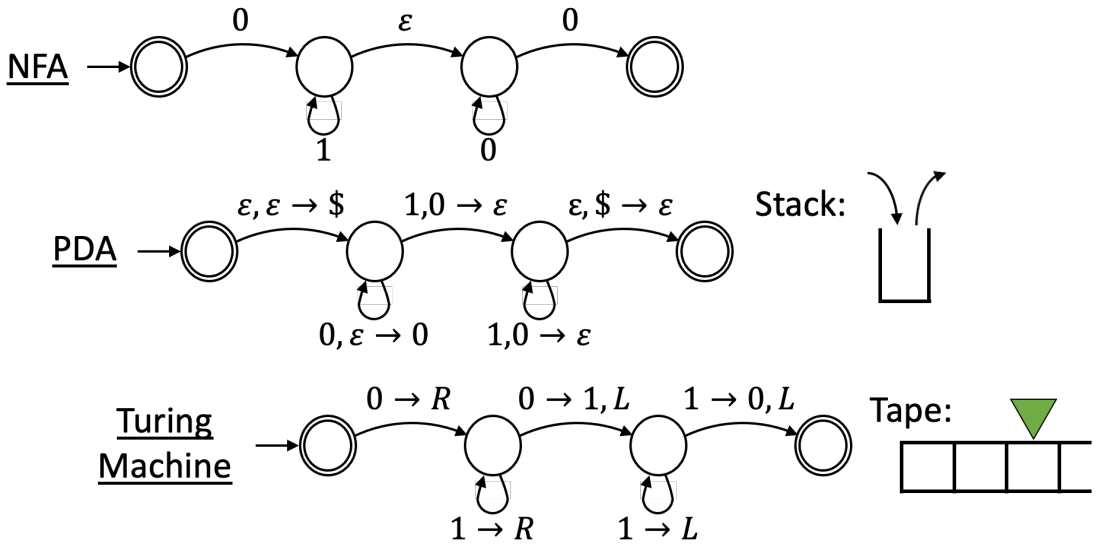
Decidability
CSCI 338

Computability: What's solvable by computers.

January → **May**

Computational Models



Complexity: What's <u>efficiently</u> solvable by computers.

<u>Goal:</u> Understand and identify fundamental limitations of computers.

Church-Turing Thesis

Intuitive notion of algorithms. = Turing Machine algorithms.

TM M: on input ω

1. If $\omega = \varepsilon$, accept. Otherwise, change first a to a 1.
2. Move right to first b and change to a 2. Reject if c or ␣ found first.
3. Move right to first c and change to a 3. Reject if a or ␣ found first.
4. Move back to first a. If it exists, loop to step 1. If not, exit loop.
5. Move right to verify no b or c exist. If so, reject. If not, accept.

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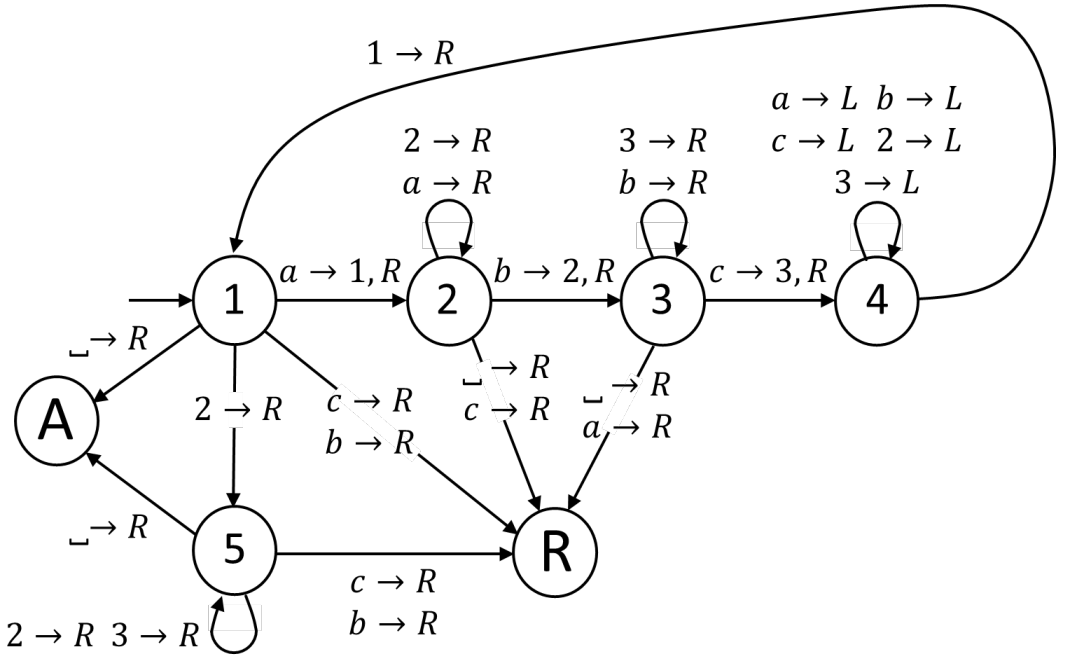

