Regular Language Wrapup

CSCI 338

Test 1 Logistics

1. During class on Friday 2/23.
2. You can bring your book and any notes you would like, but no electronic devices.
3. You may assume anything proven in class or on homeworks.
4. Four questions:
   1) Show a language is regular. (5 points)
   2) Show a language is not regular. (10 points)
   3) Show a language is regular or not regular. (5 points)
   4) Show a language is not regular. (2 points)

Consider the language $L = \{a0^n1^n : n \geq 1\} \cup \{a^k\omega : k \neq 1, \omega \in (0 \cup 1)^*\}$ over the alphabet $\Sigma = \{0,1,a\}$.

<u>Claim:</u> L satisfies the pumping lemma.

<u>Claim:</u> L is not regular.

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<u>Proof:</u> $\{a^k\omega : k \neq 1, \omega \in (0 \cup 1)^*\}$ is regular.

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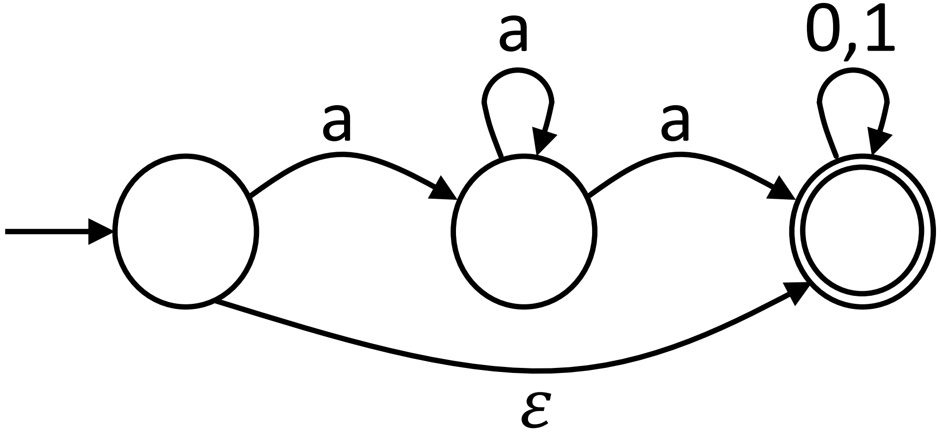

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Let $s = xyz$ where $x = \varepsilon$, $y = a$, and z is everything else.

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If y is pumped up or down, $s' \in \{a^k\omega : k \neq 1, \omega \in (0 \cup 1)^*\}$ (0 or > 1 a).

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<u>Proof:</u> $\{a^k\omega : k \neq 1, \omega \in (0 \cup 1)^*\}$ is regular.

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If y is pumped up or down, $s' \in \{a^k\omega : k \neq 1, \omega \in (0 \cup 1)^*\}$ (0 or > 1 a).
Thus, every string <u>can</u> be split into xyz that satisfy the pumping lemma conditions.

So L satisfies the pumping lemma.

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<u>Claim:</u> L is not regular.

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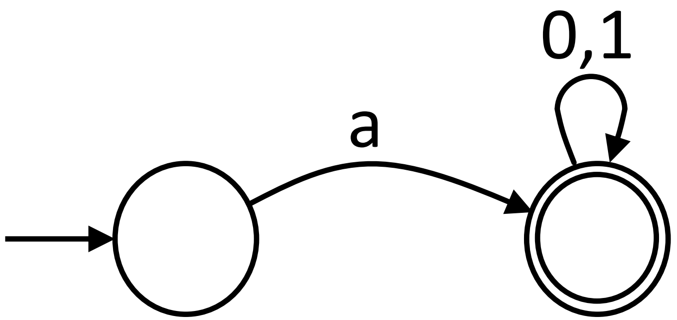
<u>Claim:</u> L is not regular.

<u>Proof:</u> $M = a(0 \cup 1)^*$ is regular.



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Proof: $M = a(0 \cup 1)^*$ is regular.

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Suppose $L \cap M$ is regular and let p be the number from the pumping lemma.

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Consider $s' = xy^0z$. Then, s' either:

- has no a (and $s' \notin L$)
- and/or now has more 1s than 0s (and $s' \notin L$).

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- has no a (and $s' \notin L$)
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This contradicts the pumping lemma, so $L \cap M$ is not regular.

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So, M is regular and $L \cap M$ is not regular. What about L?

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So, M is regular and $L \cap M$ is not regular. What about L?

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Thus, L cannot be regular.

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