Pumping Lemma
CSCI 338

Pumping Lemma Example 1

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Proof: Suppose L is regular.

Then, L must abide by the pumping lemma.

Pumping Lemma: Given a regular language L, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:

1. $xy^i z \in L, \forall i \geq 0$.
2. $|y| > 0$.
3. $|xy| \leq p$.

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<u>Proof</u>: Suppose L is regular.

Then, L must abide by the pumping lemma. I.e. There must be some number p such that any string $s \in L$, with $|s| \geq p$, …

<u>Pumping Lemma</u>: Given a regular language L, ∃ a number p **such that <u>any</u> string $s \in L$, with $|s| \geq p$,** can be divided into three pieces, $s = xyz$ satisfying:

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Proof: Suppose L is regular.

Then, $\boldsymbol{L}$ must abide by the pumping lemma. I.e. There must be some number $\boldsymbol{p}$ such that any string $\boldsymbol{s \in L}$, with $\boldsymbol{|s| \geq p}$, can be divided into three pieces, $\boldsymbol{s = xyz}$ satisfying the three conditions.

Pumping Lemma: Given a regular language L, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ **satisfying:**

1. $\boldsymbol{xy^iz \in L, \forall i \geq 0.}$
2. $\boldsymbol{|y| > 0.}$
3. $\boldsymbol{|xy| \leq p.}$

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Claim: The language $L = \{0^n 1^n : n \geq 0\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Then, L must abide by the pumping lemma. I.e. There must be some number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying the three conditions.

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Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Now, we need some appropriate string ($s \in L$ and $|s| \geq p$) that will break condition 1 when we allow multiple y's.

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Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 1^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

To break condition 1, we need to learn more about y

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<u>Proof</u>: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 1^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

$y = 0^k$ for some $k > 0$,
since ?

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$y = 0^k$ for some $k > 0$,
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$$s = \overbrace{000 \ldots 00}^{p}\overbrace{11 \ldots 111}^{p}$$

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$$x \quad y \qquad\qquad z$$

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$x \quad y \quad z$

$|xy| = p + 1 > p$

There is no possible way to partition s into xyz where both:

1. **$|xy| \leq p$**
2. **y has 1s in it.**

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$y = 0^k$ for some $k > 0$,
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Since $|xy| \leq p$, y must be in the first p characters of every string. Since the first p characters of this string are all 0, y must contain all 0s.

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Consider the string $s' = xy^2 z = 0^{p-k} 0^{2k} 1^p$

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$\Rightarrow s' \notin L$.

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Consider the string $s' = xy^2z = 0^{p-k} 0^{2k} 1^p$

But, the number of zeros $= p - k + 2k = p + k >$ number of ones $= p$.

$\Rightarrow s' \notin L$.

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Claim: The language $L = \{0^n 1^n : n \geq 0\}$ is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0$

Since $s \in L$ an $= xyz$.

$y = 0^k$ for so

$\Rightarrow s = 0^{p-k}0$

Consider the s

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But, the number of zeros $= p - k + 2k = p + k >$ number of ones $= p$.

$\Rightarrow s' \notin L$. **But the pumping lemma said this should work for every string in L!**

