Agent-Based Modeling in Electrical Energy Markets Using Dynamic Bayesian Networks

Kaveh Dehghanpour, *Student Member, IEEE*, M. Hashem Nehrir, *Life Fellow, IEEE*, John W. Sheppard, *Fellow, IEEE*, and Nathan C. Kelly, *Student Member, IEEE*

***Abstract*—Due to uncertainties in generation and load, optimal decision making in electrical energy markets is a complicated and challenging task. Participating agents in the market have to estimate optimal bidding strategies based on incomplete public information and private assessment of the future state of the market, to maximize their expected profit at different time scales. In this paper, we present an agent-based model to address the problem of short-term strategic bidding of conventional generation companies (GenCos) in a power pool. Based on the proposed model, each GenCo agent develops a private probabilistic model of the market (using dynamic Bayesian networks), employs an online learning algorithm to train the model (sparse Bayesian learning), and infers the future state of the market to estimate the optimal bidding function. We show that by using this multiagent framework, the agents will be able to predict and adapt to approximate Nash equilibrium of the market through time using local reasoning and incomplete publicly available data. The model is implemented in MATLAB and is tested on four test case systems: two generic systems with 5 and 15 GenCo agents, and two IEEE benchmarks (9-bus and 30-bus systems). Both the day-ahead (DA) and hour-ahead (HA) bidding schemes are implemented. The results show a drop in market power in the 15-agent system compared to 5-agent system, along with a Pareto superior equilibrium in the HA scheme compared to the DA scheme, which corroborates the validity of the proposed decision making model. Also, the simulations show that introduction of an HA decision making stage as an uncertainty containment tool, leads to a more stable and less volatile price signal in the market, which consequently results in flatter and improved profit curves for the GenCos.**

***Index Terms*—Dynamic Bayesian networks, multiagent systems, Nash equilibrium, sparse Bayesian learning, strategic bidding.**

I. Introduction

UNBUNDLING of the electrical power supply industry and the introduction of market-based energy management logic into power systems has created an interactive system of decision-making-agents that seek to optimize their local objectives. In this context, in oligopolistic markets (i.e., a market with a few dominant firms), privately-owned generation companies (GenCos) try to maximize their expected profit from sales of energy to the consumers by bidding strategically in the market. Thus, GenCos exercise market power to affect the state of the energy market to their benefit.

However, power markets are abounded with uncertainty: intermittent renewable energy resources, and variable electrical load are two main sources of uncertainty. Moreover, each agent, by affecting the outcome of the market, is a source of uncertainty to its competitors. Hence, the problem of optimal decision making by participating agents in the market turns out to be a complex issue that has attracted considerable scientific interest in the past decade.

Strategic bidding in pool-based energy markets has been studied in the literature using the concepts of game theory. It has been shown that the oligopolistic energy markets can be modelled using bi-level optimization problems [1], [2]. At the top level, GenCos pursue the maximization of their profit functions, while at the lower level the Independent System Operator (ISO) agent solves a social welfare maximization problem to clear the market. Thus, the higher level optimization problems are constrained by the lower level problem. If the lower level problem is convex it can be replaced by its Karush-Kuhn-Tucker optimality conditions (KKT), which results in an Equilibrium Problem with Equilibrium Constraints (EPEC), which is a system of coupled Mathematical Programs with Equilibrium Constraints (MPEC) [3], [4]. The solution of EPEC would be the Nash Equilibrium (NE) [5] of the energy market, which is a strategy profile of the agents from which none has any incentive to deviate unilaterally. The EPEC has been described analytically as a Stackelberg game with several leaders (GenCos) and one common follower (ISO) [6]. Many papers are dedicated to solving this system of nested optimization problems. Different approaches have been employed to simplify and find the equilibrium of the market: diagonalization [7], [8], binary expansion [9], [10], particle swarm optimization [11], variational inequality techniques [12], [13], information gap decision theory [14], polynomial equations [15], and the penalty interior point algorithm [6] are some of the tools that have been used in previous works. While these papers provide invaluable analytical insight on energy markets, they do not study the details of agent-based temporal learning process under uncertainty and incomplete information that leads to the market equilibrium.

Several papers have addressed the application of multiagent system (MAS) theory to energy markets. A remarkable review is presented in [16], in which the shortcomings of analytical

Manuscript received July 01, 2015; revised October 19, 2015 and December 10, 2015; accepted January 20, 2016. Date of publication February 26, 2016; date of current version October 18, 2016. This work was supported by the U.S. Department of Energy, Office of Science, Basic Energy Sciences, under Award # DE-FG02-11ER46817. Paper no. TPWRS-00937-2015.

K. Dehghanpour, M. H. Nehrir, and N. Kelly are with the Department of Electrical and Computer Engineering, Montana State University, Bozeman, MT 59717 USA (e-mail: hnehrir@ece.montana.edu; Kaveh.dehghanpour@msu.montana.edu; kellyna4@gmail.com).

J. W. Sheppard is with the Department of Computer Science, Montana State University, Bozeman, MT 59715 USA (e-mail: john.sheppard@cs.montana.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TPWRS.2016.2524678

approaches are pointed out. In [17], a rule-based naïve reinforcement learning (RL) algorithm is employed for each agent to search separately for optimal bidding strategies. In [18], an RL-based day-ahead (DA) bidding procedure for GenCos is proposed, based on discrete Markov decision processes. This method has been applied in [19] to study the effects of water shortage on electric power generation. Another RL scheme is introduced in [20] to study the dynamics of forward and spot markets. The effects of initial belief of the agents employing RL on the outcome of energy market is studied in [21]. As another RL-based technique, Q-learning is used in [22] to implement an agent-based model for energy markets. An interesting recently published work employs a multi-layer agent-based model to study the energy markets, considering the uncertain behavior of competitors [23]. This paper relies on probabilistic scenario generation and stochastic programming. Stochastic programming is also used in [24] in combination with a branch-and-bound method for uncertainty assessment in the market. Agent-based optimal decision making in energy markets using numerical sensitivity analysis is proposed in [25].

Previous works on agent-based decision making in energy markets have mostly relied on reinforcement learning and stochastic programming to develop agent-level decision making models [26]. However, a shortcoming of these works is their dependence on discretizing the space of states and actions of agents. This on one hand, might lead to suboptimal solutions and on the other hand, introduces a limitation on applicability and scalability of the models. One shortcoming of the methods that are based on stochastic programming is their dependence on probability density functions that are non-stationary and hard to obtain [27]. Also, the integration of forecasting tools has been mostly ignored.

In this paper, we present a novel probabilistic model for GenCos' optimal decision making in energy markets. The agent-based reasoning apparatus is based on a Dynamic Bayesian Network (DBN) representation [28] where Sparse Bayesian Learning (SBL) is employed for model training. DBNs provide a natural and efficient framework to study reasoning under uncertainty; DBNs can be scaled to include a large number of random variables, if necessary. Thus, they have been adopted here for modeling agents' behavior in electrical energy markets. The presented DBN-based model constitutes the *belief system* of agents on the state of the market. This belief system is updated constantly through participation in the DA and HA energy markets. Each agent, using its private belief system develops offer curves according to a Linear Supply Function Equilibrium (LSFE) model [29], [30], which provides an efficient and realistic model of behavior of GenCos in the market.

Two sources of uncertainty are considered in this paper: electrical load, and GenCo competitors behavior. The electrical demand side is assumed to be inelastic. Real load data from the Pennsylvania-New Jersey-Maryland (PJM) market is fed to the market model to simulate the temporal changes of electrical demand [31]. To represent and predict the behavior of competing GenCos, a Residual Demand Curve (RDC) [32], [33], is employed. Since the agents do not have access to the cost functions and decisions made by competitors, they have to rely on incomplete publicly available data (which is assumed to be the aggregate demand and supply curves published by the ISO) to construct their individual RDC. Both the load and RDC prediction functionalities are integrated into the DBN-based belief system of the agents.

The DBN-based decision making model provides adaptive agent behavior through variable and uncertain conditions in the market. An advantage of the proposed model is that it is based on continuous-valued variables; hence, complexities pertaining to discretization and the possibility of suboptimal solutions are avoided. Another advantage of the DBN-based model is that the introduced probabilistic decision making tool is free and independent of any assumptions on probability density functions of the variables of the energy market; hence, avoiding the shortcoming of stochastic programming.

