# Pumping Lemma

## CSCI 338

If all regular languages have property $\boldsymbol{P}$, and some new language $\boldsymbol{L}$ does not have property $\boldsymbol{P}$, then...?

If all regular languages have property $\boldsymbol{P}$, and some new language $\boldsymbol{L}$ does not have property $\boldsymbol{P}$, then $\boldsymbol{L}$ cannot be a regular language.

# Properties That Imply Language is Regular?

<u>Claim</u>: Languages that are finite in size are regular.

<u>Claim</u>: Languages where all strings have bounded size (each string has size $\leq$ some $n$) are regular.

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Proof: Since there are a finite number of strings, build a DFA for each individual string in the language.

Then connect all start states to a new start state via $\varepsilon$-transitions.

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Claim: Languages where all strings have bounded size (each string has size $\leq$ some $n$) are regular.

Proof: Since the alphabet is finite, there is a finite number of strings constructible with $n$ characters.

Thus, the language is finite and regular.

# Quest for Regular Language Properties

What do we know about non-regular languages?

- They must be infinite in size.
- They must have arbitrarily long strings in them.

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Suppose some regular language $L$ contains strings of arbitrary length (i.e. $\forall n \geq 0, \exists s \in L$ such that $|s| \geq n$). Then:

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Let $s$ be <u>any</u> string in $L$ such that $|s| \geq p$.
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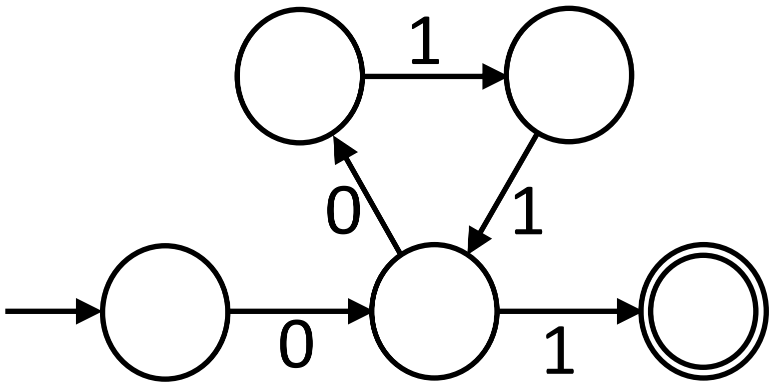


e.g. $s = 00111 \in L$

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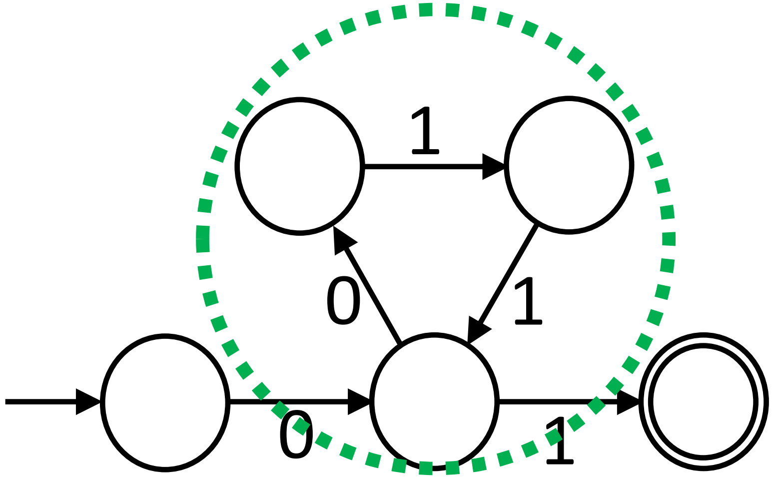


