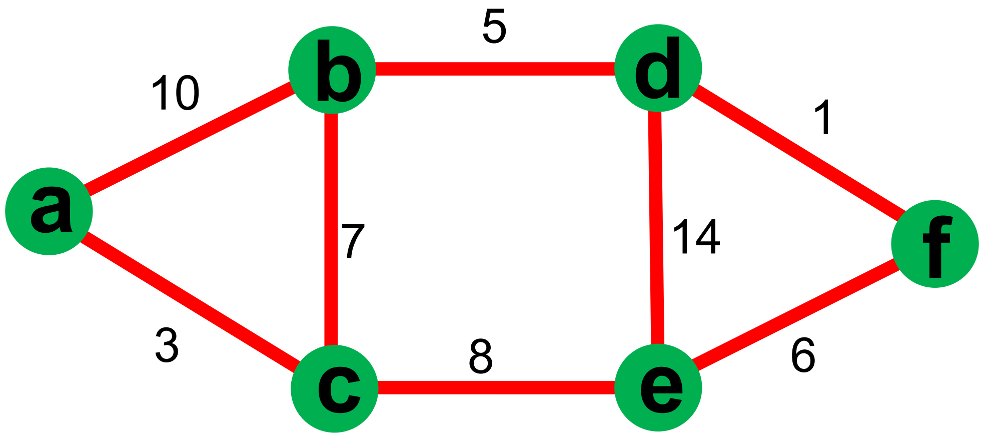

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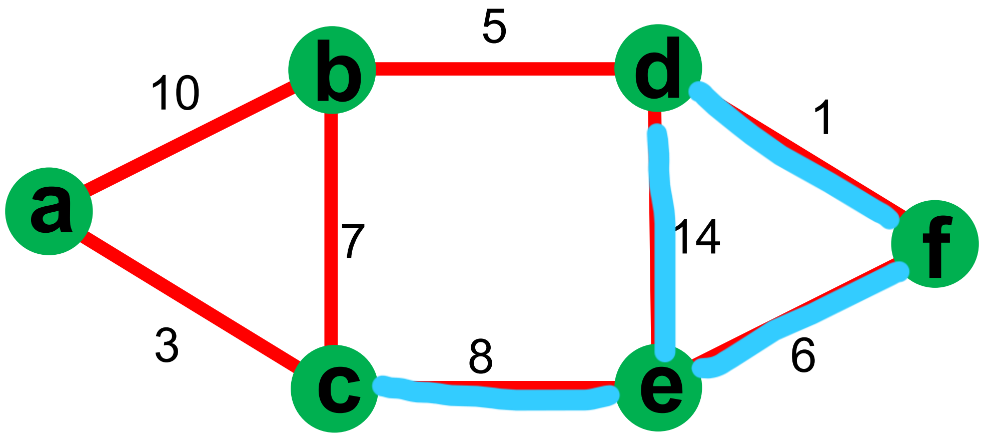

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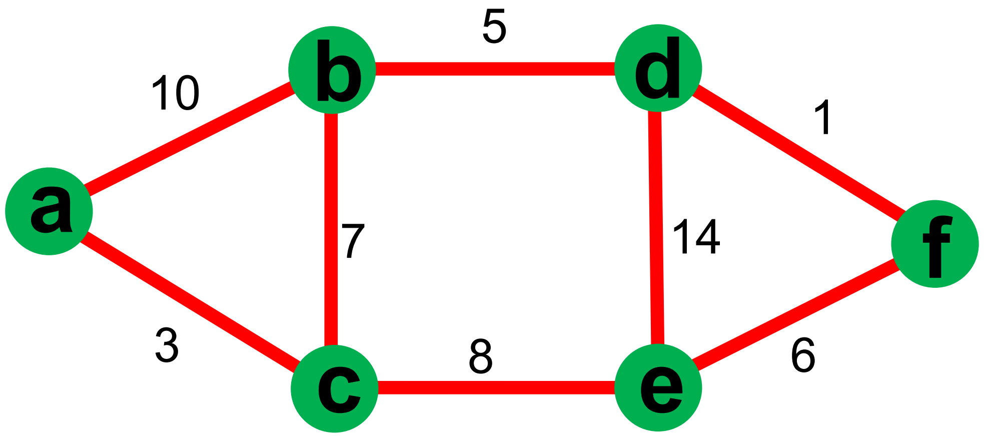

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- Cycle = Sequence of vertices connected by edge with loop(s).