To summarize, the main contributions of this paper are as follows:

— To provide an agent-based decision making tool for energy markets, using probabilistic graphical models. Specifically, a dynamic Bayesian network is selected as the tool that is most fit for the task of modeling the belief systems of GenCos, as explained later in the paper (Section III).

— To avoid discretization of decision/action space, we have employed sparse Bayesian learning to train the dynamic Bayesian networks for each agent. Also, using the proposed belief system, particle-based sampling is employed for forecasting.

The rest of the paper is constructed as follows. In Section II, the basics of the agent-based model are explained. In Section III the DBN-based belief system of GenCo agents, along with learning and inference schemes are described. Simulation results are presented and discussed in Section IV. Conclusion are reported in Sections V.

II. Structure of the Agent-Based System

The energy market under study is formulated into a hierarchical multiagent system, as shown in Fig. 1. At the top level, GenCo agents compete with each other to supply the inelastic demand by submitting optimal bidding functions. There is no direct communication link among GenCo competitors, and any interaction among them takes place through market outcome and thus, is indirect by nature. At the bottom level, the ISO agent clears the market based on the received bidding functions. The details of the agents' functions are discussed below. Also, the DA/HA bidding procedure of the GenCo agents is introduced for market modeling.

A. Genco Agents Model

The cost function (C_{gi}) of the ith GenCo agent is modeled as a quadratic function of its output power (P_i), as given in (1). Fixed cost elements are ignored. Also, the power production of the GenCo is bounded by its maximum power capacity (P^i_{max}), which defines the feasible operational region of the GenCo.

$$C_{gi}(P_i) = a_i P_i + \frac{1}{2} b_i P_i^2$$
$$0 \leq P_i \leq P^i_{max}. \tag{1}$$

The marginal cost function of the ith agent (MC_{gi}) is therefore, a linear function of output power:

$$MC_{gi}(P_i) = a_i + b_i P_i \tag{2}$$

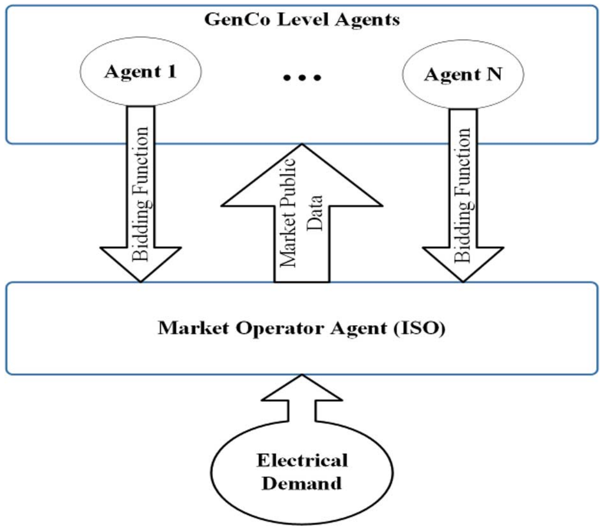


Fig. 1. Structure of the multiagent market model.

In a purely competitive market the GenCos act as price-takers and bid their marginal cost functions to the market [34], whereas in an oligopolistic market, GenCo agents submit bidding functions that deviate from their marginal cost curves, in order to maximize their expected profit. Price-making GenCos exercise some levels of market power. In this paper we have adopted LSFE model for GenCos' bidding procedure. Thus, the submitted bidding function (B_{gi}) to the ISO agent is a linear function of output power of the GenCo, as follows:

$$B_{gi}(P_i) = \hat{a}_i + \hat{b}_i P_i. \tag{3}$$

where, the coefficients $\hat{a}_i$, and $\hat{b}_i$ are assigned by the ith GenCo, to maximize its expected profit level. Therefore, the goal of the strategic bidding problem is to optimally parameterize the bidding functions of the GenCos. In this paper, we have used what is known as intercept-parameterization [21] (i.e., the slope of $B_{gi}(P_i)$ is kept equal to the slope of $MC_{gi}(P_i)$, and the intercept with the price axis is modified). The merits of intercept-parameterization are discussed in [6]. Hence, $B_{gi}(P_i)$ is determined as follows:

$$B_{gi}(P_i) = a_i + O_i + b_i P_i \tag{4}$$

where, parameter O_i in (4) determines the level of deviation of $B_{gi}(P_i)$ from $MC_{gi}(P_i)$, and is referred to as *strategic parameter*. Thus, the objective of each GenCo agent would be to maximize its profit level (Π_{gi}) at each round of bidding by modifying the strategic parameter, subject to power constraints:

$$\begin{aligned} \max_{o_i} \quad & \Pi_{gi} = \{\lambda(O_i, \mathbf{O}_{-\boldsymbol{i}}) \cdot P_i - C_{gi}(P_i)\} \\ \text{s.t.} \quad & 0 \leq P_i \leq P^i_{max} \end{aligned} \tag{5}$$

where, λ is the energy price, which is a function of both the ith GenCo's strategic parameter and vector of strategic parameters of competitors ($\mathbf{O}_{-\boldsymbol{i}}$).

To provide a base for individual GenCo's self-scheduling behavior, the concept of RDC is employed. RDC provides a measure of dependence of market status merely based on a single player's actions by fixing the behavior of competitors. Using the aggregate supply function of the market and the demand level, each GenCo can construct its individual RDC as discussed in [35]; and since the basic assumption in this paper is that the only published data to GenCos is the aggregate supply and demand functions of the market at each round, we are able to employ the concept of RDC. Note that RDC can only be constructed after the market is cleared; thus, the agents have access to RDCs of the previous rounds of auction. As a part of the optimal decision making process, one goal of the learning model proposed in this paper is to predict the RDC for the future round of bidding.

The RDC of the ith agent is a decreasing nonlinear function of the generated output power. However, in order to be able to employ RDCs in the agents' belief systems, they have to be parameterized. For that purpose, simple linear regression is used to present a linear estimation of RDC, denoted by λ_{gi}.

$$\lambda_{gi}(P_i) = \alpha_i P_i + \beta_i. \tag{6}$$

The slope of linearized RDC is negative ($\alpha_i < 0$). By intersecting $\lambda_{gi}(P_i)$ and $B_{gi}(P_i)$, the approximate clearing price (λ^*) and dispatched generation level for the ith agent (P^*_{gi}) are obtained (note that the approximation is due to using a linear representation for the RDC).

$$\lambda^* \approx \frac{a_i\alpha_i + O_i\alpha_i - b_i\beta_i}{\alpha_i - b_i} \tag{7}$$

$$P^*_{gi} \approx \frac{a_i + O_i - \beta_i}{\alpha_i - b_i} \tag{8}$$

By inserting (7) and (8) into (5) and simplifying it, we can formulate Π_{gi} as a quadratic concave function in O_i. The value of the strategic variable for the ith agent at which the unique maximum profit level (Π^{max}_{gi}) is achieved is denoted by O_{mi}, which along with Π^{max}_{gi} are obtained by solving $\frac{\partial \Pi_{gi}}{\partial O_i} = 0$. The results are as follows:

$$O_{mi} \approx \frac{\alpha_i(\beta_i - a_i)}{2\alpha_i - b_i} \tag{9}$$

$$\Pi^{max}_{gi} = \frac{(\beta_i - a_i)^2}{2(b_i - 2\alpha_i)} \tag{10}$$

As can be seen from (9), as the sensitivity of the market price to the actions of the ith agent (i.e., α_i) approaches zero, the GenCo becomes a price-taker, and therefore, O_{mi} converges to zero, resulting in truthful bidding. This is in accordance with our expectation.

While (9) provides a deterministic relationship between the optimal value of strategic variable and model parameters, it cannot be used directly for GenCo decision making. The reason for this is that the actual RDC is nonlinear; thus, using parameters of its linear approximation (i.e., α_i, and β_i) to obtain O_{mi}, leads to additional numerical errors. To overcome this problem, we have introduced the optimal strategic parameter as another random variable inside the belief system of each agent (refer to Section III). Numerical experiments show that employing probabilistic inference to obtain estimations of optimal value of strategic parameter reduces numerical errors considerably.

