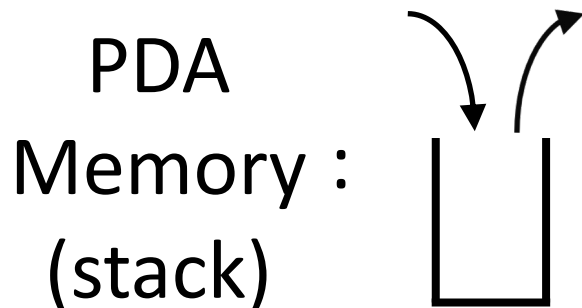


# Turing Machines

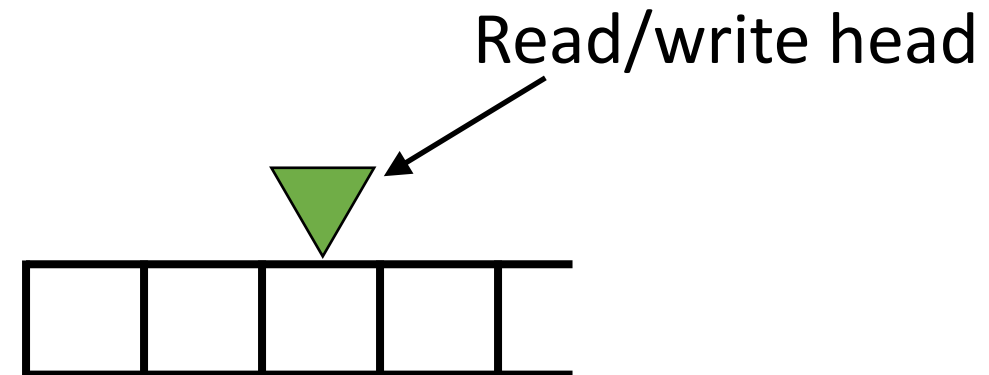
## CSCI 338

# Turing Machine

- Finite automaton with unrestricted memory (tape).
- Can read from and write to memory.
- Can access any spot in memory.
- Infinite memory.
- Start configuration: start state, input on tape, r/w head far left.
- $\exists$  states other than accept and reject.
- Accept and reject take effect immediately.



Turing  
Machine  
Memory :  
(tape)



# Turing Machine Formal Definition

TMs consist of:

1. Finite set of states,  $Q$ .
2. Finite input alphabet,  $\Sigma$  (does not blank symbol  $\sqcup$ ).
3. Finite tape alphabet,  $\Gamma$  (includes  $\sqcup$ ,  $\Sigma \subset \Gamma$ ).
4. Transition function,  $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ .
5. Start state,  $q_0 \in Q$ .
6. Accept state,  $q_{\text{accept}} \in Q$ .
7. Reject state,  $q_{\text{reject}} \in Q$ , where  $q_{\text{accept}} \neq q_{\text{reject}}$ .

# Turing Machine Example

How would you use a TM's tape to see if a string is in the language

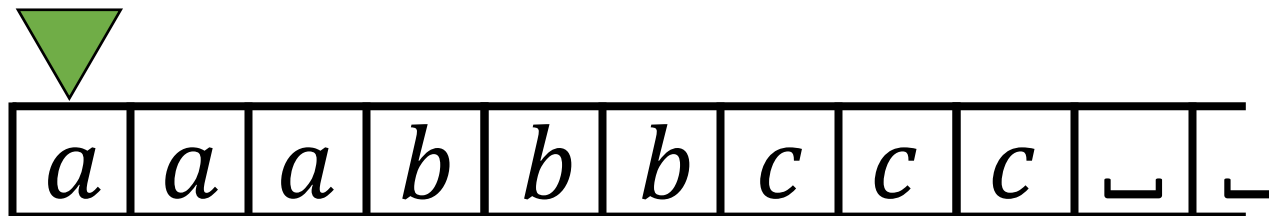
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Memory initial state: input on tape, read/write head at start.



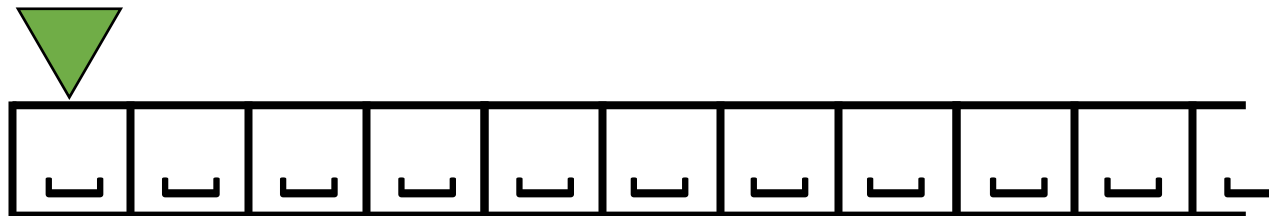
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TM M: on input  $\omega$