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e.g. $s = 0|011|1 \in L$

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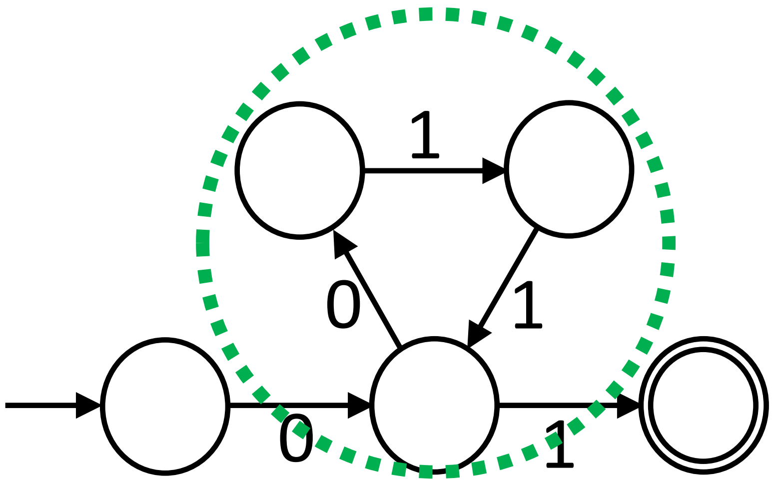


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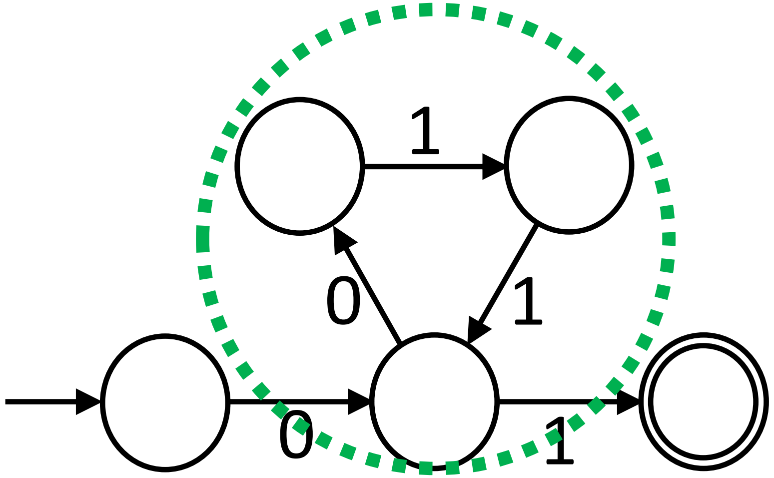


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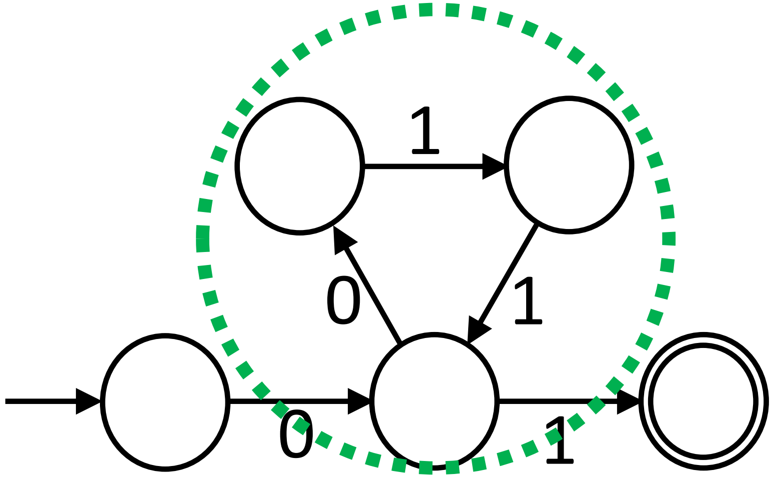


