Decidability

CSCI 338

E_{DFA}

Claim: $E_{DFA} = \{\langle A \rangle : A$ is a DFA and $L(A) = \emptyset\}$ is decidable.

Proof:

?

A language L is <u>decidable</u> if there is a TM that recognizes L and rejects everything else. (i.e. halts on all input)

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M_3 = on input $\langle A \rangle$

1. Mark start state of A.

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M$_3$ = on input $\langle A \rangle$

1. Mark start state of A.
2. Mark any state with transition coming from marked state.

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2. Mark any state with transition coming from marked state.
3. Repeat 2 until no new states are marked.

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3. Repeat 2 until no new states are marked.
4. ???? , <u>accept</u>. Otherwise, <u>reject</u>.

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1. Mark start state of A.
2. Mark any state with transition coming from marked state.
3. Repeat 2 until no new states are marked.
4. If no accept states are marked, <u>accept</u>. Otherwise, <u>reject</u>.

M$_3$ is a decider since at least one state must be added for step 2 to repeat, and there are a finite number of states.

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Claim: $EQ_{DFA} = \{\langle A, B\rangle : A$ and B are DFAs and $L(A) = L(B)\}$ is decidable.

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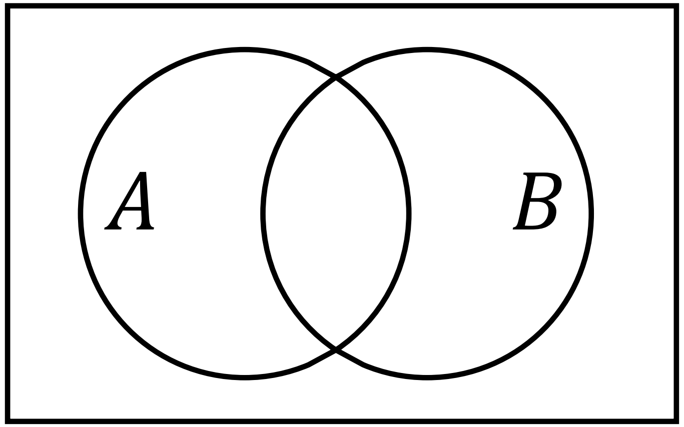
What if we tried to use E_{DFA} somehow?

If $L(A) = L(B)$, what would be empty?



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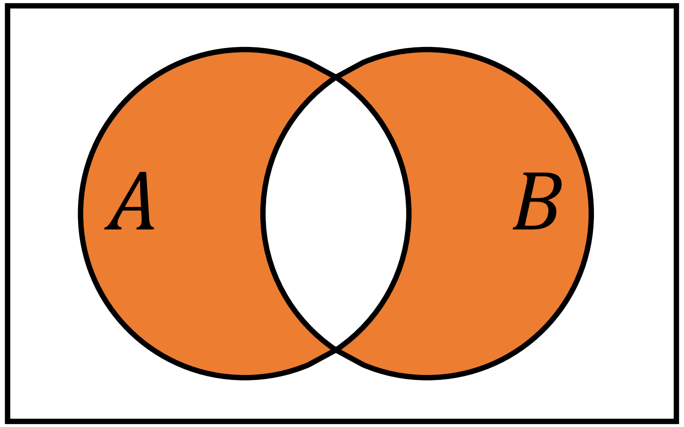

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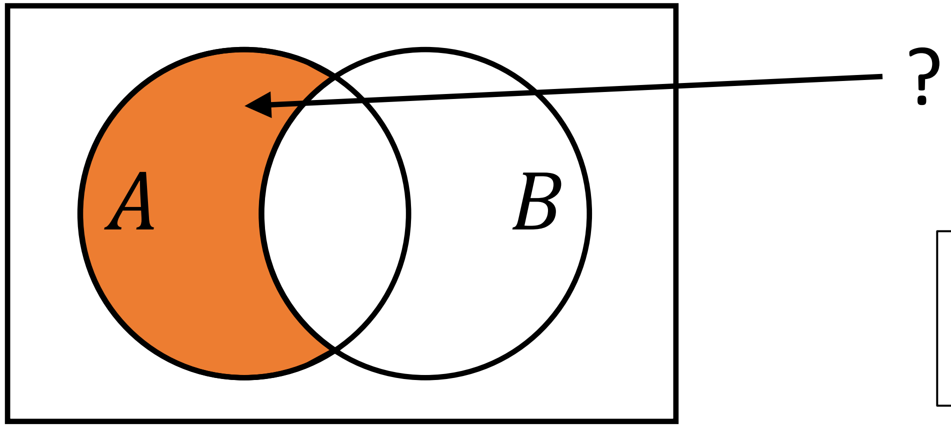

