

CSCI 232:

Data Structures and Algorithms

Minimum Spanning Tree (MST)

Reese Pearsall
Summer 2025

Announcements

Lab 6 due tomorrow

Program 3 posted. Due one week from today

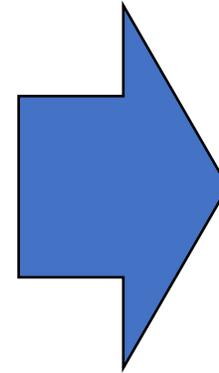
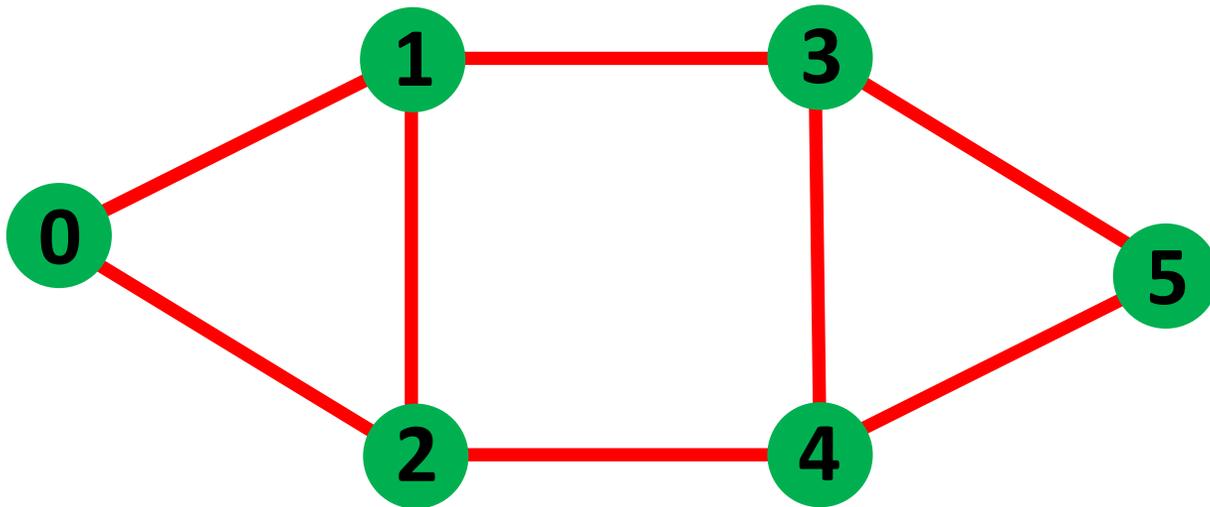
→ We will write some starting code on Wednesday



Very relevant meme for this week's lab

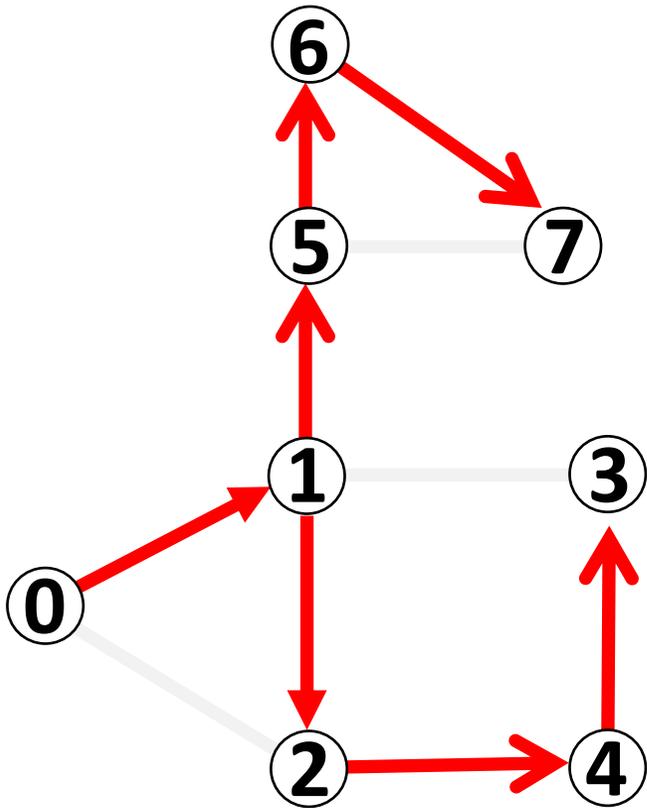
Graphs

$$G = (V, E)$$

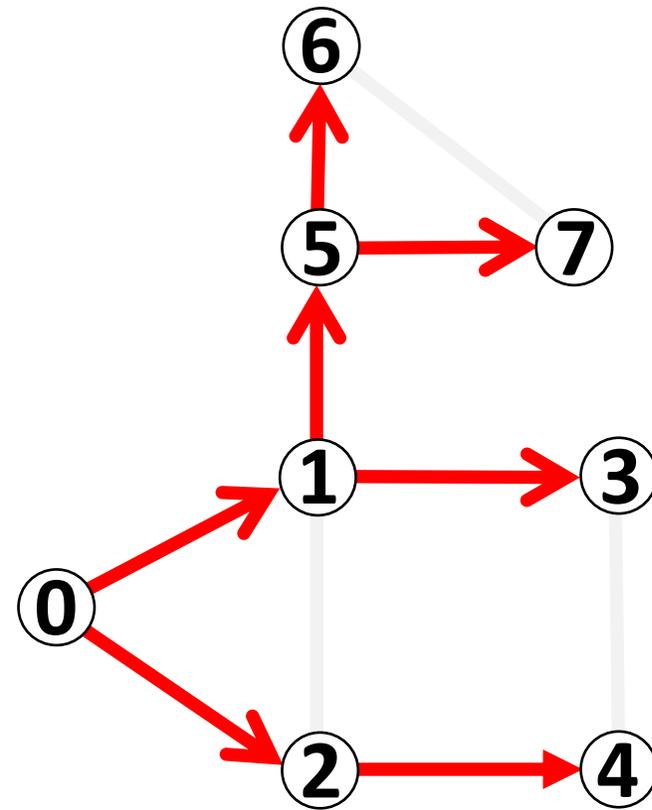


Adjacency List

0	→	{1,2}
1	→	{0,2,3}
2	→	{0,1,4}
3	→	{1,4,5}
4	→	{2,3,5}
5	→	{3,4}



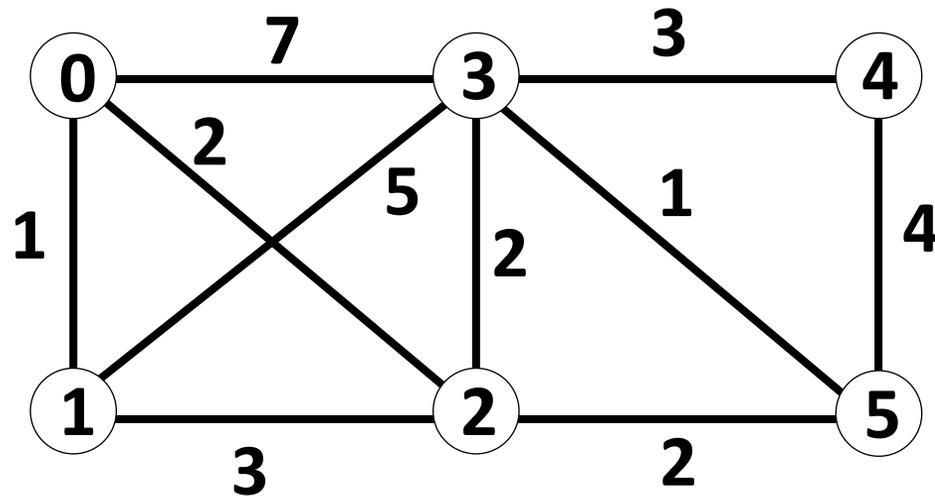
Depth First Order*:
0, 1, 2, 4, 3, 5, 6, 7



Breadth First Order*
0, 1, 2, 3, 5, 4, 6, 7

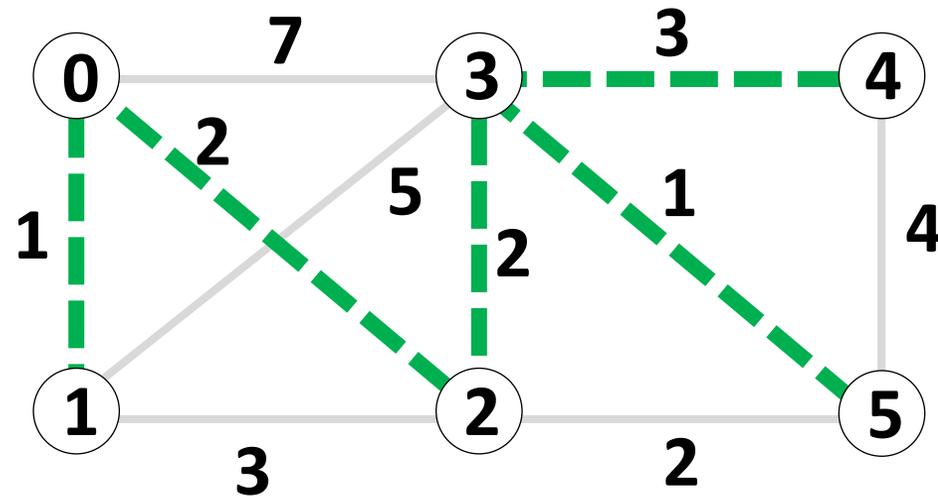
Given a starting point, DFS and BFS will visit every vertex in a graph it is a **connected** graph

Minimum Spanning Tree



Edge-weighted graph: A graph where each edge has a weight (cost).

Minimum Spanning Tree

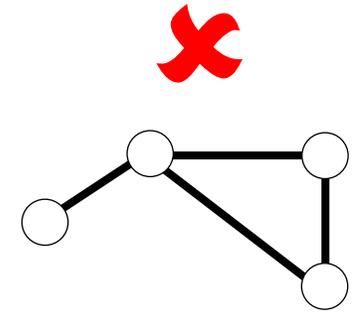
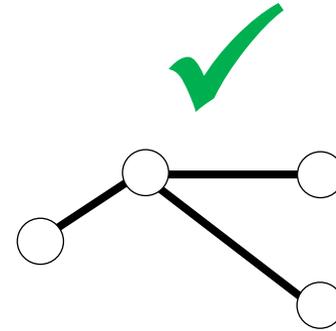


Edge-weighted graph: A graph where each edge has a weight (cost).

MST Goal: Connect all vertices to each other with a minimum weight subset of edges.

Minimum Spanning Tree

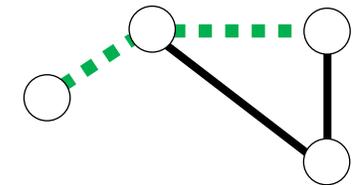
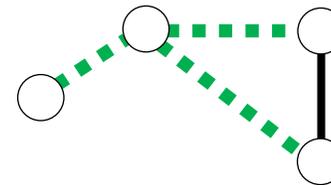
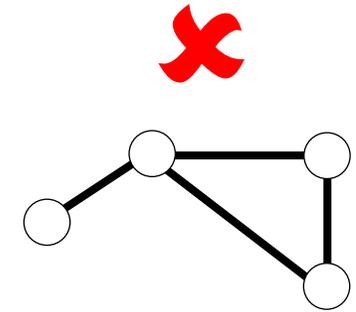
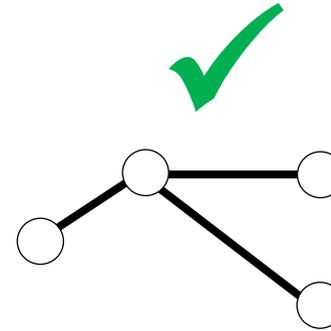
Tree – connected graph with no loops.



Minimum Spanning Tree

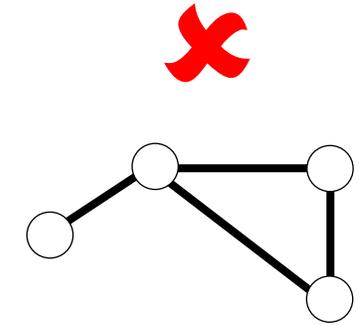
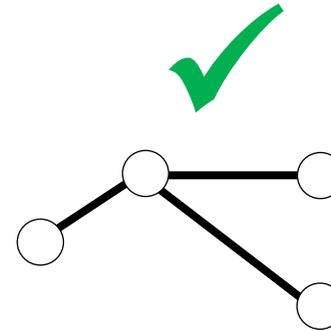
Tree – connected graph with no loops.

Spanning tree – tree that includes all vertices in a graph.

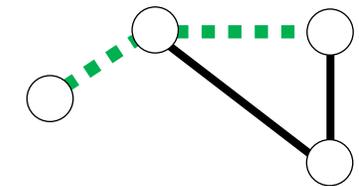
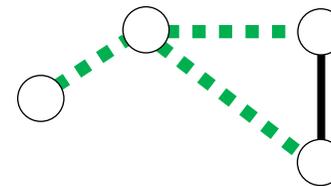


Minimum Spanning Tree

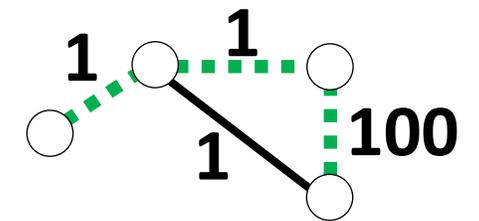
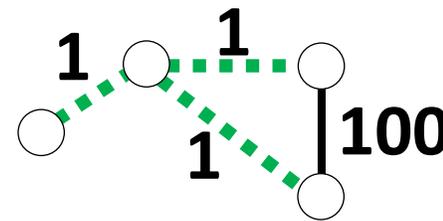
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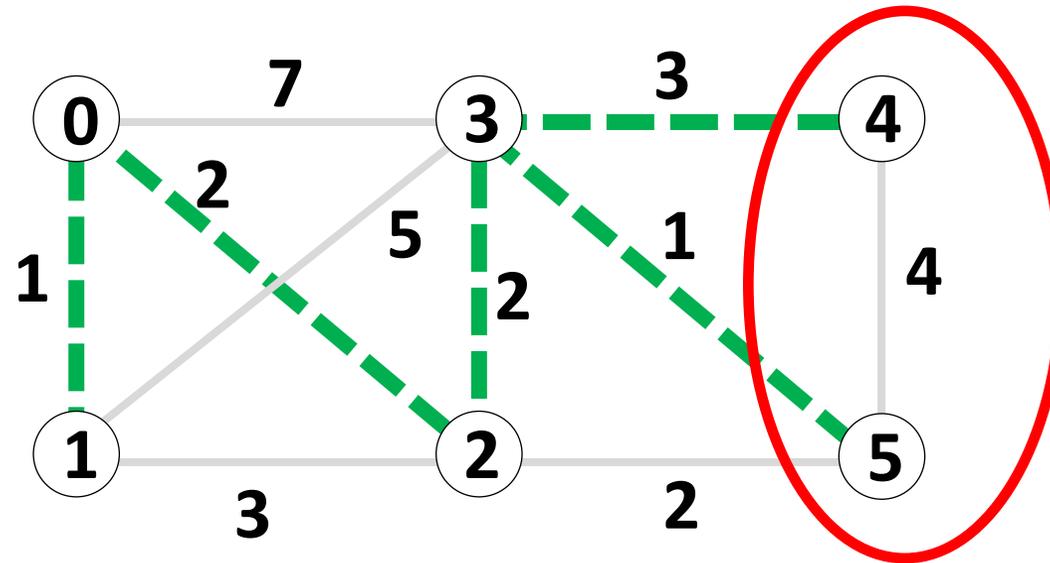
Spanning tree – tree that includes all vertices in a graph.



