NP
CSCI 338
$P$ is the set of languages that are decidable in polynomial time on a deterministic single-tape TM.
\( P \) is the set of languages that are decidable in polynomial time on a deterministic single-tape TM. To show something is in \( P \), build a polynomial time decider for it.
$NP$

$P$ is the set of languages that are decidable in polynomial time on a deterministic single-tape TM.  

**To show something is in $P$, build a polynomial time decider for it.**

$NP$  

Set of languages that have polynomial time verifiers.
Vertex Cover (VC)

Vertex Cover = \{\langle G, k \rangle: G = (V, E) is a graph and k is an integer \leq |V| such that there exists some \( V' \subseteq V \) with \(|V'| \leq k \), such that each edge in \( E \) contains an end point in \( V' \) \}
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Vertex Cover: Given graph G = (V, E) and integer k \leq |V|, is there V' \subseteq V, with |V'| \leq k, such that each edge in E contains an end point in V'?
Vertex Cover (VC)

Vertex Cover = \{ (G, k) : G = (V, E) is a graph and k is an integer \leq |V| such that there exists some V' \subseteq V with |V'| \leq k, such that each edge in E contains an end point in V' \}

Vertex Cover: Given graph G = (V, E) and integer k \leq |V|, is there V' \subseteq V, with |V'| \leq k, such that each edge in E contains an end point in V'?

Is there a VC \leq k for k = 8?
Vertex Cover (VC)

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Is there a VC \( \leq k \) for \( k = 7 \)?
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Is there a VC \(\leq k\) for \(k = 6\)?
Vertex Cover (VC)

Vertex Cover = \{ (G, k) : G = (V, E) is a graph and k is an integer \leq |V| such that there exists some \( V' \subseteq V \) with \( |V'| \leq k \), such that each edge in \( E \) contains an end point in \( V' \) \}\}

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Is there a VC ≤ k for k = 6?
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Vertex Cover: Given graph \ G = (V, E) and integer \ k \leq |V|, is there \ V' \subseteq V, with \ |V'| \leq k, such that each edge in \ E \ contains an end point in \ V'? 

Is there a VC \leq k for k = 5?
Vertex Cover (VC)

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Vertex Cover: Given graph \( G = (V, E) \) and integer \( k \leq |V| \), is there \( V' \subseteq V \), with \( |V'| \leq k \), such that each edge in \( E \) contains an end point in \( V' \)?

Is there a VC \( \leq k \) for \( k = 4 \)?
Vertex Cover (VC)

Vertex Cover = \{ (G, k) : G = (V, E) is a graph and \(k\) is an integer \(\leq |V|\) such that there exists some \(V' \subseteq V\) with \(|V'| \leq k\), such that each edge in \(E\) contains an end point in \(V'\) \}

Vertex Cover: Given graph \(G = (V, E)\) and integer \(k \leq |V|\), is there \(V' \subseteq V\), with \(|V'| \leq k\), such that each edge in \(E\) contains an end point in \(V'\)?

Is there a VC \(\leq k\) for \(k = 4\)?
Vertex Cover (VC)

Vertex Cover = \{\langle G, k \rangle: G = (V, E) is a graph and k is an integer \leq |V| such that there exists some V' \subseteq V with |V'| \leq k, such that each edge in E contains an end point in V'\}

Vertex Cover: Given graph G = (V, E) and integer k \leq |V|, is there V' \subseteq V, with |V'| \leq k, such that each edge in E contains an end point in V'?

Is there a VC \leq k for k = 4?
Vertex Cover (VC)

Vertex Cover = \{\langle G, k \rangle: G = (V, E) is a graph and k is an integer \leq |V| such that there exists some \( V' \subseteq V \) with \( |V'| \leq k \), such that each edge in \( E \) contains an end point in \( V' \)\}

Vertex Cover: Given graph \( G = (V, E) \) and integer \( k \leq |V| \), is there \( V' \subseteq V \), with \( |V'| \leq k \), such that each edge in \( E \) contains an end point in \( V' \)?

Is there a VC \( \leq k \) for \( k = 4 \)?

Decision problem:
“Yes/No” – Is there a VC \( \leq k \)?

