Incorporating Novelty Search Into Multi-Objective Cooperative Coevolution

Shahnaj Zahan Mou, Asibul Islam, and John Sheppard
Gianforte School of Computing
Montana State University
Bozeman, MT, USA
shahnajzahan.mou@student.montana.edu, asibul.islam@student.montana.edu, john.sheppard@montana.edu

Abstract*—Multi-objective evolutionary algorithms (MOEAs) are widely used for solving complex optimization problems involving conflicting objectives. While NSGA-II (Non-dominated Sorting Genetic Algorithm II) remains a strong baseline, recent approaches such as Multi-Objective Factored Evolutionary Algorithms (MOFEA) and novelty-driven search aim to improve scalability and exploration in high-dimensional decision spaces. Based on a hypothesis that introducing novelty search into MOEAs should improve the quality of returned Pareto sets, we present a systematic empirical comparison of four algorithms: NSGA-II, MOFEA, Novelty-MOEA (Multi-Objective Evolutionary Algorithm), and Novelty-MOFEA (Multi-Objective Factored Evolutionary Algorithms) on the ZDT test suite (excluding ZDT5) and all DTLZ problems. We evaluate their performance using standard quality indicators including Hypervolume (HV), Inverted Generational Distance (IGD), k-nearest neighbor distance, and Inverted Spacing (SP). Our results reveal that no single algorithm dominates across all problem types. Instead, performance depends strongly on problem characteristics such as modality, separability, and the presence of deceptive local optima. We analyze these trends and provide insights into when factored optimization and novelty mechanisms are most beneficial.

***Index Terms*—Multi-objective optimization, NSGA-II, Factored Evolutionary Algorithms, Novelty Search**

I. Introduction

Multi-objective optimization problems involve simultaneously optimizing conflicting objectives, and evolutionary algorithms are among the most widely used approaches for approximating Pareto-optimal solution sets [1], [2]. A central challenge in multi-objective evolutionary algorithms (MOEAs) is balancing convergence toward the true Pareto front with maintaining diversity in the Pareto set [3], [4]. Inadequate convergence yields suboptimal solutions, whereas insufficient diversity produces incomplete or uneven front coverage.

NSGA-II [5] remains one of the most influential MOEAs due to its efficient nondominated sorting and crowding-distance diversity preservation. However, it has been shown to struggle on problems exhibiting strong variable interactions, multimodality, or disconnected Pareto fronts, often leading to premature convergence or limited coverage [6], [7]. To mitigate these limitations, structural decomposition and enhanced exploration mechanisms have been explored. The Multi-Objective Factored Evolutionary Algorithm (MOFEA) [8] partitions decision variables into overlapping subsets that evolve cooperatively, reducing effective dimensionality and improving performance on nonseparable and biased landscapes. Separately, novelty-driven evolution [9] promotes exploration by rewarding behavioral distinctiveness rather than objective dominance, and has proven effective in escaping deception and improving coverage in quality-diversity frameworks such as MAP-Elites [10]. Despite these advances, the integration of novelty-guided exploration into multi-objective optimization, particularly in combination with structural factorization, remains largely unexplored.

In this paper, we evaluate four algorithms: NSGA-II, MOFEA, and two novel algorithms proposed in this work: a Novelty-driven MOEA and a hybrid Novelty-driven MOFEA approach. Experiments are conducted on the ZDT and DTLZ benchmark problems [6], [11], covering convex, concave, multimodal, biased, degenerate, and disconnected Pareto fronts [6]. Performance is assessed over ten independent runs using Hypervolume (HV), Inverted Generational Distance (IGD), k-nearest neighbor distance, and inverted spacing (SP). Our goal is to identify problem characteristics under which structural factorization and novelty-based selection provide measurable advantages over classical dominance-based MOEAs, and to clarify how these mechanisms affect convergence and diversity across heterogeneous multi-objective landscapes. To the best of our knowledge, this is the first work to integrate novelty search within a MOFEA framework, enabling improved exploration while preserving multi-objective convergence properties.

II. Related Work

A wide range of multi-objective evolutionary algorithms (MOEAs) has been proposed to approximate complex Pareto fronts. Foundational methods such as NSGA-II [5], SPEA2 [12], and MOEA/D [13] introduced core mechanisms including dominance-based ranking, density estimation, elitism, and decomposition-based search. NSGA-II remains widely used due to its computational efficiency and crowding-distance diversity preservation, while SPEA2 improves elitism through strength-based fitness assignment and MOEA/D enhances scalability via problem decomposition. Nevertheless, classical MOEAs often struggle on problems with deceptive regions, strong variable interactions, multimodality, or disconnected Pareto fronts [6], [14], [15].