Minimum spanning tree – spanning tree whose sum of edge costs is the minimum possible value.



Minimum Spanning Tree



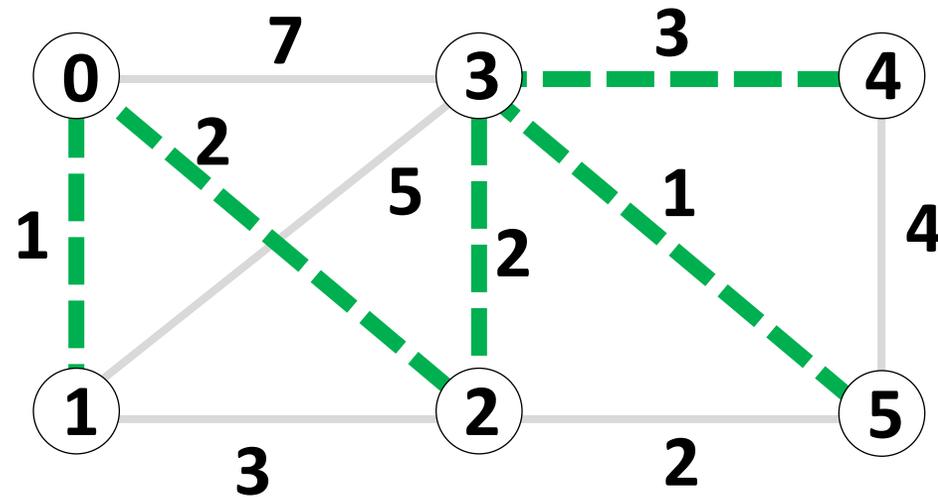
**Does it ever
make sense to
have a cycle?**

No!

Must be a tree!

MST Goal: Connect all vertices to each other with a minimum weight subset of edges.

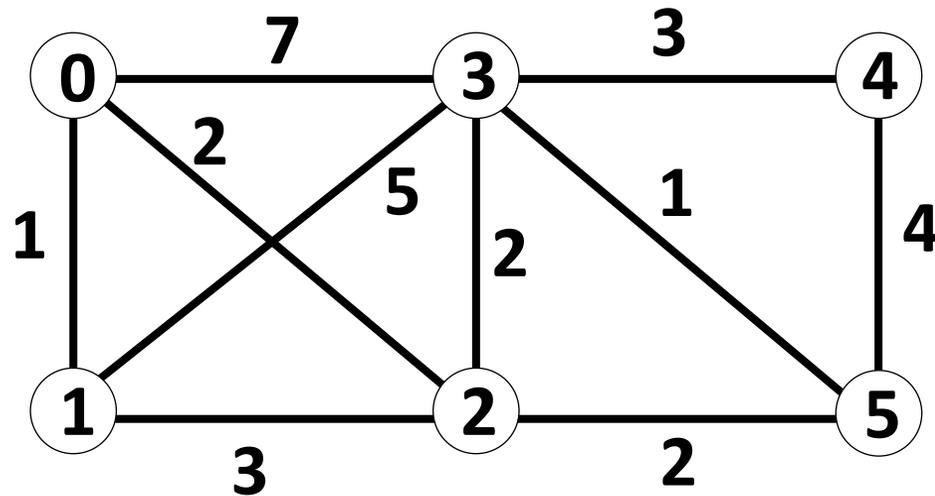
Minimum Spanning Tree



How to find MSTs?

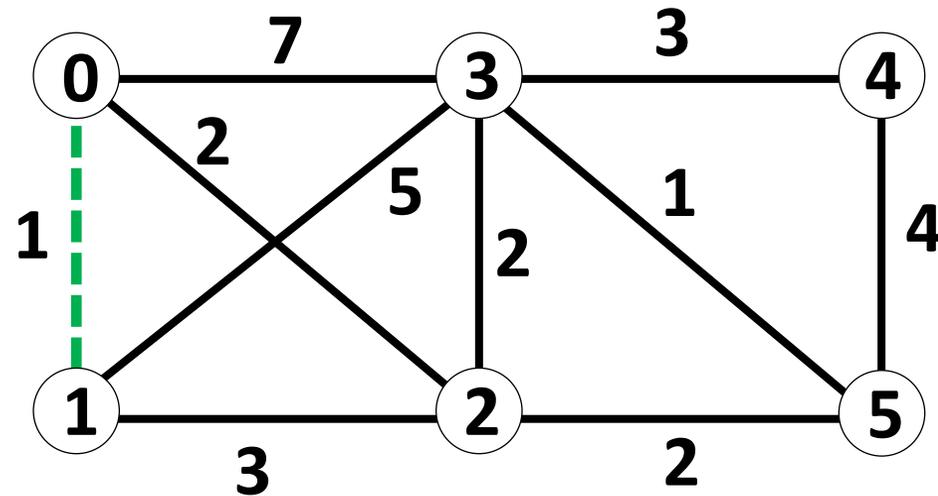
Kruskal's MST Algorithm

At each iteration, add the edge with smallest weight, that does not create a cycle.



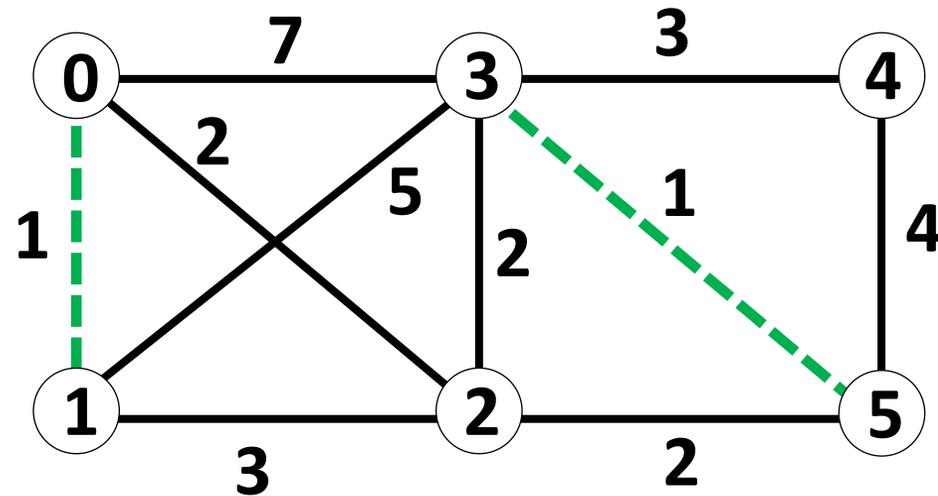
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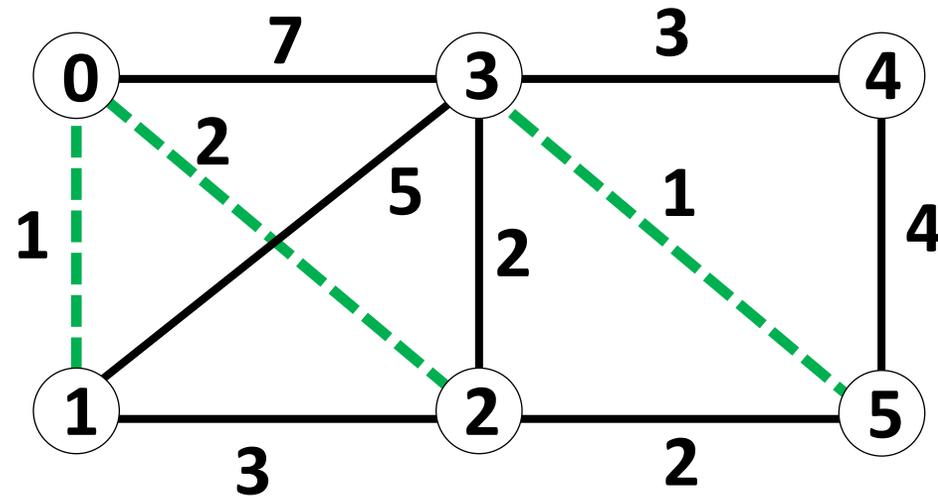
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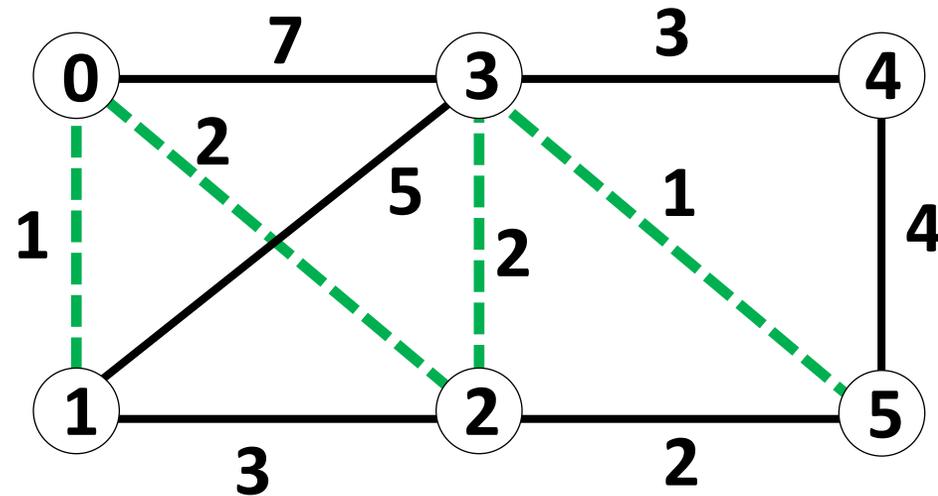
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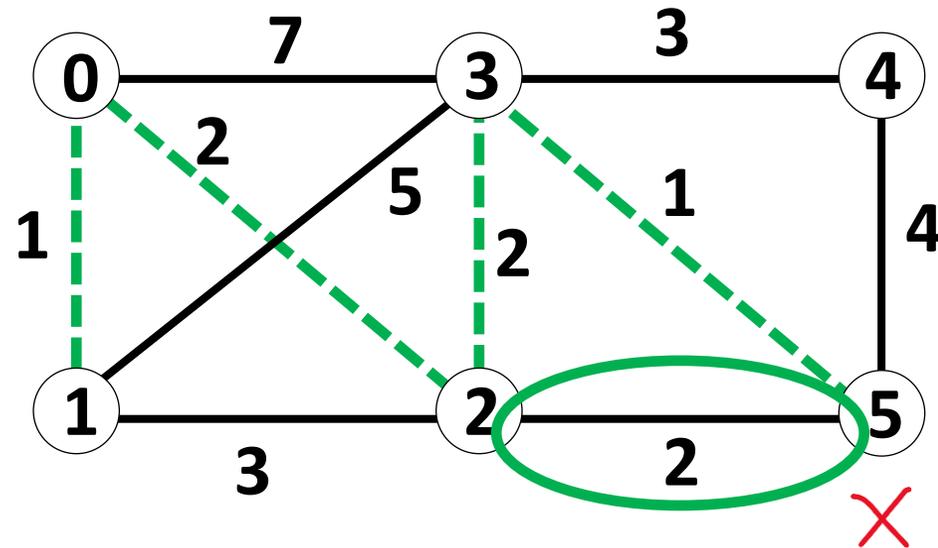
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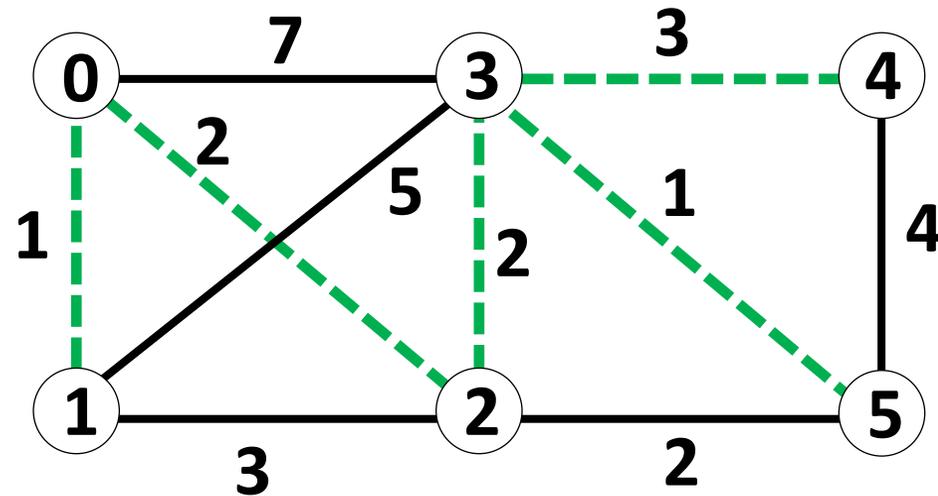
Kruskal's MST Algorithm

At each iteration, add the edge with smallest weight, that does not create a cycle.



Kruskal's MST Algorithm

At each iteration, add the edge with smallest weight, that does not create a cycle.

