Regular Expressions
CSCI 338
Regular Expressions

Rules for building regular expressions (regex):
1. Each \( e \in \Sigma \) is a regex
2. \( \{\varepsilon\} \) is a regex
3. \( \emptyset \) is a regex
4. \( (R_1 \cup R_2) \) is a regex
5. \( (R_1 \circ R_2) \), denoted \( (R_1 R_2) \) is a regex
6. \( R_1^* \) is a regex

Order of operations:
- Parentheses, star (and plus), concatenation, union.
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = ?$
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^* \cdot 0^* \cdot 1 = \{w : w \text{ contains } \geq 0 \ 1 \text{s, then } \geq 0 \ 0 \text{s, then a } 1\}$
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w \text{ contains } \geq 0 \ 1\text{s, then } \geq 0 \ 0\text{s, then a } 1\}$
- $(1 \cup 0)^*1 = ?$
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w$ contains $\geq 0$ 1s, then $\geq 0$ 0s, then a 1\}$
- $(1 \cup 0)^*1 = \{w: w$ ends in 1\}$
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^*1 = \{w: w \text{ ends in 1}\}$
- $\{w: w \text{ contains a single 1}\} = ?$
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^* 0^* 1 = \{w : w \text{ contains } \geq 0 \ 1s, \text{ then } \geq 0 \ 0s, \text{ then a } 1\}$
- $(1 \cup 0)^* 1 = \{w : w \text{ ends in } 1\}$
- $\{w : w \text{ contains a single } 1\} = 0^* 10^*$
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^*0^*1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^*1 = \{w: w \text{ ends in 1}\}$
- $\{w: w \text{ contains a single 1}\} = 0^*10^*$
- $\{w: w \text{ contains at least one 1}\} = ?$
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^*0^*1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^*1 = \{w: w \text{ ends in } 1\}$
- $\{w: w \text{ contains a single 1}\} = 0^*10^*$
- $\{w: w \text{ contains at least one 1}\} = \Sigma^*1\Sigma^*$ or $(0 \cup 1)^*1(0 \cup 1)^*$
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^* 0^* 1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^* 1 = \{w: w \text{ ends in 1}\}$
- $\{w: w \text{ contains a single 1}\} = 0^* 1 0^*$
- $\{w: w \text{ contains at least one 1}\} = \Sigma^* 1 \Sigma^*$
- $(\Sigma \Sigma)^* = ?$
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^* 0^* 1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^* 1 = \{w: w \text{ ends in 1}\}$
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- $\{w: w \text{ contains at least one 1}\} = \Sigma^* 1 \Sigma^*$
- $(\Sigma \Sigma)^* = \{w: w \text{ has even length}\}$
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a } 1\}$
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- $\{w: w \text{ contains at least one 1}\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w \text{ has even length}\}$
- $\{w: \text{ every 0 is followed by at least one 1}\} = ?$
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w$ contains $\geq 0$ 1s, then $\geq 0$ 0s, then a 1$\}$
- $(1 \cup 0)^*1 = \{w: w$ ends in 1$\}$
- $\{w: w$ contains a single 1$\} = 0^*10^*$
- $\{w: w$ contains at least one 1$\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w$ has even length$\}$
- $\{w: every 0 is followed by at least one 1$\} = 1^*(01^+)^*$
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^*1 = \{w: w \text{ ends in 1}\}$
- $\{w: w \text{ contains a single 1}\} = 0^*10^*$
- $\{w: w \text{ contains at least one 1}\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w \text{ has even length}\}$
- $\{w: \text{every 0 is followed by at least one 1}\} = 1^*(01^+)^*$
- $1^*\emptyset = ?$

By definition, $A \circ B = \{xy: x \in A, y \in B\}$
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^* 0^* 1 = \{w: w$ contains $\geq 0$ 1s, then $\geq 0$ 0s, then a 1$\}$
- $(1 \cup 0)^* 1 = \{w: w$ ends in 1$\}$
- $\{w: w$ contains a single 1$\} = 0^* 10^*$
- $\{w: w$ contains at least one 1$\} = \Sigma^* 1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w$ has even length$\}$
- $\{w: every$ 0 is followed by at least one 1$\} = 1^*(01^+)^*$
- $1^* \emptyset = \emptyset$

By definition, $A \circ B = \{xy: x \in A, y \in B\}$

Since there is no element in $\emptyset$, there cannot be any $xy$ such that $y \in \emptyset$. 
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^*1 = \{w: w \text{ ends in 1}\}$
- $\{w: w \text{ contains a single 1}\} = 0^*10^*$
- $\{w: w \text{ contains at least one 1}\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w \text{ has even length}\}$
- $\{w: \text{every 0 is followed by at least one 1}\} = 1^*(01^+)^*$
- $1^\emptyset = \emptyset$
- $1^\varepsilon = ?$

By definition, $A \circ B = \{xy: x \in A, y \in B\}$
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^*0^*1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^*1 = \{w: w \text{ ends in 1}\}$
- $\{w: w \text{ contains a single 1}\} = 0^*10^*$
- $\{w: w \text{ contains at least one 1}\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w \text{ has even length}\}$
- $\{w: \text{ every 0 is followed by at least one 1}\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$

By definition, $A \circ B = \{xy: x \in A, y \in B\}$
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^*0^*1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^*1 = \{w: w \text{ ends in 1}\}$
- $\{w: w \text{ contains a single 1}\} = 0^*10^*$
- $\{w: w \text{ contains at least one 1}\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w \text{ has even length}\}$
- $\{w: \text{ every 0 is followed by at least one 1}\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = ?$

By definition, $A^* = \{x_1x_2 \ldots x_k: k \geq 0, x_i \in A\}$
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w : w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^*1 = \{w : w \text{ ends in 1}\}$
- $\{w : w \text{ contains a single 1}\} = 0^*10^*$
- $\{w : w \text{ contains at least one 1}\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w : w \text{ has even length}\}$
- $\{w : \text{ every 0 is followed by at least one 1}\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = \varepsilon$

By definition, $A^* = \{x_1x_2 \ldots x_k : k \geq 0, x_i \in A\}$

Thus, it can append 0 elements of $\emptyset$ and get the empty string $\varepsilon$. 
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w: w \text{ contains } \geq 0 \text{ ones, then } \geq 0 \text{ zeros, then a } 1\}$
- $(1 \cup 0)^*1 = \{w: w \text{ ends in } 1\}$
- $\{w: w \text{ contains a single } 1\} = 0^*10^*$
- $\{w: w \text{ contains at least one } 1\} = \Sigma^*1\Sigma^*$
- $(\Sigma\Sigma)^* = \{w: w \text{ has even length}\}$
- $\{w: \text{ every } 0 \text{ is followed by at least one } 1\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = \varepsilon$
- $\emptyset^+ = ?$
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^* 0^* 1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^* 1 = \{w: w \text{ ends in 1}\}$
- $\{w: w \text{ contains a single } 1\} = 0^* 1 0^*$
- $\{w: w \text{ contains at least one } 1\} = \Sigma^* 1 \Sigma^*$
- $(\Sigma \Sigma)^* = \{w: w \text{ has even length}\}$
- $\{w: \text{ every 0 is followed by at least one 1}\} = 1^* (01^+)^*$
- $1^* \emptyset = \emptyset$
- $1^* \varepsilon = 1^*$
- $\emptyset^* = \varepsilon$
- $\emptyset^+ = \emptyset$