Definitions

A language is Turing recognizable if there is a TM that accepts every string in the language, and nothing not in the language.

Called a decider.

A language L is decidable if there is a TM that recognizes L and rejects everything else. (i.e. halts on all input)

Definitions

Language $L = \{w: |w| \text{ is even}\}$

Decider:

```
if (s.length() % 2 == 0) {
    return true;
} else {
    return false;
}
```

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Definitions

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Recognizer:

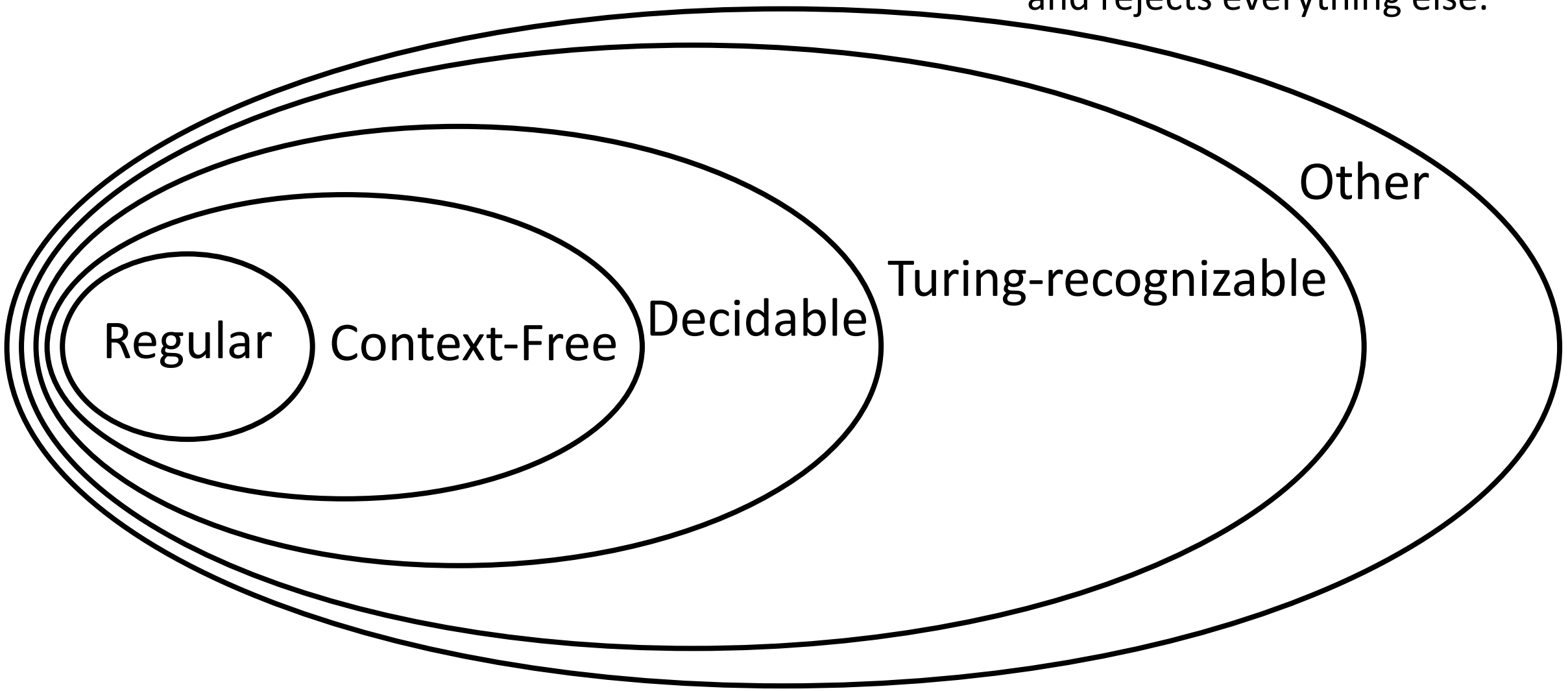
```
if (s.length() % 2 == 0) {
   return true;
} else {
   while (true) {

   }
}
```

Computability Hierarchy

Recognizable: ∃ TM that accepts everything in L, and nothing not.