With this introduction on GenCo agents' models, the step-by-step algorithm performed separately by *each agent* is as follows:

1) After the market is cleared at time t:
   - Construct the RDC for the latest round of auction using publicly available aggregate demand and supply functions.

- Use the constructed RDC to obtain the optimal value of the strategic parameter by creating a profit curve as a function of $O_i(t)$.
- Use linear regression to parameterize the RDC and obtain $\alpha_i(t)$, and $\beta_i(t)$.
- Use the obtained values of $\alpha_i(t)$, $\beta_i(t)$, $O_{mi}(t)$, and system demand at time t (denoted by $D(t)$), to update agent data history.
- Employ the updated data history and perform SBL to update the DBN-based belief system (Section III).

2) DA/HA optimal bidding for the future round of market at time $t+1$:

- Perform probabilistic particle-based forward sampling over the DBN-based belief system (Section III) to predict the values of variables ($\alpha_i(t+1)$, $\beta_i(t+1)$, optimal $O_i(t+1)$, and $D(t+1)$) for the future round of DA/HA auction.
- Construct the bidding function using the predicted value for the optimal strategic parameter. Submit the bidding function to the ISO agent.

We will show that using the procedure above, each agent is able to maximize its profit in real-time with acceptable errors. Therefore, this distributed system of interactive decision makers approaches a Nash equilibrium (i.e., each player is able to predict its best response to competitors).

B. ISO Agent Model

The task of the ISO agent is to perform an optimal generation resource allocation to supply the electrical demand on an hour-by-hour basis. Hence, the ISO runs an economic dispatching problem to maximize social welfare (here to minimize total production cost) based on the received bidding functions from GenCo agents.

$$\min_{P_1,\ldots,P_N} \quad \sum\nolimits_{i=1}^{N} (\hat{a}_i P_i + \frac{1}{2}\hat{b}_i P_i^2)$$
$$\text{s.t.} \sum\nolimits_{i=1}^{N} P_i = D, 0 \le P_i \le P_{max}^i \ (\forall i = 1,\ldots,N) \quad (11)$$

In this paper, the ISO agent uses the lambda-iteration method [36] to solve the economic dispatching problem. The inputs of the ISO agent are GenCo agents' bidding curves and the real-time electrical demand. Note that the ISO agent does not have access to GenCo agents' actual marginal cost functions. As mentioned earlier, for the electrical demand, actual hourly data (normalized by maximum available generation capacity) from the PJM market is employed [31]. After normalization, the load signal is fed directly to the model in simulations.

When the GenCos submit their offers, the ISO solves (11) for real-time load to clear the market. Then it publishes (publicly) the energy price, aggregate supply curve, and real-time demand. Details of individual GenCos' bidding functions are not exposed to competitors. Since we have ignored the effects of the transmission system on the energy market, the energy price is the same for all GenCo agents.

C. DA/HA Bidding Procedures

Note that the bidding process in the market takes place on two distinct but closely related time scales: DA market and HA market. Hence, we need to study the overall behavior of the energy market on these two scales. The mechanisms of HA/DA stages of energy markets have been described in [37] and [38]. As demonstrated in these works, as the GenCos move closer to real-time they need to compensate the power mismatch created by the errors of forecasting tools in the DA stage. In this paper, we study the interaction and effects of introducing the HA stage into the energy market, using the proposed DBN-based decision making model.

GenCo agents employ two private databases to keep their belief systems updated: the DA database and the HA database. The DA database is decomposed into 24 sections, with each section corresponding to a certain hour in the day. This has been done because of the high correlation among the load samples with a 24-hour time difference, which makes the daily load profile semi-periodic. Hence, decision making for a certain hour of the day in the DA market is based on the data history of the same hour on previous days (in this study, up to the past 300 days). Note that weekly, seasonal, and annual trends of electrical load are ignored in this paper. The HA database is composed of the HA data history of the previous hours of the market up to the most recent hours.

At each hour of the day, each GenCo agent submits two bidding functions to the market: one bidding function is submitted to participate in the next incoming hour of the same day (i.e., HA bidding), and the second bidding function is submitted for participation in the energy market of that same hour in the next day (i.e., DA bidding). Note that the market is cleared using the predicted value of the load for each stage.

According to [38], the total amount of payment that the ith GenCo receives in a multistage market R_i (i.e., a market with both DA and HA stages) is as follows:

$$R_i = P_i^{DA}\lambda^{DA} + \left(P_i^{HA} - P_i^{DA}\right)\lambda^{HA} + \left(P_i^{RT} - P_i^{HA}\right)\lambda^{RT} \quad (12)$$

where, P_i^{DA}, P_i^{HA}, and P_i^{RT} are the scheduled power of the ith unit in DA, HA, and real time markets, respectively; λ^{DA}, λ^{HA}, and λ^{RT} are the energy prices corresponding to these different stages. On the other hand, the payment amount in a single-stage market (i.e., DA stage only) is modified as follows:

$$R_i = P_i^{DA}\lambda^{DA} + \left(P_i^{RT} - P_i^{DA}\right)\lambda^{RT}. \quad (13)$$

III. DBN-Based Optimal Decision Making

Probabilistic graphical models (PGMs) offer useful tools for representing and analyzing statistical relationships and dependencies among random variables. Bayesian networks, Markov networks, Hidden Markov Models (HMM), etc. [28] are a few variations of PGMs that have been studied and used in different applications. A thorough introduction on PGMs can be found in [39].

DBNs are a category of directed PGMs that are able to capture the temporal evolution of random variables; thus, they are often a good fit to model systems with discrete-time stochastic processes [28]. DBNs can be thought of as a generalization of both static Bayesian networks and HMMs: while static Bayesian networks are a class of multivariate directed PGMs, they are unable to represent time-dependency of random variables, and are consequently not fit to model random processes. On the other hand, HMMs are a category of DBNs in which the dynamic nature of the model is preserved; however, HMMs are conventionally best fit to model univariate systems (i.e., one random variable for the hidden state chain and one for the output chain). DBNs

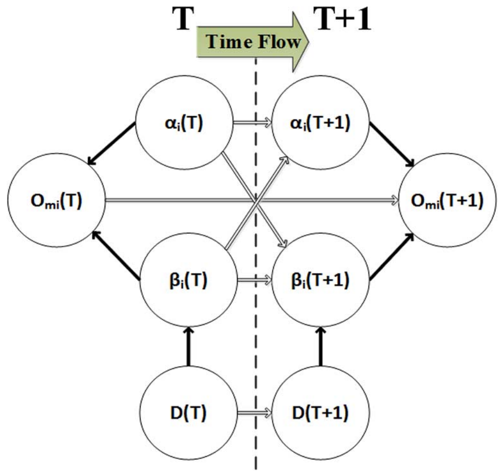


Fig. 2. DBN-based belief system of GenCo agents.

provide a general framework that address the shortcomings of static Bayesian networks and HMMs.

We model the belief system of each GenCo using DBNs. The belief system embodies each agent's probabilistic perception of its environment, which is the energy market (including the competitors, the ISO agent, and uncertain demand). Thus, each GenCo develops a DBN-based private model of the market and keeps it updated at each time step. The DBN-based models are then used by each agent to predict the future state of the market and act accordingly.

A DBN is composed of several vertices that represent random variables, and directed edges among them, which represent probabilistic dependencies. A directed edge starts at a parent vertex and ends in a child vertex (Fig. 2). The parameter of the model associated with a particular parent-child structure is equal to the conditional probability distribution function (PDF) of the random variable corresponding to the child node, given the values of random variables corresponding to its parents. Also, the DBN spans time to model the temporal changes in variables (the same as HMM). To develop a DBN-based model we have to address three issues: structure learning (i.e., determining hidden/observable random variables, edges and their direction), parameter learning (i.e., finding the conditional PDFs corresponding to parent-child structures), and inference over future events (i.e., prediction).

A. Structure Selection

Since the number of variables considered in the proposed DBN-based belief system is low, the structure of the model is selected based on experiments and using statistical measures such as mutual information (MI). However, if in future developments of the model, the number of variables grows to be large, a thorough structure learning algorithm should be implemented. The proposed structure is depicted in Fig. 2. As can be seen, the proposed DBN is a first-order Markov model with four variables at each time slice. The transition time step can be 1 hour or 24 hours, depending on the bidding horizon being HA or DA. This means that for DA bidding at a certain hour of the future day, the data history of that same hour on previous days are employed. On the other hand for HA bidding, the data for previous hours is used as the training set.