1. If  $\omega = \varepsilon$ , ?



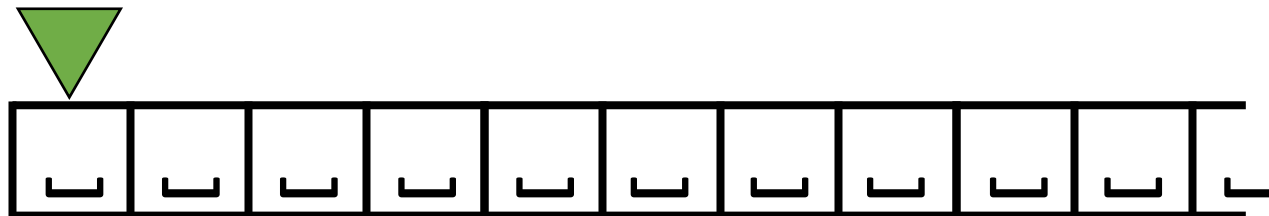
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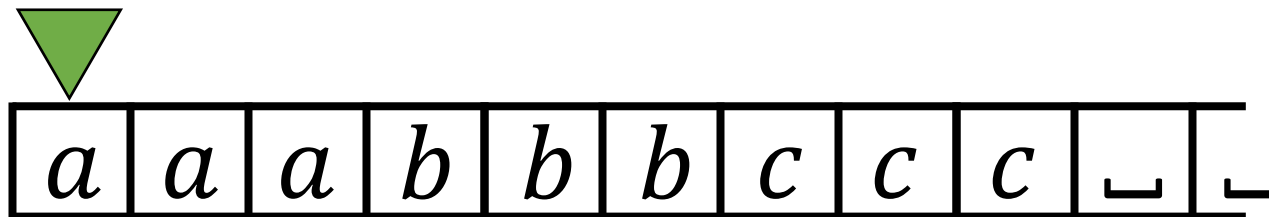
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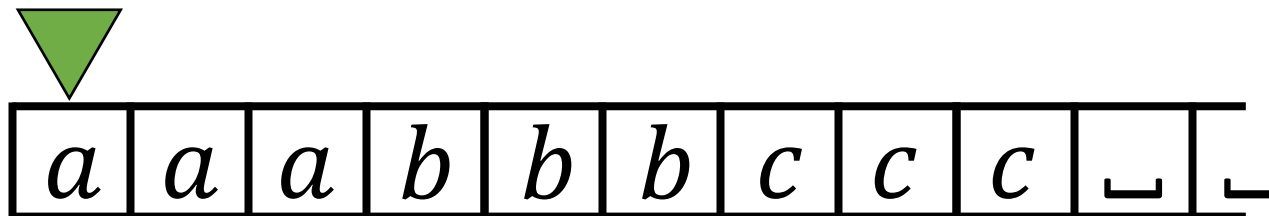
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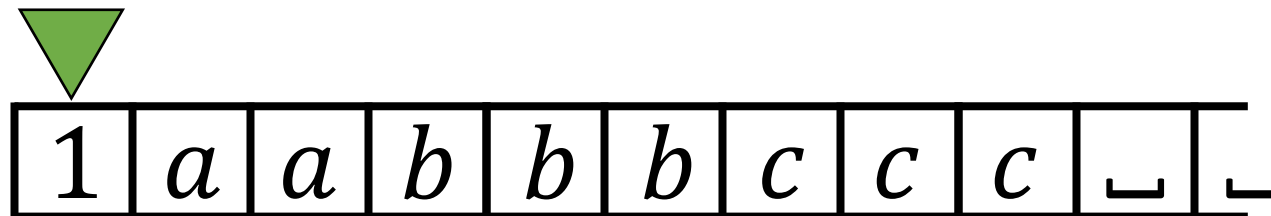
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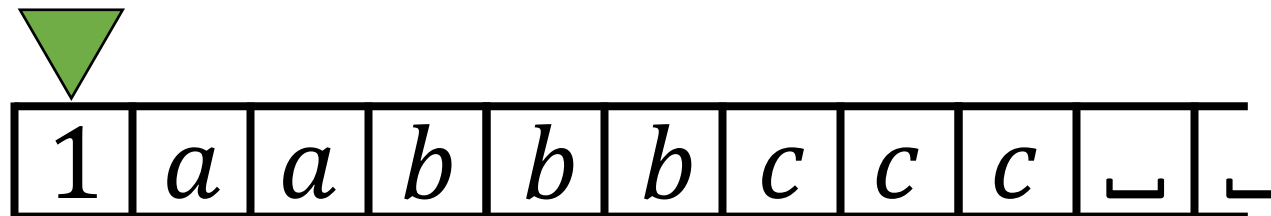
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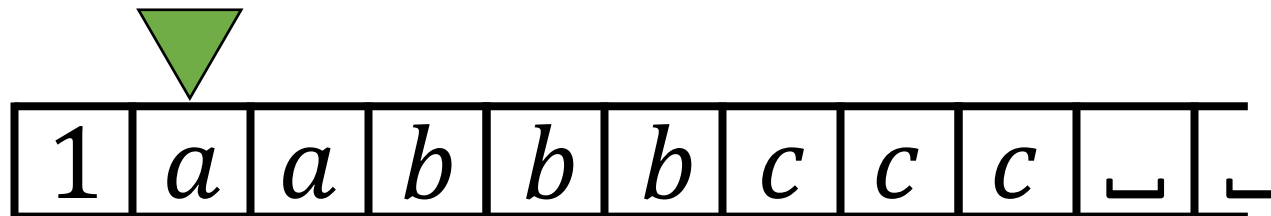