Decidable: ∃ TM that recognizes L and rejects everything else.



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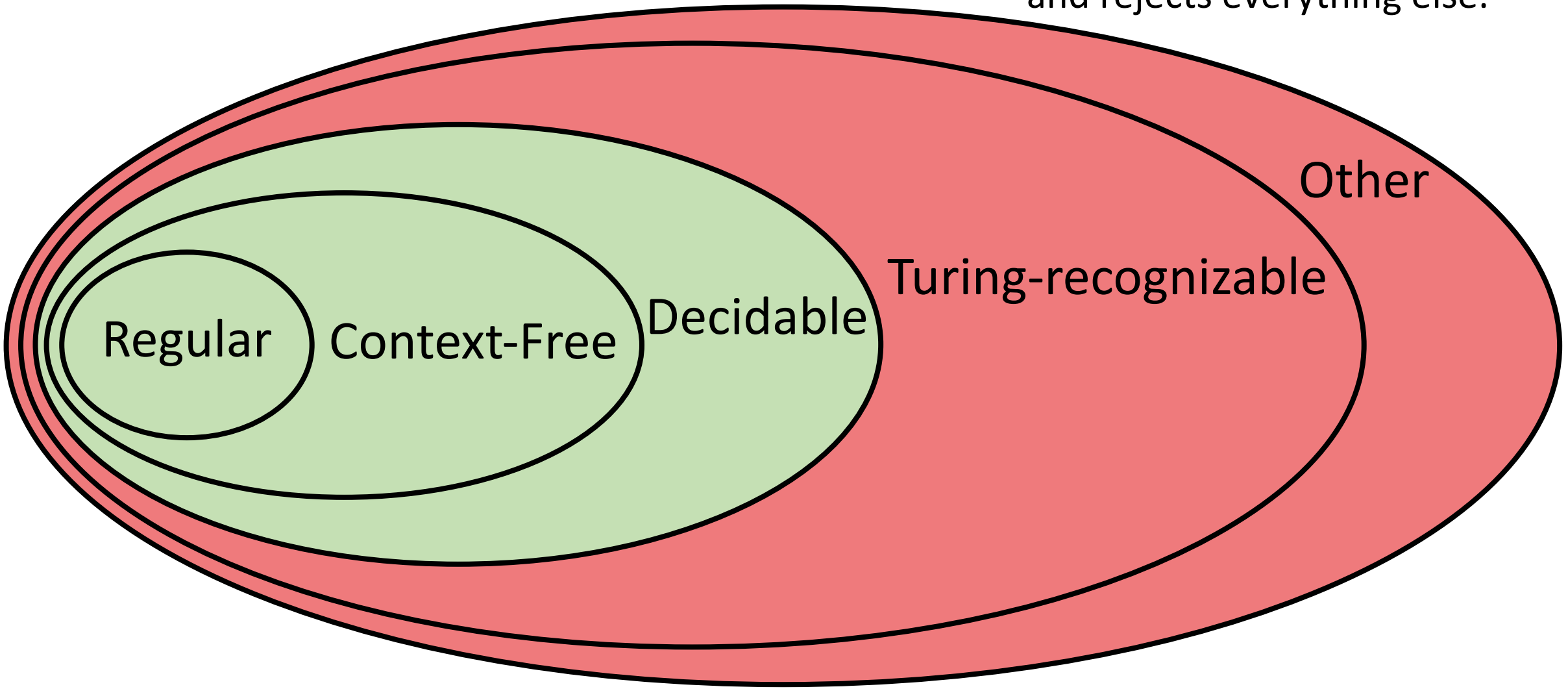