To address these limitations, factorization-based evolutionary algorithms incorporate structural decomposition into the search process. Early work on linkage learning and variable grouping

[16], [17] demonstrated that capturing inter-variable dependencies can significantly improve performance in high-dimensional and correlated landscapes. Building on this principle, Peerlinck and Sheppard proposed MOFEA [8], which evolves overlapping variable groups using local NSGA-II instances and periodically recombines partial solutions via a global archive. This cooperative design has shown improved performance on problems with strong linkage and nonlinear interactions [15].

In parallel, novelty search provides an alternative exploration mechanism by rewarding behavioral distinctiveness rather than objective dominance [18]. This idea has influenced quality-diversity algorithms such as MAP-Elites [10], which aim to discover diverse sets of high-performing solutions. Recent studies have further examined the benefits and limitations of such approaches in complex domains including robotics and control [19].

While dominance-based MOEAs, factorization, and novelty-driven search have each been studied extensively in isolation, their combined effect within a unified multi-objective framework and across the full ZDT and DTLZ benchmark suites remains largely unexplored. To address this gap, we compare NSGA-II, MOFEA, and the two proposed algorithms, Novelty-MOEA and Novelty-MOFEA, to analyze how optimization pressure, structural decomposition, and novelty-driven exploration jointly influence convergence and diversity across heterogeneous multi-objective landscapes.

III. Methods

This section describes the four algorithms evaluated in this study. We first summarize the baseline methods, followed by the design of the proposed novelty-based extensions.

A. Baseline Algorithms

1) NSGA-II: As mentioned, NSGA-II [5] is one of the most widely used MOEAs; therefore, we chose it as a the baseline method to compare against. In each generation of NSGA-II, offspring are generated via crossover and mutation, merged with the parent population, ranked into nondominated fronts, and truncated using crowding distance to maintain population size. NSGA-II performs well on smooth and well-structured Pareto fronts [3], but often suffers from premature convergence or poor diversity on problems with strong variable interactions, multimodality, or disconnected fronts.

2) MOFEA: The Multi-Objective Factored Evolutionary Algorithm (MOFEA) [8] decomposes the decision vector into overlapping subsets of variables, called *factors*, which are evolved independently using local NSGA-II instances. This factorization reduces the effective search dimensionality and enables the algorithm to capture variable interactions that hinder traditional cooperative co-evolutionary methods.

In this study, MOFEA employs a linear sliding-window factorization strategy, providing partial overlap between neighboring factors to capture cross-variable dependencies. Larger windows allow modeling longer-range interactions, while smaller windows reduce computational cost. The algorithm follows an iterative *compete-and-share* cycle in which each factor population evolves independently using NSGA-II, partial solutions are periodically recombined into full decision vectors, a global nondominated archive maintains high-quality candidates, and archive information is shared back into factor populations to guide subsequent local search.

B. Proposed Novelty-Driven Algorithms

This work introduces two new algorithms that integrate novelty-guided exploration into multi-objective optimization:

- Novelty-MOEA: a multi-objective evolutionary algorithm that promotes exploration by prioritizing behaviorally novel solutions during parent selection while preserving elitist environmental replacement.
- Novelty-MOFEA: a hybrid approach that embeds novelty-guided selection within the MOFEA factor-level evolution.

Both methods aim to counteract premature convergence by explicitly promoting exploration of underrepresented regions of the Pareto front while preserving elitist environmental selection.

1) Novelty-MOEA: Algorithm 1 summarizes the procedure of the Novelty-MOEA. The Novelty-MOEA is a population-based multi-objective evolutionary algorithm that integrates explicit behavior-space exploration into the evolutionary search process. The method promotes discovery of underrepresented regions of the Pareto set by prioritizing individuals that exhibit high behavioral novelty while preserving convergence through elitist environmental selection.

Behavior descriptors are defined as objective vectors, and novelty is computed as the average Euclidean distance to the k nearest neighbors among the combined current first nondominated front and a bounded novelty archive [18]. At each generation, the most novel individuals are inserted into the archive, and parent selection is restricted to the first front using novelty-biased tournaments.

Offspring are generated using simulated binary crossover (SBX) and polynomial mutation [20], [21]. Environmental selection follows NSGA's elitist nondominated sorting and crowding-distance replacement.