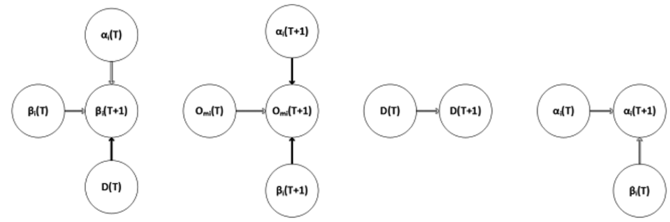


Fig. 3. Local parent-child structures within the DBN.

The random variables at each time slice of the belief system of ith agent are: D (electrical demand), β_i (intercept of agent's linearized RDC with price axis), α_i (slope of agent's linearized RDC), and O_{mi} (optimal value of strategic parameter). As shown in Fig. 2, the electrical demand is not affected by market conditions (i.e., price-insensitivity); therefore, the value of load at the future time slice $(D(T+1))$ is affected only by the demand level at present time slot $D(T)$. Note that each agent employs its private DBN-based belief system in the decision-making process.

Since to the best of our knowledge, this structure has the best performance among the candidates, and the GenCos are assumed to be rational agents (i.e., the chances of making mistakes are ignored), all GenCos will use the same structure for decision making.

B. Parameter Learning

Now that the structure of the graphical model is selected, the problem of parameterization should be addressed. Each local parent-child structure of the DBN (Fig. 3) represents a conditional PDF that has to be determined online. Note that all of the variables of the model are considered to be continuous; that is, to maintain model precision and avoid difficulties pertaining to discretization, we keep the original continuous nature of the system.

As mentioned previously, Sparse Bayesian Learning (SBL) [40] is employed to parametrize the DBN-based belief system. Thus, when the database of the agents are updated (i.e., when ISO publishes data to agents), SBL is applied to each of the local structures in Fig. 3, for the individual belief system of each separate agent to keep the overall belief systems updated.

SBL is a kernel-based learning algorithm which is also known as "relevance vector machine" [40]. The goal in SBL is to calculate the weights of the kernel functions, which for this project are selected to be Gaussians. A great advantage of SBL is its "sparsity"—only a subset of kernels have non-zero weights. As the learning process evolves, an increasing number of kernels tend to have weights with practically zero value. Therefore, using a pruning operation, we can omit the kernels that are not relevant to the learning process. The deletion of irrelevant kernels is a mechanism within SBL that prevents overfitting.

Given our training set $\{\boldsymbol{x_n}, t_n\}_{n=1}^{N}$ in which $\boldsymbol{x_n}$ is the set of explanatory variables (i.e., parents) and t_n is the target variable (i.e., child variable), the kernel-based representation of target variable is shown below:

$$t_n = \sum_{i=1}^{N} w_i \cdot K(x_n, x_i) + w_0 + \epsilon_n. \tag{14}$$

Here w_i's are the weights of the kernel functions $K(x_n, x_i)$, and ϵ_n is a noise process, which is assumed to be a zero mean

Gaussian process with variance σ^2. Using this model, the objective of SBL is to estimate the weights of the kernel functions and the variance of the noise process. Note that the likelihood of the dataset can be written as a Gaussian function (since we have assumed that the distribution of noise is Gaussian):

$$p\left(\mathbf{t}|\mathbf{w},\sigma^2\right) = (2\pi\sigma^2)^{\frac{-N}{2}} \exp\left\{-\frac{1}{2\sigma^2}\|\mathbf{t}-\mathbf{\Phi}\mathbf{w}\|^2\right\} \quad (15)$$

where $\mathbf{\Phi}$ is called the design matrix:

$$\mathbf{\Phi} = \begin{bmatrix} 1 & K(x_1,x_1) & K(x_1,x_2) & \dots & K(x_1,x_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & K(x_n,x_1) & K(x_n,x_2) & \dots & K(x_n,x_n) \end{bmatrix} \quad (16)$$

The prior distributions over the weights are selected to be independent zero-mean Gaussians:

$$p(\mathbf{w}|\boldsymbol{\alpha}) = \prod\nolimits_{i=0}^{N} \mathcal{N}\left(w_i|0,\alpha_i^{-1}\right) \quad (17)$$

where, $\boldsymbol{\alpha}$ is the vector of hyperparameters, and is equal to the set of inverse variances of the kernel weights. As α_i gets larger and larger during the learning process, the corresponding kernel function gets more and more irrelevant and can be eliminated. To make predictions, we will be needing the posterior of the unknown parameters and hyperparameters given the observed target vector. The posterior is decomposed using the chain rule, as follows:

$$p\left(\mathbf{w},\boldsymbol{\alpha},\sigma^2|\mathbf{t}\right) = p\left(\mathbf{w}|\mathbf{t},\boldsymbol{\alpha},\sigma^2\right) p\left(\boldsymbol{\alpha},\sigma^2|\mathbf{t}\right). \quad (18)$$

The first term on the right hand side of (18) is the posterior distribution over the weights and can be calculated analytically. Using Bayes rule, the posterior is formulated into a Gaussian distribution with covariance matrix Σ and mean vector μ [40]. The second term on the right hand side of (18) can be replaced with a delta function that is nonzero only at the most probable values for the hyperparameters (σ^2_{MP} and $\boldsymbol{\alpha}_{MP}$). Values of σ^2_{MP} and $\boldsymbol{\alpha}_{MP}$ have been calculated using expectation-maximization-based recursive estimations, to maximize the marginal likelihood function (different update rules are discussed in [40].) Thus, the parameter of a local structure in the DBN-based belief system with child variable X, and its parents $\mathbf{Pa}(X)$ is a Gaussian distribution, obtained as follows:

$$p\left(X|\mathbf{Pa}(X)\right) \sim \mathcal{N}\left(\boldsymbol{\mu}^{\mathrm{T}}\mathbf{\Phi}\left(\mathbf{Pa}(X)\right), \sigma^2_{MP} + \mathbf{\Phi}\left(\mathbf{Pa}(X)\right)^T \mathbf{\Sigma}\mathbf{\Phi}\left(\mathbf{Pa}(X)\right)\right). \quad (19)$$

C. Inference

After the conclusion of the learning procedure at each time step, the agents perform probabilistic inference over variables of a future time slice. In other words, they predict the future state of the market to estimate the optimal value of the strategic variable for the future round of auction (next hour or next day).

In this paper, inference is accomplished using a particle-based forward sampling method [39]. The values of the variables at the current time slice are initialized as evidence (primary particles). Then, using the learned model parameters (i.e., conditional PDFs) we move along the directed edges of the DBN to obtain particles of the future time slice. This process is repeated 500 times. The mean of obtained particles is used as the predicted values for the variables in the future time slice. When the GenCos obtain an estimation of $O_{mi}(T+1)$ through probabilistic inference, they modify their bidding function accordingly and submit it to the ISO agent.

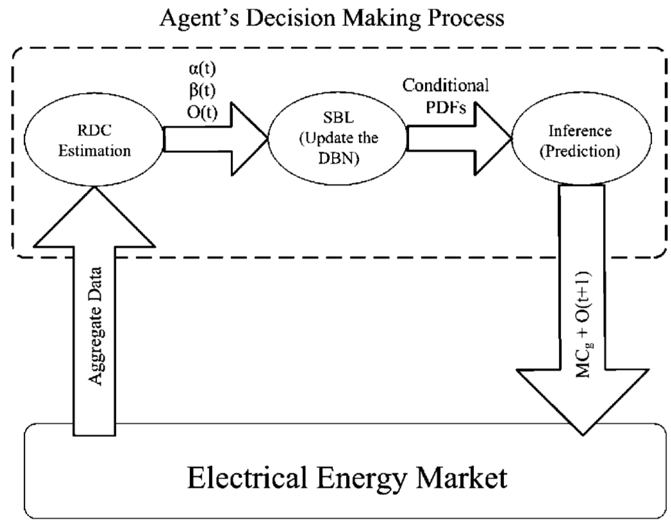


Fig. 4. GenCo's agent-based model.