A_{DFA}

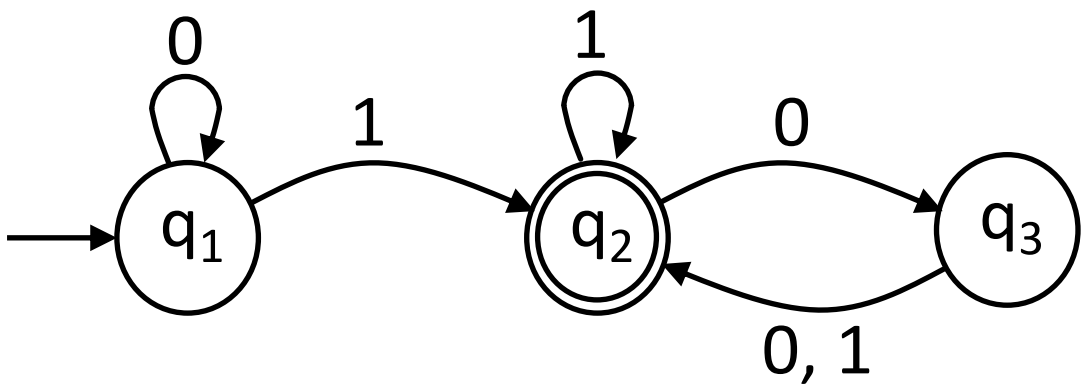
Claim: $A_{DFA} = \{\langle B, \omega \rangle : B$ is a DFA that accepts string $\omega\}$ is a decidable language.

A_{DFA}

Denotes string encoding of some object

Claim: $A_{DFA} = \{\langle B, \omega \rangle \mid B$ is a DFA that accepts string $\omega\}$ is a decidable language.

DFA Formal Definition



$Q = \{q_1, q_2, q_3\}$

$\Sigma = \{0,1\}$

δ:

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

Start state = q_1

$F = \{q_2\}$

```
private String[] states;
private char[] alphabet;
private HashMap<String, HashMap<Character, HashSet<String>>> transitions;
private String startState;
private String[] acceptStates;
public String name;
```

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Language decidability	Equivalent To	Computational problem decidability

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M_1 = on input $\langle B, \omega \rangle$

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1. ?

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1. Run B on ω.
2. ?

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M_1 is a decider, because ?

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M_1 is a decider, because all DFAs halt on all input.

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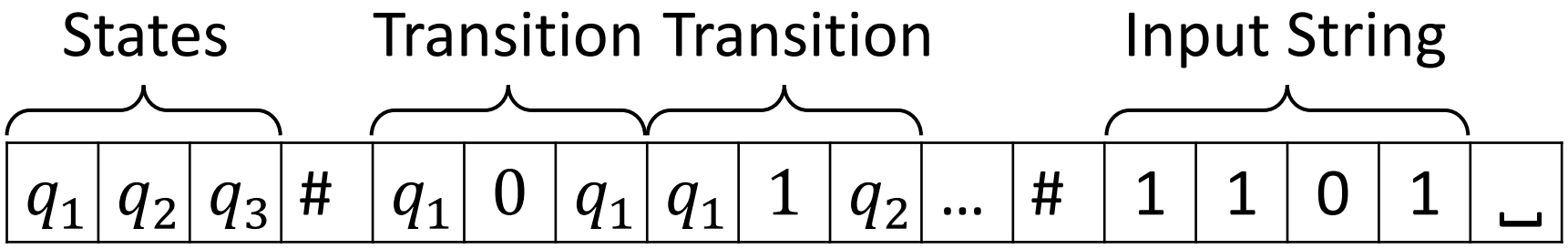

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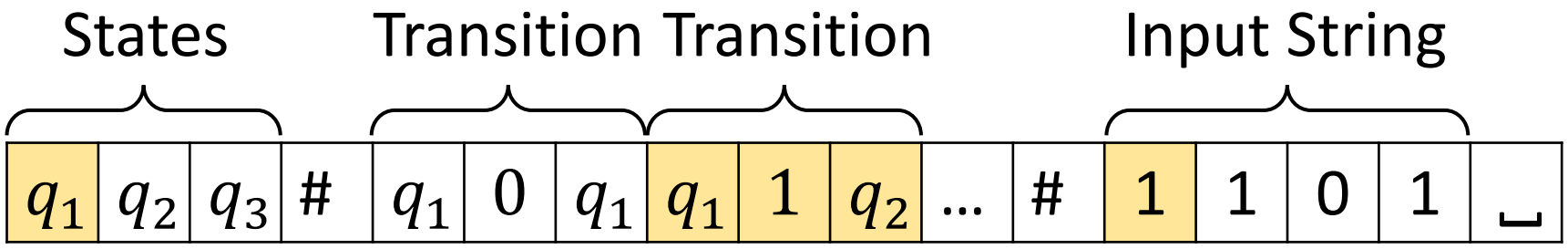

Mark current state.
Mark current character.
Find applicable transition.
Update state/character.