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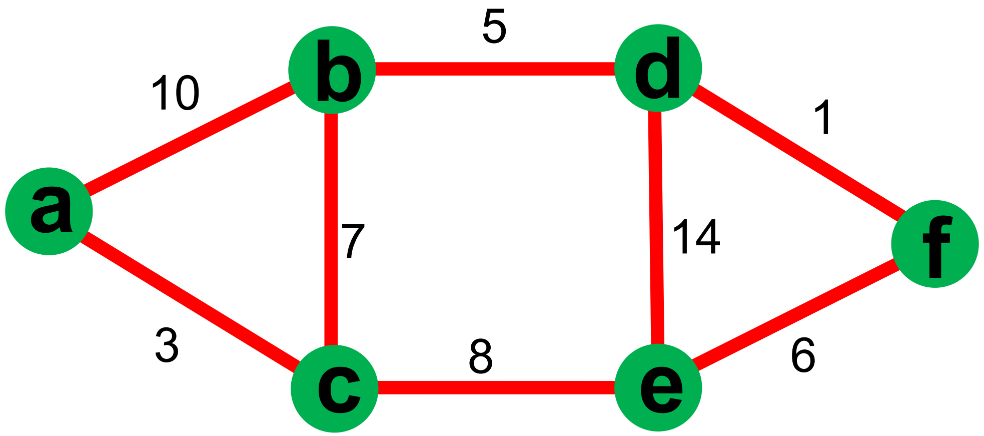

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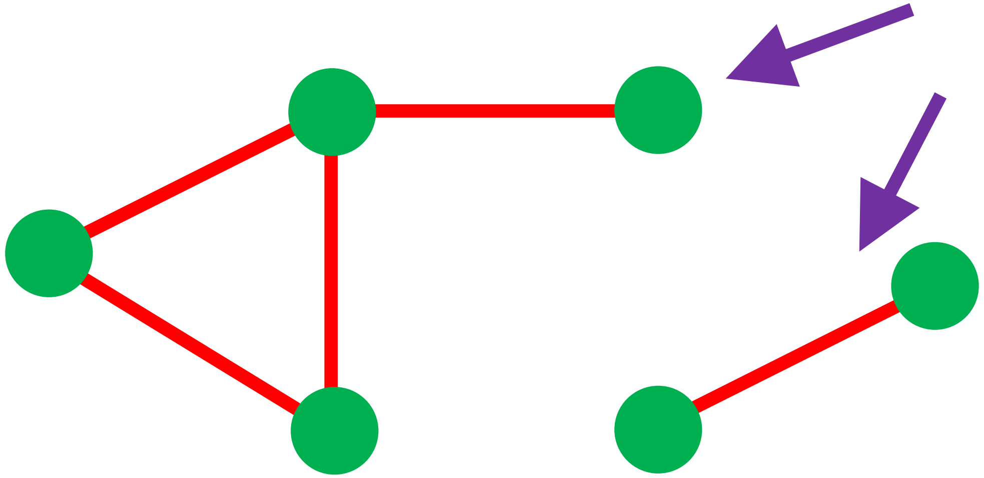
- Connected Graph = Graph that has a path between every vertex pair.

