Beyond Regular
CSCI 338
Computational Models

NFA

0
ε
0

1
0
Computational Models

NFA

0 \rightarrow 0
\varepsilon \rightarrow \varepsilon
0 \rightarrow \varepsilon

PDA

\varepsilon, \varepsilon \rightarrow \$
1,0 \rightarrow \varepsilon
\varepsilon, \$ \rightarrow \varepsilon
0, \varepsilon \rightarrow 0
1,0 \rightarrow \varepsilon

Stack:
Pushdown Automata (PDA)

Transition Notation:
- Input character, pop
- Stack push
- Stack pop
Pushdown Automata (PDA)

$\varepsilon, \varepsilon \rightarrow \$\n
$1,0 \rightarrow \varepsilon$

$\varepsilon, \$ \rightarrow \varepsilon$

$a, b \rightarrow c$ If ‘$a$’ is read from input and ‘$b$’ is on top of stack, pop ‘$b$’ and push ‘$c$’.

$\varepsilon, b \rightarrow c$ Without reading input, if ‘$b$’ is on top of stack, pop ‘$b$’ and push ‘$c$’.

$a, \varepsilon \rightarrow c$ If ‘$a$’ is read, push ‘$c$’.

$a, b \rightarrow \varepsilon$ If ‘$a$’ is read and ‘$b$’ is on top of stack, pop ‘$b$’.

Stack: $s = 0011$
Pushdown Automata (PDA)

$\varepsilon, \varepsilon \rightarrow \$ \quad 1,0 \rightarrow \varepsilon \quad \varepsilon, \$ \rightarrow \varepsilon

$a, b \rightarrow c$ If ‘$a$’ is read from input and ‘$b$’ is on top of stack, pop ‘$b$’ and push ‘$c$’.

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$a, b \rightarrow \varepsilon$ If ‘$a$’ is read and ‘$b$’ is on top of stack, pop ‘$b$’.

Stack: $s = 0011$
Pushdown Automata (PDA)

If 'a' is read from input and 'b' is on top of stack, pop 'b' and push 'c'.

Without reading input, if 'b' is on top of stack, pop 'b' and push 'c'.

If 'a' is read, push 'c'.

If 'a' is read and 'b' is on top of stack, pop 'b'.

Stack: $s = 0011$
Pushdown Automata (PDA)

If ‘a’ is read from input and ‘b’ is on top of stack, pop ‘b’ and push ‘c’.

If ‘a’ is read, push ‘c’.

If ‘a’ is read and ‘b’ is on top of stack, pop ‘b’.

Stack: "s = 0011"
Pushdown Automata (PDA)

- $\epsilon, \epsilon \rightarrow \$ 
- $1,0 \rightarrow \epsilon 
- \epsilon, \$ \rightarrow \epsilon 
- 0, \epsilon \rightarrow 0 
- 1,0 \rightarrow \epsilon 
- a, b \rightarrow c$ If ‘a’ is read from input and ‘b’ is on top of stack, pop ‘b’ and push ‘c’.
- $\epsilon, b \rightarrow c$ Without reading input, if ‘b’ is on top of stack, pop ‘b’ and push ‘c’.
- $a, \epsilon \rightarrow c$ If ‘a’ is read, push ‘c’.
- $a, b \rightarrow \epsilon$ If ‘a’ is read and ‘b’ is on top of stack, pop ‘b’.

Stack: $s = 0011$
Pushdown Automata (PDA)

- $\varepsilon, \varepsilon \rightarrow \$$
- $1,0 \rightarrow \varepsilon$
- $\varepsilon, \$$ \rightarrow \varepsilon$

- $a, b \rightarrow c$ If ‘a’ is read from input and ‘b’ is on top of stack, pop ‘b’ and push ‘c’.
- $\varepsilon, b \rightarrow c$ Without reading input, if ‘b’ is on top of stack, pop ‘b’ and push ‘c’.
- $a, \varepsilon \rightarrow c$ If ‘a’ is read, push ‘c’.
- $a, b \rightarrow \varepsilon$ If ‘a’ is read and ‘b’ is on top of stack, pop ‘b’.