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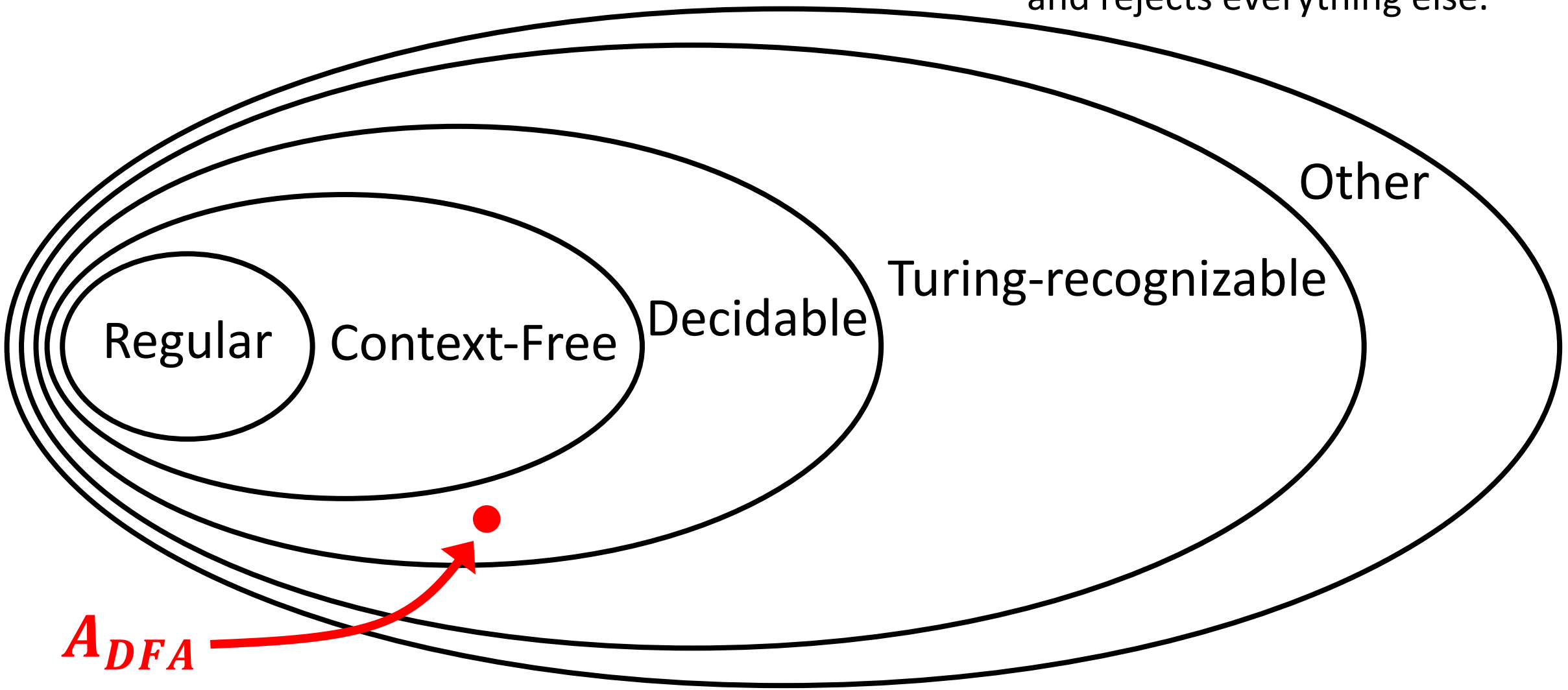

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Claim: $A_{NFA} = \{\langle C, \omega \rangle : C \text{ is an NFA that accepts string } \omega\}$ is a decidable language.

Proof:

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Claim: $A_{NFA} = \{\langle C, \omega \rangle : C$ is an NFA that accepts string $\omega\}$ is a decidable language.

Proof:

M_2 = on input $\langle C, \omega \rangle$

1. Convert C to an equivalent DFA B.

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M_2 = on input $\langle C, \omega \rangle$

1. Convert C to an equivalent DFA B.
2. Run M_1 (TM from first example) on $\langle B, \omega \rangle$.

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