Graphs



Two Connected Components

Vertices (or Nodes)
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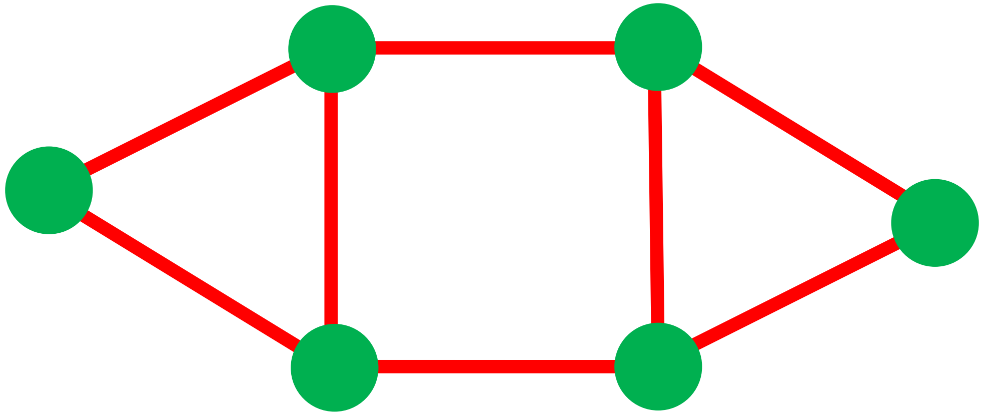

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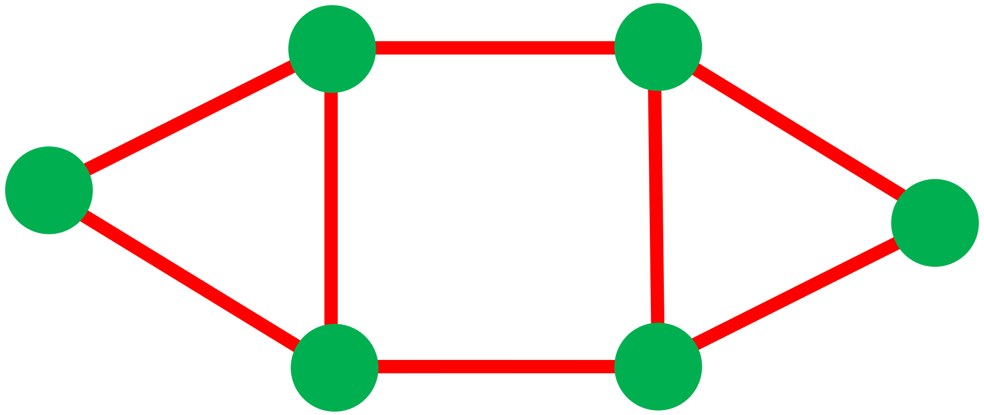
- Degree of a vertex = $\deg(v)$ = # of edges touching it (undirected).

Graphs



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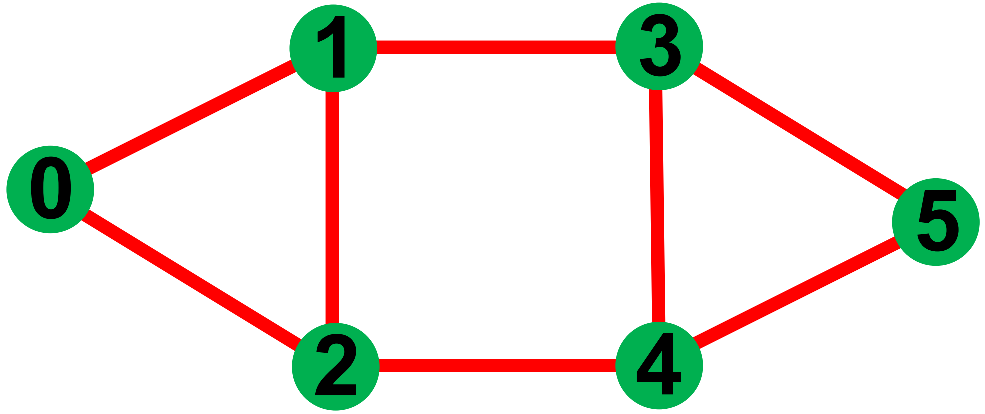
What are some operations we may want to perform on a graph?

- Add vertices/edges.
- Find path between vertex pair.
- Is graph connected?
- Find degree of vertex.
- Is the graph simple?
- Get number of vertices/edges.
- Get neighbors of vertex.
- Is there a cycle?
- Find max degree of graph.

How can we represent a graph in a computer?