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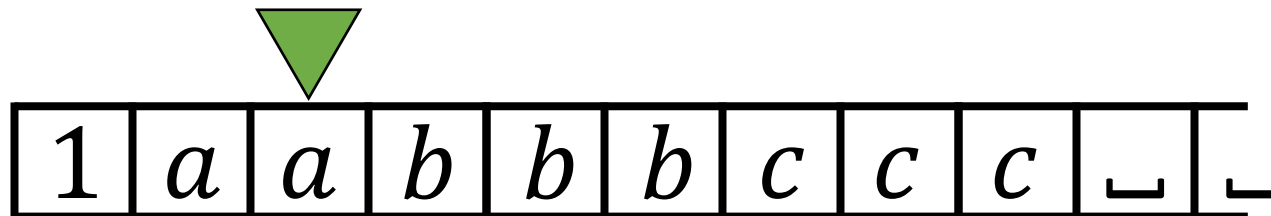
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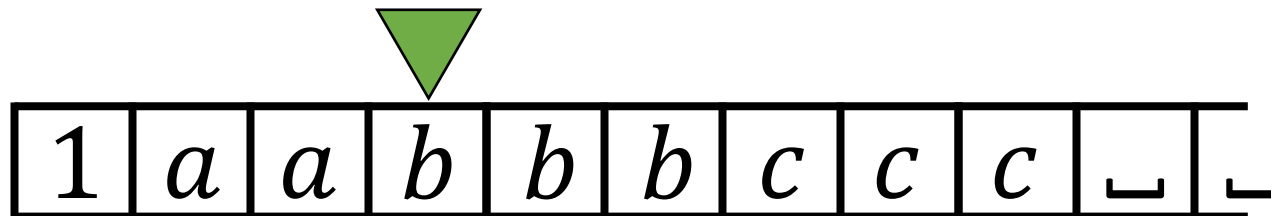
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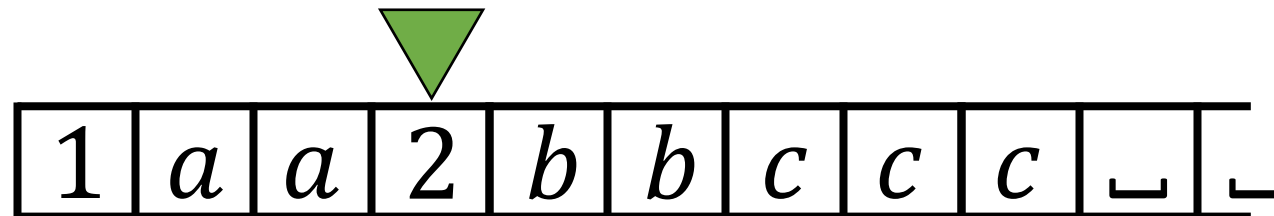
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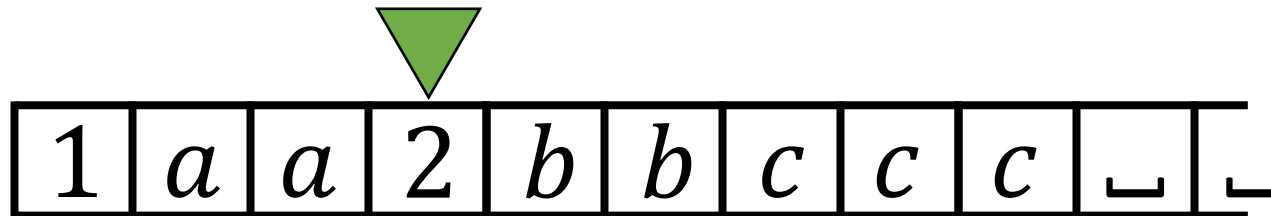
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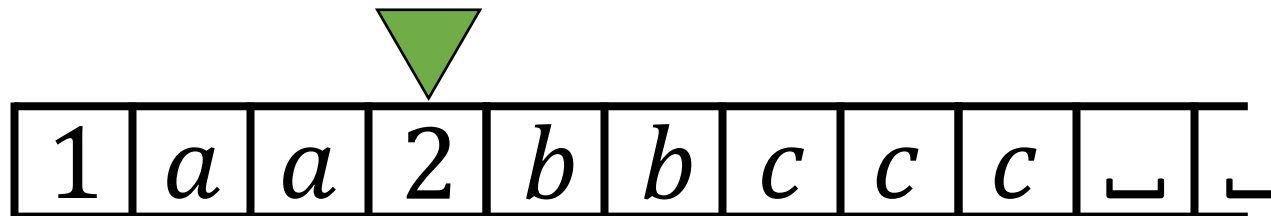
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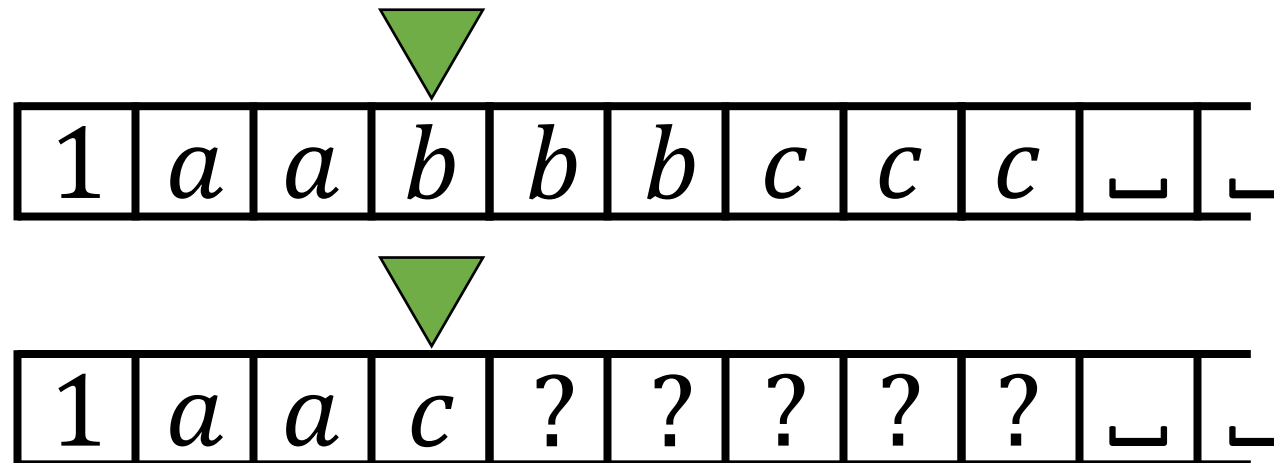