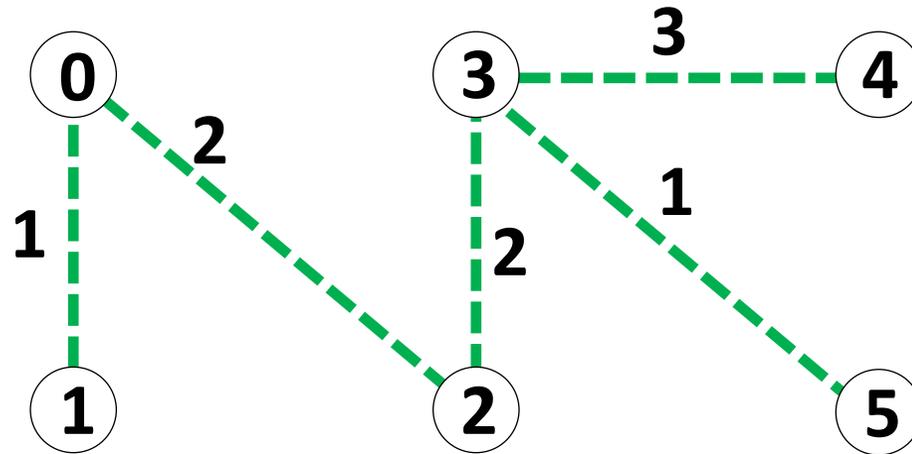


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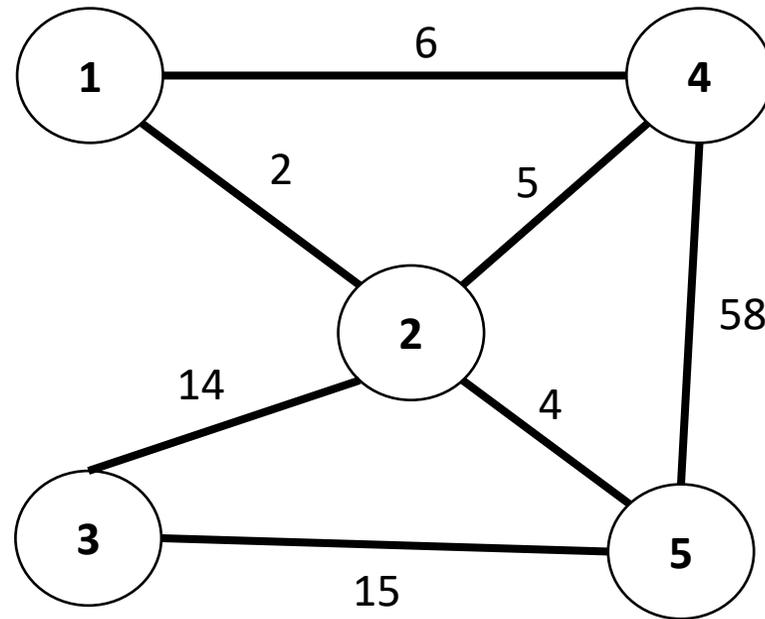
MST = [0, 1], [0, 2], [2,3], [3,5], [3,4]

Total Cost = 9



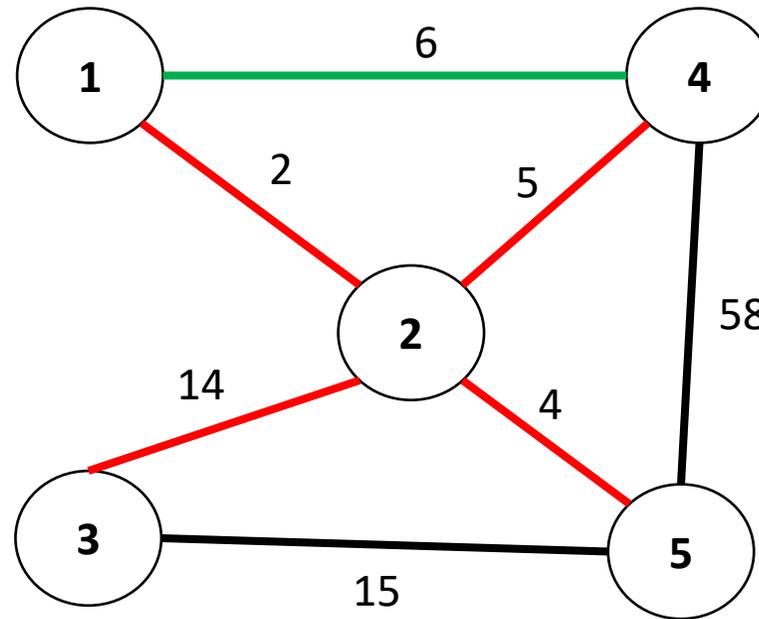
MST vs Shortest Path

MST and shortest path are two different problems, and sometimes that shortest path will not be part of the MST



MST vs Shortest Path

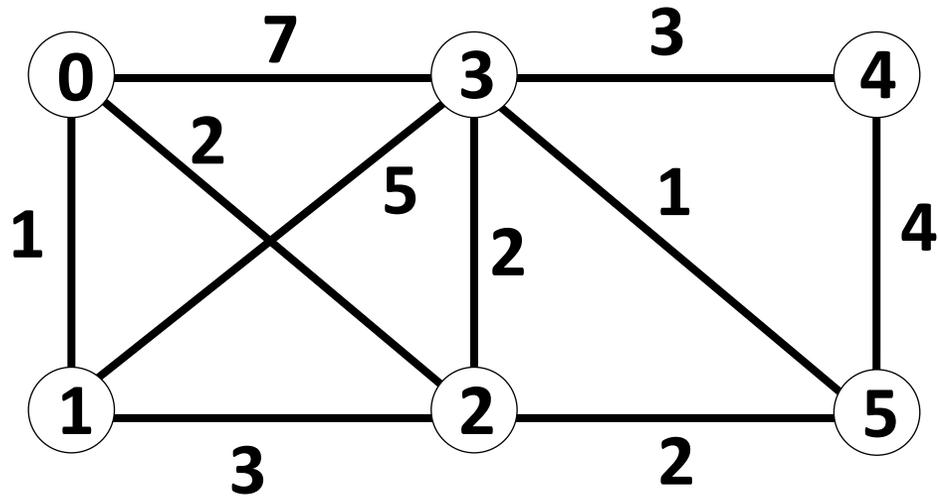
MST and shortest path are two different problems, and sometimes that shortest path will not be part of the MST



MST Cost = 25

Shortest Path from 1 to 4 = 6

Weighted Graph



```
public class Edge {  
  
    private int vertex1;  
    private int vertex2;  
  
    private int weight;  
  
    public int[] getVertices()  
  
    public int getWeight()  
  
    public String toString()  
  
    public boolean equals()
```

```
public kruskalsAlgorithm(WeightedGraph graph) {
```

```
}
```

```
public kruskalsAlgorithm(WeightedGraph graph) {  
    HashSet<Edge> mst = new HashSet<>();
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```
    // Get the set of edges.
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```
    HashSet<Edge> Edges = graph.getEdges();
```

```
}
```

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public kruskalsAlgorithm(WeightedGraph graph) {  
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    // Get the set of edges, in order of increasing weight.  
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public kruskalsAlgorithm(WeightedGraph graph) {  
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    // Get the set of edges, in order of increasing weight.  
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    // Get the set of edges, in order of increasing weight.  
    PriorityQueue<Edge> edgeQueue = new PriorityQueue<>();  
    for (Edge edge : graph.getEdges()) {  
        edgeQueue.add(edge);  
    }  
  
}
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public kruskalsAlgorithm(WeightedGraph graph) {  
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    PriorityQueue<Edge> edgeQueue = new PriorityQueue<>();  
    for (Edge edge : graph.getEdges()) {  
        edgeQueue.add(edge);  
    }  
    // Run Kruskal's algorithm.  
  