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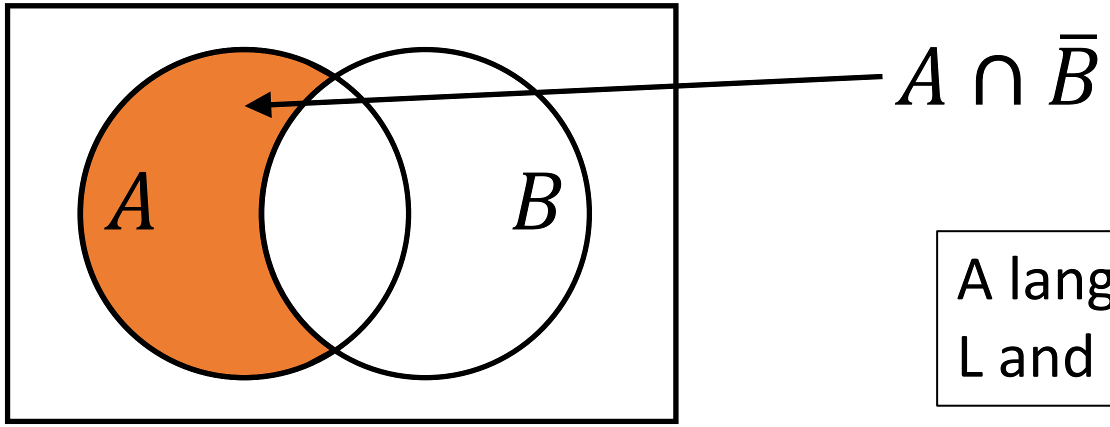

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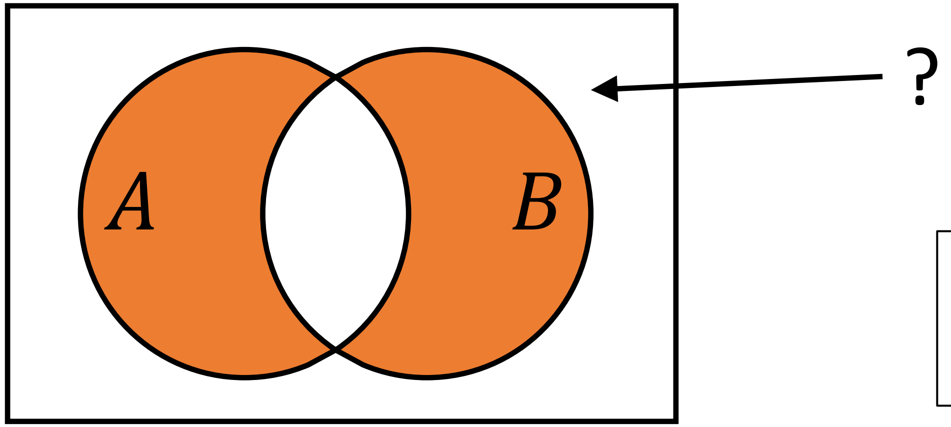