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

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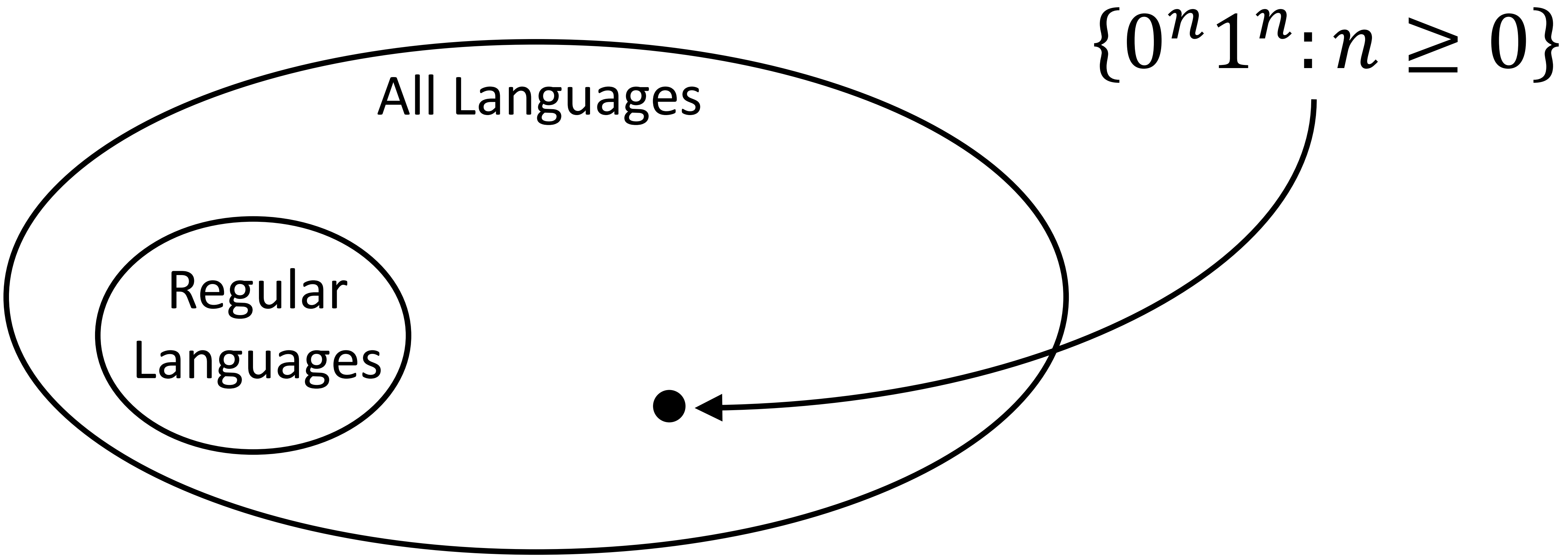
$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

DFA/NFA Limitations



Pumping Lemma Proof Blueprint

Claim: Some language L is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = \textbf{?}$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

?

Consider the string $s' = xy^{?}z = \textbf{?}$

?

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Proof Blueprint

<u>Claim</u>: Some language L is not regular.

<u>Proof</u>: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s =$ **?**. **1 – Select s that will work with $s \in L$ and $|s| \geq p$**

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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Consider the string $s' = xy^{?}z =$ **?**

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Proof Blueprint

Claim: Some language L is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s =$ **?**. **1 – Select s that will work with $\boldsymbol{s \in L}$ and $\boldsymbol{|s| \geq p}$**

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

? **2 – Find some conditions that $\boldsymbol{y}$ must meet**

Consider the string $s' = xy^{?}z =$ **?**

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

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<u>Claim</u>: Some language L is not regular.

<u>Proof</u>: Suppose L is regular. Let p be the number from the pumping lemma.

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Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

? **2 – Find some conditions that $\boldsymbol{y}$ must meet**

3 – Select an $\boldsymbol{i}$ (number of times to repeat $\boldsymbol{y}$)

Consider the string $s' = xy^{?}z =$ **?**

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

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Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

? **2 – Find some conditions that $\boldsymbol{y}$ must meet**

3 – Select an $\boldsymbol{i}$ (number of times to repeat $\boldsymbol{y}$)

Consider the string $s' = xy^{?}z =$ **?** **4 – Show what $\boldsymbol{s'}$ equals**

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Proof Blueprint

Claim: Some language L is not regular.

Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s =$?. **1 – Select s that will work with $s \in L$ and $|s| \geq p$**

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

? **2 – Find some conditions that y must meet**

3 – Select an i (number of times to repeat y)

Consider the string $s' = xy^{?}z =$? **4 – Show what s' equals**

? **5 – Show s' is not in L**

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.