}
```



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    }  
  
    while (!edgeQueue.isEmpty()) {  
        Edge edge = edgeQueue.poll();  
  
    }  
}
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**Need to check if adding
edge adds a loop!**

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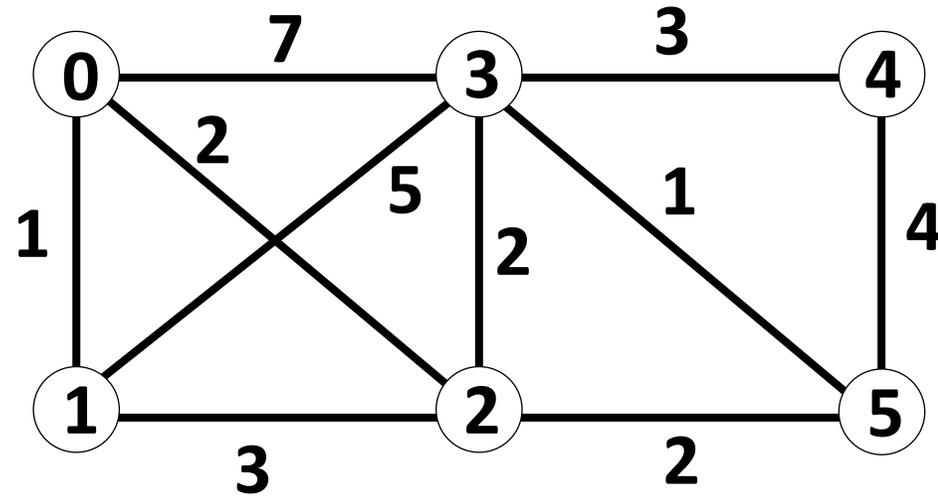
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How?

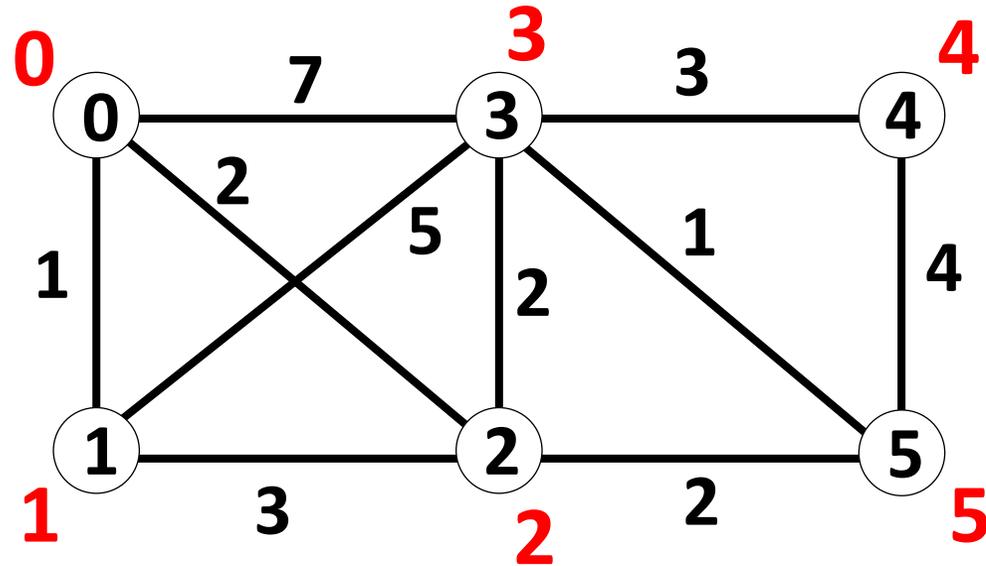
```
    }  
}
```

Cycle Finding



**How can we
determine if adding an
edge puts in a cycle?**

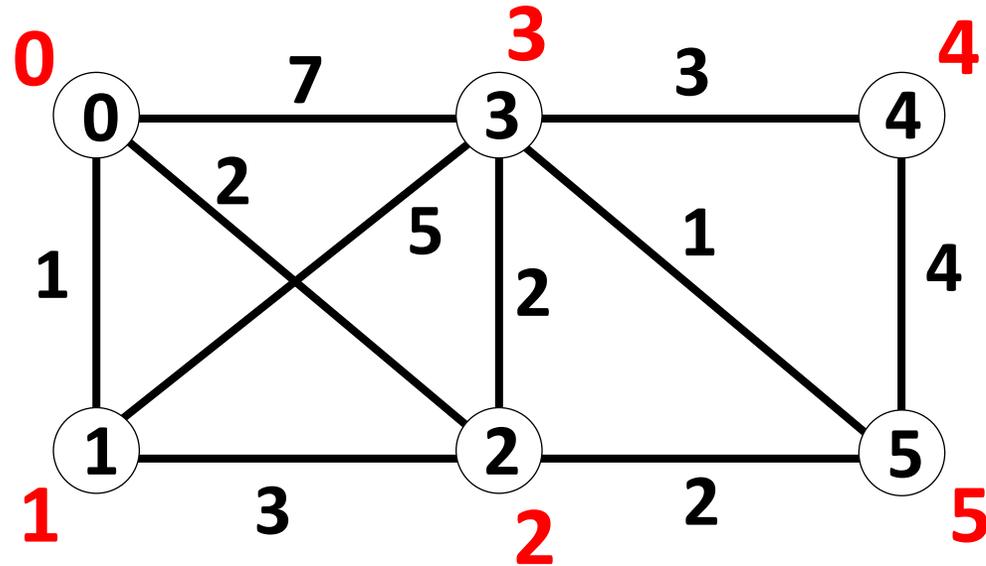
Cycle Finding



**Connected component
(in the tree) marker.**

Rules: Only add edge if vertices have different connected component markers.

Cycle Finding

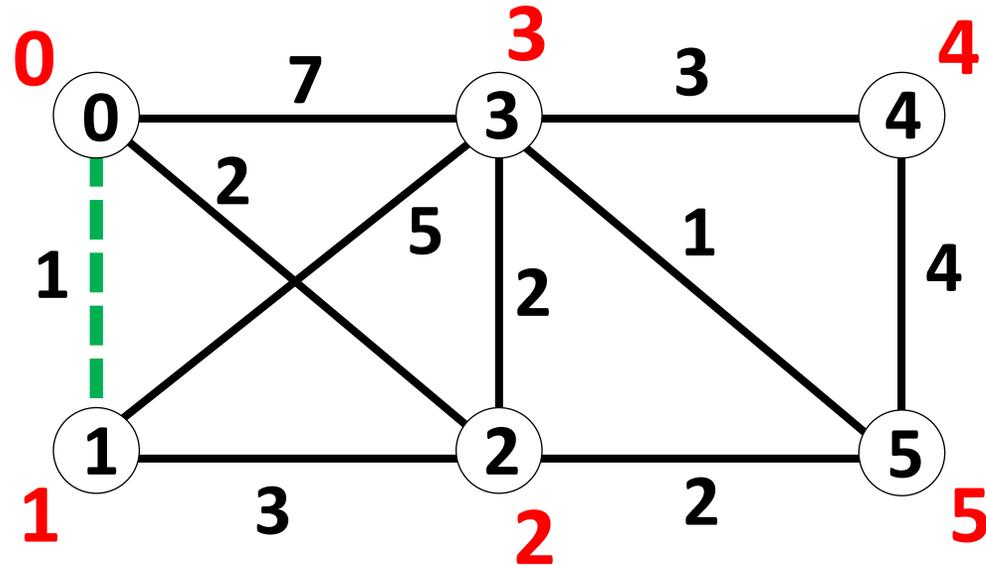


**Connected component
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To add edge, pick one vertex's marker and change all vertices with the other marker.

Cycle Finding

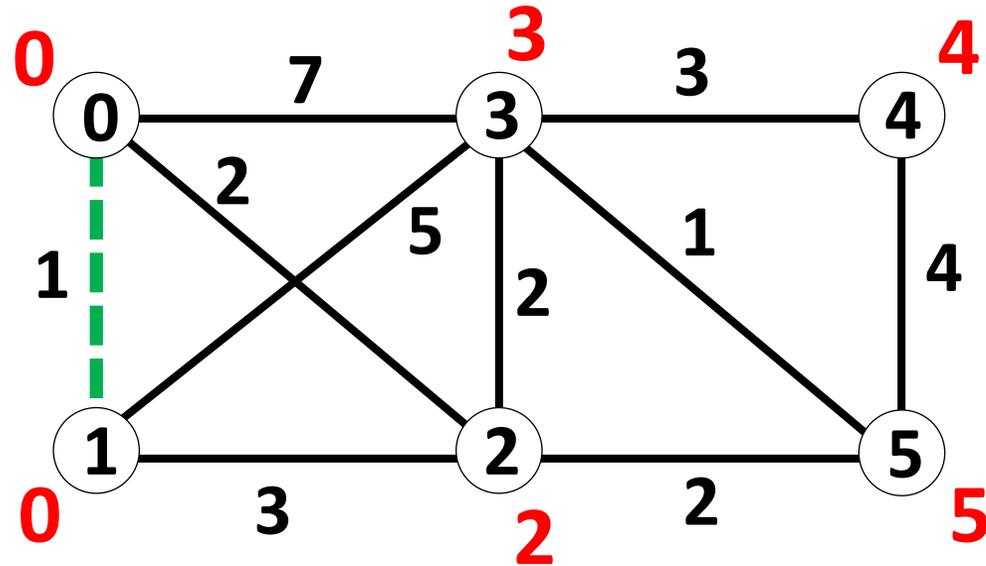


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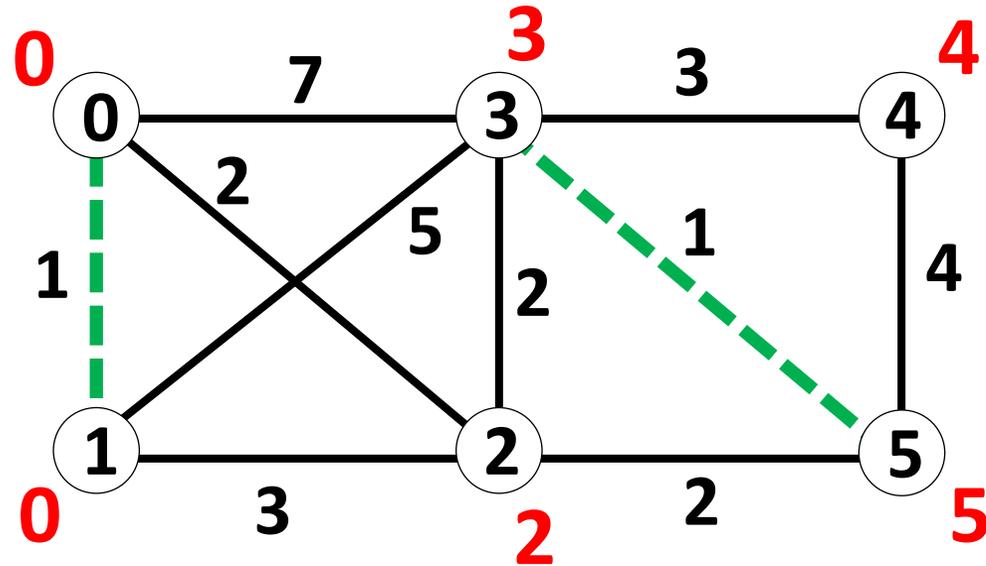


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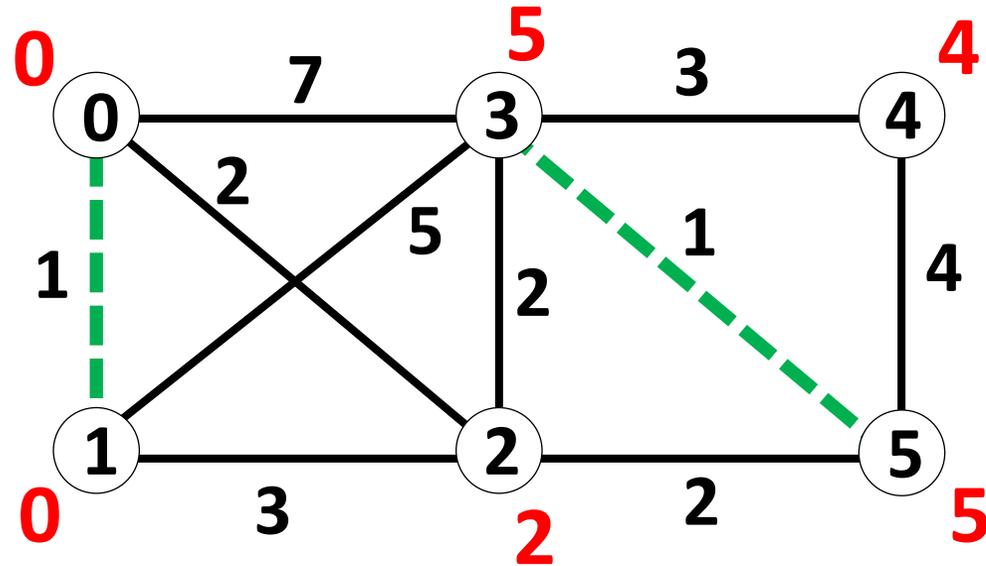


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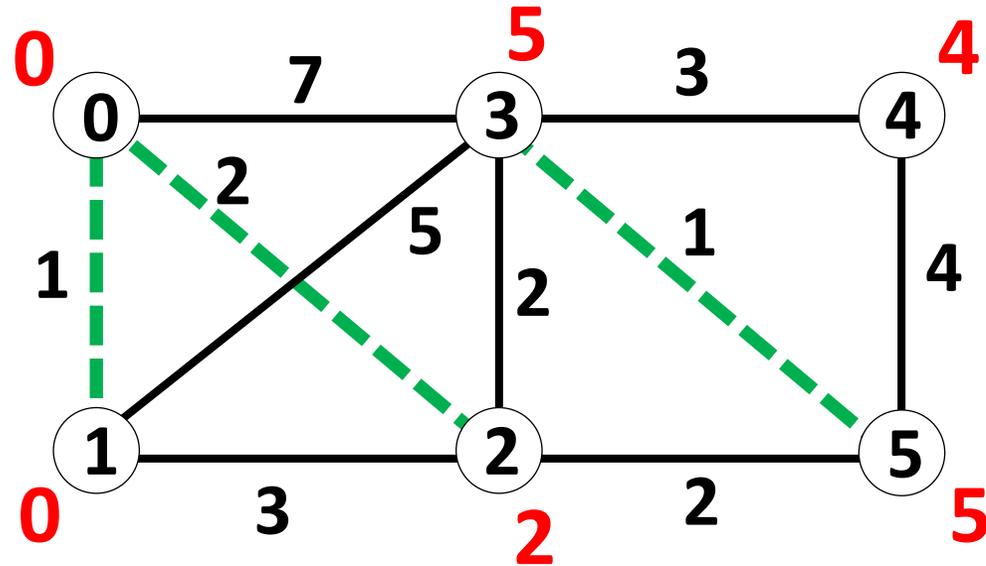


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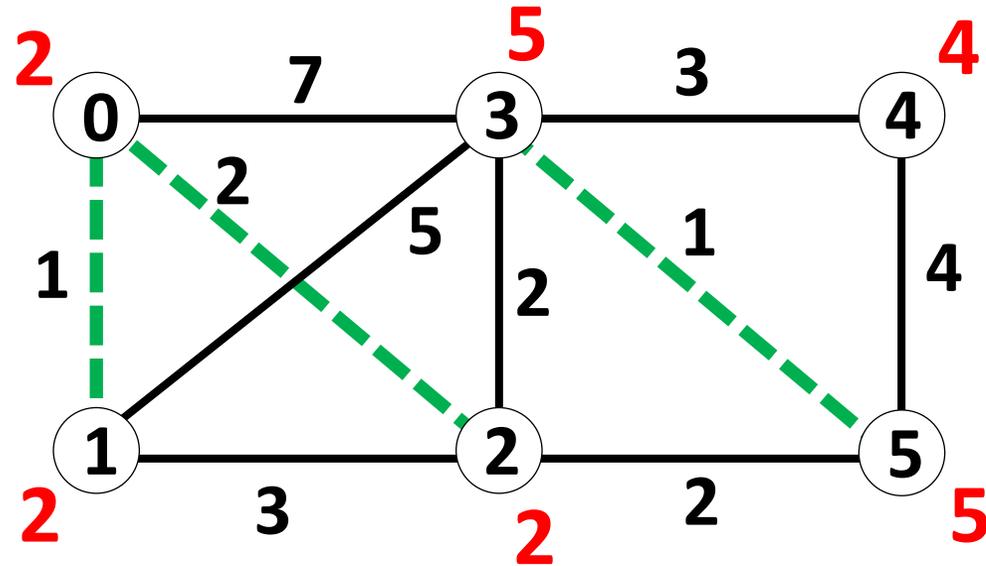


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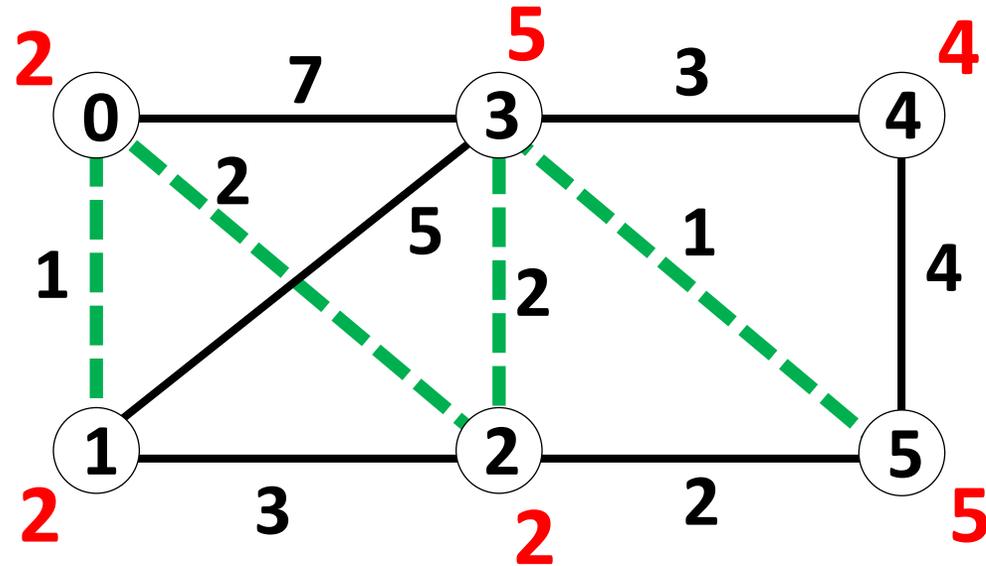


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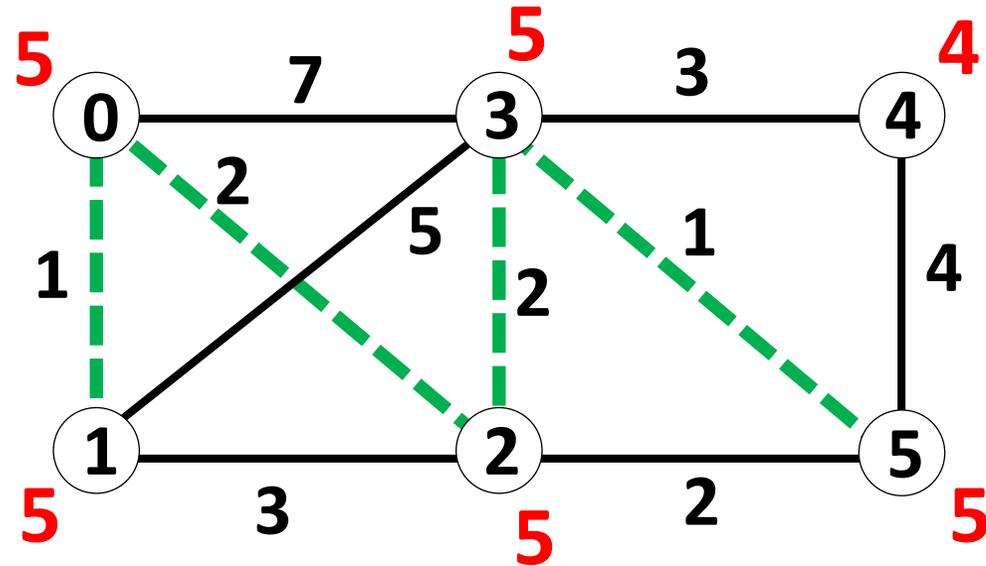


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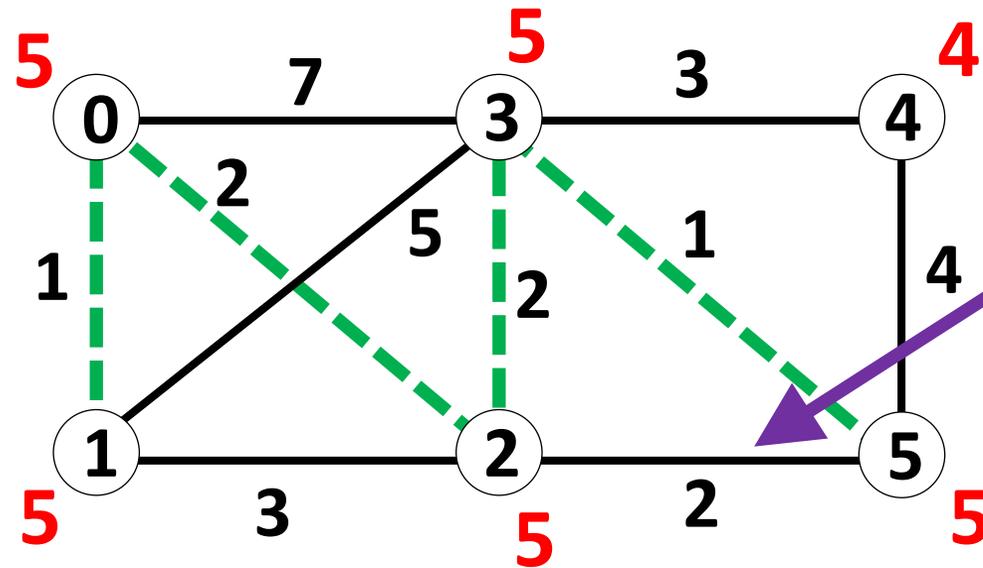


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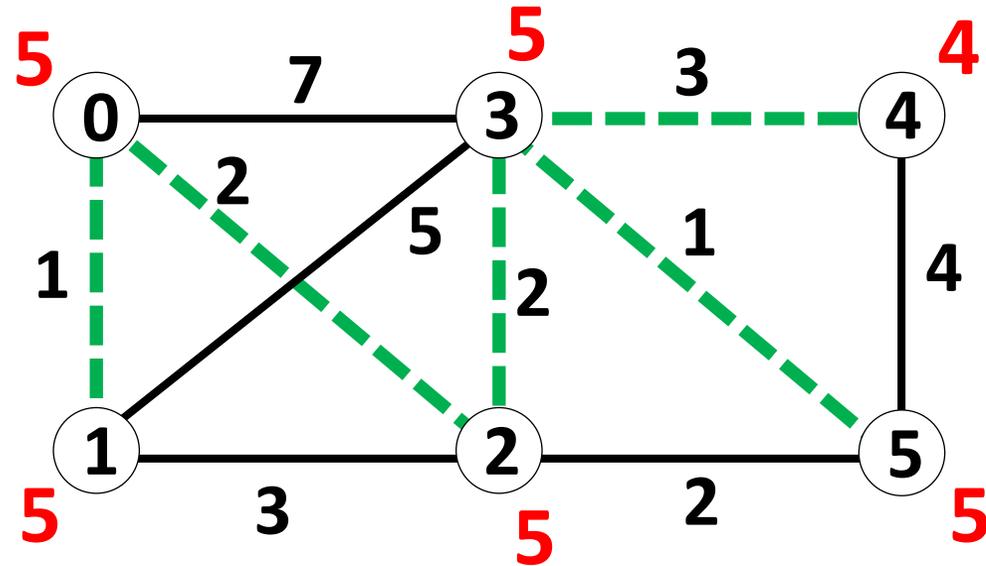
Why is adding this edge a bad idea?

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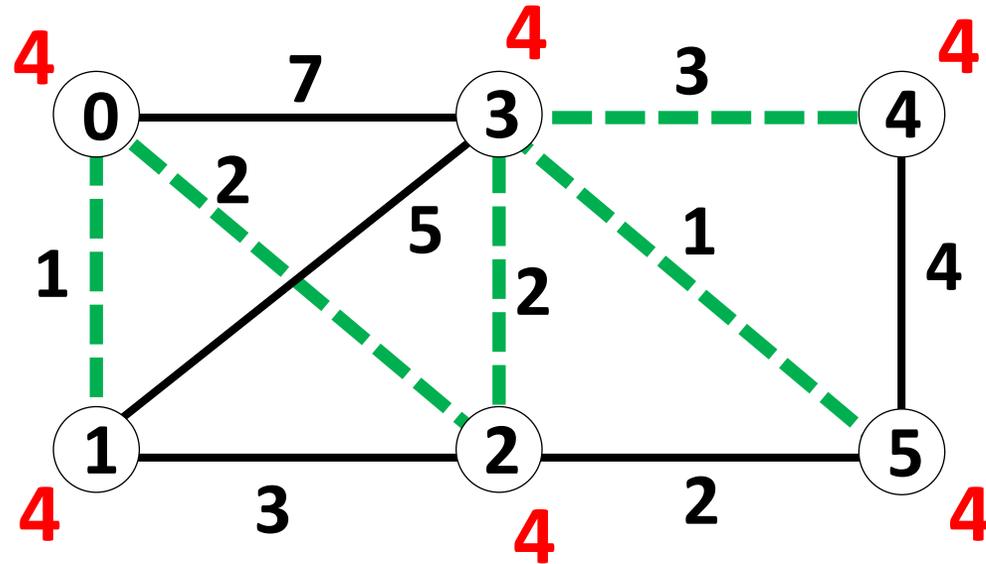


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    HashSet<Edge> mst = new HashSet<>();
```

```
    PriorityQueue<Edge> edgeQueue = new PriorityQueue<>();  
    for (Edge edge : graph.getEdges()) {  
        edgeQueue.add(edge);  
    }
```

```
    while (!edgeQueue.isEmpty()) {  
        Edge edge = edgeQueue.poll();  
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**Need to check if adding
edge adds a loop!**

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public kruskalsAlgorithm(WeightedGraph graph) {
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    int[] connectedComponentMarker = new int[graph.getNumVertices()];

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    while (!edgeQueue.isEmpty()) {
        Edge edge = edgeQueue.poll();
        if (connectedComponentMarker[edge.getVertices()[0]]
            != connectedComponentMarker[edge.getVertices()[1]]) {

        }
    }
}
}
}

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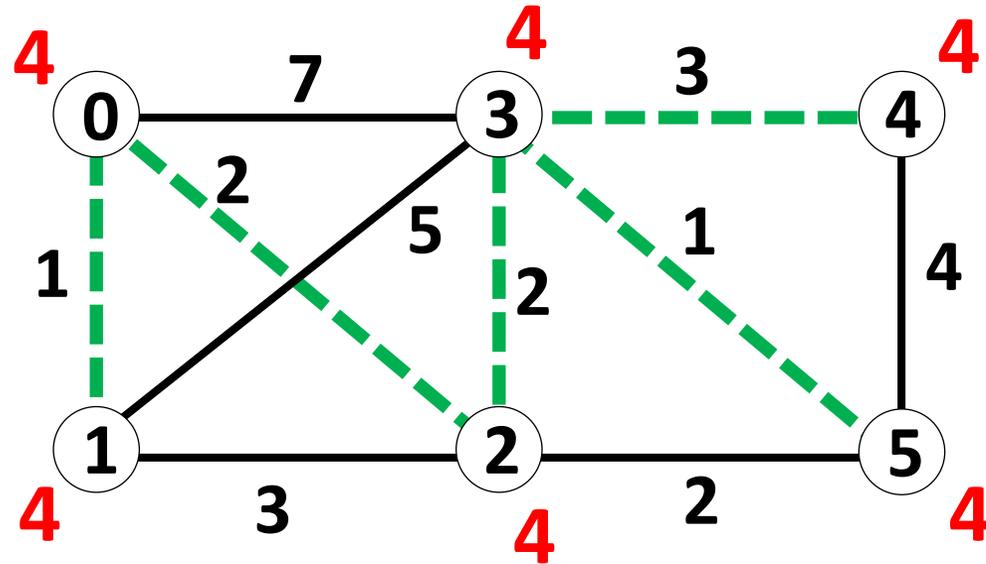
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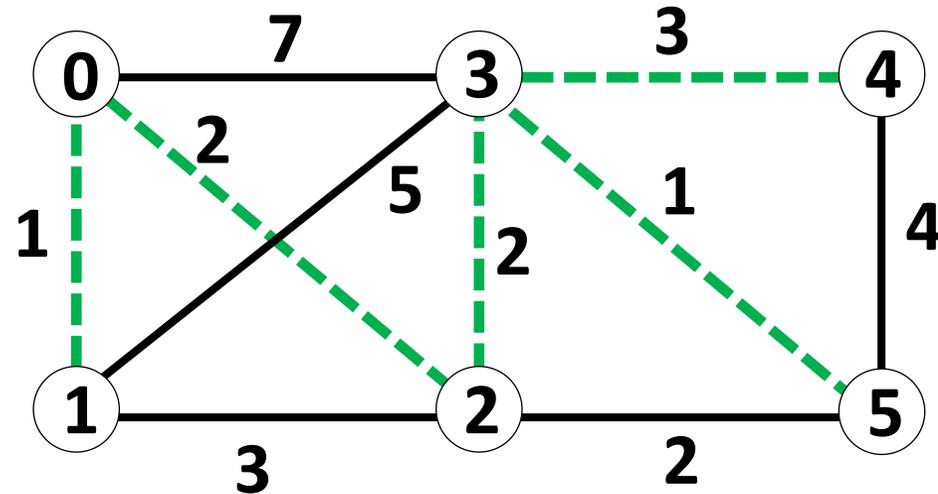
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Kruskal's MST Algorithm

At each iteration, add smallest weight edge that doesn't create a cycle.



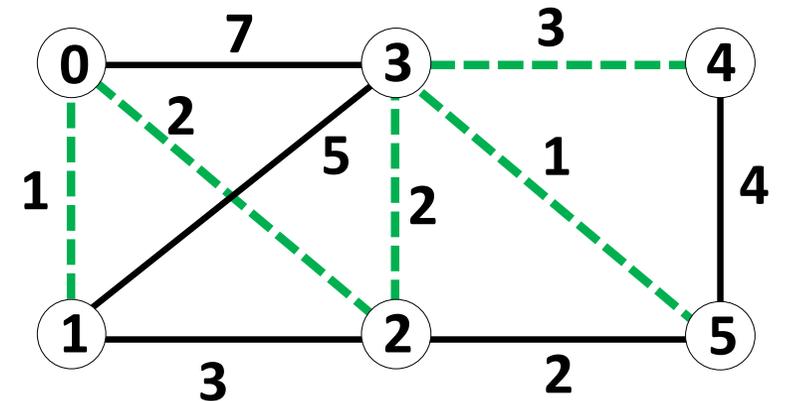
Is the solution valid (i.e., is the output a spanning tree)?

Is the solution optimal (i.e., is the output a minimum spanning tree)?

Kruskal's MST Algorithm

At each iteration, add smallest weight edge that doesn't create a cycle.

Proof of validity: Let $G = (V, E)$ be connected, and $T \subseteq E$ be the set of edges resulting from Kruskal's algorithm.

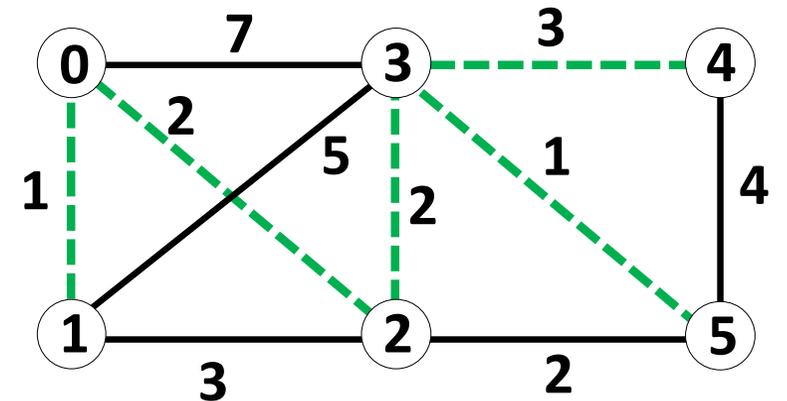


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T is a tree because...?

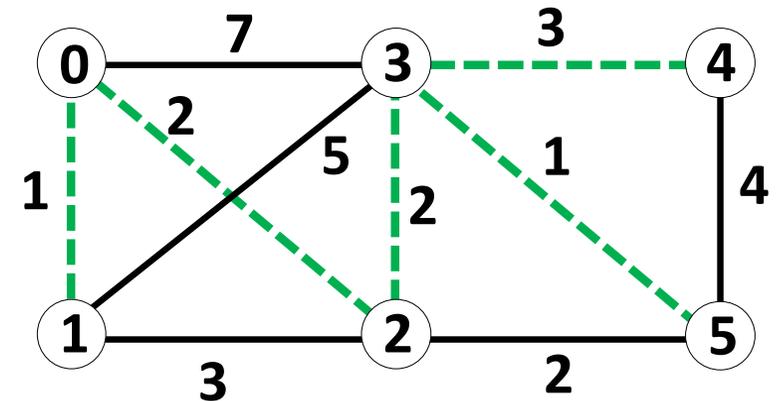