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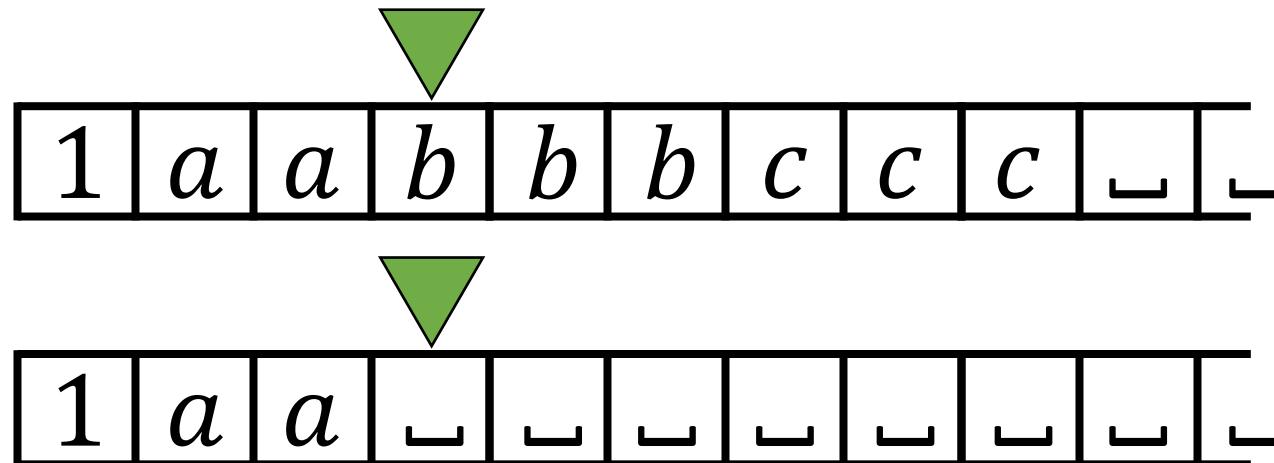
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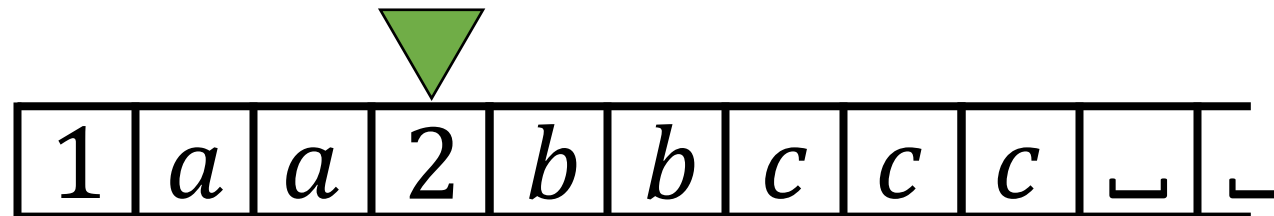
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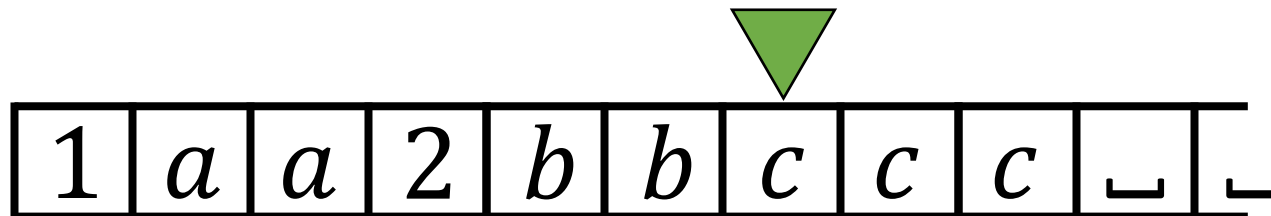
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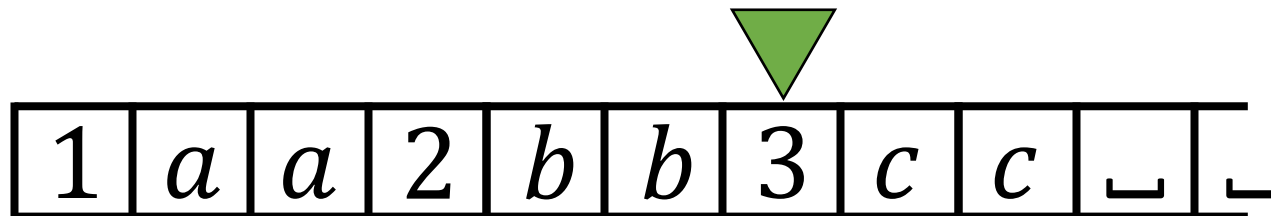
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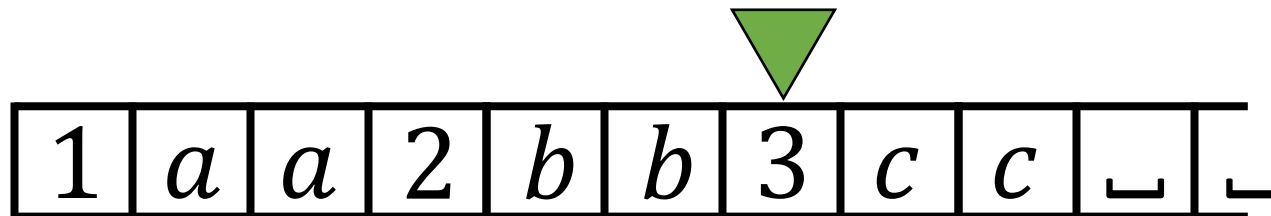
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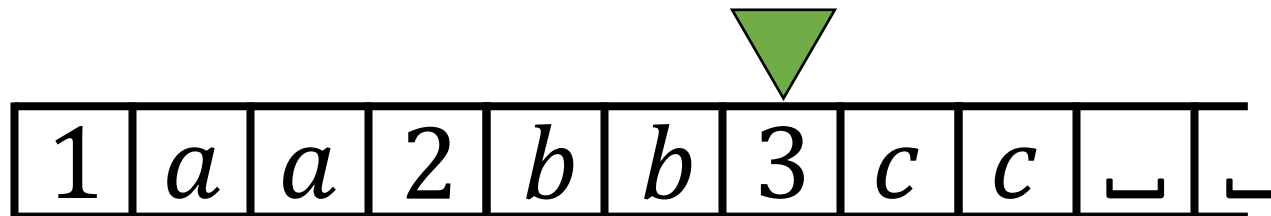