Stack: $s = 0011$
Pushdown Automata (PDA)

- $\varepsilon, \varepsilon \rightarrow \$\$
- $1,0 \rightarrow \varepsilon$
- $\varepsilon, \$ \rightarrow \varepsilon$
- $0, \varepsilon \rightarrow 0$
- $1,0 \rightarrow \varepsilon$

$a, b \rightarrow c$ If ‘$a$’ is read from input and ‘$b$’ is on top of stack, pop ‘$b$’ and push ‘$c$’.

$\varepsilon, b \rightarrow c$ Without reading input, if ‘$b$’ is on top of stack, pop ‘$b$’ and push ‘$c$’.

$a, \varepsilon \rightarrow c$ If ‘$a$’ is read, push ‘$c$’.

$a, b \rightarrow \varepsilon$ If ‘$a$’ is read and ‘$b$’ is on top of stack, pop ‘$b$’.

Stack: $s = 0011$
Pushdown Automata (PDA)

- $\varepsilon, \varepsilon \rightarrow \$\n- $1,0 \rightarrow \varepsilon$
- $\varepsilon, \$ \rightarrow \varepsilon$
- $0, \varepsilon \rightarrow 0$
- $1,0 \rightarrow \varepsilon$
- $a, b \rightarrow c$
  - If ‘$a$’ is read from input and ‘$b$’ is on top of stack, pop ‘$b$’ and push ‘$c$’.
- $\varepsilon, b \rightarrow c$
  - Without reading input, if ‘$b$’ is on top of stack, pop ‘$b$’ and push ‘$c$’.
- $a, \varepsilon \rightarrow c$
  - If ‘$a$’ is read, push ‘$c$’.
- $a, b \rightarrow \varepsilon$
  - If ‘$a$’ is read and ‘$b$’ is on top of stack, pop ‘$b$’.

Stack: $s = 0011$
Pushdown Automata (PDA)

- $\varepsilon, \varepsilon \rightarrow \$\$
- $1, 0 \rightarrow \varepsilon$
- $\varepsilon, \$ \rightarrow \varepsilon$

- If 'a' is read from input and 'b' is on top of stack, pop 'b' and push 'c'.
- Without reading input, if 'b' is on top of stack, pop 'b' and push 'c'.
- If 'a' is read, push 'c'.
- If 'a' is read and 'b' is on top of stack, pop 'b'.

Stack: $0011$

$\varepsilon, \varepsilon \rightarrow \$
$1, 0 \rightarrow \varepsilon$
$\varepsilon, \$ \rightarrow \varepsilon$

$0, \varepsilon \rightarrow 0$
$1, 0 \rightarrow \varepsilon$

$\varepsilon, \varepsilon \rightarrow \$
$1, 0 \rightarrow \varepsilon$
$\varepsilon, \$ \rightarrow \varepsilon$

$s = 0011$
Pushdown Automata (PDA)

If ‘a’ is read from input and ‘b’ is on top of stack, pop ‘b’ and push ‘c’.

Without reading input, if ‘b’ is on top of stack, pop ‘b’ and push ‘c’.

If ‘a’ is read, push ‘c’.

If ‘a’ is read and ‘b’ is on top of stack, pop ‘b’.

Stack:  
\[
\begin{array}{c}
0 \\
$ \\
\end{array}
\]

\[s = 0011\]
Pushdown Automata (PDA)

- $\varepsilon, \varepsilon \rightarrow \$\n- $1,0 \rightarrow \varepsilon$
- $\varepsilon, \$ \rightarrow \varepsilon$
- $0, \varepsilon \rightarrow 0$
- $1,0 \rightarrow \varepsilon$
- $a, b \rightarrow c$ If ‘$a$’ is read from input and ‘$b$’ is on top of stack, pop ‘$b$’ and push ‘$c$’.
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- $a, \varepsilon \rightarrow c$ If ‘$a$’ is read, push ‘$c$’.
- $a, b \rightarrow \varepsilon$ If ‘$a$’ is read and ‘$b$’ is on top of stack, pop ‘$b$’.