2) Novelty-MOFEA: Algorithm 2 summarizes the procedure of the Novelty-MOFEA. This algorithm extends MOFEA by integrating novelty-guided parent selection at the factor level. Within each factor subpopulation, novelty is computed for rank-1 individuals using k-nearest-neighbor distances in objective space, and a bounded novelty archive preserves historically diverse behaviors across generations.

Parent selection is biased toward individuals that are both nondominated and behaviorally novel, thereby balancing objective-driven convergence with sustained exploration. After local factor evolution, partial solutions are recombined into full candidates, evaluated globally, and stored in a nondominated archive, whose contents are shared back into factor populations to maintain global coordination. This design is particularly effective for multimodal and disconnected landscapes such as ZDT3 and DTLZ7, where purely dominance-based search often converges prematurely to limited regions of the Pareto front.

Algorithm 1: Novelty-MOEA

Input: Population size N, max generations G, crossover prob. p_c, mutation prob. p_m, distribution indices η_c, η_m, neighborhood size k, archive max size $A_{\max}$, reference Pareto front
Output: Final non-dominated front F_1

// Initialization
$P \leftarrow$ RandomPopulation(N)
Evaluate all individuals in P
Fronts $\leftarrow$ FastNonDominatedSort(P)
$F_1 \leftarrow$ first front of Fronts
Archive $\leftarrow \emptyset$
Initialize convergence monitor
for $g = 1$ **to** G **do**
    // Novelty computation on F_1
    foreach $x \in F_1$ **do**
        $\mathbf{b}(x) \leftarrow f(x)$
    UpdateNovelty(F_1, Archive, k)
    Add top $\lceil N/20 \rceil$ most novel individuals from F_1 to Archive
    $O \leftarrow \emptyset$
    while $|O| < N$ **do**
        $p_1 \leftarrow$ NoveltyTournament(F_1)
        $p_2 \leftarrow$ NoveltyTournament(F_1)
        $(c_1, c_2) \leftarrow$ SBX(p_1, p_2, p_c, η_c)
        Mutate(c_1, p_m, η_m), Mutate(c_2, p_m, η_m)
        Evaluate(c_1), Evaluate(c_2)
        Add children to O
    $U \leftarrow P \cup O$
    Fronts $\leftarrow$ FastNonDominatedSort(U)
    AssignCrowdingDistances(Fronts)
    $P \leftarrow$ FillByRankThenCrowding($Fronts, N$)
    $F_1 \leftarrow$ first front of P
    // Early-stopping check
    if *ConvergenceMonitor indicates stagnation* **then**
        break
return F_1

Algorithm 2: Novelty-MOFEA

Input: Objective function F, factor sets $\mathcal{F} = \{S_1, \ldots, S_K\}$, population size N, max iterations $T_{\max}$, neighborhood size k
Output: Global nondominated archive $\mathcal{A}$

Initialize novelty archive $A \leftarrow \emptyset$;
Initialize global archive $\mathcal{A} \leftarrow \emptyset$;
for $j \leftarrow 1$ **to** K **do**
    Initialize factor population $P^{(j)}$;
for $t \leftarrow 1$ **to** $T_{\max}$ **do**
    for $j \leftarrow 1$ **to** K **do**
        Embed partial solutions of $P^{(j)}$ into full candidates using archive $\mathcal{A}$;
        Evaluate objective vectors of embedded candidates;
        Identify first nondominated front $F_1^{(j)}$;
        Compute novelty scores using k-NN distances over $F_1^{(j)} \cup A$;
        Select parents using novelty-biased tournament;
        Apply crossover and mutation on variables in S_j;
        Update factor population $P^{(j)}$;
        Update novelty archive A;
    Recombine factor populations into full solutions;
    Evaluate full solutions;
    Update global archive $\mathcal{A}$;
    for $j \leftarrow 1$ **to** K **do**
        Project archive solutions into factor S_j and replace worst individuals in $P^{(j)}$;
    if *archive stagnates or evaluation budget is exhausted* **then**
        break;
return $\mathcal{A}$;

IV. Benchmark Problems

We evaluated all four algorithms on the ZDT and DTLZ benchmark suites [6], [11]. Unless stated otherwise, all decision variables were limited to the domains of $x_i \in [0, 1]$, and the dimensionality was fixed to $n = 30$.

a) ZDT Suite: The ZDT problems are two-objective test functions designed to exhibit diverse Pareto-front geometries and search-space difficulties, including convex (ZDT1), concave (ZDT2), disconnected (ZDT3), highly multimodal (ZDT4), and biased/nonuniform density distributions (ZDT6). These properties allow evaluation of algorithm robustness under smooth, multimodal, and discontinuous landscapes [11].