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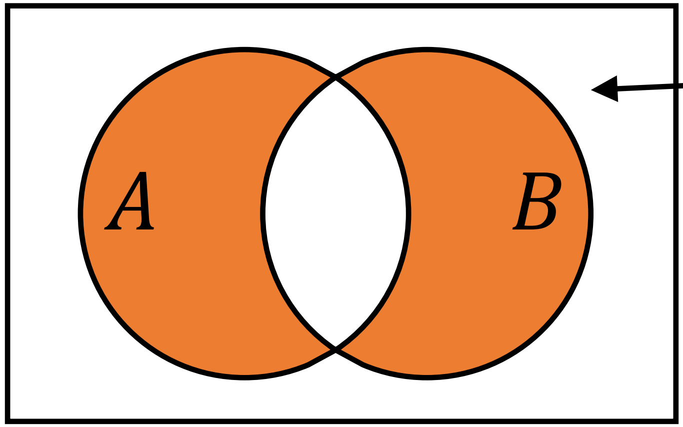

$(A \cap \bar{B}) \cup (\bar{A} \cap B)$

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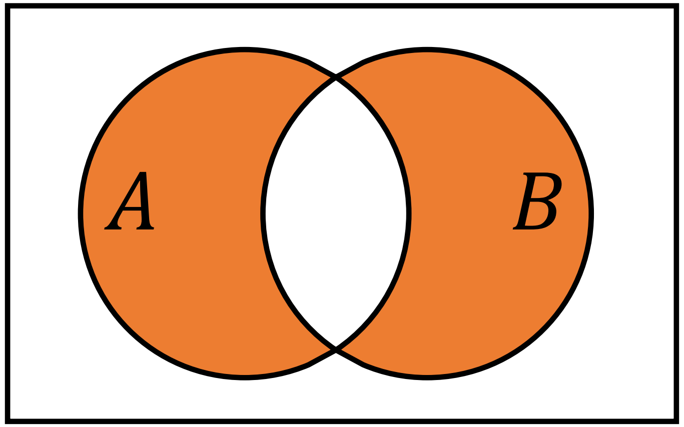
Proof:

M_4 = on input $\langle A, B\rangle$



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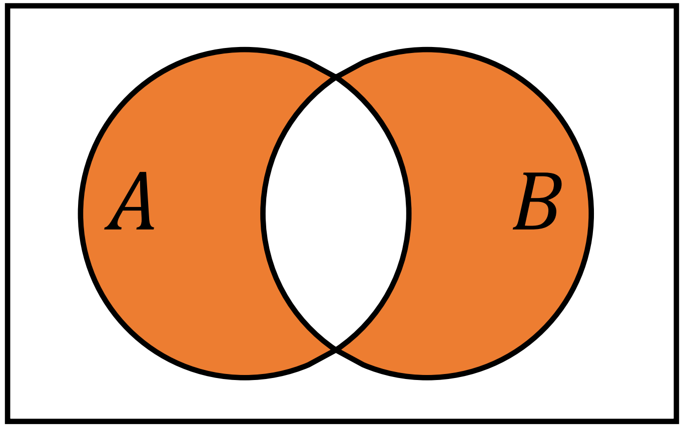
1. Construct DFA C as $(A \cap \bar{B}) \cup (\bar{A} \cap B)$.



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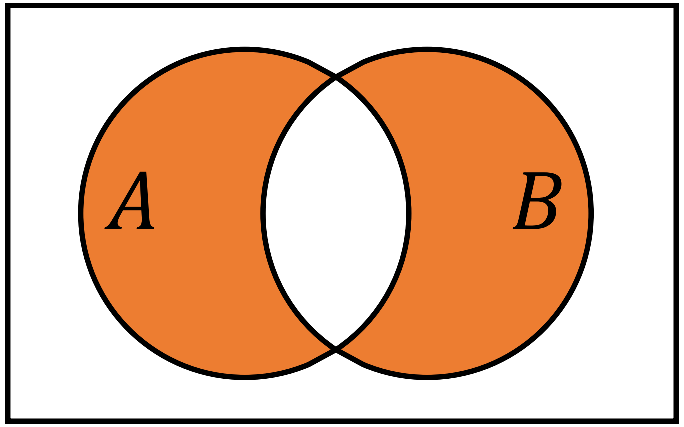
1. Construct DFA C as $\overbrace{(A \cap \bar{B}) \cup (\bar{A} \cap B)}^{\textbf{Details???}}$.



