Pumping Lemma
CSCI 338

Pumping Lemma Example 4

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<u>Proof</u>:

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Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s =$ **?**.

Since $s \in L$ and $|s| \geq p$, the conditions of the pumping lemma must hold for $s = xyz$.

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Consider the string $s' = xy^{?}z =$ **?**

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$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

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Consider $s = 0^p 1^{p+1}$.

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Pumping Lemma: Given a regular language L, $\exists$ a number p such that any string $s \in L$, with $|s| \geq p$, **can be divided into three pieces, $s = xyz$** satisfying:

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Goal: Pick an s such that repeating y (no matter what y is) is guaranteed (at some point) to make #0's equal #1's

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Proof: Suppose L is regular. Let p be the number from the pumping lemma.

Consider $s = 0^p 1^{p+\alpha}$. $\boldsymbol{\alpha = ?}$

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$y = 0^k$ for some $k > 0$

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$y = 0^k$ for some $k > 0 \implies s = 0^{p-k} 0^k 1^{p+\alpha}$

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If #0's = #1's, then…

If we can find an $\boldsymbol{i}$ such that #0's = #1's, we have contradicted the pumping lemma.

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So, α needs to be evenly divisible by k for all possible $0 < k \leq p$. Let $\alpha =$?

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$$y = 0^k \text{ for some } k > 0 \Longrightarrow s = 0^{p-k} 0^k 1^{p+p!}$$

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Consider the string $s' = xy^{p!/_k+1} z = 0^{p-k} 0^{\left(p!/_k+1\right)k} 1^{p+p!}$

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$y = 0^k$ for some $k > 0 \Longrightarrow s = 0^{p-k} 0^k 1^{p+p!}$

Consider the string $s' = xy^{p!/_k+1} z = 0^{p-k} 0^{\left(p!/_k+1\right)k} 1^{p+p!}$

#0's $= p - k + p! + k = p + p! =$ #1's

$\Rightarrow s' \notin L$, which is a contradiction of the pumping lemma.

Therefore, the language is not regular.

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Regular Operations

Regular languages are closed under:

- Complement

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- Union

  $A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$
- Intersection

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- Star

  $A^* = \{x_1 x_2 \dots x_k : k \geq 0 \text{ and each } x_i \in A\}$

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If $\bar{L}$ was regular, so would $0^n 1^n$
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