Kruskal's MST Algorithm

At each iteration, add smallest weight edge that doesn't create a cycle.

Proof of validity: Let $G = (V, E)$ be connected, and $T \subseteq E$ be the set of edges resulting from Kruskal's algorithm.

T is a tree because it is connected (otherwise we could have added more edges without creating cycles) and there are no cycles.



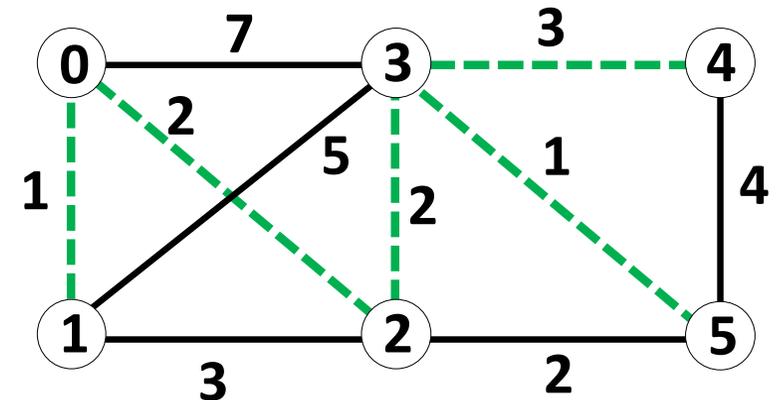
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T spans G because...?



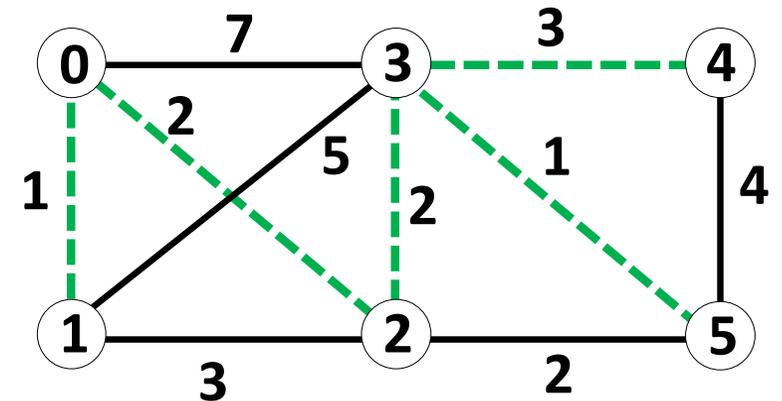
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T is a tree because it is connected (otherwise we could have added more edges without creating cycles) and there are no cycles.

T spans G because if it did not, we could have added more edges to connected unreachable nodes without creating cycles.



Kruskal's MST Algorithm

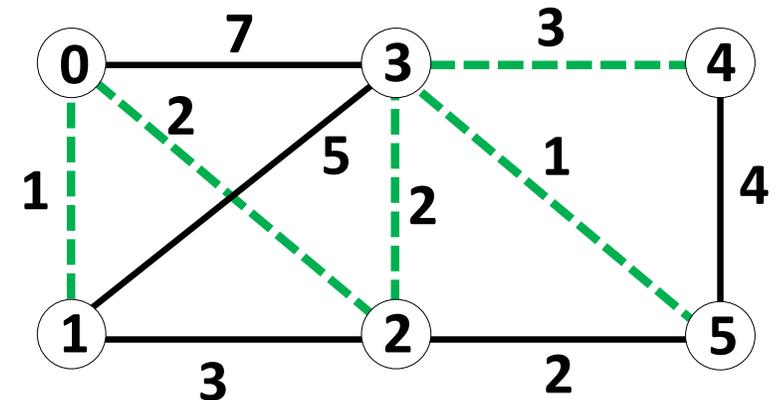
At each iteration, add smallest weight edge that doesn't create a cycle.

Proof of validity: Let $G = (V, E)$ be connected, and $T \subseteq E$ be the set of edges resulting from Kruskal's algorithm.

T is a tree because it is connected (otherwise we could have added more edges without creating cycles) and there are no cycles.

T spans G because if it did not, we could have added more edges to connected unreachable nodes without creating cycles.

$\therefore T$ is a spanning tree of G



Kruskal's MST Algorithm

At each iteration, add smallest weight edge that doesn't create a cycle.

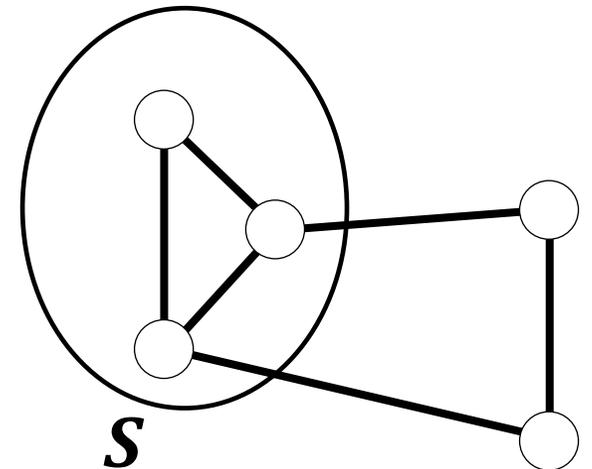
Proof of optimality: Let $G = (V, E)$ be connected, and $T \subseteq E$ be the set of edges resulting from Kruskal's algorithm.

T is an MST because???

MST Cut Property

Lemma: Suppose that S is a subset of nodes from $G = (V, E)$. Then, the cheapest edge e between S and $V \setminus S$ is part of every MST.

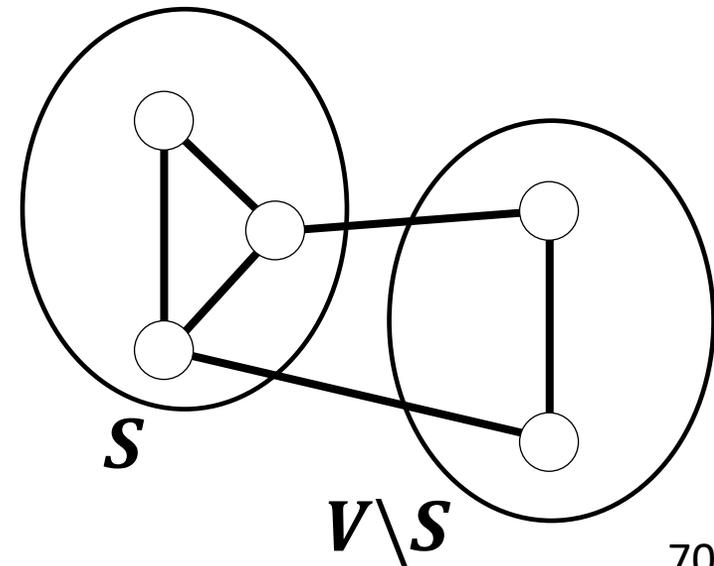
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MST Cut Property

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Proof:



MST Cut Property

Lemma: Suppose that S is a subset of nodes from $G = (V, E)$. Then, the cheapest edge e between S and $V \setminus S$ is part of every MST.

Proof: Any MST of G must include some edge between S and $V \setminus S$ (otherwise it would not be a tree).

Let e be the cheapest edge between S and $V \setminus S$.

Suppose T is a ST that does not include e . $T \cup \{e\}$ must have a cycle and that cycle must have another edge e' between S and $V \setminus S$.

Remove e' to form $T' = T \cup \{e\} \setminus \{e'\}$.

T' is a tree (removing edge from cycle cannot disconnect graph)

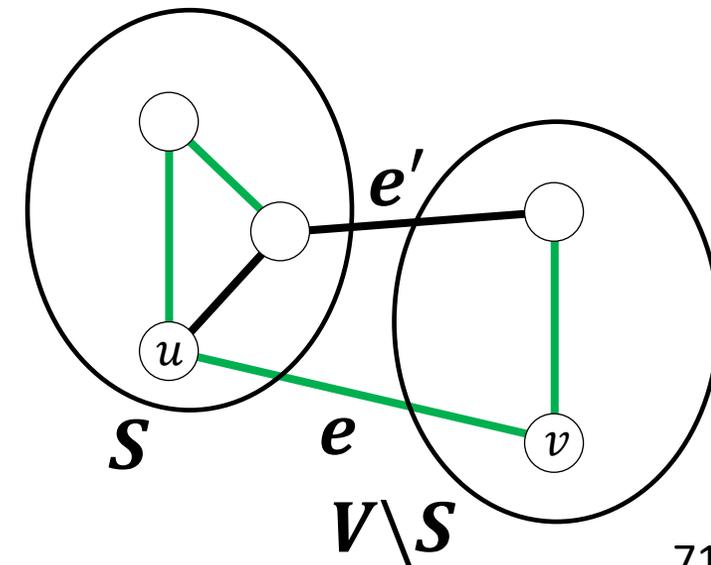
T' spans V (same number of edges as ST T)

$\text{weight}(T') = \text{weight}(T) + \text{weight}(e) - \text{weight}(e')$.

$\Rightarrow \text{weight}(T') < \text{weight}(T)$, since $\text{weight}(e) < \text{weight}(e')$.

$\Rightarrow T'$ is a cheaper ST.

So, e is part of every MST.

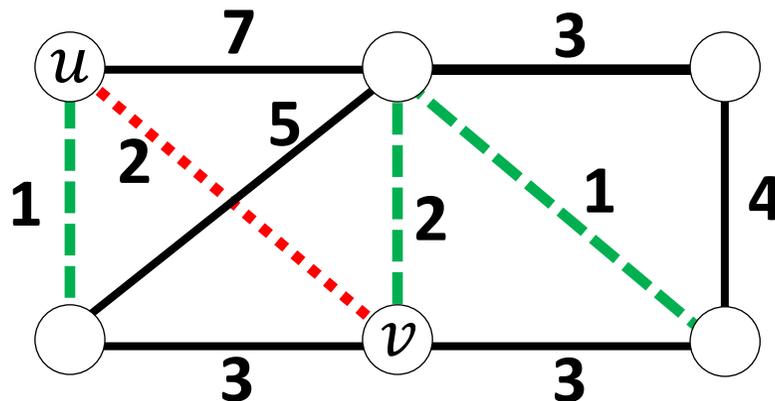


Kruskal's MST Algorithm

At each iteration, add smallest weight edge that doesn't create a cycle.

Proof of optimality: Let $G = (V, E)$ be connected, and $T \subseteq E$ be the set of edges resulting from Kruskal's algorithm.

Consider the iteration that edge $e = (u, v)$ is added by Kruskal's algorithm. Let S be the set of v and nodes connected to v . Clearly $v \in S$ and $u \in V \setminus S$ (otherwise adding e would have created a cycle). We are picking the cheapest such edge (otherwise the cheaper edge would have been selected since it would not have created a cycle either). By the cut property lemma, this edge must be part of the MST.



Lemma: The cheapest edge between $S \subseteq V$ and $V \setminus S$ is part of every MST.

