CSCI 246: Assignment 7
Due: April 27, 2026

Name: _____

Problem 1 (6 points). Let $G = \langle V, E \rangle$ be a graph where
$V = \{a, b, c, d, e\}$ and
$E = \{(a, a), (a, b), (b, c), (c, a), (c, d), (d, e), (e, d)\}$.

A. What are the order and size of G?

B. Compute the out-degree for each vertex.

C. Is G regular? Why or why not.

D. Is G complete?

E. Is G strongly connected? Weakly connected?

F. Identiy all strongly connected components of G.

Problem 2 (6 points). Let $G = \langle V, E \rangle$ be a graph where
$V = \{1, 2, 3, 4\}$ and
$E = \{(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (3, 4)\}$.

A. Draw G

B. Find a spanning subgraph of G that is weekly connected but has fewer edges than G.

C. Compute the induced subgraph of $V' = \{1, 2, 3\}$.

D. List a clique of maximum size.

E. List an independent set of maximum size.

D. Compute the complement of G.

Problem 3 (5 points). Let $G = \langle V, E \rangle$ be a graph where
$V = \{a, b, c, d, e\}$ and
$E = \{(a, b), (b, c), (c, a), (c, d), (d, e), (e, d)\}$.

A. List a walk from a to e.

B. List a path from a to e.

C. Is there a path from e to a. Why or why not?

D. List all strongly connected components of G.

E. List all weakly connected components of G.

Problem 4 (3 points). Let $G = \langle V, E \rangle$ be a graph where
$V = \{a, b, c, d, e\}$ and
$E = \{(a, b), (b, c), (c, d), (d, b), (d, e)\}$.
Assume that G is undirected (i.e., that an all edges are symmetric).

A. Identify all cut vertices.

B. Identify all cut edges.

C. Explain the criteria for a vertex (resp. edge) to be a cut vertex (resp. edge).

Problem 5 (10 points). Consider the cycle graph of 5 vertices C_5.

A. Compute a coloring of C_5 with as few colors as possible.

B. What is the chromatic number of C_5?

C. Is C_5 bipartite?

D. Repeat parts **A**-**C** for C_6.

E. Conjecture which cycle graphs are bipartite.

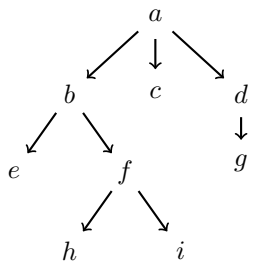
Problem 6 (6 points). Consider the bipartite graph $G = \langle X \cup Y, E \rangle$ where
$X = \{1, 2, 3\}$, $Y = \{a, b, c\}$, and
$E = \{(1, a), (1, b), (2, b), (2, c), (3, a)\}$.

A. Find any matching of G.

B. List a maximum matching of G.

C. Is there matching that is maximal but not maximum? Explain why or why not.

Problem 7 (8 points). Consider the tree to the right:



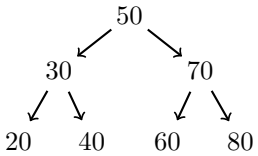
A. Which is the root node? Which are interior nodes? Leaf nodes?

B. What is the parent node of h? Children node of B? Siblings of C?

C. What is the size of the tree? Height of the tree?

D. List the level of each node.

Problem 8 (8 points). Consider the binary search tree to the right:



A. What is the search path to determine 60 is in the BST?

B. What is the search path to determine 35 is not in the BST?

C. What is the search path to determine 80 is in the search tree?

D. What is the search path to determine 10 is not in the search tree?

Problem 9 (8 points). For each of the following statements about binary trees determine if the statement is true or provide a counter example.

A. Every full binary tree is complete.

B. Every complete binary tree is balanced.

C. Every balanced binary tree is full.

D. Every binary search tree is balanced.