Stack: $s = 0011$
Pushdown Automata (PDA)

- $(\epsilon, \epsilon \rightarrow \$)$
- $(1,0 \rightarrow \epsilon)$
- $(\epsilon, \$ \rightarrow \epsilon)$
- $(0, \epsilon \rightarrow 0)$
- $(1,0 \rightarrow \epsilon)$

**Rules:**

- $a, b \rightarrow c$  If ‘$a$’ is read from input and ‘$b$’ is on top of stack, pop ‘$b$’ and push ‘$c$’.
- $\epsilon, b \rightarrow c$  Without reading input, if ‘$b$’ is on top of stack, pop ‘$b$’ and push ‘$c$’.
- $a, \epsilon \rightarrow c$  If ‘$a$’ is read, push ‘$c$’.
- $a, b \rightarrow \epsilon$  If ‘$a$’ is read and ‘$b$’ is on top of stack, pop ‘$b$’.

Stack: $s = 0011$
Pushdown Automata (PDA)

\[
\begin{align*}
\epsilon, \epsilon & \rightarrow \$ \\
1, 0 & \rightarrow \epsilon \\
\epsilon, \$ & \rightarrow \epsilon \\
0, \epsilon & \rightarrow 0 \\
1, 0 & \rightarrow \epsilon \\
\end{align*}
\]

\[a, b \rightarrow c\]  If ‘a’ is read from input and ‘b’ is on top of stack, pop ‘b’ and push ‘c’.

\[\epsilon, b \rightarrow c\]  Without reading input, if ‘b’ is on top of stack, pop ‘b’ and push ‘c’.

\[a, \epsilon \rightarrow c\]  If ‘a’ is read, push ‘c’.

\[a, b \rightarrow \epsilon\]  If ‘a’ is read and ‘b’ is on top of stack, pop ‘b’.

Stack: \[s = 0011\]
Pushdown Automata (PDA)

If ‘a’ is read and ‘b’ is on top of stack, pop ‘b’ and push ‘c’.

Without reading input, if ‘b’ is on top of stack, pop ‘b’ and push ‘c’.

If ‘a’ is read, push ‘c’.

If ‘a’ is read and ‘b’ is on top of stack, pop ‘b’.

Stack: $\text{s} = 0011$
If ‘a’ is read from input and ‘b’ is on top of stack, pop ‘b’ and push ‘c’.

Without reading input, if ‘b’ is on top of stack, pop ‘b’ and push ‘c’.

If ‘a’ is read, push ‘c’.

If ‘a’ is read and ‘b’ is on top of stack, pop ‘b’.

Stack:

$s = 0011$
Pushdown Automata (PDA)

\[
\begin{align*}
\varepsilon, \varepsilon &\rightarrow \$ \\
1,0 &\rightarrow \varepsilon \\
\varepsilon, \$ &\rightarrow \varepsilon \\
0, \varepsilon &\rightarrow 0 \\
1,0 &\rightarrow \varepsilon \\
a, b &\rightarrow c &\text{If ‘a’ is read from input and ‘b’ is on top of stack, pop ‘b’ and push ‘c’.} \\
\varepsilon, b &\rightarrow c &\text{Without reading input, if ‘b’ is on top of stack, pop ‘b’ and push ‘c’.} \\
a, \varepsilon &\rightarrow c &\text{If ‘a’ is read, push ‘c’.} \\
a, b &\rightarrow \varepsilon &\text{If ‘a’ is read and ‘b’ is on top of stack, pop ‘b’.} \\
\end{align*}
\]

Stack: \[s = 0011\]