Optimization problem:
“Best” – What is the smallest VC?
Vertex Cover (VC)

Claim: \( VC \in NP \)

Proof:

Decider: Is \( \langle G, k \rangle \in VC \)?
Vertex Cover (VC)

Claim: VC $\in \mathcal{NP}$

Proof:

Vertex Cover: Given graph $G = (V, E)$ and integer $k \leq |V|$, is there $V' \subseteq V$, with $|V'| \leq k$, such that each edge in $E$ contains an end point in $V'$?

Decider: Is $\langle G, k \rangle \in VC$?

Verifier: Is $\langle G, k \rangle \in VC$, given a candidate solution?
Vertex Cover (VC)

Claim: VC ∈ NP

Proof:

Decider: Is ⟨G, k⟩ ∈ VC?

Verifier: Is ⟨G, k⟩ ∈ VC, given a candidate solution?
Vertex Cover (VC)

Claim: VC $\in \mathcal{NP}$

Proof:

Build a polynomial time verifier.

Vertex Cover: Given graph $G = (V, E)$ and integer $k \leq |V|$, is there $V' \subseteq V$, with $|V'| \leq k$, such that each edge in $E$ contains an end point in $V'$?
Vertex Cover (VC)

Claim: VC ∈ \(NP\)

Proof:

Build a polynomial time verifier.

\[ M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

1. ???.
Vertex Cover (VC)

Claim: VC ∈ 𝑁𝑃

Proof:
Build a polynomial time verifier.

\[ M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]
1. Test if \( |V'| \leq k \), reject if not.
Vertex Cover (VC)

Claim: \( VC \in NP \)

Proof:
Build a polynomial time verifier.

\[ M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

1. Test if \(|V'| \leq k\), reject if not.
2. ???
Vertex Cover (VC)

Claim: VC ∈ NP

Proof:

Build a polynomial time verifier.

\[ M = \text{on input } \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

1. Test if \(|V'| \leq k\), reject if not.
2. For each edge \(e = (a, b)\) in \(E\),
   2.1 ??
Vertex Cover (VC)

Claim: VC ∈ NP

Proof:

Build a polynomial time verifier.

\[ M = \text{on input } \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

1. Test if \( |V'| \leq k \), reject if not.
2. For each edge \( e = (a, b) \) in \( E \),
   2.1 Test if \( a \in V' \) or \( b \in V' \), ???.
**Vertex Cover (VC)**

**Claim:** $\text{VC} \in \mathcal{N}P$

**Proof:**

Build a polynomial time verifier.

\[
M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V.
\]

1. Test if $|V'| \leq k$, **reject** if not.
2. For each edge $e = (a, b)$ in $E$,
   2.1 Test if $a \in V'$ or $b \in V'$, **reject** if neither.

**Vertex Cover:** Given graph $G = (V, E)$ and integer $k \leq |V|$, is there $V' \subseteq V$, with $|V'| \leq k$, such that each edge in $E$ contains an end point in $V'$?
**Vertex Cover (VC)**

Claim: VC ∈ NP

Proof:

Build a polynomial time verifier.

\[ M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

1. Test if \(|V'| \leq k\), reject if not.
2. For each edge \(e = (a, b)\) in \(E\),
   2.1 Test if \(a \in V'\) or \(b \in V'\), reject if neither.
3. ???.
**Vertex Cover (VC)**

**Claim:** VC ∈ NP

**Proof:**

Build a polynomial time verifier.

\[ M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

1. Test if \(|V'| \leq k\), reject if not.
2. For each edge \(e = (a, b)\) in \(E\),
   2.1 Test if \(a \in V'\) or \(b \in V'\), reject if neither.
3. accept.
Vertex Cover (VC)

Claim: VC ∈ NP

Proof:

Build a polynomial time verifier.

$\mathcal{M} = \text{on input } \langle G, k, V' \rangle$, where $V'$ is a subset of $V$.

$O(\cdot) \rightarrow 1. \text{ Test if } |V'| \leq k, \text{ reject if not.}$

2. For each edge $e = (a, b)$ in $E$,
   2.1 Test if $a \in V'$ or $b \in V'$, reject if neither.

3. accept.
Vertex Cover (VC)

Claim: VC ∈ NP

Proof:
Build a polynomial time verifier.