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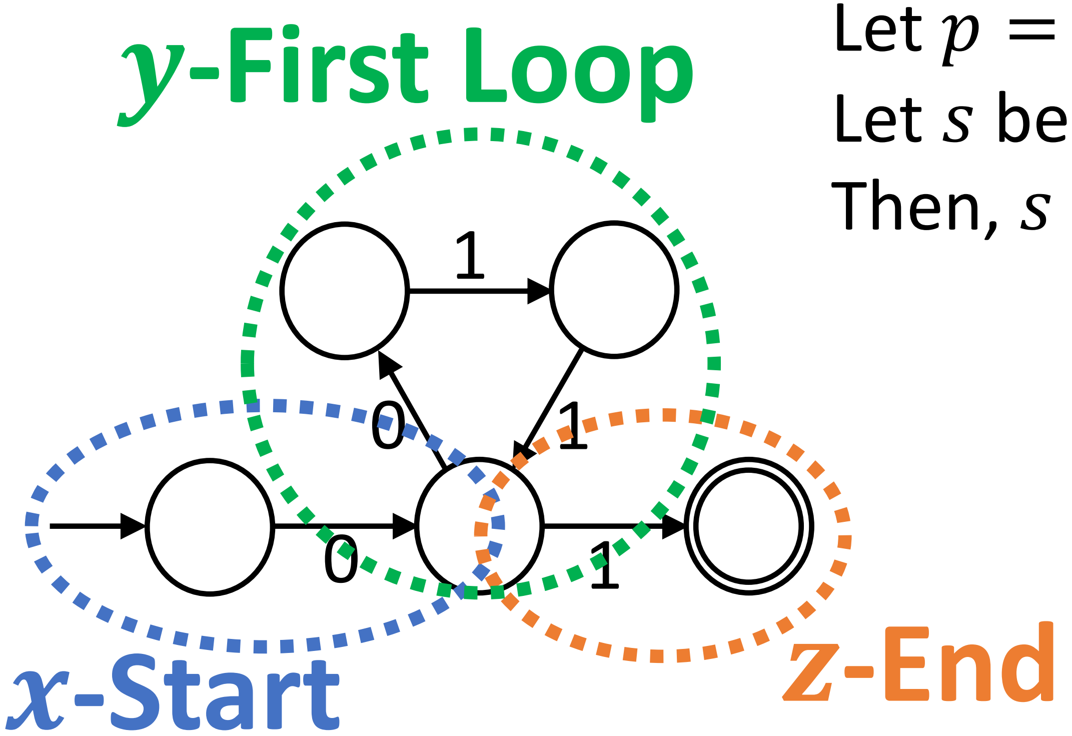


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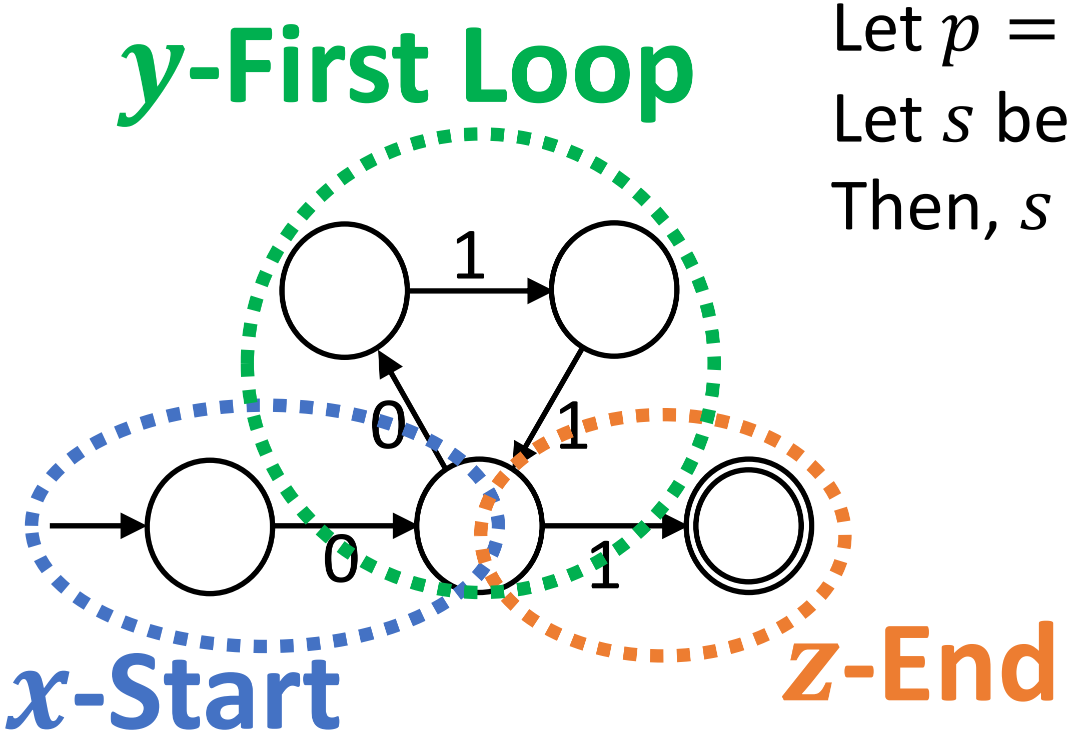