b) DTLZ Suite: The DTLZ problems extend benchmark evaluation to scalable objective spaces and include a broad range of structural characteristics: multimodality (DTLZ1, DTLZ3), spherical fronts (DTLZ2), biased variable mappings (DTLZ4), degenerate Pareto fronts (DTLZ5), nonseparable decision variables (DTLZ6), and disconnected fronts (DTLZ7). Together, they provide a comprehensive testbed for analyzing algorithm behavior under increasing dimensionality and complex objective interactions [6].

c) Remark on ZDT5: ZDT5 operates on a binary decision space and requires discrete genetic operators and factor architectures. Since MOFEA and the proposed novelty-driven algorithms rely on real-valued search, arithmetic recombination, and continuous projection, ZDT5 is excluded from the experimental evaluation in this work and left for future extension to discrete optimization domains.

V. Experimental Setup

A. Problem Configuration

All problems were evaluated using $n = 30$ decision variables. ZDT problems used two objectives and DTLZ problems used three objectives. Each algorithm was executed for 10 independent runs with fixed random seeds.

NSGA-II and the proposed Novelty-MOEA used a population size of 1000 for all problems. MOFEA and Novelty-MOFEA employed factor-level populations based on problem complexity: 300 for DTLZ1, 250 for ZDT4 and ZDT6, and 200 for all remaining problems. The number of outer FEA iterations was set to 500 for DTLZ problems and 300 for ZDT problems. Novelty-MOFEA used identical budgets to MOFEA to enable direct comparison. We note that comparing number of fitness evaluations between factored and non-factored methods is difficult, given the fact FEA does "partial" fitness evaluations. Nevertheless, the experiments were designed such that computational burden was comparable across methods.

B. MOFEA Factorization Settings

MOFEA and Novelty-MOFEA employed a linear overlapping factor architecture [8] with GROUP_SIZE $= 10$ and OFFSET $= 5$. GROUP_SIZE defines the number of decision variables per factor, while OFFSET controls the shift between consecutive factors and thus the degree of overlap. This sliding-window configuration preserves variable linkage while reducing subproblem dimensionality and enabling information sharing between neighboring factors. These parameters were fixed in this study but remain tunable for adaptive architectures.

C. Evaluation Metrics

For each run, the final nondominated solution set was recorded. Performance was evaluated using Hypervolume (HV) [22], Inverted Generational Distance (IGD), k-nearest neighbor distance, and inverted spacing (SP) [23]. Results are reported as mean $\pm$ standard deviation over 10 runs.

HV measures the objective-space volume dominated by the approximation set, capturing both convergence and coverage. IGD measures the average distance from points on the true Pareto front to the obtained solution set, reflecting convergence quality. The k-nearest neighbor distance estimates local solution density using Euclidean distances in objective space. SP quantifies spacing uniformity as the standard deviation of nearest-neighbor distances.

D. Statistical Hypothesis Testing

Statistical significance was assessed using the Wilcoxon signed-rank test [24]. For each benchmark and performance indicator, pairwise comparisons were conducted between all algorithm pairs (NSGA-II, MOFEA, Novelty-MOEA, and Novelty-MOFEA) using paired runs with identical seeds. The null hypothesis of equal median performance was tested at $\alpha = 0.05$.

All algorithms were implemented within a unified experimental framework with identical operators and evaluation procedures. Random number generation was controlled using the predefined seed set to ensure reproducibility. Since NSGA-II serves as the base evolutionary framework across all methods, performance differences can be attributed to the proposed mechanisms rather than variations in the underlying optimizer.