```

public kruskalsAlgorithm(WeightedGraph graph) {
    HashSet<Edge> mst = new HashSet<>();

    int[] connectedComponentMarker = new int[graph.getNumVertices()];
    for (int i = 0; i < connectedComponentMarker.length; i++) {
        connectedComponentMarker[i] = i;
    }

    PriorityQueue<Edge> edgeQueue = new PriorityQueue<>();
    for (Edge edge : graph.getEdges()) {
        edgeQueue.add(edge);
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    while (!edgeQueue.isEmpty()) {
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            mst.add(edge);
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    for (Edge edge : graph.getEdges()) {
        edgeQueue.add(edge);
    } O(E log E)

```

```

while (!edgeQueue.isEmpty()) { O(E) O(E * V)
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        for (int i = 0; i < connectedComponentMarker.length; i++) { O(V)
            if (connectedComponentMarker[i] == oldMarker) { O(1)
                connectedComponentMarker[i] = newMarker; O(1)
            }
        }
    }
}

```

```

public kruskalsAlgorithm(WeightedGraph graph) { V = # of vertices      E = # of edges
    HashSet<Edge> mst = new HashSet<>(); O(1)

    int[] connectedComponentMarker = new int[graph.getNumVertices()]; O(V)
    for (int i = 0; i < connectedComponentMarker.length; i++) {
        connectedComponentMarker[i] = i;
    }
}

```

```

PriorityQueue<Edge> edgeQueue = new PriorityQueue<>(); O(1)
for (Edge edge : graph.getEdges()) {
    edgeQueue.add(edge);
}

```

O(E log E)

```

while (!edgeQueue.isEmpty()) { O(E)
    Edge edge = edgeQueue.poll(); O(1)*
    if (connectedComponentMarker[edge.getVertices()[0]]
        != connectedComponentMarker[edge.getVertices()[1]]) {
        mst.add(edge); O(1)
        int newMarker = connectedComponentMarker[edge.getVertices()[0]]; O(1)
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        }
    }
}

```

O(E * V)

```
public kruskalsAlgorithm(WeightedGraph graph) { V = # of vertices      E = # of edges
```

```
    HashSet<Edge> mst = new HashSet<>(); O(1)
```

```
    int[] connectedComponentMarker = new int[graph.getNumVertices()]; O(V)
```

```
    for (int i = 0; i < connectedComponentMarker.length; i++) {  
        connectedComponentMarker[i] = i;      O(V)      O(V)  
    }
```

```
    PriorityQueue<Edge> edgeQueue = new PriorityQueue<>(); O(1)      O(E log E)
```

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    for (Edge edge : graph.getEdges()) {  
        edgeQueue.add(edge);      O(E log E)  
    }
```

```
    while (!edgeQueue.isEmpty()) { O(E)      O(E * V)  
        Edge edge = edgeQueue.poll(); O(1)*  
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                if (connectedComponentMarker[i] == oldMarker) { O(1)  
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                }            }        }    }
```

public kruskalsAlgorithm(WeightedGraph graph) { $V = \# \text{ of vertices}$ $E = \# \text{ of edges}$

HashSet<Edge> mst = new HashSet<>(); $O(1)$

int[] connectedComponentMarker = new int[graph.getNumVertices()]; $O(V)$ $O(V)$

for (int i = 0; i < connectedComponentMarker.length; i++) {
 connectedComponentMarker[i] = i; } $O(V)$ $O(V)$