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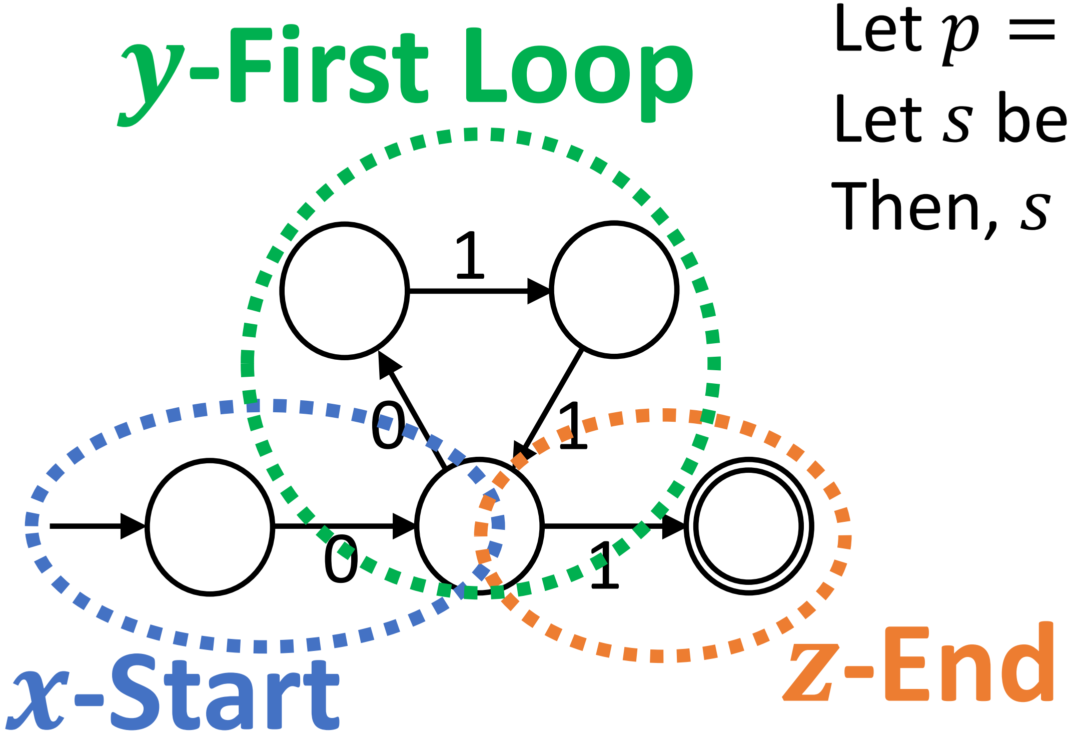