D. The Decision Making Process

The DBN-based decision making process involves the two functionalities of learning and prediction (i.e., inference). Upon receiving the latest data samples from the most recent round of auction, first, each agent updates the conditional probability density functions corresponding to parent-child structures of the DBNs, employing SBL. These updated conditional PDFs represent the statistical affinities among the variables (i.e., α, β, D, and O). The final goal of the decision making problem is to find or estimate the optimal course of action for the future round of auction, at the HA or the DA look-ahead windows. Hence, the updated conditional PDFs are used to achieve this goal. Given the latest samples for the variables at the current time step, the future samples for the variables are extracted using a forward sampling method, employing the learned conditional PDFs. Considering the structure of the DBN (Fig. 2), the current samples of variables α, β, D serve as inputs to the decision making process and are used to generate the predicted samples for the same variables. Finally, the predicted samples for the input variables (at time $t+1$) are employed to estimate the optimal course of action for the incoming round of auction (i.e., strategic parameter, O). The sampling process is performed repeatedly, as explained in the previous subsection, and the mean of the samples for the strategic parameter, O, is used as the forecasted optimal action, which determines the deviation from the marginal cost curve when bidding in the market. The algorithmic overview of the overall decision making process is presented in Section II of the paper. The agent-based decision making process is illustrated in Fig. 4, for one GenCo agent.

IV. Numerical Experiments and Results

The proposed decision making model is tested in MATLAB environment for four test cases: two generic systems with 5 and 15 GenCo agents, corresponding to energy markets with high and low market share concentration, and two IEEE benchmarks (9-bus and 30-bus systems). Both HA and DA bidding schemes are implemented in all the test cases. Thus, the behavior of the proposed model on different time scales and different market concentration levels is studied.

After the market is cleared, real-time optimal values of strategic parameters of each agent are obtained using real time RDC for each GenCo. The results are then compared with DBN-based DA/HA predictions to assess the performance of the decision making model.

A. Case Study I

Considering an HA bidding scheme for the system with 5 agents (cost parameters given in Table II, Appendix I), the optimal real-time value of strategic parameter for one of the agents is depicted in Fig. 5(a), along with its DBN-based estimation. As can be seen, the DBN-based estimation follows the real-time optimal value with satisfactory precision (mean absolute error (MAE) of 3.83%). This implies that the DBN-based belief system can be employed for prediction of the result of HA market and optimal acting accordingly. Also, the results of the prediction of the other three variables of the belief system $(\alpha, \beta,\ and\ D)$ and their real-time values are shown in Fig. 5(b), (c), and (d), respectively. The MAE level of predictions for these variables are below 3%.

The rest of the GenCo agents in the system show similar performance. As the real-time average optimal value of the strategic parameter approaches zero, the MAE increases (since the error normalizer tends to shrink to near zero values). This implies that, for agents that are not exercising market power (i.e., no tangible deviation from marginal cost), the DBN-based decision making procedure becomes less reliable. However, practically the average error values of O_{mi} prediction for all the agents are similar and around \$0.3/MWh.

Since the actions of all the agents are nearly optimal, the system approaches NE. Thus, the DBN-based belief system enables the GenCos to predict the real-time market equilibrium and modify their actions to maximize their profit.

B. Case Study II

The decision making model is also tested on the 15-agent system for a DA bidding scheme (cost parameters given in Table II, Appendix I). As mentioned before, for DA bidding on a certain hour of the next day, the agents use the data history of the same hour on previous days to train their DBN-based private belief systems. Hence, each agent has 24 DBNs corresponding to each hour of the day. In Fig. 6(a), the result of prediction of O_{mi} is presented for one of the agents, on a certain hour of the day. The spikes on the curve correspond to an increase in the DA load profile for that hour of the day (Fig. 6(b)). This implies that, as the demand increases, the GenCo's incentive to deviate from its marginal cost curve grows. The profit profile of the agent is depicted in Fig. 6(c). As shown here, the DBN-based strategic bidding leads to an increase in the real-time profit level and approximately reaches the real-time maximum possible profit. The DBN-based DA demand prediction outcome is shown in Fig. 6(b). Compared to HA load estimation with MAE of 2.34%, the MAE level of DA load forecasting increases to 7%.

Here again, all the agents show similar performance in prediction and decision-making; and since each agent is maximizing its profit at the same time with acceptable precision, the

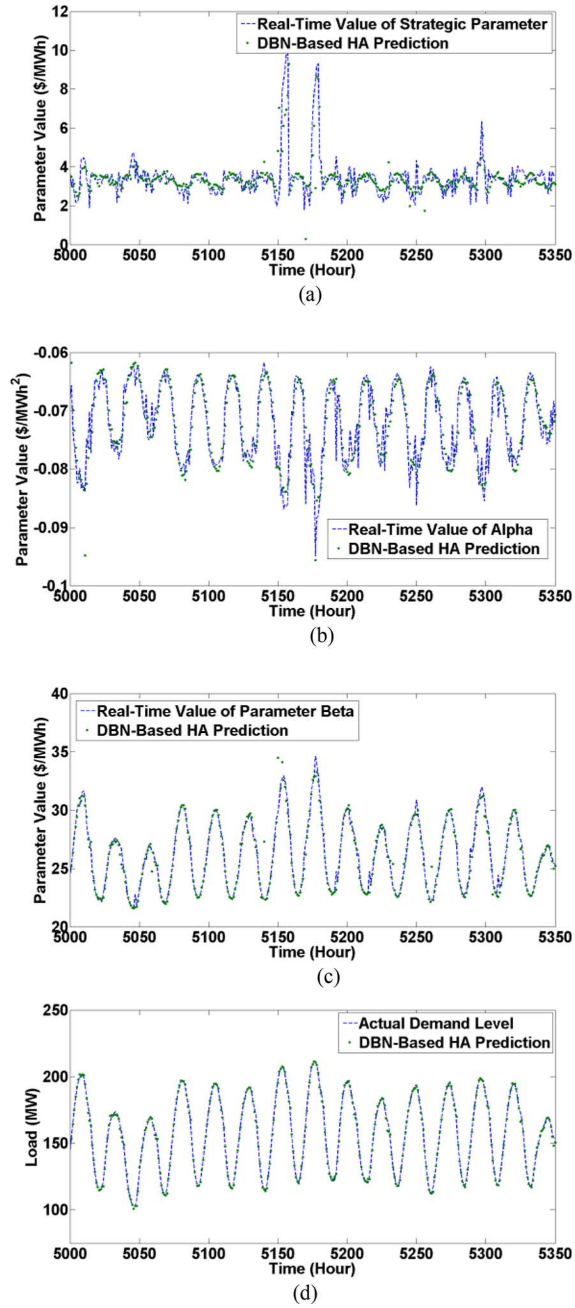


Fig. 5. Comparing the outcome of the DBN-based decision making with optimal real-time outcome of the market for one GenCo agent (HA bidding—system with 5 agents). (a) Strategic parameter, (b) Alpha parameter, (c) Beta parameter, (d) Actual demand level.

DBN-based distributed decision-making has led the multiagent system to approach its NE.