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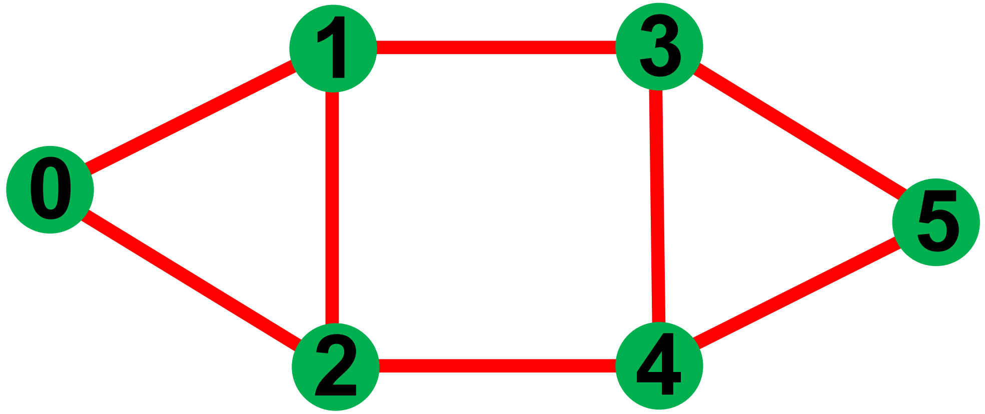
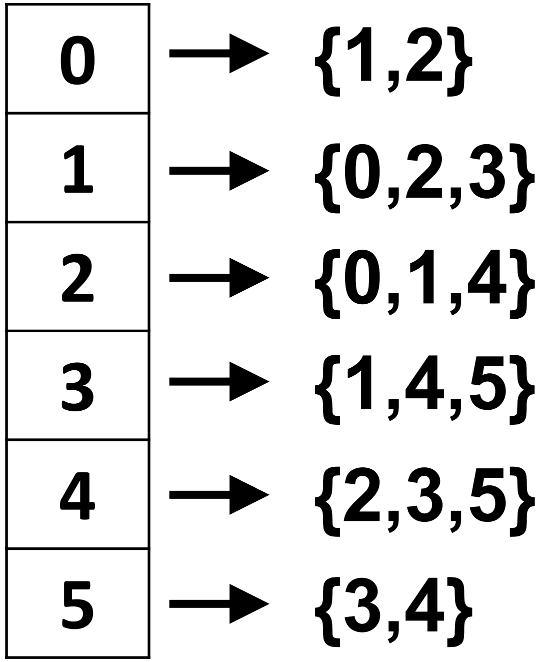


1. Adjacency List



0 → {1,2}

1 → {0,2,3}

2 → {0,1,4}

3 → {1,4,5}

4 → {2,3,5}

5 → {3,4}

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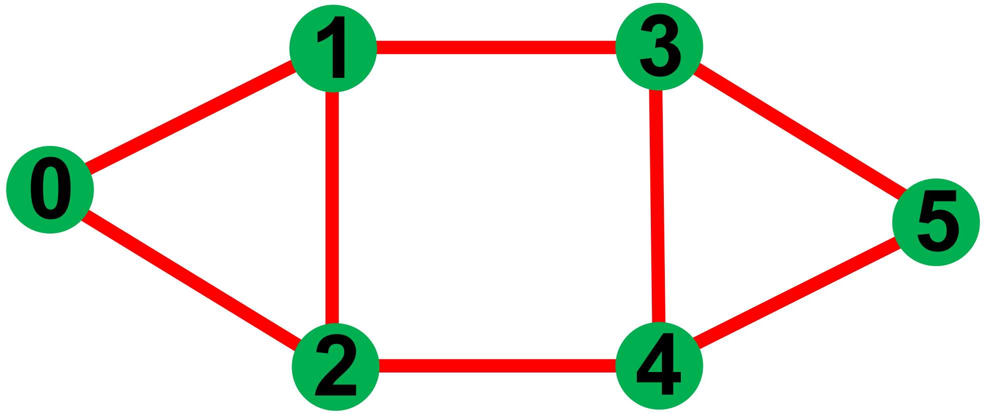