TABLE I
RESULTS OF EXPERIMENTS ON ZDT AND DTLZ BENCHMARK PROBLEMS

Problem	Algorithm	M	HV		IGD		k-NN Dist		SP↓
			mean↑	std↓	mean↓	std↓	mean↑	std↓	
ZDT1	NSGA-II	2	0.8761	0.0000	0.0004	0.0000	0.0011	0.0005	0.0007
	MOFEA	2	0.8750	0.0001	0.0013	0.0000	0.0019	0.0015	0.0019
	Novelty-MOEA	2	0.8790	0.0042	0.0004	0.0000	0.0011	0.0006	0.0007
	Novelty-MOFEA	2	0.8747	0.0001	0.0013	0.0000	0.0024	0.0014	0.0018
ZDT2	NSGA-II	2	0.5428	0.0000	0.0004	0.0000	0.0011	0.0005	0.0007
	MOFEA	2	0.5424	0.0000	0.0008	0.0000	0.0012	0.0008	0.0011
	Novelty-MOEA	2	0.5480	0.0023	0.0005	0.0000	0.0011	0.0005	0.0007
	Novelty-MOFEA	2	0.5415	0.0001	0.0014	0.0001	0.0024	0.0013	0.0017
ZDT3	NSGA-II	2	0.8813	0.0000	0.0003	0.0000	0.0008	0.0004	0.0004
	MOFEA	2	0.8540	0.0001	0.0015	0.0001	0.0013	0.0011	0.0013
	Novelty-MOEA	2	0.8522	0.0051	0.0003	0.0000	0.0008	0.0004	0.0004
	Novelty-MOFEA	2	0.8719	0.0004	0.0017	0.0001	0.0016	0.0011	0.0013
ZDT4	NSGA-II	2	0.8763	0.0000	0.0003	0.0000	0.0007	0.0003	0.0005
	MOFEA	2	0.8747	0.0000	0.0043	0.0044	0.0019	0.0013	0.0017
	Novelty-MOEA	2	0.8826	0.0079	0.0004	0.0001	0.0007	0.0004	0.0005
	Novelty-MOFEA	2	0.7944	0.0839	0.1399	0.2700	0.0194	0.0159	0.0183
ZDT6	NSGA-II	2	0.5068	0.0000	0.0009	0.0000	0.0011	0.0005	0.0007
	MOFEA	2	0.5045	0.0007	0.0436	0.0002	0.0020	0.0014	0.0019
	Novelty-MOEA	2	0.5067	0.0000	0.0007	0.0001	0.0011	0.0070	0.0071
	Novelty-MOFEA	2	0.5054	0.0003	0.0015	0.0002	0.0020	0.0018	0.0023
DTLZ1	NSGA-II	3	1.3062	0.0003	0.0258	0.0007	0.0194	0.0143	0.0217
	MOFEA	3	1.3069	0.0001	0.0128	0.0003	0.0244	0.0139	0.0217
	Novelty-MOEA	3	1.3310	0.0000	0.0031	0.0058	0.0011	0.0037	0.0039
	Novelty-MOFEA	3	1.3137	0.0002	0.0071	0.0007	0.0131	0.0092	0.0144
DTLZ2	NSGA-II	3	0.7776	0.0005	0.0195	0.0012	0.0172	0.0102	0.0150
	MOFEA	3	0.7686	0.0008	0.0347	0.0011	0.0323	0.0178	0.0267
	Novelty-MOEA	3	0.7504	0.0075	0.0163	0.0008	0.0133	0.0090	0.0126
	Novelty-MOFEA	3	0.7672	0.0009	0.0347	0.0009	0.0320	0.0179	0.0264
DTLZ3	NSGA-II	3	0.7824	0.0008	0.0195	0.0008	0.0162	0.0107	0.0156
	MOFEA	3	0.7685	0.0005	0.0345	0.0006	0.0322	0.0177	0.0263
	Novelty-MOEA	3	0.6905	0.0355	0.0017	0.0003	0.0009	0.0032	0.0033
	Novelty-MOFEA	3	0.7684	0.0004	0.0314	0.0016	0.0323	0.0181	0.0261
DTLZ4	NSGA-II	3	0.7790	0.0004	0.0193	0.0003	0.0174	0.0103	0.0152
	MOFEA	3	0.7689	0.0008	0.0348	0.0009	0.0313	0.0200	0.0278
	Novelty-MOEA	3	0.7639	0.0014	0.0177	0.0007	0.0143	0.0086	0.0124
	Novelty-MOFEA	3	0.7699	0.0006	0.0339	0.0007	0.0316	0.0192	0.0283
DTLZ5	NSGA-II	3	0.4420	0.0000	0.0006	0.0000	0.0014	0.0007	0.0010
	MOFEA	3	0.7685	0.0008	0.0348	0.0010	0.0314	0.0184	0.0264
	Novelty-MOEA	3	0.4419	0.0000	0.0007	0.0000	0.0015	0.0007	0.0010
	Novelty-MOFEA	3	0.4411	0.0001	0.0015	0.0001	0.0032	0.0018	0.0028
DTLZ6	NSGA-II	3	1.1439	0.0085	0.1842	0.0316	0.0080	0.0061	0.0093
	MOFEA	3	1.1824	0.0000	0.3037	0.0009	0.0053	0.0054	0.0080
	Novelty-MOEA	3	1.1492	0.0109	0.0671	0.0229	0.0056	0.0063	0.0077
	Novelty-MOFEA	3	1.1824	0.0000	0.0036	0.0005	0.0047	0.008	0.0105
DTLZ7	NSGA-II	3	0.4446	0.0001	0.1059	0.0014	0.0114	0.0073	0.0110
	MOFEA	3	0.4293	0.0012	0.1495	0.0042	0.0229	0.0137	0.0197
	Novelty-MOEA	3	0.4217	0.0093	0.0820	0.0018	0.0114	0.0081	0.0108
	Novelty-MOFEA	3	0.4313	0.0011	0.0408	0.0013	0.0226	0.0146	0.0201