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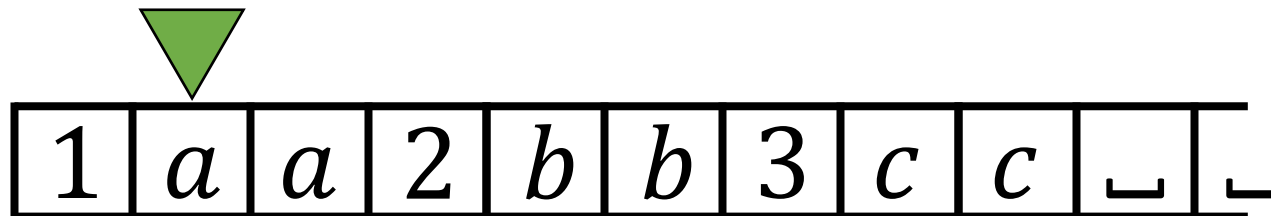
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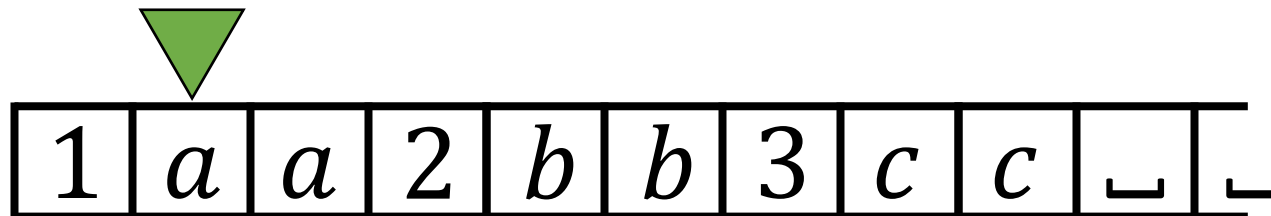
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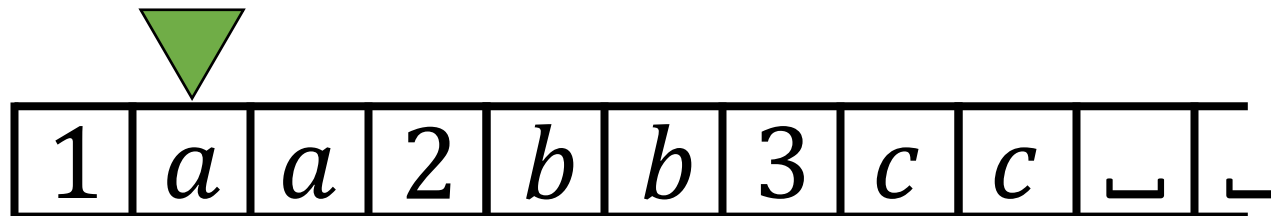
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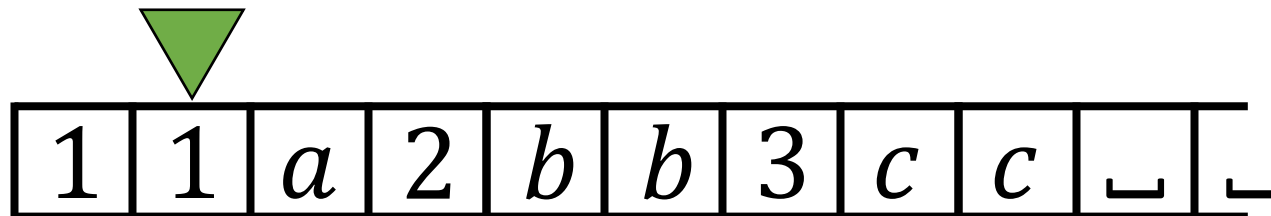
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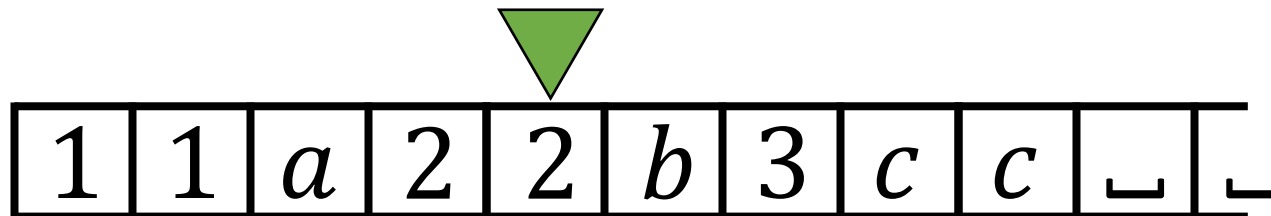
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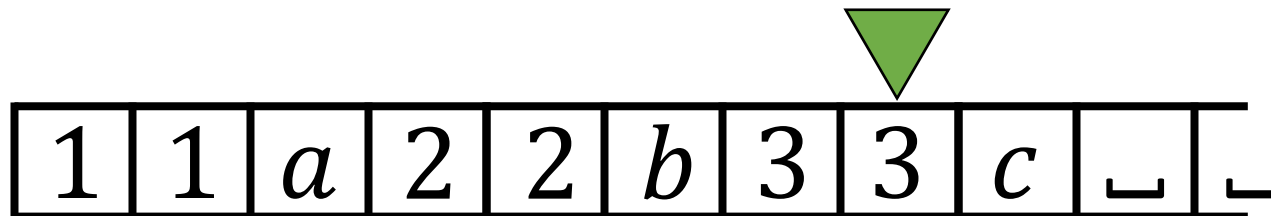
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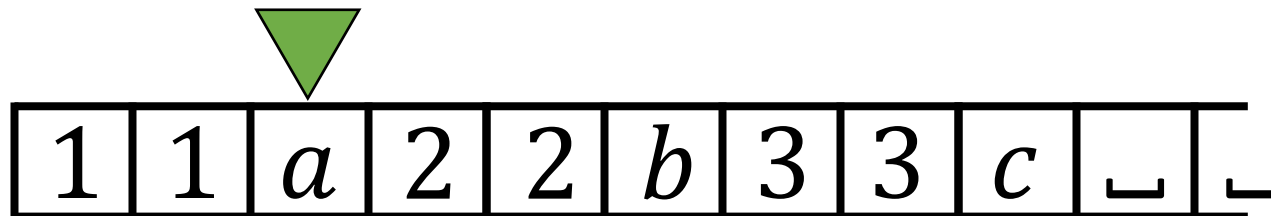