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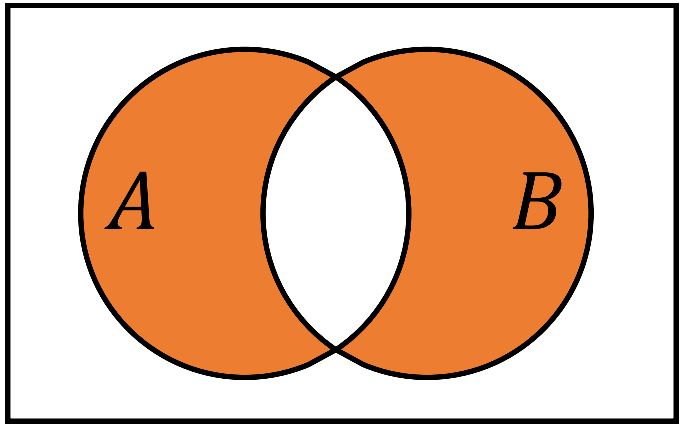
1. Construct DFA C as $(A \cap \bar{B}) \cup (\bar{A} \cap B)$.
2. Run E_{DFA} Decider on $\langle C\rangle$.



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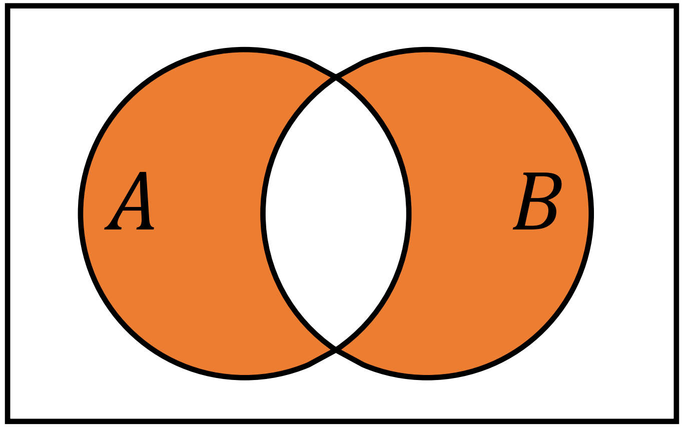
M$_4$ = on input $\langle A, B\rangle$

1. Construct DFA C as $(A \cap \bar{B}) \cup (\bar{A} \cap B)$.
2. Run E_{DFA} Decider on $\langle C\rangle$.
3. Accept/Reject?



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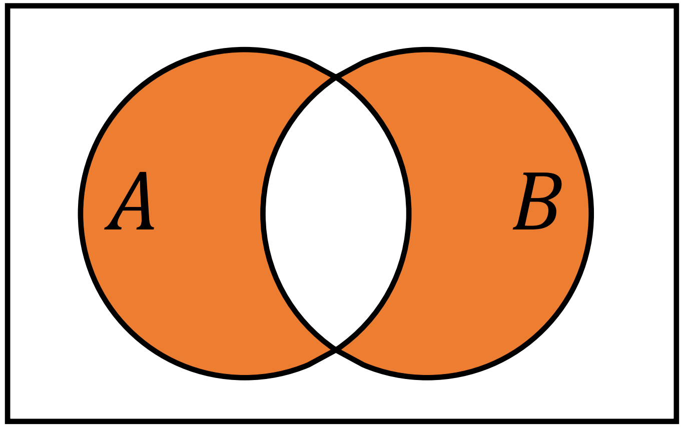
M_4 = on input $\langle A, B\rangle$

1. Construct DFA C as $(A \cap \bar{B}) \cup (\bar{A} \cap B)$.
2. Run E_{DFA} Decider on $\langle C\rangle$.
3. If Decider accepts, accept. If Decider rejects, reject.



M_4 is a decider since constructing C halts and the E_{DFA} Decider is a decider.

A language L is decidable if there is a TM that recognizes L and rejects everything else. (i.e. halts on all input)

$INFINITE_{DFA}$

Claim: $INFINITE_{DFA} = \{\langle A\rangle : A \text{ is a DFA and } |L(A)| = \infty\}$ is decidable.

Proof:

?

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