VI. RESULTS AND ANALYSIS

This section discusses and analyzes the relative performance of NSGA-II, MOFEA, Novelty-MOEA, and Novelty-MOFEA across the ZDT and DTLZ benchmark suites using Hypervolume (HV), IGD, k-nearest neighbor distance, and inverted spacing (SP). The results can be found in Table I.

A. Performance on ZDT Problems

ZDT1 (convex front) and ZDT2 (concave front) exhibit highly similar performance trends. In both cases, Novelty-MOEA achieves the highest hypervolume, indicating superior convergence. NSGA-II attains the lowest spacing values, reflecting strong local uniformity, while Novelty-MOFEA consistently provides the strongest global diversity, achieving the highest kNN distances.

Statistical testing shows that MOFEA-based variants differ significantly from NSGA-II across most metrics, with additional significant differences observed among the alternative methods. For Novelty-MOEA, improvements in HV, SP, and IGD are significant, whereas differences in kNN distance are not significant in either problem. These results suggest that novelty-driven selection primarily enhances convergence on smooth Pareto fronts while preserving local neighborhood density.

ZDT3 introduces a disconnected Pareto front. NSGA-II achieves the highest HV, indicating strong convergence on individual segments. Spacing differences between NSGA-II and Novelty-MOEA are not statistically significant, while kNN and IGD show significant differences. In contrast, Novelty-MOFEA achieves higher diversity with significant differences across most metrics. Additional pairwise comparisons among the alternative methods also show significant differences, indicating varied behavior across approaches. These results suggest that dominance-based search favors convergence on fragmented fronts, whereas novelty-enhanced approaches improve coverage across separated regions.

ZDT4 is highly multimodal. Novelty-MOEA attains the highest HV, outperforming NSGA-II and demonstrating improved escape from local optima. In contrast, Novelty-MOFEA exhibits degraded performance with unstable convergence behavior. Statistical testing confirms significant differences across most metrics between NSGA-II and the alternative methods, while some pairwise comparisons among MOFEA-based variants (e.g., IGD between MOFEA and Novelty-MOFEA) are not significant.

ZDT6 exhibits strong objective-space bias and non-uniform solution density. NSGA-II achieves the highest HV and lowest spacing, indicating strong local precision. Novelty-MOFEA provides higher global diversity, but statistical testing shows no significant differences between NSGA-II and Novelty-MOFEA for HV, kNN, and SP, while IGD differs significantly. Similarly, Novelty-MOEA does not significantly alter local neighborhood density relative to NSGA-II, and several comparisons among alternative methods are not significant. These results suggest that ZDT6 imposes a performance ceiling where novelty and factorization offer limited advantage over dominance-based selection.

B. Performance on DTLZ Problems

The results on the DTLZ benchmarks show wider performance differences than those observed on the ZDT problems due to increased dimensionality and structural complexity. On the highly multimodal DTLZ1 and DTLZ3 problems, most algorithmic comparisons differ significantly across metrics. For DTLZ1, MOFEA and novelty-based variants differ significantly from NSGA-II in most metrics, with some exceptions such as spacing in the comparison with MOFEA. For DTLZ3, most pairwise comparisons are significant, although differences between MOFEA and Novelty-MOFEA for HV, kNN, and SP are not significant.

On DTLZ2 and DTLZ4, alternatives differ significantly from NSGA-II across all metrics. However, comparisons between MOFEA and Novelty-MOFEA show no significant differences for several diversity-related metrics, indicating similar behavior between these two factorization-based approaches.

DTLZ5, which features a degenerate Pareto front, strongly favors factorization. MOFEA achieves statistically significant improvements over NSGA-II across all metrics. Novelty-MOEA shows no significant difference in spacing relative to NSGA-II, while all other metrics differ significantly. Pairwise comparisons among alternative methods further indicate performance differences driven by factorization.