1. Adjacency List

0	→	{1,2}
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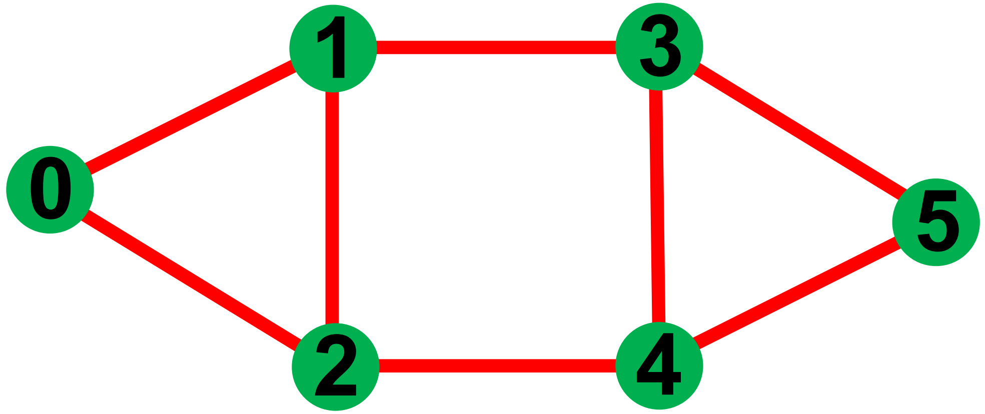
2	→	{0,1,4}
3	→	{1,4,5}
4	→	{2,3,5}
5	→	{3,4}

2. Adjacency Matrix

	0	1	2	3	4	5
0	F	T	T	F	F	F
1	T	F	T	T	F	F
2	T	T	F	F	T	F
3	F	T	F	F	T	T
4	F	F	T	T	F	T
5	F	F	F	T	T	F

How can we represent a graph in a computer?



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2. Adjacency Matrix

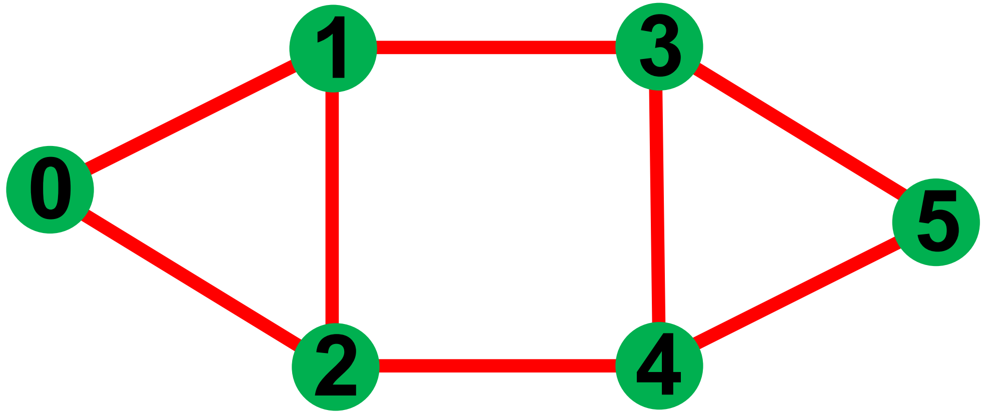
	0	1	2	3	4	5
0	F	T	T	F	F	F
1	T	F	T	T	F	F
2	T	T	F	F	T	F
3	F	T	F	F	T	T
4	F	F	T	T	F	T
5	F	F	F	T	T	F

3. Objects

```
public class Node {
    private Set<Node> neighbors;

    …
}
```

How can we represent a graph in a computer?



