Regular Expressions
CSCI 338

Regular Expressions

Rules for building regular expressions (regex):

1. Each $e \in \Sigma$ is a regex
2. $\{\varepsilon\}$ is a regex
3. $\emptyset$ is a regex
4. $(R_1 \cup R_2)$ is a regex
5. $(R_1 \circ R_2)$, denoted $(R_1 R_2)$ is a regex
6. ${R_1}^*$ is a regex

R_1 and R_2 are regexs

Order of operations:

- Parentheses, star (and plus), concatenation, union.

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 =$?

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- $1^*0^*1 = \{w\!: w$ contains ≥ 0 1s, then ≥ 0 0s, then a $1\}$

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- $(1 \cup 0)^*1 = \{w\colon w \text{ ends in } 1\}$

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- $\{w : w \text{ contains a single } 1\} =$?

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- $\{w\colon w \text{ contains a single } 1\} = 0^*10^*$

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- $\{w\colon w$ contains at least one $1\} =$?

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- $\{w\colon w \text{ contains at least one } 1\} = \Sigma^*1\Sigma^*$ or $(0 \cup 1)^*1(0 \cup 1)^*$

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- $(\Sigma\Sigma)^* = \{w\colon w \text{ has even length}\}$

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- $\{w$: every 0 is followed by at least one $1\} =$?

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- $\{w\!: \text{every } 0 \text{ is followed by at least one } 1\} = 1^*(01^+)^*$

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- $\{w : \text{every } 0 \text{ is followed by at least one } 1\} = 1^*(01^+)^*$
- $1^*\emptyset =$?

By definition, $A \circ B = \{xy : x \in A, y \in B\}$

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- $1^*\emptyset = \emptyset$

By definition, $A \circ B = \{xy\colon x \in A, y \in B\}$
Since there is no element in $\emptyset$, there cannot be any xy such that $y \in \emptyset$.

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- $1^*\varepsilon = 1^*$

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- $\{w$: every 0 is followed by at least one $1\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = ?$

By definition, $A^* = \{x_1x_2 \ldots x_k : k \geq 0, x_i \in A\}$

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- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = \varepsilon$

By definition, $A^* = \{x_1x_2 \ldots x_k\!: k \geq 0, x_i \in A\}$
Thus, it can append 0 elements of $\emptyset$ and get the empty string ε.

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- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = \varepsilon$ $\quad\quad\quad\quad$ $\emptyset^+ = ?$

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- $\emptyset^* = \varepsilon$ $\quad\quad\quad$ $\emptyset^+ = \emptyset$