$|V| = n.$

$M = \text{on input } \langle G, k, V' \rangle, \text{ where } V' \text{ is a subset of } V.$

$O(1)$

1. Test if $|V'| \leq k$, reject if not.
2. For each edge $e = (a, b)$ in $E$,
   2.1 Test if $a \in V'$ or $b \in V'$, reject if neither.
3. accept.
Vertex Cover (VC)

Claim: \( VC \in NP \)

Proof:

Build a polynomial time verifier.

\( |V| = n. \)

\( M = \) on input \( \langle G, k \rangle, V' \rangle \), where \( V' \) is a subset of \( V \).

\( O(1) \rightarrow 1. \) Test if \( |V'| \leq k \), reject if not.

\( O(?) \rightarrow 2. \) For each edge \( e = (a, b) \) in \( E \),

2.1 Test if \( a \in V' \) or \( b \in V' \), reject if neither.

3. accept.
Vertex Cover (VC)

Claim: VC $\in \mathbb{NP}$

Proof:

Build a polynomial time verifier.

$M$ on input $G = (V, E)$, $k \leq |V|$, $V'$, where $V'$ is a subset of $V$.

1. Test if $|V'| \leq k$, reject if not.

2. For each edge $e = (a, b)$ in $E$,
   2.1 Test if $a \in V'$ or $b \in V'$, reject if neither.

3. accept.

Vertex Cover: Given graph $G = (V, E)$ and integer $k \leq |V|$, is there $V' \subseteq V$, with $|V'| \leq k$, such that each edge in $E$ contains an end point in $V'$?

At most, how many edges are in a graph with $n$ vertices?

$|V| = O(1)$

What graph has the most number of edges?

Complete graph (every pair of vertices have an edge).

How many edges does a complete graph with $n$ vertices have?

$O(?)$

How many edges leave each vertex? $n - 1$

How much does that all add up to? $n(n - 1)$

Did we double count any edges? Yes

So how many edges are there? $\frac{n(n-1)}{2} \in O(n^2)$
Claim: VC ∈ NP

Proof:

Build a polynomial time verifier.

\[ |V| = n. \]

\[ M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

\[ O(1) \rightarrow 1. \text{ Test if } |V'| \leq k, \text{ reject if not.} \]

\[ O(n^2) \rightarrow 2. \text{ For each edge } e = (a, b) \text{ in } E, \]

\[ \quad 2.1 \text{ Test if } a \in V' \text{ or } b \in V', \text{ reject if neither.} \]

\[ 3. \text{ accept.} \]
Vertex Cover (VC)

Claim: VC ∈ NP

Proof:

Build a polynomial time verifier.

\[ |V| = n. \]

\[ M = \text{on input } \langle G, k \rangle, V', \text{ where } V' \text{ is a subset of } V. \]

\[ \mathcal{O}(1) \rightarrow 1. \text{ Test if } |V'| \leq k, \text{ reject if not.} \]

\[ \mathcal{O}(n^2) \rightarrow 2. \text{ For each edge } e = (a, b) \text{ in } E, \]

\[ \mathcal{O}(?) \rightarrow 2.1 \text{ Test if } a \in V' \text{ or } b \in V', \text{ reject if neither.} \]

\[ 3. \text{ accept.} \]
Vertex Cover (VC)

Claim: VC ∈ NP

Proof:

Build a polynomial time verifier.

|V| = n. M = on input 〈(G, k), V’〉, where V’ is a subset of V.

\[O(1)\rightarrow 1. \text{Test if } |V'| \leq k, \text{ reject if not.}\]

\[O(n^2)\rightarrow 2. \text{For each edge } e = (a, b) \text{ in } E,\]

\[O(n)\rightarrow 2.1 \text{Test if } a \in V' \text{ or } b \in V', \text{ reject if neither.}\]

3. accept.
Vertex Cover (VC)

Claim: VC ∈ NP

Proof:
Build a polynomial time verifier.

|V| = n.

\[ M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

\[ O(1) \rightarrow 1. \text{ Test if } |V'| \leq k, \text{ reject if not.} \]

\[ O(n^2) \rightarrow 2. \text{ For each edge } e = (a, b) \text{ in } E, \]

\[ O(n) \rightarrow 2.1 \text{ Test if } a \in V' \text{ or } b \in V', \text{ reject if neither.} \]

\[ O(?) \rightarrow 3. \text{ accept.} \]
Vertex Cover (VC)