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2. Adjacency Matrix

	0	1	2	3	4	5
0	F	T	T	F	F	F
1	T	F	T	T	F	F
2	T	T	F	F	T	F
3	F	T	F	F	T	T
4	F	F	T	T	F	T
5	F	F	F	T	T	F

3. Objects

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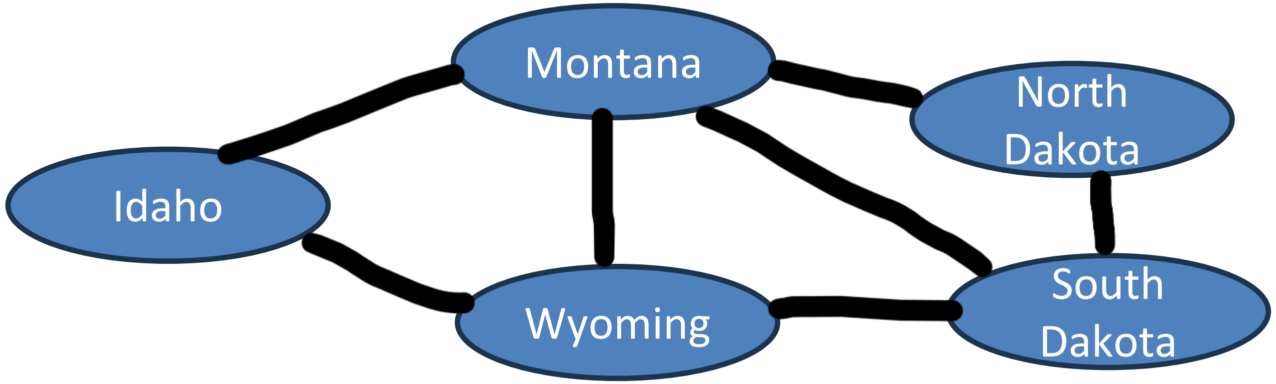
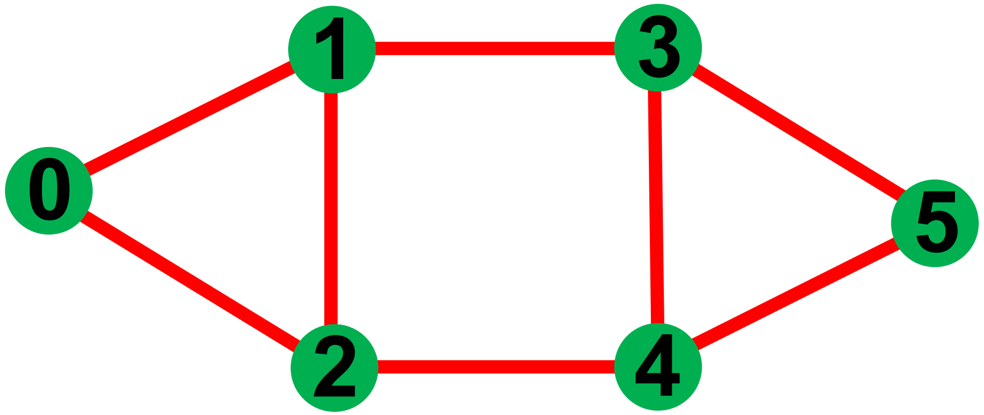
1. Adjacency Lists





Montana: [North Dakota, South Dakota, Wyoming, Idaho]
Idaho: [Montana, Wyoming]
Wyoming: [Idaho, Montana, South Dakota]
North Dakota: [Montana, South Dakota]
South Dakota: [Wyoming, Montana, North Dakota]

?

0	→	{1,2}
1	→	{0,2,3}
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4	→	{2,3,5}
5	→	{3,4}

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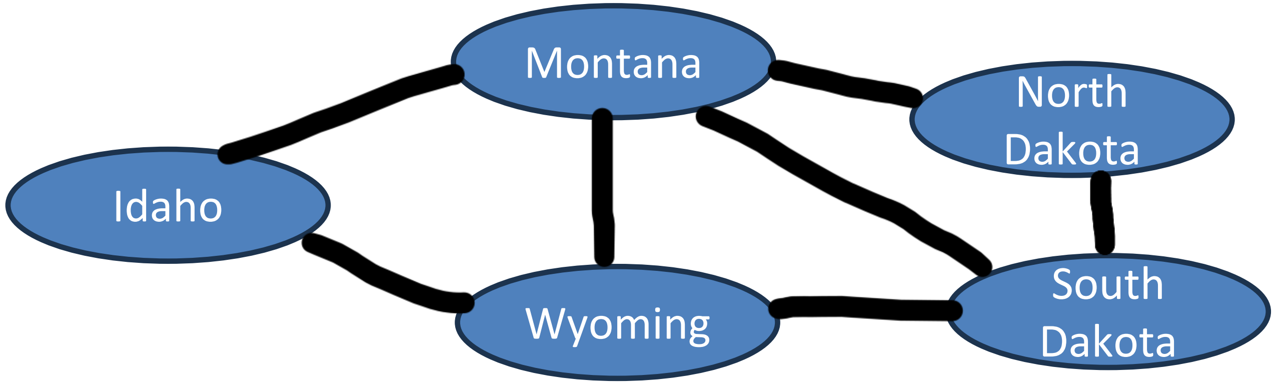