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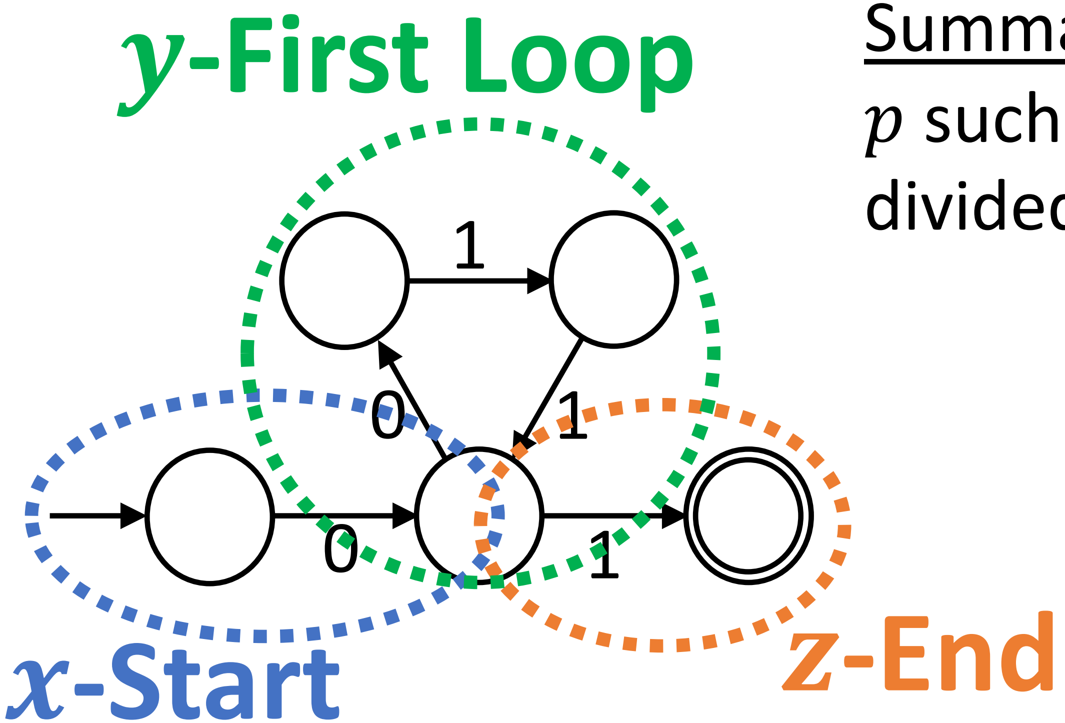


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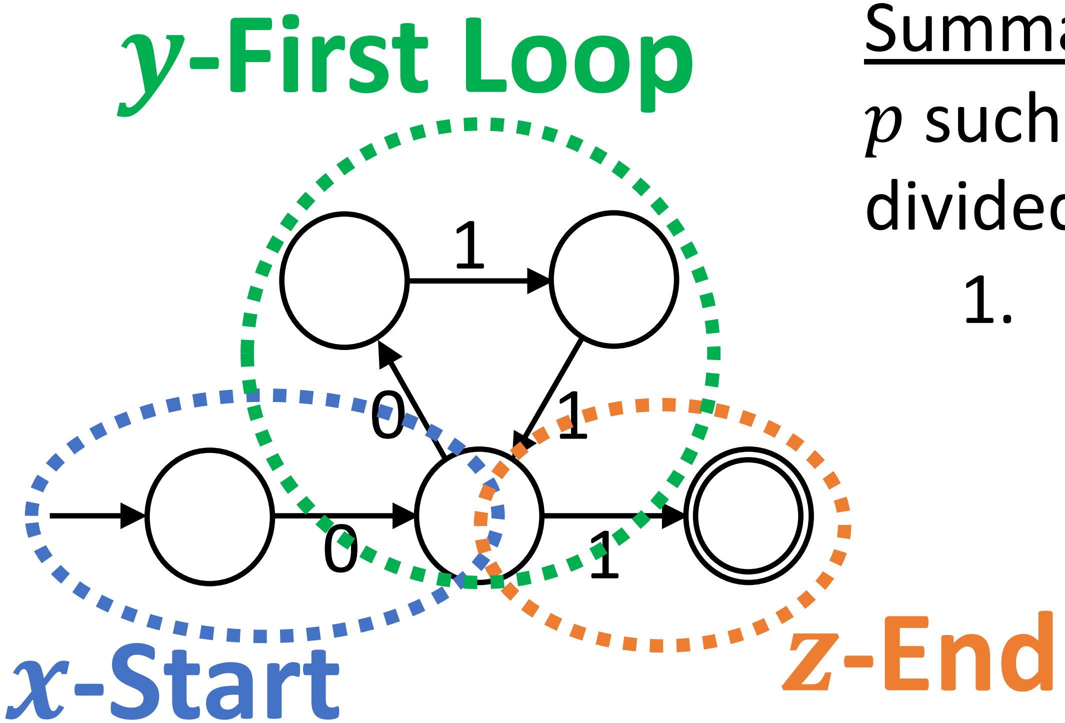


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1. $xy^iz \in L, \forall i \geq 0$.

From our previous argument.