Claim: VC ∈ NP

Proof:
Build a polynomial time verifier.

\[ |V| = n. \]
\[ M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

\[ O(1) \rightarrow 1. \text{ Test if } |V'| \leq k, \text{ reject if not.} \]

\[ O(n^2) \rightarrow 2. \text{ For each edge } e = (a, b) \text{ in } E, \]

\[ O(n) \rightarrow 2.1 \text{ Test if } a \in V' \text{ or } b \in V', \text{ reject if neither.} \]

\[ O(1) \rightarrow 3. \text{ accept.} \]
Vertex Cover (VC)

Claim: \( VC \in NP \)

Proof:

Build a polynomial time verifier.

\( |V| = n. \)

\( M = \) on input \( \langle G, k \rangle, V' \rangle \), where \( V' \) is a subset of \( V \).

\[ O(1) \rightarrow 1. \) Test if \( |V'| \leq k \), reject if not.

\[ O(n^2) \rightarrow 2. \) For each edge \( e = (a, b) \) in \( E \),

\[ O(n) \rightarrow 2.1 \) Test if \( a \in V' \) or \( b \in V' \), reject if neither.

\[ O(1) \rightarrow 3. \) accept.

For \( |V| = n \), \( M \) runs in \( O(n^3) \) time, therefore \( VC \in NP \).
$NP$

$P$ is the set of languages that are decidable in polynomial time on a deterministic single-tape TM.

To show something is in $P$, build a polynomial time decider for it.

$NP$ is the set of languages that have polynomial time verifiers.
$\mathbf{NP}$

$\mathbf{P}$ is the set of languages that are decidable in polynomial time on a deterministic single-tape TM. 

To show something is in $\mathbf{P}$, build a polynomial time decider for it.

$\mathbf{NP} \begin{cases} \text{Set of languages that have polynomial time verifiers.} \\
\text{Set of languages that are decidable by nondeterministic polynomial time TMs.} \end{cases}$
$NP$ is the set of languages that are decidable in polynomial time on a deterministic single-tape TM.

To show something is in $P$, build a polynomial time decider for it.

Set of languages that have polynomial time verifiers.

Set of languages that are decidable by nondeterministic polynomial time TMs.

Nondeterministic polynomial time decider:

1. Pick a potential solution.
2. Verify its correctness.
Vertex Cover (VC)

Claim: $VC \in NP$

Proof:

$M = \text{on input } \langle G, k, V' \rangle$, where $V'$ is a subset of $V$.

1. Test if $|V'| \leq k$, reject if not.
2. For each edge $e = (a, b)$ in $E$,
   2.1 Test if $a \in V'$ or $b \in V'$,
      reject if neither.
3. accept.
**Vertex Cover (VC)**

Claim: VC ∈ NP

Proof:

\[
M = \text{on input } \langle G, k \rangle, \text{ where } V' \text{ is a subset of } V. \\
1. \text{ Test if } |V'| \leq k, \text{ reject if not.} \\
2. \text{ For each edge } e = (a, b) \text{ in } E, \\
   2.1 \text{ Test if } a \in V' \text{ or } b \in V', \\
   \text{ reject if neither.} \\
3. \text{ accept.}
\]

---

Vertex Cover: Given graph \( G = (V, E) \) and integer \( k \leq |V| \), is there \( V' \subseteq V \), with \( |V'| \leq k \), such that each edge in \( E \) contains an end point in \( V' \)?
$P$ versus $NP$

$P$ is the set of languages that are decidable in polynomial time on a deterministic single-tape TM. 

To show something is in $P$, build a polynomial time decider for it.

$NP$ is the set of languages that have polynomial time verifiers. 

Set of languages that are decidable by nondeterministic polynomial time TMs.

To show something is in $NP$, build a polynomial time verifier (or nondeterministic decider) for it.
$P$ versus $NP$

$P \overset{?}{=} NP$

Solvable in polynomial time

Verifiable in polynomial time
$P$ versus $NP$

Can all problems that are verifiable in polynomial time be solved in polynomial time?
Independent Set (IS)

Independent Set: Given a graph $G = (V, E)$ and integer $k \leq |V|$, is there a subset $V'$ of size $\geq k$, such that no two vertices $\in V'$ are adjacent?
Independent Set (IS)

Independent Set: Given a graph $G = (V, E)$ and integer $k \leq |V|$, is there a subset $V'$ of size $\geq k$, such that no two vertices $\in V'$ are adjacent?