C. Case Study III

The proposed decision making model is tested on the IEEE 9-bus system (consisting of three agents), as illustrated in Fig. 7. The cost data for the three GenCo agents in this system are provided in the Appendix I [41]. The final output variable (i.e., the strategic parameter) of the DBN for one of the agents of this system (with $a_i = \$1$/MWh, $b_i = \$0.24$/MWh^2, and $P^i_{max} = 120$ MW) as a function of time is depicted in Fig. 8. The MAE level is 4.95% and 11.24% for the HA and DA look-ahead times,

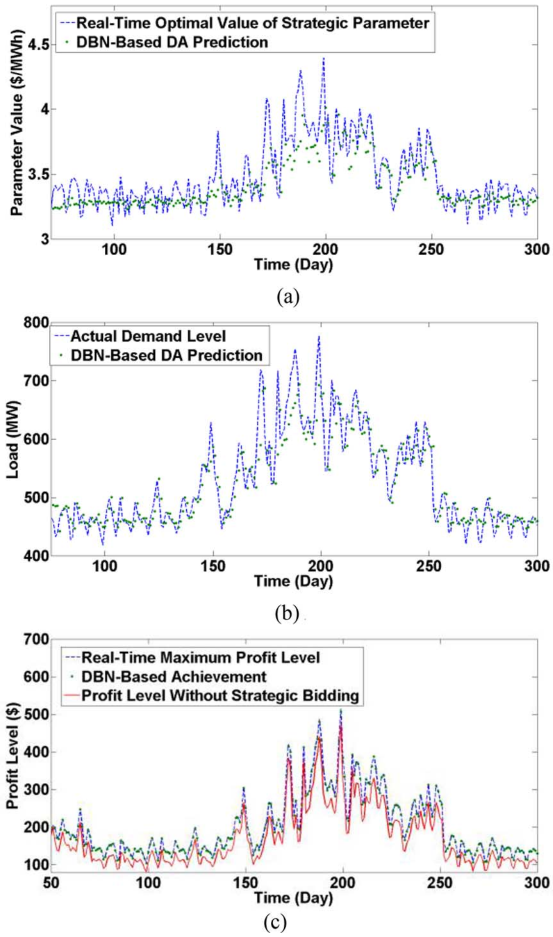


Fig. 6. Comparing the outcome of the DBN-based decision making with optimal real-time outcome of the market for one GenCo agent (DA bidding—system with 15 agents). (a) Strategic parameter, (b) Actual demand level, (c) Profit level.

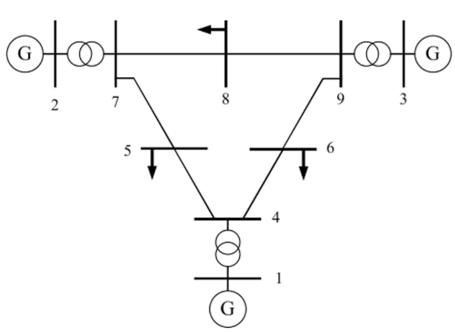


Fig. 7. Structure of IEEE 9-bus system.

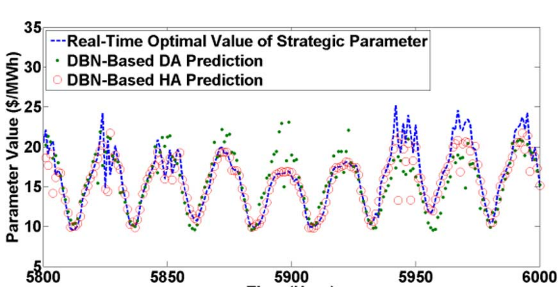


Fig. 8. Comparing the outcome of DBN-based decision making with optimal real-time values of the market for one agent in IEEE 9-bus system.

TABLE I
DETAILS OF THE DECISION MAKING PROCESS

	α_i ($/MWh^2)	β_i ($/MWh)	D (MW)	O_i ($/MWh)
t = 5900 h	-0.1091	76.3	435	16.36
HA prediction for t + 1 = 5901 h	-0.1095	73.1	415	15.7
Real-time values for t + 1 = 5901 h	-0.1081	72.8	433	15.67

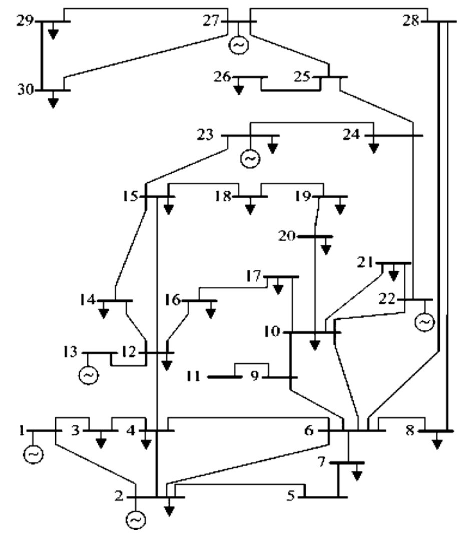
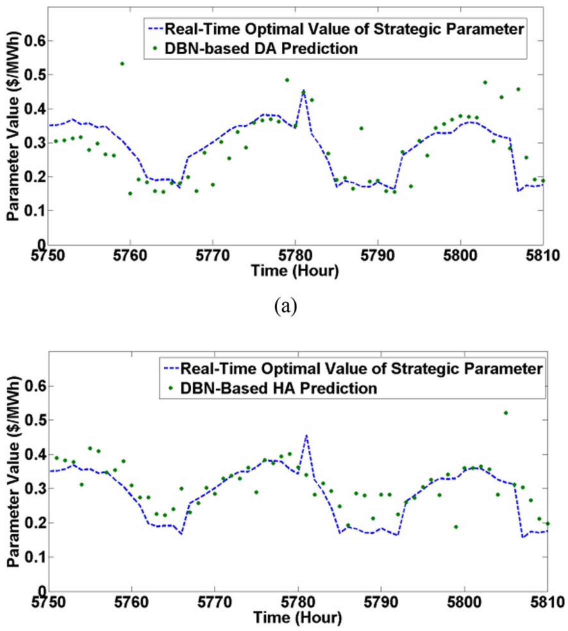


Fig. 9. Structure of IEEE 30-bus system [42].

respectively. Note that Fig. 8 shows the optimal course of action to be taken by the agent at each time step for the HA and DA cases. Hence, the agent determines its bidding function based on the predicted value of the strategic parameter at each time step. To further clarify the decision making process, we have shown the samples of the variables for the agent in Table I, at $t = 5900$ h and predictions for $t + 1 = 5901$ h. The samples at time t are used for updating the DBN and forecasting the values of the variables at time $t + 1$. The actual values of the variables at time $t + 1$ are given in the last row of the table. As can be seen, the HA predicted values for time $t + 1$ are close to their actual real-time values that are obtained after market clearance at that specific time.

D. Case Study IV

The DBN-based decision making scheme is implemented on the IEEE 30-bus system, shown in Fig. 9; the data for the system is given in [42]. The cost parameters of the generators (GenCos) are given in Table II, Appendix I [43]. In Fig. 10(a), and Fig. 10(b) the results of the DBN-based prediction of the strategic parameter for the DA and HA stages are shown, for one of the agents. The MAE of estimation is 10% and 15% for the HA and DA markets, respectively. Note that while the MAE has increased compared to the previous two test cases (due to lower mean value of strategic parameter), the absolute value of the error is in the same range (around \$0.1/MWh to \$0.3/MWh). As expected, the accuracy of the DBN-based decision making procedure in the HA stage has improved compared to the DA stage. The other five agents show similar accuracy in estimating their optimal strategic bidding functions.



(a)

(b)

Fig. 10. Comparing the outcome of the DBN-based decision making with optimal real-time outcome of the market for one GenCo agent (DA and HA markets—IEEE 30-bus system). (a) Strategic parameter (DA), (b) Strategic parameter (HA).

E. Comparing the DA and HA Bidding Schemes

Since the uncertainty pertaining to load prediction is diminished in the HA decision-making, the equilibrium of the HA market is Pareto superior for the GenCos, compared to the DA market. Thus, the GenCos can make additional profit by modifying their DA bidding functions on the HA stage (as they move closer to real-time dispatching). We have compared the profit levels of the GenCo agents in case studies I and IV for the DA and DA + HA markets. In the 5-agent system of case study I, an *average* annual profit level growth of $90,000 is observed for each agent in the DA + HA market, compared to the DA market.

The results of the simulation in the forth case study (IEEE 30-bus system) show that introducing the HA stage into the market leads to more stable and less volatile prices, as depicted in Fig. 11. Also, the HA stage reduces the volatility of the profit streams of the GenCos. Compared to the single-stage market (i.e., DA only), the mean total profit of the agents for the multistage case $(\mathrm{DA}+\mathrm{HA})$ has increased from $8,800 to $10,000 in a period of 60 hours. Hence, a multistage procedure is capable of decreasing the risk and improving the stability of the energy market.

