Functions: Group Exercises

CSCI 246

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Problem 1. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. Determine if the following relations from A to B are functions. Briefly justify your answer.

A. $R_1 = \{(1, a), (2, b), (3, c)\}$.

Yes, because R_1 has exactly one element of the form (x, y) for each $x \in A$.

B. $R_2 = \{(1, a), (1, b), (2, c), (3, a)\}$.

No, because R_2 contains two pairs whose first element is 1 (i.e., $(1, a)$ and $(1, b)$).

C. $R_3 = \{(1, a), (2, b)\}$.

No, because R_3 does not contain any element whose first element is 3 (i.e., $(3, x)$ for some $x \in B$).

D. $R_4 = \{(1, a), (2, a), (3, a)\}$.

Yes, because R_4 has exactly one element of the form (a, b) for each $a \in A$.

Problem 2. Let $f : A \to B$ where $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$ and $f = \{(1, a), (2, c), (3, c), (4, b)\}$.

A. What is the domain of f? dom $f = A = \{1, 2, 3, 4\}$

B. What is the codomain of f? co-dom $f = B = \{a, b, c, d\}$

C. What is the image (or range) of f? im $f = \{y : (x, y) \in f\} = \{a, b, c\}$

D. Which elements of B are not in the image of f? $B -$ (im f) $= \{d\}$

Problem 3. For each function, determine if f has an inverse function f^{-1}. If it does, find f^{-1} and verify by computing $f(f^{-1}(x))$.

A. Let $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = 2x + 3$.

Yes, $f^{-1}(y) = \frac{y-3}{2}$. For any x, $f(f^{-1}(x)) = 2(\frac{x-2}{2}) + 3 = x$; thus f and f^{-1} are inverse functions.

B. Let $f : \mathbb{N} \to \mathbb{N}$ with $f(x) = |x|$.

Yes, $f^{-1}(y) = y$. For any x, $f(f^{-1}(x)) = |x|$ and note that for $n \geq 0$ which is true for all natural numbers, $|x| = x$. So for $f : \mathbb{N} \to \mathbb{N}$, f^-1 and f are inverse functions.

C. Let $f : \mathbb{Z} \to \mathbb{N}$ with $f(x) = |x|$.

No, since f's domain is the integers (i.e., $f : \mathbb{Z} \to \mathbb{N}$), there is no inverse function for f.
Specifically, because $f(x) = f(-x)$ for any integer x, we know that f^-1 cannot be a function.

Problem 4. For each function determine if the function is *one-to-one*, *onto*, and/or *bijective*.

A. $f : \mathbb{N} \to \mathbb{N}$ with $f(x) = |x|$.

The function f is both one-to-one and onto, and is thus bijective.

B. $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x^3 + 4$.

The function f is both one-to-one and onto, and is thus bijective.

C. $f : \mathbb{Z} \to \mathbb{Z}$ with $f(x) = x^3 + 4$.

The function f is one-to-one but not onto, and is thus not bijective.

D. $f : \mathbb{Z} \to \mathbb{N}$ with $f(x) = |x|$.

The function f is onto but not one-to-one, and is thus not bijective.

Problem 5. Let A and B be finite sets. How many functions $f : A \to B$ exists for:

A. $|A| = 3$ and $|B| = 5$.

There are a total of $|A|^{|B|} = 3^5 = 243$ functions $f : A \to B$.

B. $A = \emptyset$ and $B \neq \emptyset$.

There is a single such function $f : A \to B$ (i.e., $f = \emptyset$). Note that $|A|^{|B|} = x^0$ for $x \neq 0$, and thus $x^0 = 1$.

C. $|A| = 5$ and $|B| = 2$.

There are a total of $|A|^{|B|} = 5^2 = 25$ functions $f : A \to B$.

Problem 6. Let $R \subseteq A \times A$ be a relation on A. Prove that if R is an equivalence relation and R is a function, then R is the identity function $f : A \to A$ with $f(x) = x$.

Proof.

Let R be both an equivalence relation on A and a function from A to A.
Let f be the identity function on A (i.e., $\{(a, a) : a \in A\}$)
Since R is an equivalence relation, we know that for any $a \in A$, $(a, a) \in R$.
Thus, clearly $f \subseteq R$.
Now, we must show that $R \subseteq f$.
Let $(a, b) \in R$ be any pair of elements related by R.
Since R is reflexive we know that $(a, a) \in R$ (or in function notation $R(a) = a$).
Additionally, since R is a function, we know that there is a unique b such that $R(a) = b$ for any $a \in A$.
Since both $R(a) = b$ and $R(a) = a$, we know that $b = a$.
Thus, $(a, b) = (a, a)$. And clearly $(a, a) \in f$.
Thus we may conclude that $R \subseteq f$ as required.
Since both $R \subseteq f$ and $f \subseteq R$ we have $R = f$.
That is R is the identify function as required. □