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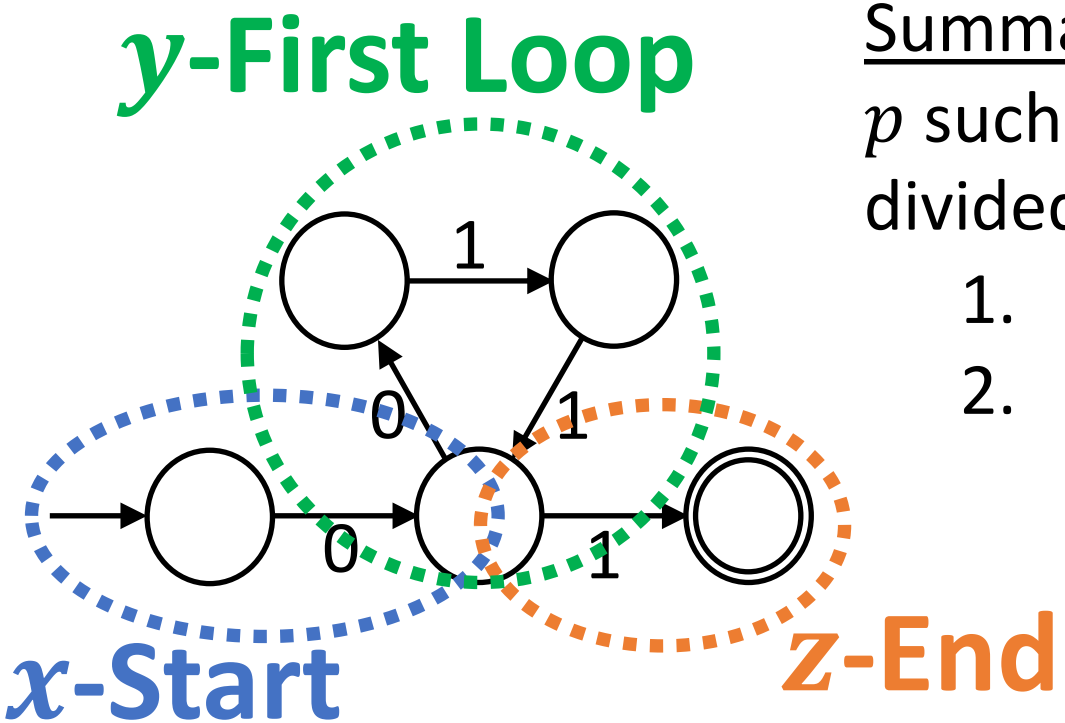


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1. $xy^iz \in L, \forall i \geq 0.$
2. $|y| > 0.$

Since $|s| \geq p$, we must have repeated states.

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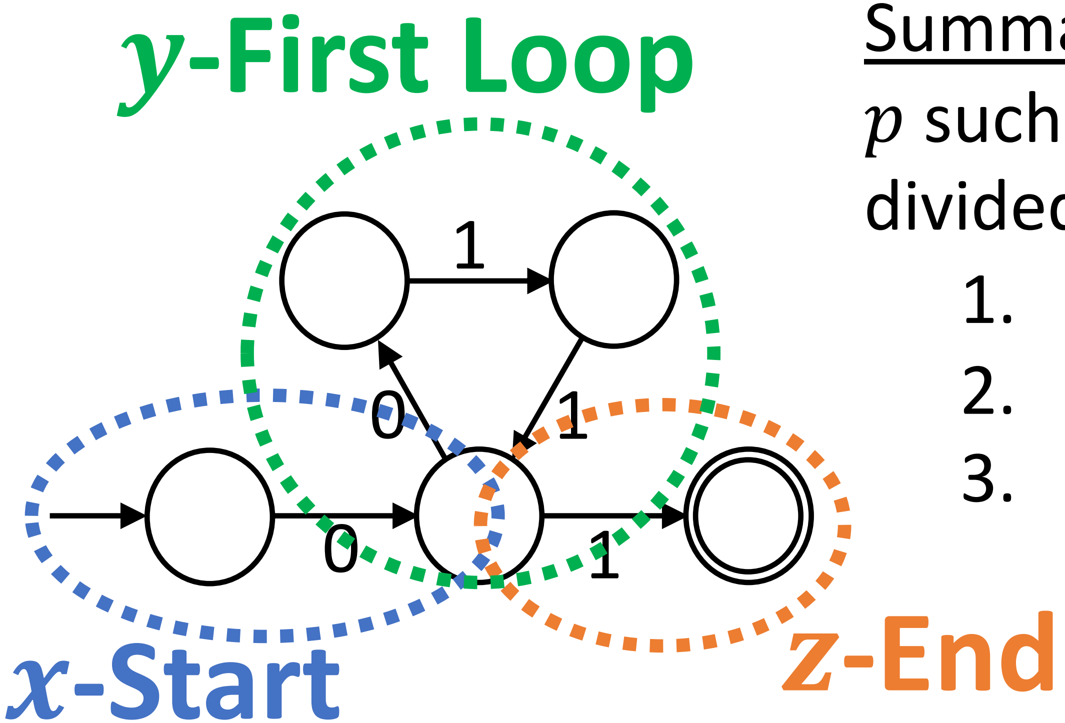