On the nonseparable DTLZ6 problem, factorized methods achieve significant improvements over NSGA-II in most metrics, though HV differences between NSGA-II and Novelty-MOEA are not significant. Several comparisons between MOFEA and Novelty-MOFEA are also not significant for diversity-related metrics, indicating comparable performance between these methods.

Finally, on DTLZ7, most comparisons are statistically significant; however, Novelty-MOEA does not differ significantly from NSGA-II in kNN distance and spacing. Additional pairwise comparisons show that novelty preserves local structure while improving global exploration relative to other methods.

C. Effect of Factorization and Novelty Integration

The empirical results reveal complementary but context-dependent strengths between structural factorization and novelty-based exploration. Factorization generally benefits problems with strong variable interactions, degenerate or high-dimensional Pareto manifolds, and requirements for broad global coverage, as observed across DTLZ2–DTLZ6 and in the diversity-oriented behavior on ZDT1–ZDT3. These trends are supported by improvements in diversity metrics and IGD across several benchmarks.

In contrast, Novelty-MOEA is most effective in highly multimodal landscapes such as ZDT4 and DTLZ1, where it facilitates escape from local optima and preserves spacing on fragmented fronts such as DTLZ7. Statistical testing indicates that these improvements are significant in several cases, particularly for convergence-related metrics.

The hybrid Novelty-MOFEA approach exhibits mixed behavior: while it inherits diversity advantages on structured problems, it degrades in highly multimodal settings, indicating that excessive exploration across factor subspaces can disrupt coordinated convergence. Pairwise comparisons further show that performance differences between MOFEA and Novelty-MOFEA are not always significant, highlighting the sensitivity of hybridization to problem structure.

D. Limitations and Future Work

While the proposed novelty-driven algorithms demonstrate competitive performance across a wide range of benchmark problems, several directions remain for further investigation.

The effectiveness of novelty pressure and structural factorization is highly problem-dependent: excessive novelty can degrade convergence in highly multimodal landscapes, while rigid factorization may be suboptimal when strong cross-variable interactions or smooth Pareto manifolds dominate. These observations are supported by statistical analysis, which shows that performance differences vary across problem classes and metrics. This motivates the need for adaptive mechanisms that balance novelty and dominance-based selection to improve robustness across diverse Pareto-front geometries.

In addition, the factorization strategy used in this study is limited to a fixed linear overlapping grouping scheme. Alternative architectures, including nonuniform group sizes, adaptive offsets, learned linkage structures, and problem-specific decompositions, may better suit different problem classes. Identifying effective factor architectures remains an open research question and is receiving increasing attention (e.g., [25], [26]).

Finally, extending the proposed frameworks to combinatorial optimization domains is a promising direction. This includes developing suitable factor architectures and local fitness models for evaluation on combinatorial benchmarks such as ZDT5.

VII. Conclusion

This paper compared NSGA-II, MOFEA, Novelty-MOEA, and Novelty-MOFEA on the ZDT and DTLZ benchmark suites using multiple convergence and diversity indicators. The results show that novelty-guided selection is particularly effective on highly multimodal and fragmented landscapes, where dominance-based methods are prone to premature convergence, leading to improved convergence reliability and, in several cases, improved solution distributions. These trends are supported by statistical analysis across multiple performance metrics.

MOFEA demonstrated advantages on problems characterized by strong variable interactions, degeneracy, and nonseparability, where structural decomposition enables more coordinated search and improved global diversity. Novelty-MOFEA exhibited variability, retaining diversity benefits on structured problems but degrading on highly multimodal landscapes, indicating sensitivity to problem structure and factorization design.

Overall, these findings highlight the complementary roles of structural decomposition and novelty-driven exploration in multi-objective optimization and motivate future research on adaptive balancing strategies, alternative factor architectures, and extensions to discrete optimization domains.

References

[1] C. Fonseca and P. Fleming, "Genetic algorithms for multiobjective optimization: Formulation discussion and generalization," in *Proceedings of the 5th International Conference on Genetic Algorithms*, 02 1999, pp. 416–423.

[2] C. A. Coello Coello, G. B. Lamont, and D. A. Van Veldhuizen, *Evolutionary Algorithms for Solving Multi-Objective Problems*. Springer, 2007.

[3] K. Deb, *Multi-Objective Optimization Using Evolutionary Algorithms*. John Wiley & Sons, 2001.

[4] J. Knowles and D. Corne, "On metrics for comparing nondominated sets," in *Proceedings of the Congress on Evolutionary Computation (CEC)*, 2002, pp. 711–716.