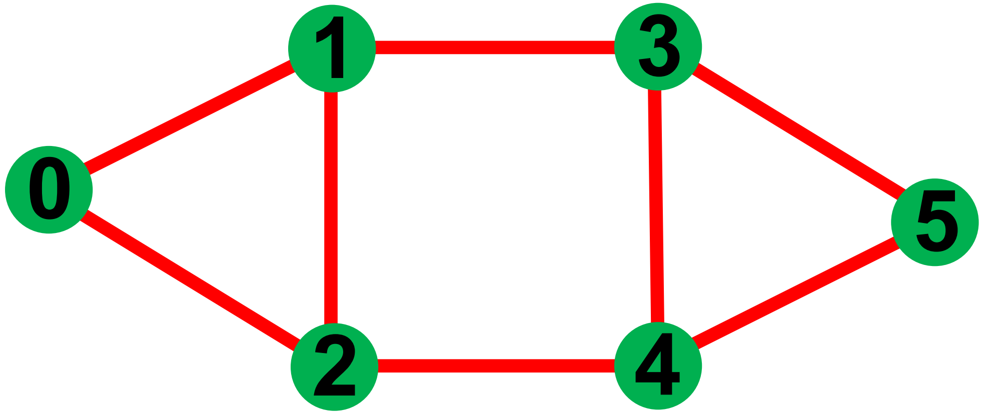

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North Dakota: [Montana, South Dakota]
South Dakota: [Wyoming, Montana, North Dakota]

HashMap<String, LinkedList<String>>
HashMap<String, LinkedList<Edge>>

0	→	{1,2}
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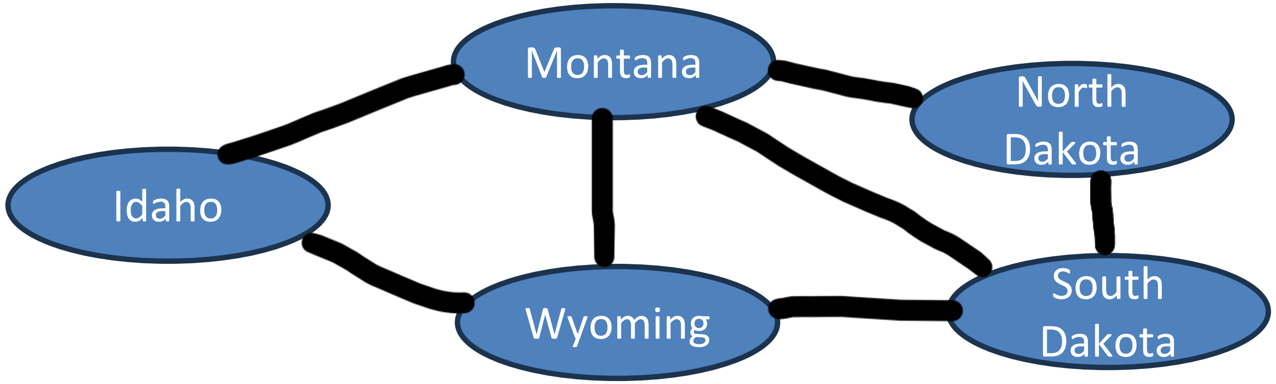

