Pumping Lemma

CSCI 338

Pumping Lemma Example 2

Claim: The language $L = \{0^i 1^j : i > j\}$ is not regular.

Proof: ?

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Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = ?$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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Consider the string $s' = xy^{?}z = ?$

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

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<u>Proof</u>: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^{p+1} 1^p$.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
$\Longrightarrow s = 0^{p-k+1} 0^k 1^p$

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> **Pumping Lemma**: Given a regular language L, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, can be divided into three pieces, $s = xyz$ satisfying:
>
> 1. $xy^i z \in L, \forall i \geq 0.$
> 2. $|y| > 0.$
> 3. $|xy| \leq p.$

Proof: Suppose L is reg

Consider $s = 0^{p+1}1^p$.

Since $s \in L$ and $|s| \geq$

$y = 0^k$ for some $k >$
$\Rightarrow s = 0^{p-k+1}0^k 1^p$

Consider the string $s' = xy^{?}z =$ **?**

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Consider the string $s' = xy^0 z = 0^{p-k+1} 1^p$

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Consider the string $s' = xy^0 z = 0^{p-k+1} 1^p$

But, $k > 0 \Longrightarrow p - k + 1 \leq p$. I.e. the number of 0s is not larger than the number of 1s.

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

Pumping Lemma Example 3

Claim: The language $L = \{w\colon w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular.

Proof:

E.g. 1101011 is a palindrome
1001 is a palindrome
10100 is not

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Consider the string $s' = xy^2z = 0^{p-k}0^{2k}0^p$

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Consider the string $s' = xy^2 z = 0^{p-k} 0^{2k} 0^p = 0^{2p+k}$

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Consider the string $s' = xy^3z =$

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Consider $s = 0^p 0^p$.

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$y = 0^k$ for some $k > 0$, since $|xy| \leq p$
$\Longrightarrow s = 0^{p-k} 0^k 0^p$

Consider the string $s' = xy^4z =$

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$y = 0^k$ for some $k > 0$, since $|xy| \leq p$

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

No matter what we set i to,
s' will always be a palindrome.

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Does this mean L is regular?

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Does this mean L is regular?

NO! It could be, but perhaps we just chose a poor string s.

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But, $k > 0 \Longrightarrow p + k \neq p$. I.e. More 0s before the 1 than after $\Rightarrow s'$ is not a palindrome

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