F. Market Power Analysis

As the number of competing GenCos grow (keeping the total generation capacity fixed), the average share of each GenCo, i.e., market concentration (which is measured by Hirschmann-Herfindahl Index (HHI)) falls [38]. In our study, the value of HHI for the 5-agent system is 0.2, while it drops to 0.072 when the number of GenCos increases to 15. On the other hand, the average value of the optimal strategic parameter decreases from $1.886/MWh, to $0.835/MWh, as the number of GenCos grows from 5 to 15; this indicates a drop in

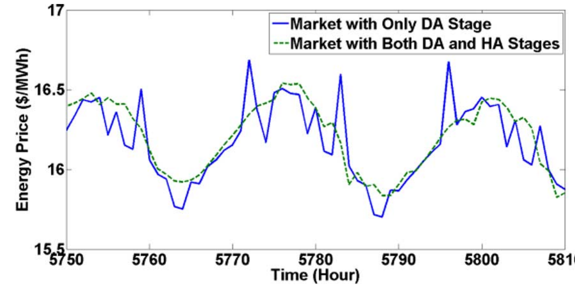


Fig. 11. Comparing the energy price in single-stage and multistage energy markets (IEEE 30-bus system).

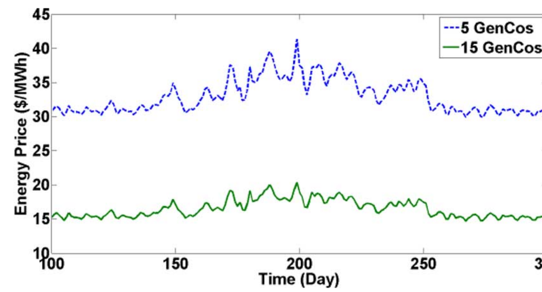


Fig. 12. Comparing energy prices in systems with 5 and 15 agents.

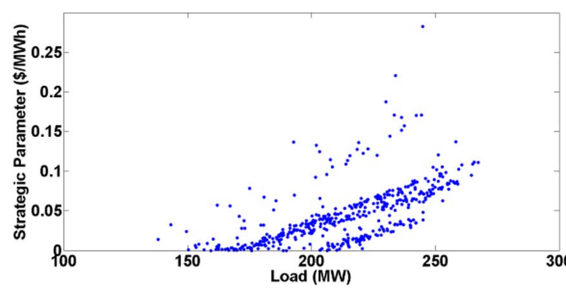


Fig. 13. Sensitivity of an agent's market power to inelastic electrical demand level.

GenCos' incentive to exercise market power. In Fig. 12, energy prices of the 5-agent and 15-agent systems are compared. In addition to an increase in average price in the 5-agent system ($32.95/MWh compared to $16.44/MWh for the 15-agent system), the standard deviation of the energy price has also grown ($2.37/MWh, compared to $1.24/MWh for the 15-agent system), suggesting higher price volatility for the system with lower number of GenCos. Thus, the model confirms the expected drop in market power exercise as the number of GenCos grow. Also, in both systems, the average correlation level of electrical demand and the optimal strategic variable is high and almost the same (0.729 in the 15-agent system, and 0.765 in the 5-agent model). The scatter diagram of optimal strategic parameter and electrical demand for one of the GenCos in the IEEE 30-bus system is shown in Fig. 13. The cost parameters of this agent are: $a_i = \$14.75/\mathrm{MWh}$, $b_i = \$0.0175/\mathrm{MWh}^2$, and $P^i_{max} = 80$ MW, as shown in Table II in the Appendix. As expected, an increase in electrical demand results in excessive market power exercise by GenCos.

The agent-based model shows two sources of volatility in the energy market: the errors in load forecasting, and the low number of GenCo agents. While the former is caused by limitation of forecasting tools in containing the uncertainty of the system (as shown in comparing single-stage and multistage markets), the latter corresponds to direct market power exercise by dominant firms in the market.

V. Conclusion

In this paper, an agent-based optimal decision making tool is designed using dynamic Bayesian networks. Employing sparse Bayesian learning, each agent trains its private belief system to predict the optimal course of action to be taken in future rounds of the market. This distributed decision-making model is tested in MATLAB on markets with high and low share concentrations (5 and 15 GenCos) and on two distinct time scales (HA/DA). Also, the model was tested on two IEEE benchmarks (9-bus and 30-bus systems). Numerical results show that, by using the proposed decision making model, the agents are able to predict the market equilibrium in advance, with acceptable errors. Thus, based on the proposed probabilistic model, the multiagent system approaches Nash equilibrium through distributed decision-making under incomplete information. The DBN-based belief system can be expanded easily to model more complex decision-making situations.

Appendix I
Cost Data for GenCo Agents of Case Studies

TABLE II
Data for Case Studies

	a_i (\$/MWh)	b_i (\$/MWh^2)	P_{max}^i (MW)
Cost Data For Case Study I			
GenCo1	10	0.25	216
GenCo2	16	0.19	193
GenCo3	20	0.1	224
GenCo4	6	0.15	211
GenCo5	22	0.05	206
Cost Data For Case Study II			
GenCo1	10	0.25	55
GenCo2	16	0.19	40
GenCo3	20	0.1	40
GenCo4	8	0.15	50
GenCo5	22	0.05	45
GenCo6	18	0.25	60
GenCo7	16	0.2	65
GenCo8	14	0.1	70
GenCo9	11	0.13	75
GenCo10	10	0.09	90
GenCo11	10	0.2	75
GenCo12	12	0.1	80
GenCo13	10	0.1	80
GenCo14	6	0.01	110
GenCo15	7	0.02	100
Cost Data For Case Study III [41]			
GenCo1	5	0.22	250
GenCo2	1.2	0.17	300
GenCo3	1	0.245	120
Cost Data For Case Study IV [43]			
GenCo1	15	0.02	80
GenCo2	14.75	0.0175	80
GenCo3	16	0.025	50
GenCo4	14	0.0625	50
GenCo5	16	0.025	30
GenCo6	15.25	0.0083	55

References

[1] T. Li and M. Shahidehpour, "Strategic bidding of transmission-constrained GENCOs with incomplete information," *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 437–447, Feb. 2005.

[2] J. D. Weber and T. J. Overbye, "A two-level optimization problem for analysis of market bidding strategies," in *Proc. IEEE Power Eng. Soc. Summer Meeting*, Jul. 18–22, 1999, vol. 2, pp. 682–687.

[3] Z. Q. Luo, J. S. Pang, and D. Ralph, *Mathematical Programs With Equilibrium Constraints*. New York, NY: Cambridge Univ. Press, 1996.

[4] S. A. Gabriel, A. J. Conejo, J. D. Fuller, B. F. Hobbs, and C. Ruiz, *Complementarity Modeling in Energy Markets*. New York, NY: Springer, 2013.

[5] M. E. Khodayar and M. Shahidehpour, "Optimal strategies for multiple participants in electricity markets," *IEEE Trans. Power Syst.*, vol. 29, no. 2, pp. 986–987, Mar. 2014.

[6] B. F. Hobbs, C. B. Metzler, and J. S. Pang, "Strategic gaming analysis for electric power systems: An MPEC approach," *IEEE Trans. Power Syst.*, vol. 15, no. 2, pp. 638–645, May 2000.

[7] E. G. Kardakos, C. K. Simoglou, and A. G. Bakirtzis, "Short-term electricity market simulation for pool-based multi-period auctions," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2526–2535, Aug. 2013.

[8] X. Hu and D. Ralph, "Using EPECs to model bi-level games in restructured electricity markets with locational prices," *Oper. Res.*, vol. 5, no. 5, pp. 809–827, Sep. 2007.

[9] M. V. Pereira, S. Granville, M. H. C. Fampa, R. Dix, and L. A. Barrsos, "Strategic bidding under uncertainty: A binary expansion approach," *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 180–188, Feb. 2005.

[10] A. G. Bakirtzis, N. P. Ziogos, A. C. Tellidou, and G. A. Bakirtzis, "Electricity producer offering strategies in day-ahead energy market with step-wise offers," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 1804–1818, Nov. 2007.

[11] G. Zhang, G. Zhang, Y. Gao, and J. Lu, "Competitive strategic bidding optimization in electricity markets using bi-level programming and swarm technique," *IEEE Trans. Ind. Electron.*, vol. 58, no. 6, pp. 2138–2146, Jun. 2011.