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Since there have to be repeated states within the first $p$ transitions.

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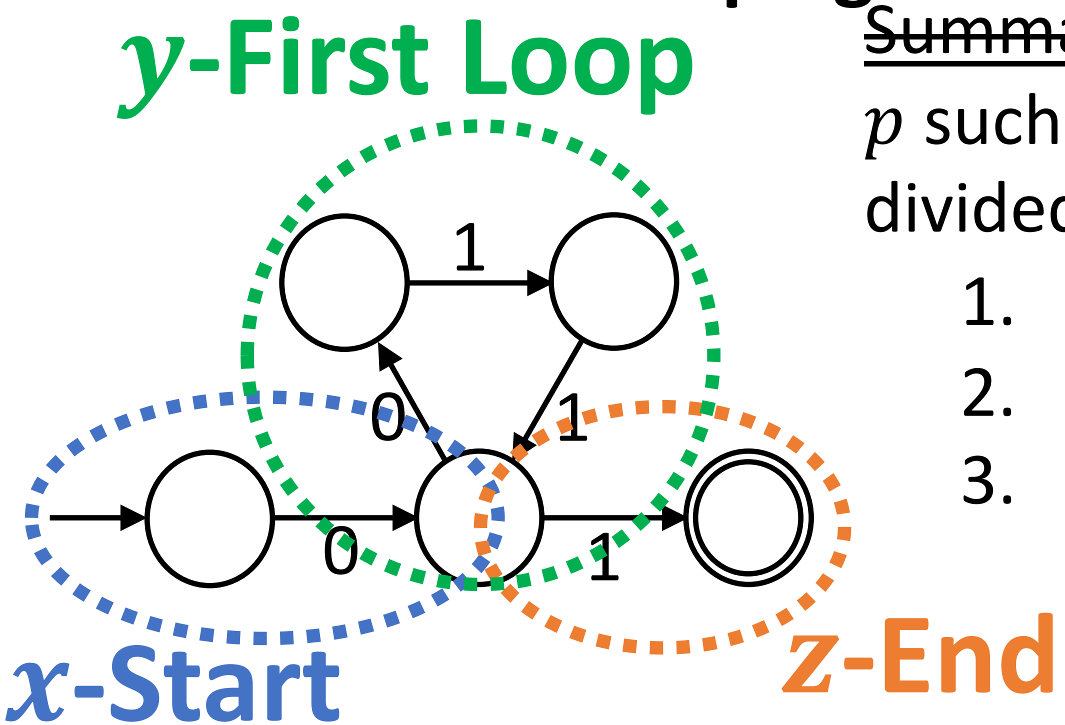
**Pumping Lemma**

