**CSCI 246**: Final Exam (Take-Home)
**Due**: May 6, 2026

**Name**: ______________________________

*This exam is open-note but must be completed individually. Show all work for full credit. You may use results proven in class or on homework assignments.*

**Problem 1 (6 points).** Rewrite each of the following statements in if-then form. Then determine whether the statement is true or false. If false, provide a counter-example.

**A**. The product of two odd numbers is odd.

**B**. The sum of two primes is even.

**C**. Every multiple of 6 is a multiple of 3.

**Problem 2 (8 points).** Prove or disprove the following statement:
For all integers $a$ and $b$, if $a^2 + b^2$ is even, then $a + b$ is even.

**Problem 3 (8 points).** Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6, 7\}$, and $C = \{5, 6, 7, 8, 9\}$.

**A**. Compute $A \cap (B \cup C)$.

**B**. Compute $(A \cap B) \cup (A \cap C)$.

**C**. Are the answers to parts **A** and **B** the same? State the general set identity this illustrates and explain briefly why it holds.

**D**. Compute $|\mathcal{P}(A \cap B)|$.

**Problem 4 (6 points).** Translate each English statement into a symbolic statement using quantifiers. Clearly define any predicates you introduce and state the domain of each quantified variable. Then prove or disprove the statement.

**A**. For every integer $n$, there exists an integer $m$ such that $n + m = 0$.

**B**. There exists a real number $x$ such that $x^2 < x$.

**Problem 5 (8 points).** Define the relation $R$ on $\mathbb{Z}$ by: $a \; R \; b$ if and only if $a - b$ is divisible by 5.

**A**. Prove that $R$ is an equivalence relation.

**B**. List the equivalence classes of $R$ and describe the partition of $\mathbb{Z}$ induced by $R$.

**C**. Are 17 and 42 in the same equivalence class? Justify your answer.

**Problem 6 (6 points).**

**A**. A committee of 5 people is to be chosen from a group of 8 men and 6 women. How many committees are possible if the committee must contain at least 2 women?

**B**. How many 10-bit strings contain exactly 4 ones?

**C**. How many paths are there from $(0, 0)$ to $(5, 3)$ on a grid if you can only move right or up one unit at a time?

**Problem 7 (6 points).** For each statement, identify the most natural proof technique (direct proof, proof by contrapositive, proof by contradiction, or proof by induction) and briefly justify your choice. No proof required.

**A**. If $3n + 2$ is odd, then $n$ is odd.

**B**. For all $n \geq 1$, $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$.

**C**. There is no largest prime number.

**Problem 8 (8 points).** Prove by induction that for all $n \geq 1$:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

**Problem 9 (8 points).** A computer system processes jobs. The number of pending jobs $a_n$ at the start of hour $n$ satisfies the recurrence:

$$a_n = 5a_{n-1} - 6a_{n-2}$$

with initial conditions $a_0 = 1$ and $a_1 = 4$.

**A**. Find the closed-form solution for $a_n$.

**B**. Use your closed form to compute $a_5$.

**Problem 10 (6 points).** Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(n) = 2n + 1$.

**A**. Is $f$ injective? Prove or disprove.

**B**. Is $f$ surjective? Prove or disprove.

**C**. Now let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = 2x + 1$. Is $g$ a bijection? Justify your answer.

**Problem 11 (8 points).**

**A**. Rank the following functions in order of increasing growth rate:

$$n!, \quad n^2, \quad \log n, \quad 2^n, \quad n \log n, \quad n^3, \quad 1$$

**B**. For each pair, determine whether $f(n) \in O(g(n))$, $f(n) \in \Omega(g(n))$, and/or $f(n) \in \Theta(g(n))$. Briefly justify.

1. $f(n) = 5n^3 + 2n$ and $g(n) = n^3$

2. $f(n) = 2^n$ and $g(n) = n^{100}$

3. $f(n) = \log(n^2)$ and $g(n) = \log n$

**Problem 12 (6 points).** A software company tests code with an automated tool. The tool detects a bug when one exists 90% of the time. However, the tool also produces false alarms (reports a bug when none exists) 8% of the time. Suppose that 5% of code submissions actually contain a bug.

**A**. Define the relevant events and state their probabilities.

**B**. If the tool reports a bug, what is the probability that the code actually contains a bug? Use Bayes' Theorem.

**C**. Let $X$ be the number of true bugs detected out of 4 independent submissions, each containing a bug. Compute $E(X)$ and $\text{Var}(X)$.

**Problem 13 (12 points).** Let $G = \langle V, E\rangle$ be a directed graph where
$V = \{a, b, c, d, e, f\}$ and
$E = \{(a, b), (b, c), (c, a), (c, d), (d, e), (e, f), (f, d)\}$.

**A**. What are the order and size of $G$?

**B**. Compute the in-degree and out-degree of each vertex.

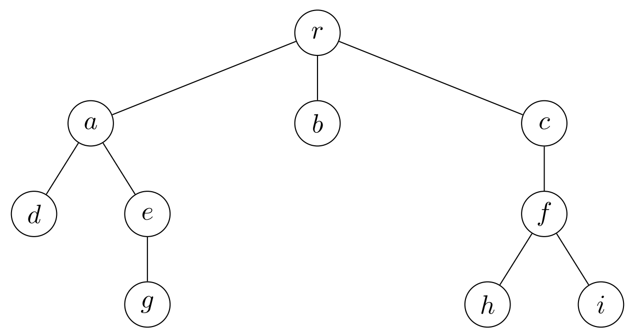
**C**. List a path from $a$ to $f$.

**D**. Is there a path from $f$ to $a$? Why or why not?

**E**. Identify all strongly connected components of $G$.

**F**. Is $G$ weakly connected? Justify your answer.

**Problem 14 (8 points).** Consider the following rooted tree:




**A**. Identify the root, all interior nodes, and all leaf nodes.

**B**. What is the height of the tree? List the level of each node.

**C**. What is the chromatic number of this tree? Justify your answer.

**D**. Is this tree a binary tree? Is it a complete binary tree? Explain.

— **End of Exam** —