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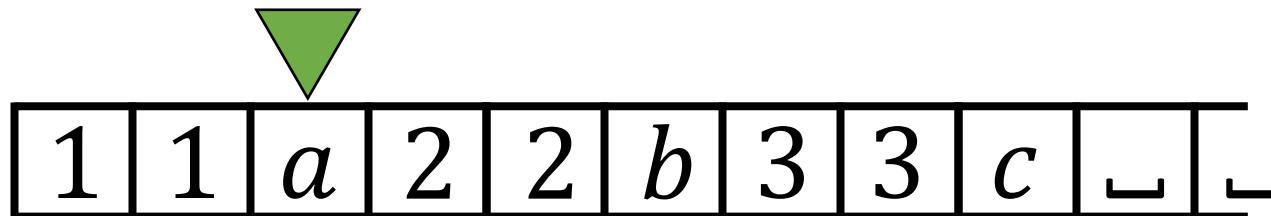
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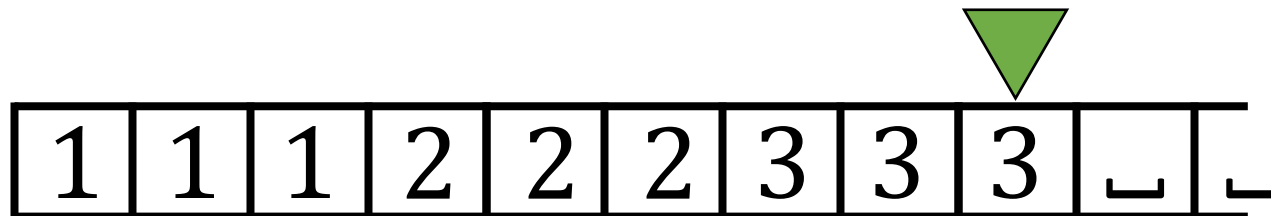
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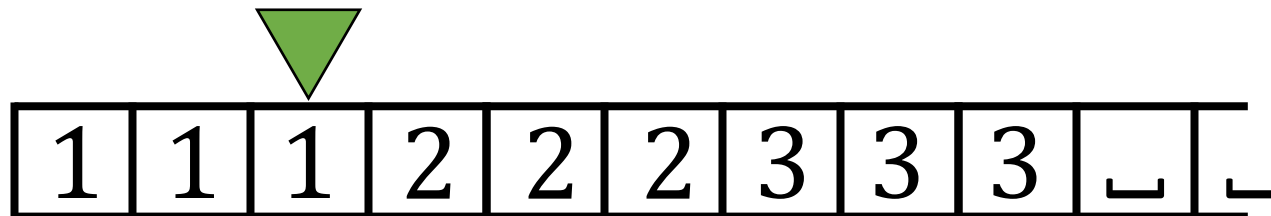
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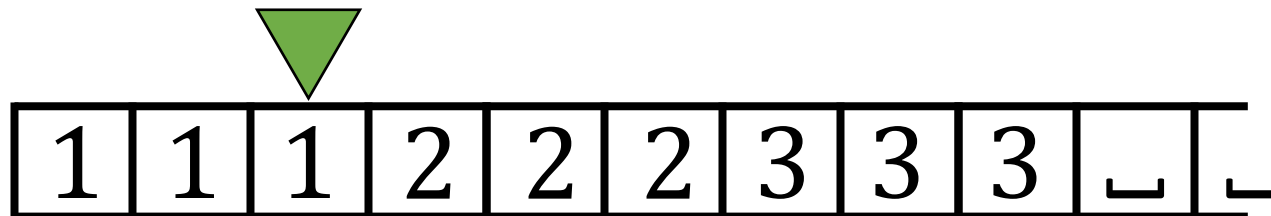
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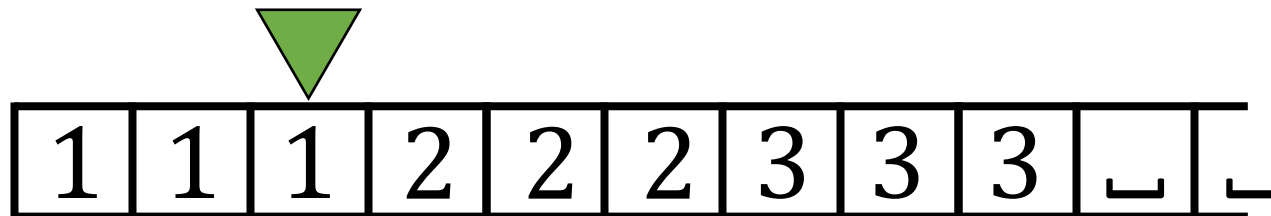
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5. ?