PriorityQueue<Edge> edgeQueue = new PriorityQueue<>(); $O(1)$ $O(E \log E)$

for (Edge edge : graph.getEdges()) {
 edgeQueue.add(edge); } $O(E \log E)$

while (!edgeQueue.isEmpty()) { $O(E)$ $O(E * V)$
 Edge edge = edgeQueue.poll(); $O(1)^*$
 if (connectedComponentMarker[edge.getVertices()[0]] $O(1)$
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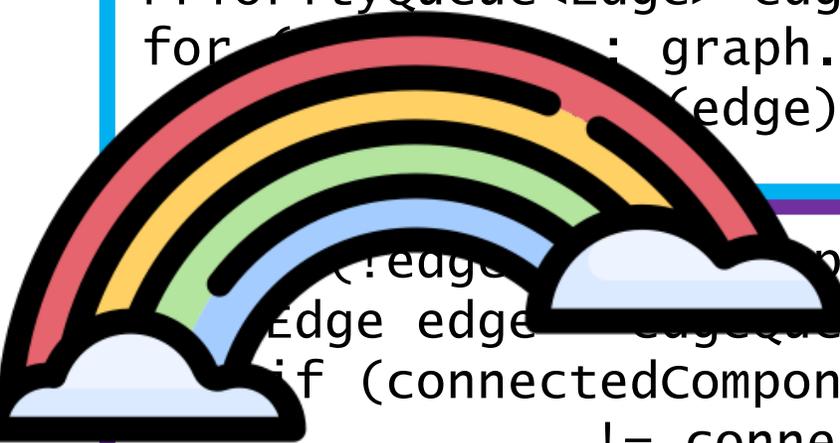
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                }
            }
        }
    }
}

```



$$O(E * V) + O(E \log E) + O(V) + O(V) + O(1)$$

$$O(E * V) + O(E \log E) + O(V) + O(V) + O(1)$$



Dominant Factor

$$O(E * V)$$



Dominant Factor

At most, how many edges are in an undirected graph with n vertices?
What graph has the most number of edges?

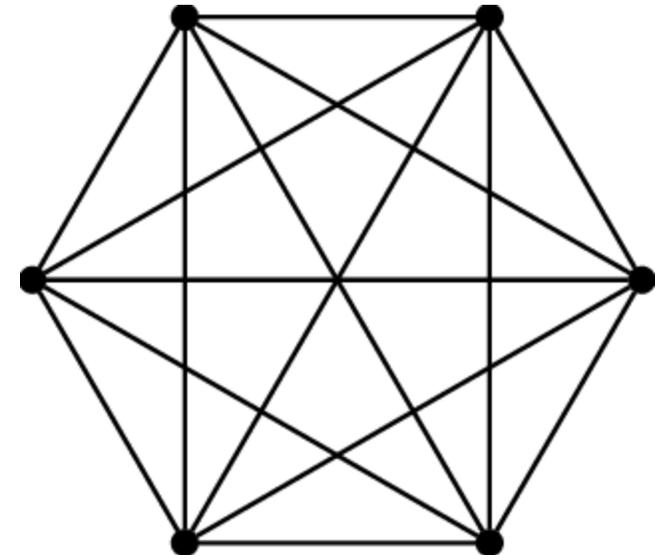
$$O(E * V)$$



Dominant Factor

At most, how many edges are in an undirected graph with n vertices?
What graph has the most number of edges?

A complete graph



$$O(E * V)$$

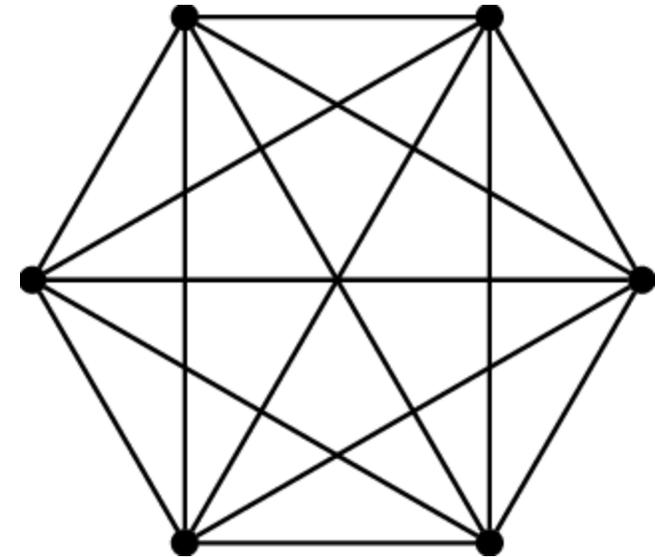


Dominant Factor

At most, how many edges are in an undirected graph with n vertices?
What graph has the most number of edges?

A complete graph

How many edges does a complete graph with n vertices have?



$$O(E * V)$$

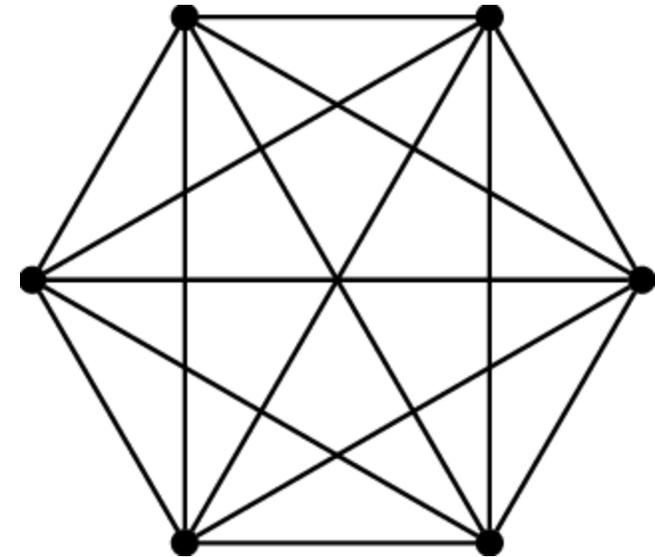


Dominant Factor

At most, how many edges are in an undirected graph with n vertices?
What graph has the most number of edges?

A complete graph

How many edges does a complete graph with n vertices have?
How many edges leave each vertex? $n-1$



$$O(E * V)$$



Dominant Factor

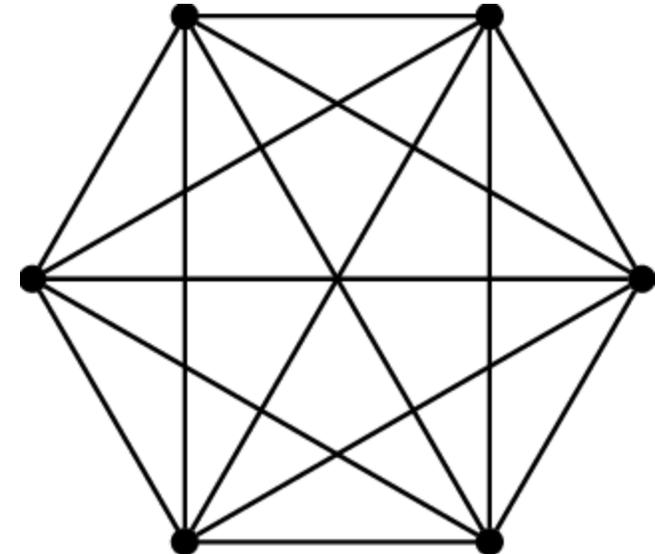
At most, how many edges are in an undirected graph with n vertices?
What graph has the most number of edges?

A complete graph

How many edges does a complete graph with n vertices have?

How many edges leave each vertex? $n-1$

How much does that all add up to? $n * (n-1)$



$$O(E * V)$$



Dominant Factor

At most, how many edges are in an undirected graph with n vertices?
What graph has the most number of edges?

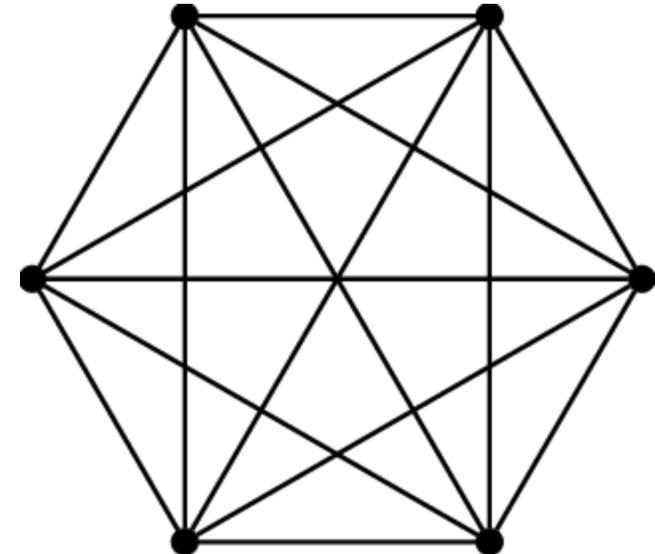
A complete graph

How many edges does a complete graph with n vertices have?

How many edges leave each vertex? $n-1$

How much does that all add up to? $n * (n-1)$

Did we double count any edges? Yes



$$O(E * V)$$



Dominant Factor

At most, how many edges are in an undirected graph with n vertices?
What graph has the most number of edges?

A complete graph

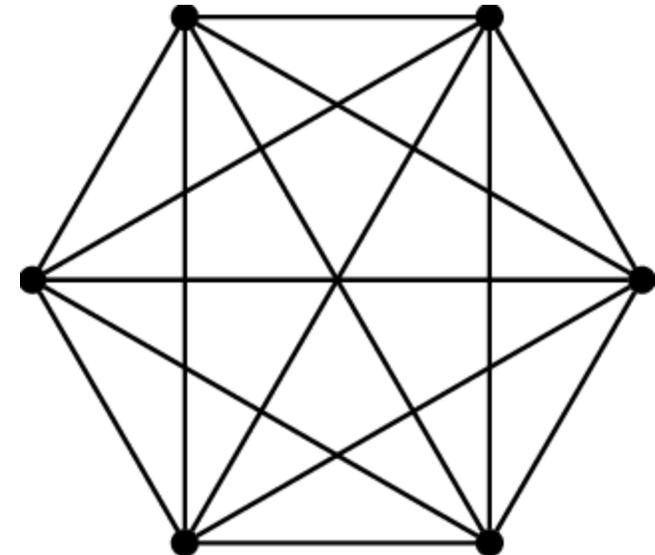
How many edges does a complete graph with n vertices have?

How many edges leave each vertex? $n-1$

How much does that all add up to? $n * (n-1)$

Did we double count any edges? Yes

So how many edges are there?



$$O(E * V)$$



Dominant Factor

At most, how many edges are in an undirected graph with n vertices?
What graph has the most number of edges?

A complete graph

How many edges does a complete graph with n vertices have?

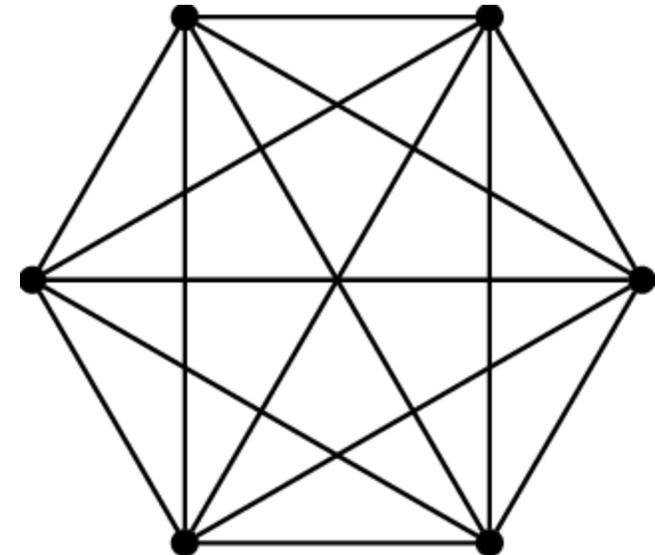
How many edges leave each vertex? $n-1$

How much does that all add up to? $n * (n-1)$

Did we double count any edges? Yes

So how many edges are there?

$$\frac{n(n-1)}{2}$$



$$O(E * V)$$



Dominant Factor

At most, how many edges are in an undirected graph with n vertices?
What graph has the most number of edges?

A complete graph

How many edges does a complete graph with n vertices have?

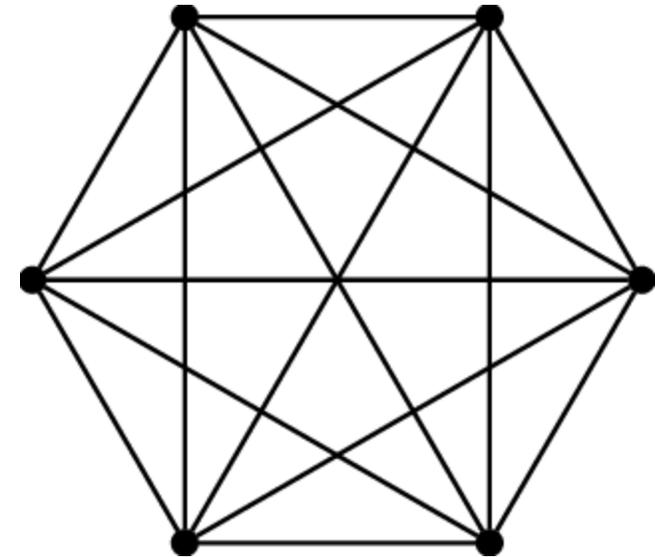
How many edges leave each vertex? $n-1$

How much does that all add up to? $n * (n-1)$

Did we double count any edges? Yes

So how many edges are there?

$$\frac{n(n-1)}{2} \rightarrow \frac{1}{2}(n^2 - n) \in O(n^2)$$



$$O(E * V) \quad O(n^2 * n)$$

Running time of **our** Kruskal's algorithm



Dominant Factor

$$O(n^3)$$

At most, how many edges are in an undirected graph with **n** vertices?
What graph has the most number of edges?

A complete graph

How many edges does a complete graph with **n** vertices have?

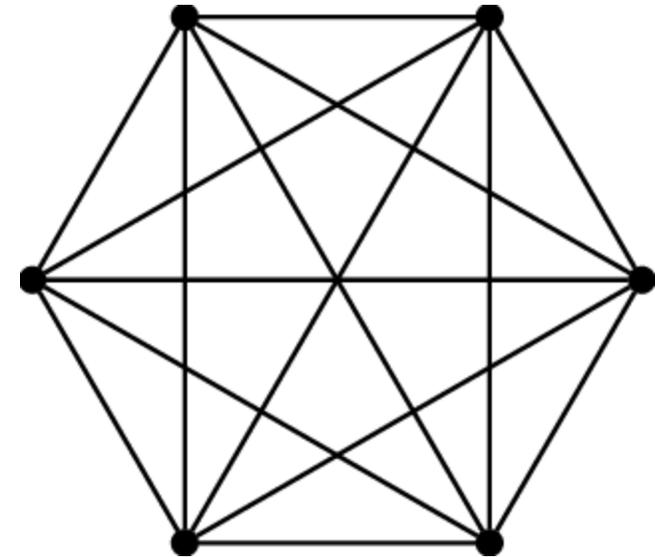
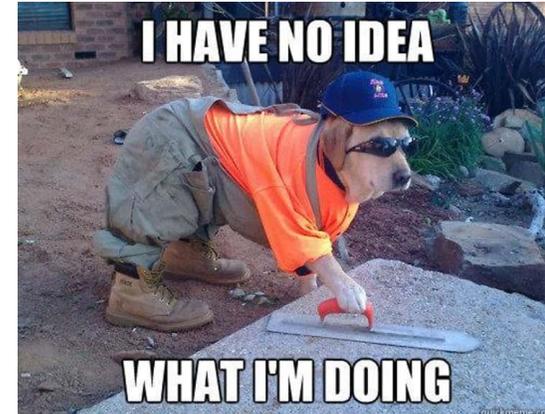
How many edges leave each vertex? $n-1$

How much does that all add up to? $n * (n-1)$

Did we double count any edges? Yes

So how many edges are there?

$$\frac{n(n-1)}{2} \rightarrow \frac{1}{2}(n^2 - n) \in O(n^2)$$



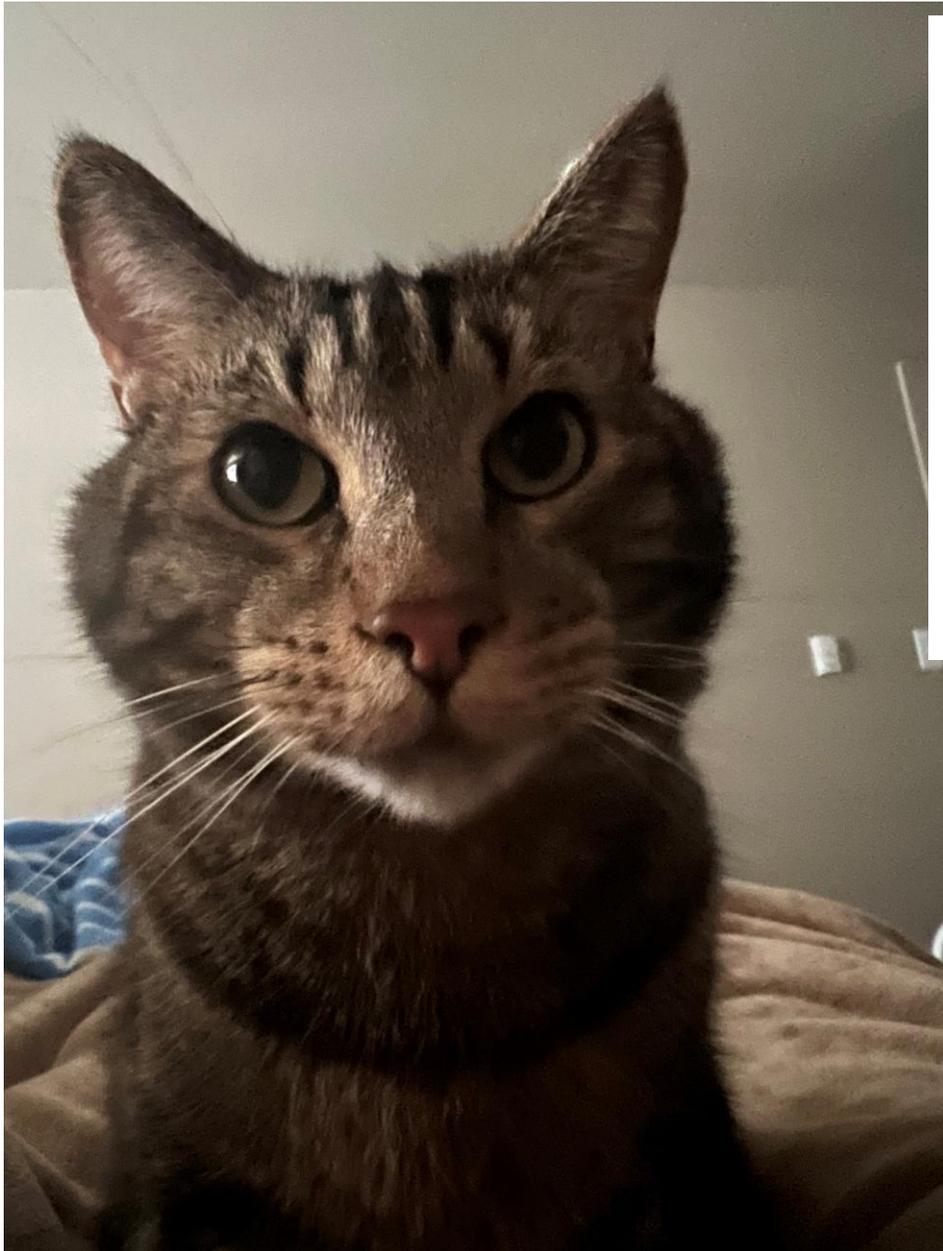
There are many ways to write Kruskal's algorithm

MST-KRUSKAL(G, w)