[5] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: Nsga-ii," in *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, 2002, pp. 182–197.

[6] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable test problems for evolutionary multiobjective optimization," in *Evolutionary Multiobjective Optimization*, A. Abraham, L. C. Jain, and R. Goldberg, Eds. London: Springer, 2005, pp. 105–145.

[7] B. Li, J. Li, K. Tang, and X. Yao, "Many-objective evolutionary algorithms: A survey," *ACM Computing Surveys*, vol. 48, no. 1, pp. 1–35, Sep. 2015.

[8] A. Peerlinck and J. W. Sheppard, "Multi-objective factored evolutionary optimization and the multi-objective knapsack problem," in *Proceedings of the IEEE Congress on Evolutionary Computation (CEC)*, 2022, pp. 1–8.

[9] J. Lehman and K. O. Stanley, "Abandoning objectives: Evolution through the search for novelty alone," *Evolutionary Computation*, vol. 19, no. 2, pp. 189–223, Jun. 2011.

[10] J. Pugh, L. Soros, and K. O. Stanley, "Quality diversity: A new frontier for evolutionary computation," *Frontiers in Robotics and AI*, vol. 3, Jul. 2016.

[11] E. Zitzler, K. Deb, and L. Thiele, "Comparison of multiobjective evolutionary algorithms: Empirical results," *Evolutionary Computation*, vol. 8, no. 2, pp. 173–195, 2000.

[12] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the strength Pareto evolutionary algorithm for multiobjective optimization," in *Proceedings of the EUROGEN Conference*, 2001, pp. 95–100.

[13] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," in *Proceedings of the IEEE Congress on Evolutionary Computation (CEC)*, 2007, pp. 112–119.

[14] K. C. Tan, T. H. Lee, and E. Khor, "Evolutionary algorithms for multi-objective optimization: Performance assessments and comparisons," *Artificial Intelligence Review*, vol. 17, pp. 251–290, 2002.

[15] M. Zhou, M. Cui, D. Xu, S. Zhu, Z. Zhao, and A. Abusorrah, "Evolutionary optimization methods for high-dimensional expensive problems: A survey," *IEEE/CAA Journal of Automatica Sinica*, vol. 11, no. 5, pp. 1092–1105, 2024.

[16] G. R. Harik, F. G. Lobo, and K. Sastry, "Linkage learning via probabilistic modeling in the extended compact genetic algorithm (ECGA)," in *Scalable Optimization via Probabilistic Modeling*, M. Pelikan, K. Sastry, and E. CantúPaz, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2006, pp. 39–61.

[17] K. Li, M. N. Omidvar, K. Deb, and X. Yao, "Variable interaction in multi-objective optimization problems," in *Proceedings of Parallel Problem Solving from Nature – PPSN XIV*. Springer, 2016, pp. 399–409.

[18] J. Lehman and K. O. Stanley, "Novelty search and the problem with objectives," in *Genetic Programming Theory and Practice IX*. Springer, 2011, pp. 37–56.

[19] S. Batra, B. Tjanaka, S. Nikolaidis, and G. Sukhatme, "Quality diversity for robot learning: Limitations and future directions," in *Companion of the Genetic and Evolutionary Computation Conference (GECC)*, 2024, pp. 587–590.

[20] K. Deb and R. B. Agrawal, "Simulated binary crossover for continuous search space," *Complex Systems*, vol. 9, no. 2, pp. 115–148, 1995.

[21] K. Deb, "An efficient constraint handling method for genetic algorithms," *Computer Methods in Applied Mechanics and Engineering*, vol. 186, no. 2, pp. 311–338, 2000.

[22] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach," *IEEE transactions on Evolutionary Computation*, vol. 3, no. 4, pp. 257–271, 2002.

[23] J. R. Schott, "Fault tolerant design using single and multicriteria genetic algorithm optimization," Master's thesis, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, 1995.

[24] F. Wilcoxon, "Individual comparisons by ranking methods," *Biometrics Bulletin*, vol. 1, no. 6, pp. 80–83, 1945.

[25] S. Strasser, J. Sheppard, and S. Butcher, "A formal approach to deriving factored evolutionary algorithm architectures," in *Proceedings of the IEEE Symposium Series on Computational Intelligence (SSCI)*, 2017.

[26] E. Pryor, A. Peerlinck, and J. Sheppard, "A study in overlapping factor decomposition for cooperative co-evolution," in *Proceedings of the IEEE Symposium Series on Computational Intelligence (SSCI)*, 2021.