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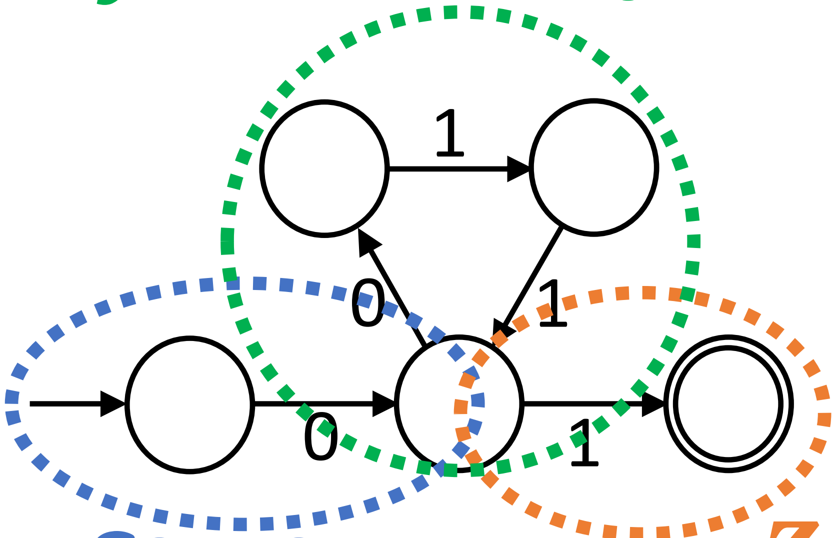

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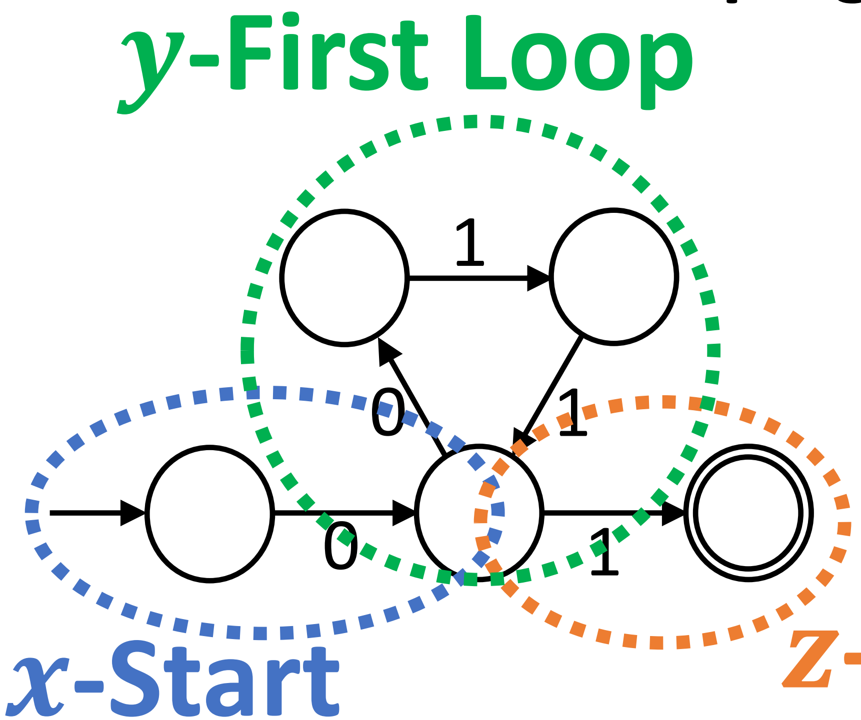
**The Pumping Lemma is our property that all regular languages must have. So, if some language does not have that property, it cannot be a regular language.**

**Pumping Lemma**

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Pumping Lemma User Manual:

1. The pumping lemma says all regular languages have property $\boldsymbol{P}$.
2. If we can show a language does not have property $\boldsymbol{P}$, then it cannot be regular.

# Non-Regularity Proofs

Pumping Lemma: **Given a regular language $L$**, $\exists$ a number $p$ such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:

1. $xy^i z \in L, \forall i \geq 0.$
2. $|y| > 0.$
3. $|xy| \leq p.$

1. Suppose language is regular.

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1. Suppose language is regular.
2. Select $p$ from pumping lemma.

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Pumping Lemma: Given a regular language $L$, $\exists$ a number $p$ **such that any string $\boldsymbol{s} \in \boldsymbol{L}$, with $|\boldsymbol{s}| \geq \boldsymbol{p}$**, can be divided into three pieces, $s = xyz$ satisfying:

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4. Determine what $y$ must consist of.

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2. $|y| > 0.$
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2. $|y| > 0$.
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1. Suppose language is regular.
2. Select $p$ from pumping lemma.
3. Carefully select $s \in L$ and $|s| \geq p$.
4. Determine what $y$ must consist of.
5. Make new string by selecting $i$.
6. Show new string is not in language.