

# Big-O Notation: Group Exercises

CSCI 246

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**Problem 1.** Match each term with the corresponding description.

- |                             |            |   |
|-----------------------------|------------|---|
| A. $f(n)$ is $O(g(n))$      | $\swarrow$ | 1. $f(n)$ is <i>eventually</i> bounded below by a multiple of $g(n)$          |
| B. $f(n)$ is $\Omega(g(n))$ | $\searrow$ | 2. $f(n)$ is <i>eventually</i> bounded above and below by multiples of $g(n)$ |
| C. $f(n)$ is $\Theta(g(n))$ | $\swarrow$ | 3. $f(n)$ is <i>eventually</i> bounded above by a multiple of $g(n)$          |

**Problem 2.** Which of the following statements are true?

- A.  $\sqrt{x}$  is  $O(x^{\frac{3}{4}})$       **true**
- B.  $n \log 4n$  is  $\Omega(n)$       **true**
- C.  $n \log 4n$  is  $O(n^{1+\epsilon})$  for  $0 < \epsilon$       **true**
- D.  $3n^2 - 1000n + 20$  is  $\Omega(n^3)$       **false**
- E.  $3n^2 - 1000n + 20$  is  $\Omega(1)$       **true**

**Problem 3.** Show that each of the below statements are true.

A.  $\log_2 n$  is  $\Theta(\log_{10} n)$

For all  $n$  we have  $\log_2 n = (\log_2 10) \log_{10} n$ .

B.  $n^5 - n^4 + n^3$  is  $O(n^6)$

For all  $n \geq 0$ ,  $n^5 - n^4 + n^3 \leq n^6$ .

C.  $n!$  is  $\Omega(4^n)$

For all  $n \geq 9$ ,  $4^n \leq n!$ .

D.  $\frac{1}{n}$  is  $O(1)$

For all  $n \geq 1$ ,  $\frac{1}{n} \leq 1$ .

**Problem 4.** Prove that if  $f(n)$  is  $O(g(n))$  and  $f(n)$  is  $\Omega(g(n))$ , then  $f(n)$  is  $\Theta(g(n))$ .

*Proof.*

Since  $f(n)$  is  $O(g(n))$ , there must be some  $n_0$  and  $0 < c_0$ , such that for all  $n \geq n_0$ ,  $f(n) \leq c_0g(n)$ .  
Similarly, since  $f(n)$  is  $\Omega(g(n))$ , there must be some  $n_1$  and  $c_1$ , such that all  $n \geq n_1$ ,  $c_1g(n) \leq f(n)$ .  
Let  $n' = \max(n_0, n_1)$ . Clearly for each  $n \geq n'$ , we have  $c_1g(n) \leq f(n) \leq c_0g(n)$ .  
Thus,  $f(n)$  is  $\Theta(g(n))$ . □

**Problem 5.** Prove that  $f(n)$  is  $O(g(n))$  if and only if  $g(n)$  is  $\Omega(f(n))$ .

*Proof.*

We proceed to prove both cases.

**Case  $\Rightarrow$ :** If  $f(n)$  is  $O(g(n))$ , then  $g(n)$  is  $\Omega(f(n))$ .

By assumption,  $f(n)$  is  $O(g(n))$ .

Thus, there is some  $n_0$  and  $0 < c$ , such that for all  $n \geq n_0$ ,  $f(n) \leq cg(n)$ .

Clearly, we also have for all  $n \geq n_0$ ,  $\frac{1}{c}f(n) \leq g(n)$ .

Therefore,  $g(n)$  is  $\Omega(f(n))$ .

**Case  $\Leftarrow$ :** If  $g(n)$  is  $\Omega(f(n))$ , then  $f(n)$  is  $O(g(n))$ .

By assumption,  $g(n)$  is  $\Omega(f(n))$ .

Thus, there is some  $n_0$  and  $0 < c$ , such that for all  $n \geq n_0$ ,  $cf(n) \leq g(n)$ .

Clearly, we also have for all  $n \geq n_0$ ,  $f(n) \leq \frac{1}{c}g(n)$ .

Therefore,  $f(n)$  is  $O(g(n))$ .

Since we have proven both directions, we may conclude that  $f(n)$  is  $O(g(n))$  iff  $g(n)$  is  $\Omega(f(n))$ . □

**Problem 6.** Prove that  $n^n$  is  $\Omega(n!)$ .

*Proof.*

We begin by proving via induction that  $0 < n \Rightarrow n! \leq n^n$ .

**Base Case:**  $n = 1$ .

Clearly,  $n! = 1! = 1 \leq 1 = 1^1 = n^n$ ; i.e.,  $n! < n^n$  for  $n = 1$ .

**Inductive Case:**  $1 < n = m + 1$  with  $m! \leq m^m$ .

By the inductive hypothesis we have  $m! \leq m^m$ .

Necessarily, we have  $n! = n(n-1)! = n(m!)$ .

We also have  $n^n = n^{m+1} = n(n^m)$ .

Note that since  $0 \leq m < n$ , we also know that  $m^m \leq n^m$ .

Combining, we have  $m! \leq m^m \leq n^m$ .

Since  $0 < n$ , we can multiply all terms by  $n$  to conclude that  $n(m!) \leq n(m^m) \leq n(n^m)$ .

Since  $n! = n(m!)$  and  $n^n = n(n^m)$  we can conclude that  $n! \leq n^n$ .

We can now conclude that for all  $n \geq 1$ , that  $n! \leq n^n$ .

If we let  $n_0 = 1$  and  $c = 1$ , then clearly for each  $n \geq n_0$ , we have  $(1)n! \leq n^n$ .

Thus by definition, we have  $n^n$  is  $\Omega(n!)$ . □