[12] I. Atzeni, L. O. Ordonez, G. Scutari, D. P. Palomar, and J. R. Fonollosa, "Noncooperative day-ahead bidding strategies for demand-side expected cost minimization with real-time adjustment: A GNEP approach," *IEEE Trans. Signal Process.*, vol. 62, no. 9, pp. 2397–2412, May 1, 2014.

[13] J. Y. Wei and Y. Smeers, "Spatial oligopolistic electricity models with Cournot generators and regulated transmission prices," *Oper. Res.*, vol. 47, no. 1, pp. 102–112, 1999.

[14] M. Kazemi, B. Mohammadi-Ivatloo, and M. Ehsan, "Risk-constrained strategic bidding of GenCos considering demand response," *IEEE Trans. Power Syst.*, vol. 30, no. 1, pp. 376–384, Jan. 2015.

[15] Y. Yang, Y. Zhang, F. Li, and H. Chen, "Computing all Nash equilibria of multiplayer games in electricity markets by solving polynomial equations," *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 81–91, Feb. 2012.

[16] A. Weidlich and D. Veit, "A critical survey of agent-based wholesale electricity market models," *Energy Econ.*, vol. 30, no. 4, pp. 1728–1759, 2008.

[17] J. Bower and D. W. Bunn, "Model-based comparisons of pool and bilateral markets for electricity," *Energy J.*, pp. 1–29, 2000.

[18] V. Nanduri and T. K. Das, "A reinforcement learning model to assess market power under auction-based energy pricing," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 85–95, Feb. 2007.

[19] V. Nanduri and I. Saavedra-Antonilez, "A competitive Markov decision process model for the energy-water-climate change nexus," *Appl. Energy*, vol. 111, pp. 186–198, Nov. 2013.

[20] J. J. Sanchez, D. W. Bunn, E. Centeno, and J. Barquin, "Dynamics in forward and spot electricity markets," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 582–591, May 2009.

[21] A. Banal-Estanol and A. R. Micola, "Behavioural simulations in spot electricity markets," *Eur. J. Oper. Res.*, vol. 214, no. 1, pp. 147–159, Oct. 2011.

[22] N. P. Yu, C. C. Liu, and J. Price, "Evaluation of market rules using a multi-agent system method," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 470–479, Feb. 2010.

[23] M. Shafie-khah and J. P. S. Catalao, "A stochastic multi-layer agent-based model to study electricity market participants behavior," *IEEE Trans. Power Syst.*, vol. 30, no. 2, pp. 867–881, Mar. 2015.

[24] M. Shafie-khah, M. P. Moghaddam, and M. K. Sheikh-El-Eslami, "Development of a virtual power market model to investigate strategic and collusive behavior of market players," *Energy Policy*, vol. 61, pp. 717–728, Oct. 2013.

[25] M. Mahvi and M. M. Ardehali, "Optimal bidding strategy in a competitive electricity market based on agent-based approach and numerical sensitivity analysis," *Energy*, vol. 36, no. 11, pp. 6367–6374, Nov. 2011.

[26] R. S. Sutton and G. B. Andrew, *Reinforcement Learning: An Introduction*. Cambridge, MA, USA: MIT Press, 1998, vol. 1, no. 1.

[27] L. Fan, J. Wang, R. Jiang, and Y. Guan, "Min-max regret bidding strategy for thermal generator considering price uncertainty," *IEEE Trans. Power Syst.*, vol. 29, no. 5, pp. 2169–2179, Sep. 2014.

[28] K. P. Murphy, "Dynamic Bayesian networks: Representation, inference and learning," Ph.D. dissertation, Univ. California, Berkeley, CA, USA, 2002.

[29] M. Ventosa, A. Baillo, A. Ramos, and M. Rivier, "Electricity market modeling trends," *Energy Policy*, vol. 33, no. 7, pp. 897–913, 2005.

[30] A. G. Petoussis, X. P. Zhang, S. G. Petoussis, and K. R. Gofrey, "Parameterization of linear supply functions in nonlinear AC electricity market equilibrium models—Part I: Literature review and equilibrium algorithm," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 650–658, May 2013.

[31] "PJM market data," [Online]. Available: http://www.pjm.com.

[32] G. Aneiros, J. M. Vilar, R. Cao, and A. M. S. Roque, "Functional prediction for residual demand in electricity spot markets," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4201–4208, Nov. 2013.

[33] A. Baillo, M. Ventosa, M. Rivier, and A. Ramos, "Optimal offering strategies for generation companies operating in electricity spot markets," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 745–753, May 2004.

[34] F. Wen and A. K. David, "Optimal bidding strategies and modeling of imperfect information among competitive generators," *IEEE Trans. Power Syst.*, vol. 16, no. 1, pp. 15–21, Feb. 2001.

[35] S. Vazquez, P. Rodilla, and C. Batlle, "Residual demand models for strategic bidding in European power exchanges: revisiting the methodology in the presence of a large penetration of renewables," *Electr. Power Syst. Res.*, vol. 108, pp. 178–184, 2014.

[36] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*. New York, NY, USA: Wiley, 1996.

[37] A. Botterud, Z. Zhou, J. Wang, J. Sumaili, H. Keko, J. Mendes, R. J. Bessa, and V. Miranda, "Demand dispatch and probabilistic wind power forecasting in unit commitment and economic dispatch: A case study of Illinois," *IEEE Trans. Sustain. Energy*, vol. 4, no. 1, pp. 250–261, Jan. 2013.

[38] D. R. Biggar and M. R. Hesamzadeh, *The Economics of Electricity Markets*. New York, NY, USA: Wiley, 2014.

[39] D. Koller and N. Friedman, *Probabilistic Graphical Models: Principles and Techniques*. Cambridge, MA, USA: MIT Press, 2009.

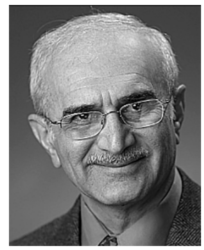
[40] M. E. Tipping, "Sparse Bayesian learning and the relevance vector machine," *J. Mach. Learn. Res.*, vol. 1, pp. 211–244, 2001.

[41] Y. Liu, H. Xin, Z. Qu, and D. Gan, "A distributed solution to real-time economic dispatch problem under power flow congestion," in *Proc. 2015 IEEE Power and Energy Soc. General Meeting*, 2015, pp. 1–5.

[42] "IEEE 30-bus system data," [Online]. Available: http://www.al-roomi.org/component/content/article?id=25:30-bus-system.

[43] "Cost parameters for GenCo agents of the IEEE 30-bus system," [Online]. Available: http://een.iust.ac.ir/profs/jadid/SCPM.pdf.

Kaveh Dehghanpour (S'14) received the B.S. and M.S. degrees from the University of Tehran, Tehran, Iran, in 2011, and 2013, respectively. He is currently pursuing the Ph.D. degree in the Electrical and Computer Engineering Department at Montana State University, Bozeman, MT, USA. His research interests include electrical energy markets, multi-agent systems, and machine learning.



M. Hashem Nehrir (S'68–M'71–SM'89–F'10–LF'13) received the B.S., M.S., and Ph.D. degrees from Oregon State University, Corvallis, OR, USA, in 1969, 1971, and 1978, respectively, all in electrical engineering.

He is a Professor with the Department of Electrical and Computer Engineering, Montana State University, Bozeman, MT, USA. His research interests include modeling and control of power systems, alternative energy power generation systems, and application of intelligent controls to power systems. He is the author of three textbooks and an author or coauthor of numerous technical papers.

John W. Sheppard (M'86–SM'97–F'07) received a B.S. in Computer Science from Southern Methodist University in 1983 and an M.S. and Ph.D. in Computer Science from Johns Hopkins University in 1990 and 1997 respectively.

He is a College of Engineering Distinguished Professor of Computer Science at Montana State University. He is also an Adjunct Professor in the Department of Computer Science at Johns Hopkins University. He performs research in probabilistic graphical models, Bayesian methods, distributed evolutionary optimization, and integrated system health management.

Nathan C. Kelly (S'14) received the B.Sc. degree in electrical engineering from Michigan State University, East Lansing, MI, USA, in 2009. He is currently working towards the M.Sc. degree in electrical engineering at Montana State University, Bozeman, MT, USA.

His research interests are in renewable power systems.