```
o(1)  1  A = ∅
o(v)  2  for each vertex v ∈ G.V
      3    MAKE-SET(v)
o(E log E) 4  sort the edges of G.E into nondecreasing order by weight w
          5  for each edge (u, v) ∈ G.E, taken in nondecreasing order by weight
          6    if FIND-SET(u) ≠ FIND-SET(v)
          7      A = A ∪ {(u, v)}
          8      UNION(u, v)
o(v log v) 9  return A
```

$E > V$ in a complete graph (worst case), so we can get Kruskal's algorithm to run in **$O(E \log E)$** time

Running time of finding an
MST: **$O(E \log E)$**



Complexity [\[edit\]](#)

For a graph with E edges and V vertices, Kruskal's algorithm can be shown to run in time $O(E \log E)$ time, with simple data structures. Here, O expresses the time in [big O notation](#), and \log is a [logarithm](#) to any base (since inside O -notation logarithms to all bases are equivalent, because they are the same up to a constant factor). This time bound is often written instead as $O(E \log V)$, which is equivalent for graphs with no isolated vertices, because for these graphs $V/2 \leq E < V^2$ and the logarithms of V and E are again within a constant factor of each other.

To achieve this bound, first sort the edges by weight using a [comparison sort](#) in $O(E \log E)$ time. Once sorted, it is possible to loop through the edges in sorted order in constant time per edge. Next, use a [disjoint-set data structure](#), with a set of vertices for each component, to keep track of which vertices are in which components. Creating this structure, with a separate set for each vertex, takes V operations and $O(V)$ time. The final iteration through all edges performs two find operations and possibly one union operation per edge. These operations take [amortized time](#) $O(\alpha(V))$ time per operation, giving worst-case total time $O(E \alpha(V))$ for this loop, where α is the extremely slowly growing [inverse Ackermann function](#). This part of the time bound is much smaller than the time for the sorting step, so the total time for the algorithm can be simplified to the time for the sorting step.

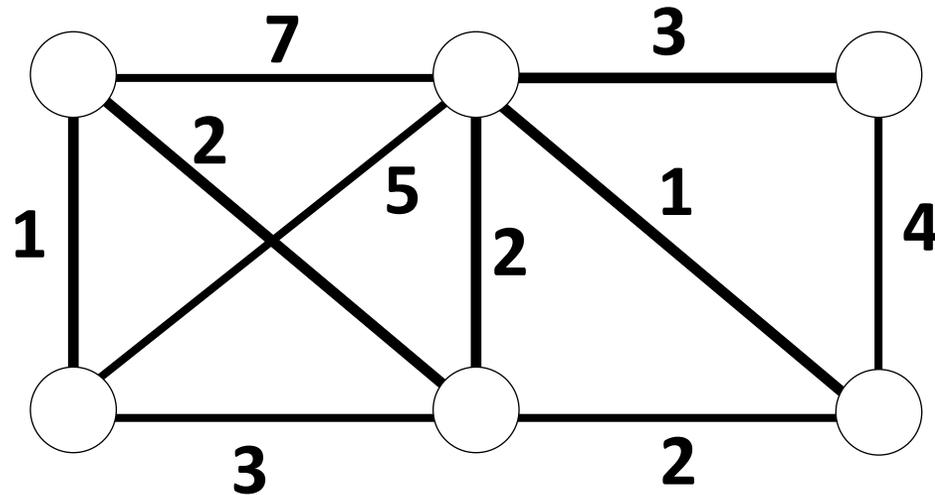
In cases where the edges are already sorted, or where they have small enough integer weight to allow [integer sorting](#) algorithms such as [counting sort](#) or [radix sort](#) to sort them in linear time, the disjoint set operations are the slowest remaining part of the algorithm and the total time is $O(E \alpha(V))$.

**Ok, take a deep
breath, the
analysis is over**

There is another prominent algorithm that can also compute an MST

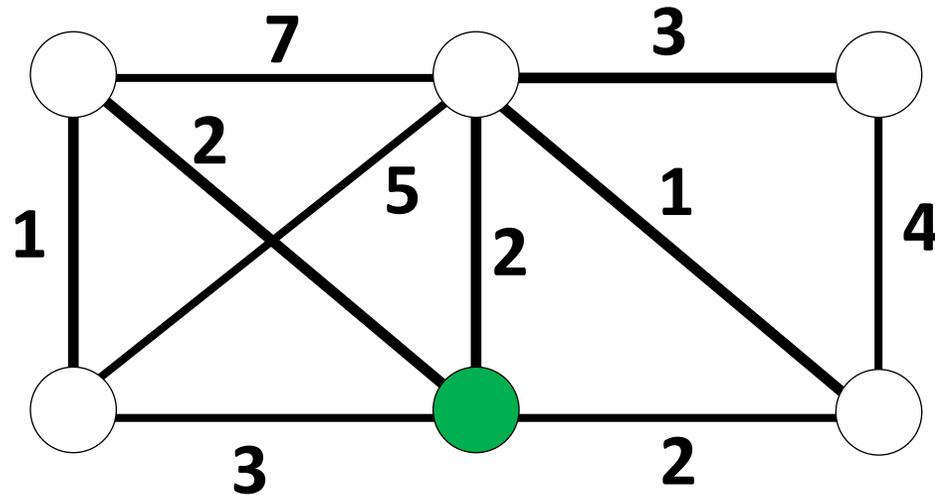
Prim's MST Algorithm

Beginning at any node, add the node that can be connected as cheaply as possible to the tree we are building.



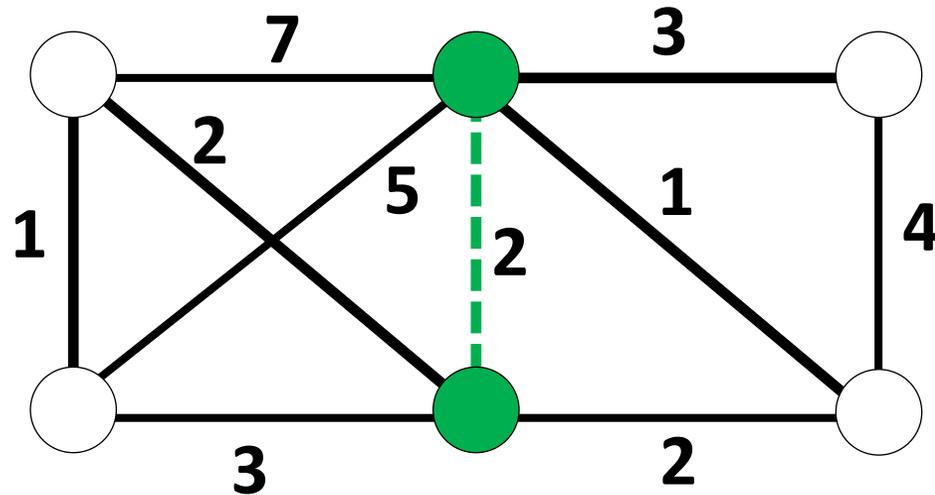
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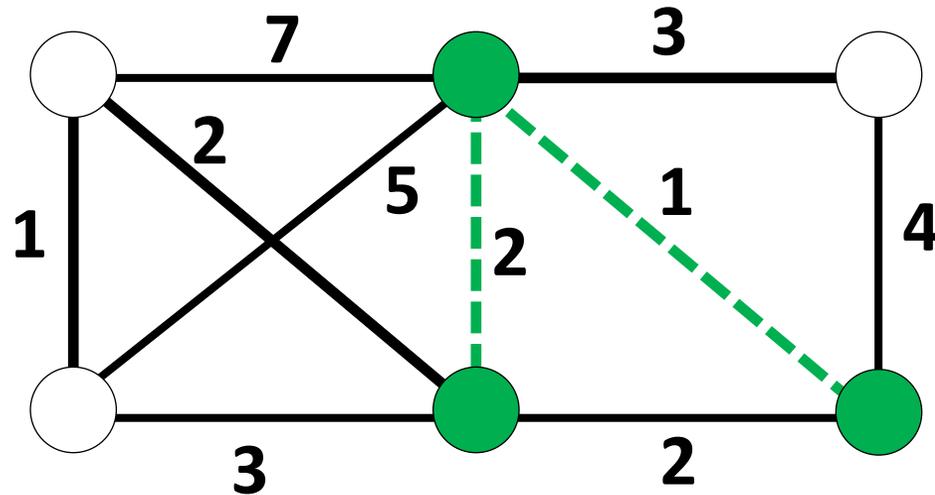
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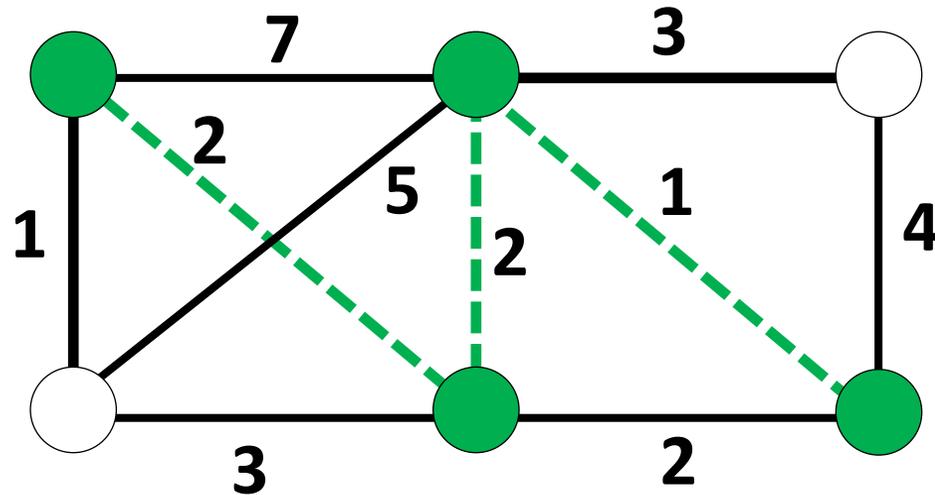
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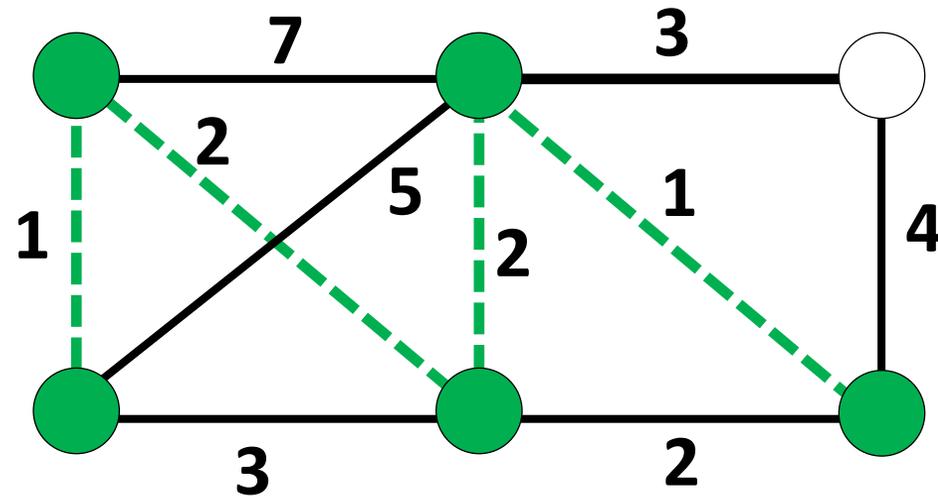
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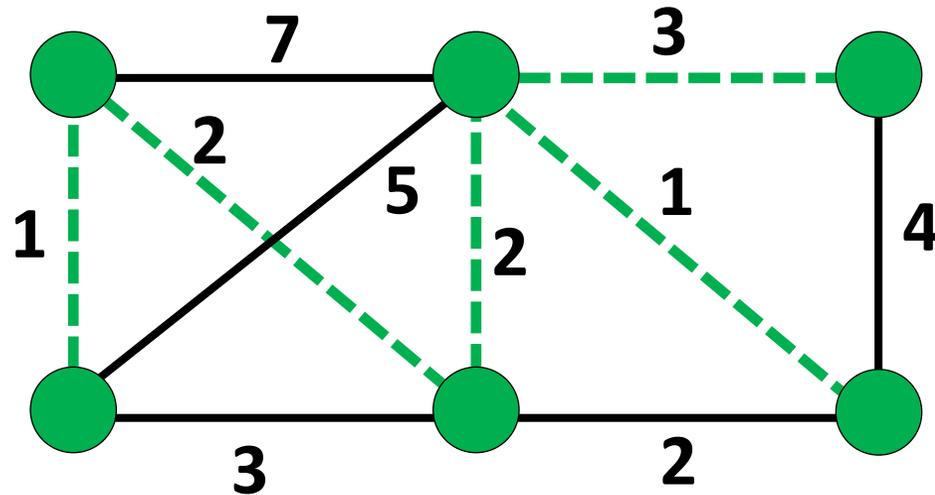
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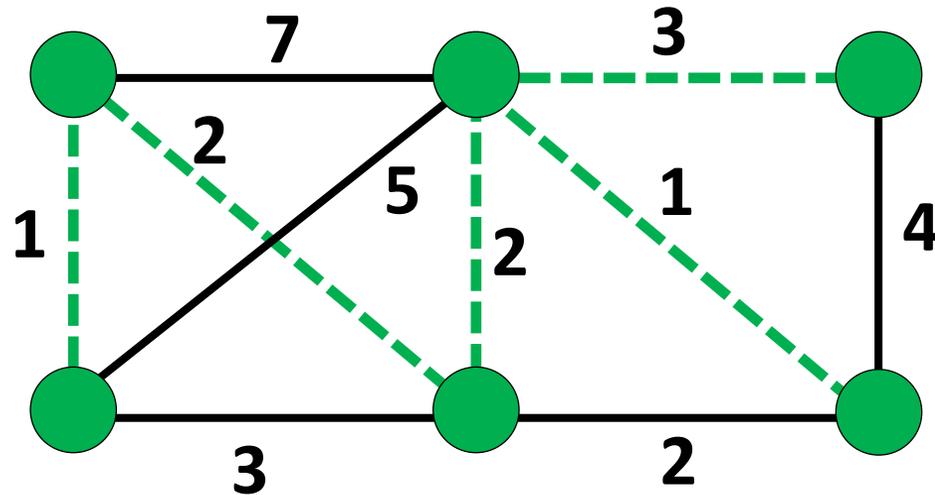
Prim's MST Algorithm

Beginning at any node, add the node that can be connected as cheaply as possible to the tree we are building.



Prim's MST Algorithm

Beginning at any node, add the node that can be connected as cheaply as possible to the tree we are building.



We will use Kruskal's algorithm in this class, but you should at least be aware of what Prim's is

Prim's Algorithm has the same running time