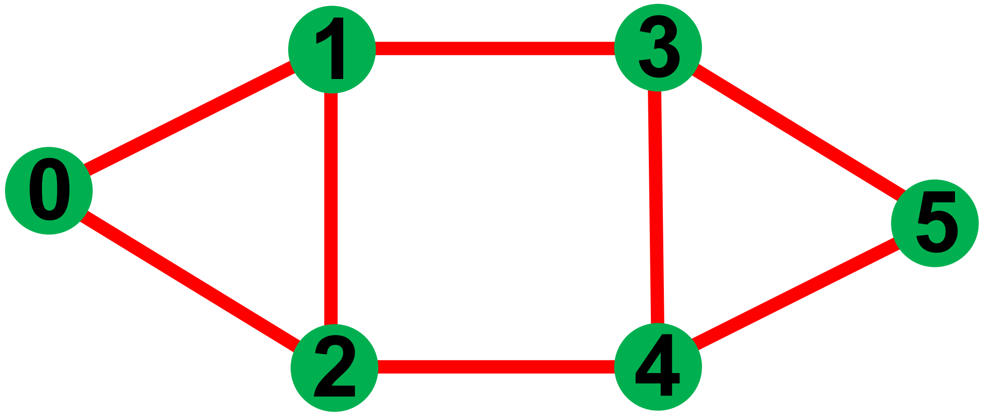

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HashMap<String, LinkedList<String>>
HashMap<String, LinkedList<Edge>>

0	→	{1,2}	?
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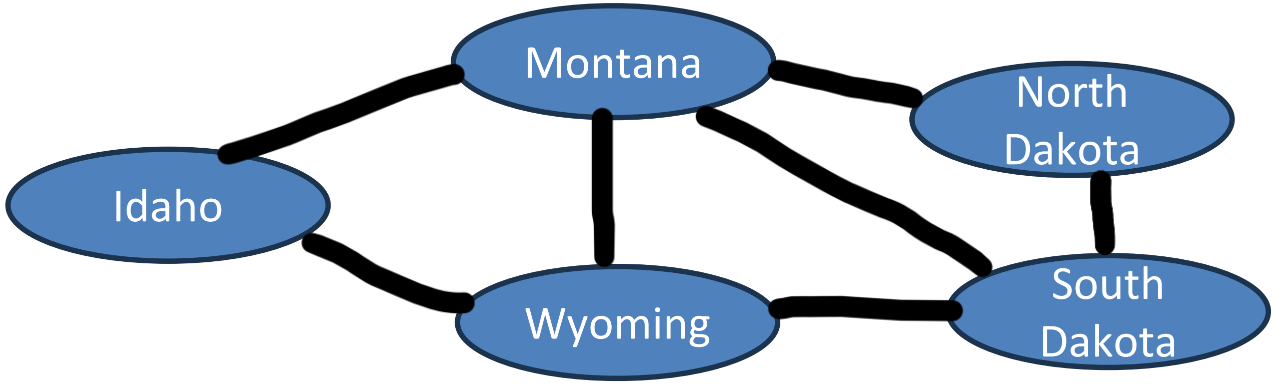

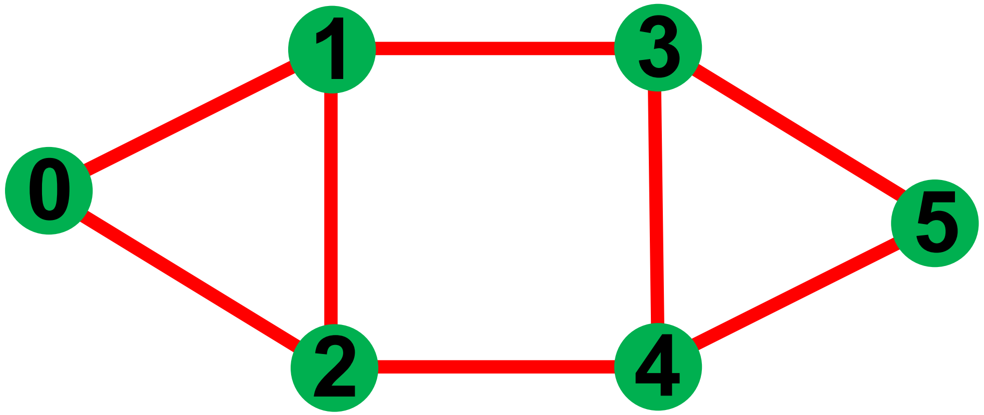

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Array of Linked Lists