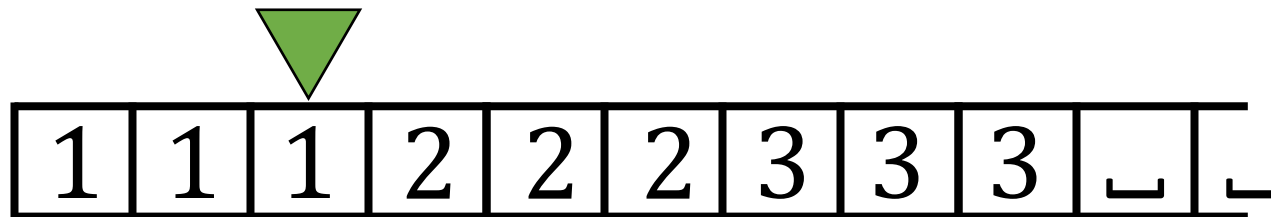
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5. Move right to verify no  $b$  or  $c$  exist. If so, reject. If not, accept.



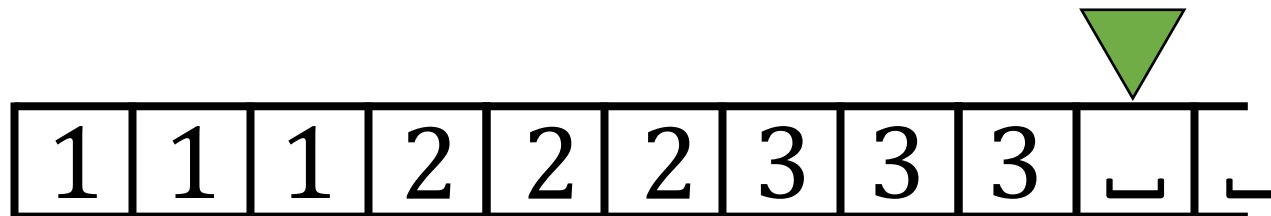
# Turing Machine Example

How would you use a TM's tape to see if a string is in the language

$$L = \{a^n b^n c^n : n \geq 0\}?$$

TM M: on input  $\omega$

1. If  $\omega = \varepsilon$ , accept. Otherwise, change first  $a$  to a 1. (reject if anything else found.)
2. Move right to first  $b$  and change to a 2. Reject if  $c$  or  $\_$  found first.
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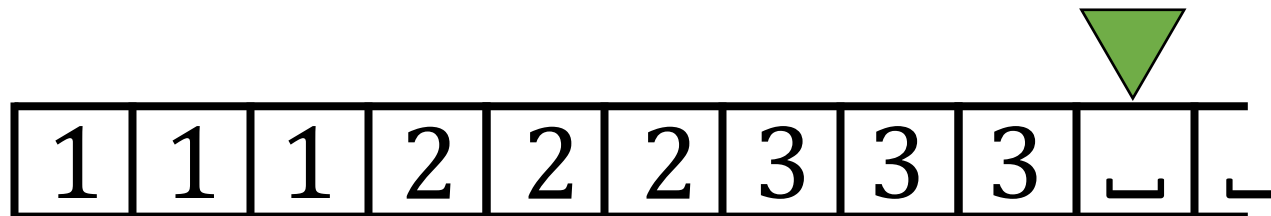
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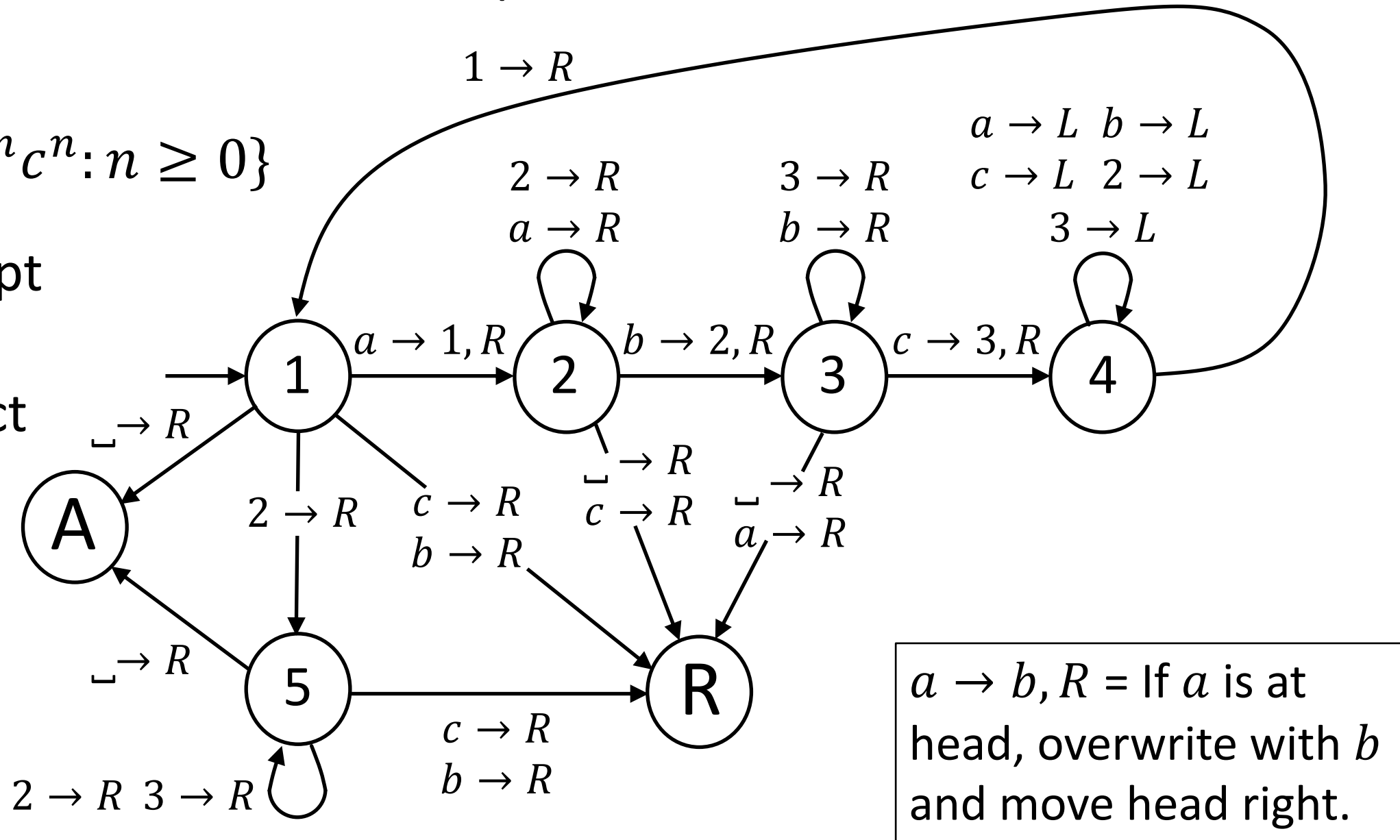


