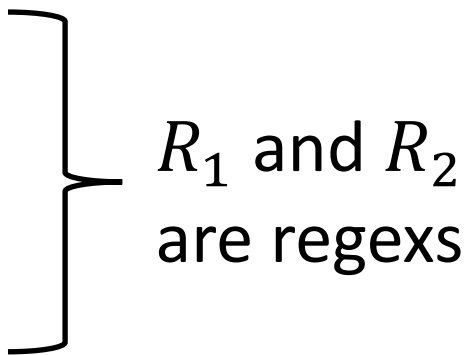


Regular Expressions

CSCI 338

Regular Expressions

Rules for building regular expressions (regex):

1. Each $e \in \Sigma$ is a regex
 2. $\{\varepsilon\}$ is a regex
 3. \emptyset is a regex
 4. $(R_1 \cup R_2)$ is a regex
 5. $(R_1 \circ R_2)$, denoted $(R_1 R_2)$ is a regex
 6. R_1^* is a regex
- 
- R_1 and R_2
are regexs

Order of operations:

- Parentheses, star (and plus), concatenation, union.

Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = ?$

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- $\{w: w \text{ contains a single 1}\} = 0^*10^*$

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- $\{w: w \text{ contains at least one 1}\} = \Sigma^*1\Sigma^* \text{ or } (0 \cup 1)^*1(0 \cup 1)^*$

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- $1^*\emptyset = ?$

By definition, $A \circ B = \{xy: x \in A, y \in B\}$

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- $1^*\emptyset = \emptyset$

By definition, $A \circ B = \{xy: x \in A, y \in B\}$
Since there is no element in \emptyset , there cannot be any xy such that $y \in \emptyset$.

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By definition, $A^* = \{x_1x_2 \dots x_k: k \geq 0, x_i \in A\}$

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- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = \varepsilon$

By definition, $A^* = \{x_1x_2 \dots x_k: k \geq 0, x_i \in A\}$
Thus, it can append 0 elements of \emptyset and get the empty string ε .

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Suppose that $\Sigma = \{0,1\}$.

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- $\{w: \text{every 0 is followed by at least one 1}\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = \varepsilon$ $\emptyset^+ = ?$

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