# Turing Machine Example

$$L = \{a^n b^n c^n : n \geq 0\}$$

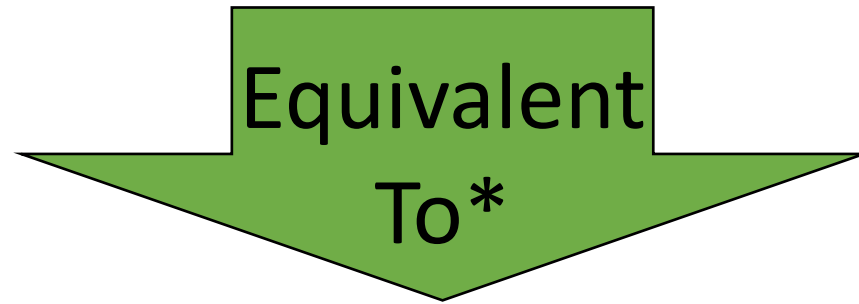
**A** = Accept

**R** = Reject





# Turing Machines – Equivalent Models

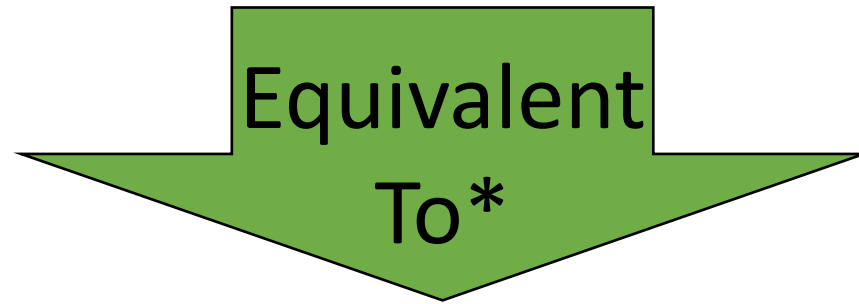


Turing Machines

\* I.e. Can solve the same problems.

# Turing Machines – Equivalent Models

Multi-tape TM



Turing Machines

\* I.e. Can solve the same problems.

# Turing Machines – Equivalent Models

Multi-tape TM

Non-deterministic TM



Turing Machines

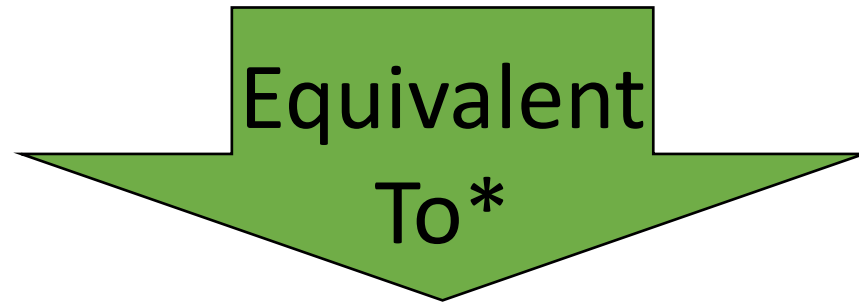
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# Turing Machines – Equivalent Models

Multi-tape TM

Non-deterministic TM

Quantum TM



Turing Machines

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# Turing Machines – Equivalent Models

Multi-tape TM

$\lambda$ -Calculus

Non-deterministic TM

General Recursive Functions

Quantum TM



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Turing Machines

# Turing Machines – Equivalent Models

Multi-tape TM

$\lambda$ -Calculus

Non-deterministic TM

General Recursive Functions

Quantum TM



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## Turing Machines

Actually, ALL computational models that allow unrestricted access to unlimited memory are equivalent to TMs (with basic assumptions) !

# Church-Turing Thesis

Intuitive notion  
of algorithms. = Turing Machine  
algorithms.

# Church-Turing Thesis

Intuitive notion  
of algorithms.

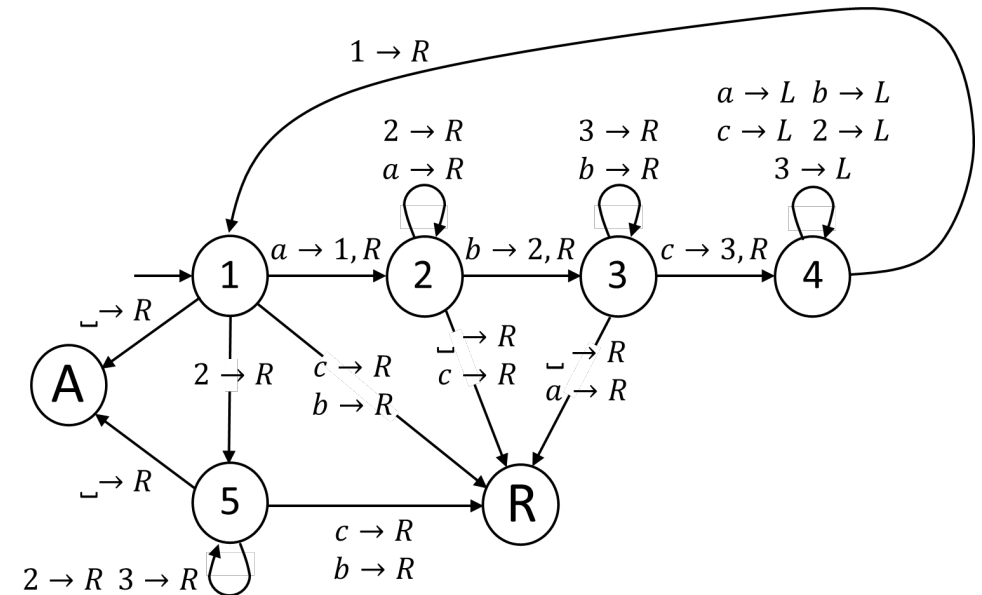
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=





# Church-Turing Thesis

Intuitive notion  
of algorithms.  $=$  Turing Machine  
algorithms.

TM M: on input  $\omega$

1. If  $\omega =$
2. Move r
3. Move r
4. Move b
5. Move r

**Takeaway: A description of the operation of a Turing Machine (i.e. an algorithm) is sufficient in place of a formal Turing Machine definition.**