Is there an IS $\leq k$ for $k = 1$?
Independent Set (IS)

Independent Set: Given a graph $G = (V, E)$ and integer $k \leq |V|$, is there a subset $V'$ of size $\geq k$, such that no two vertices $\in V'$ are adjacent?

Is there an IS $\leq k$ for $k = 1$?

Yes! Any vertex by itself is an IS!
Independent Set (IS)

Independent Set: Given a graph $G = (V, E)$ and integer $k \leq |V|$, is there a subset $V'$ of size $\geq k$, such that no two vertices $\in V'$ are adjacent?

What is the optimal (i.e., largest) independent set?
Independent Set (IS)

Independent Set: Given a graph $G = (V, E)$ and integer $k \leq |V|$, is there a subset $V'$ of size $\geq k$, such that no two vertices $\in V'$ are adjacent?

What is the optimal (i.e., largest) independent set?
Claim: IS \(\in NP\)

Proof:

Independent Set (IS)

Independent Set: Given a graph \(G = (V, E)\) and integer \(k \leq |V|\), is there \(V' \subseteq V\), with \(|V'| \geq k\), such that no two vertices \(\in V'\) are adjacent?
Independent Set (IS)

Claim: \( \text{IS} \in \mathcal{NP} \)

Proof:

Build a polynomial time verifier.

\[ M = \text{on input } \langle \langle G, k \rangle, ?? \rangle \]
Independent Set (IS)

Claim: IS ∈ \(NP\)

Proof:

Build a polynomial time verifier.

\[ M = \text{on input } \langle G, k \rangle, V', \text{ where } V' \text{ is a subset of } V. \]
Independent Set (IS)

Claim: IS ∈ NP

Proof:

Build a polynomial time verifier.

\[ M = \text{on input } \langle G, k, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

1. ????
Independent Set (IS)

Claim: IS $\in \mathcal{NP}$

Proof:
Build a polynomial time verifier.

$M = \text{on input } \langle (G, k), V' \rangle, \text{ where } V' \subseteq V.$

1. Test if $|V'| \geq k$, reject if not.

2. ???
Independent Set (IS)

Claim: $\text{IS} \in \mathcal{NP}$

Proof:

Build a polynomial time verifier.

$$M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V.$$ 

1. Test if $|V'| \geq k$, reject if not.
2. For each pair of vertices $v_1, v_2$ in $V'$,
   
   2.1 ???
Independent Set (IS)

Claim: IS ∈ NP

Proof:

Build a polynomial time verifier.

\[ M = \text{on input } \langle (G, k), V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

1. Test if \(|V'| \geq k\), reject if not.
2. For each pair of vertices \(v_1, v_2\) in \(V'\),
   2.1 Test if \((v_1, v_2) \in E\) and reject if it is.
3. ???

Independent Set: Given a graph \(G = (V, E)\) and integer \(k \leq |V|\), is there \(V' \subseteq V\), with \(|V'| \geq k\), such that no two vertices \(\in V'\) are adjacent?
Independent Set (IS)

Claim: IS ∈ NP

Proof:

Build a polynomial time verifier.

\[ M = \text{on input } \langle (G, k), V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

1. Test if \(|V'| \geq k\), reject if not.

2. For each pair of vertices \(v_1, v_2\) in \(V'\),
   
   2.1 Test if \((v_1, v_2) \in E\) and reject if it is.

3. accept.
Independent Set (IS)

Claim: IS ∈ \( NP \)

Proof:

Build a polynomial time verifier.

\( |V| = n. \)

\( M = \) on input \( \langle G, k \rangle, V' \), where \( V' \) is a subset of \( V \).

\( O(\cdot) \)

1. Test if \( |V'| \geq k \), reject if not.

2. For each pair of vertices \( v_1, v_2 \) in \( V' \),

   2.1 Test if \( (v_1, v_2) \in E \) and reject if it is.

3. accept.
Independent Set (IS)

Claim: IS \(\in\) \(N_P\)

Proof:

Build a polynomial time verifier.

\(M = \text{on input } \langle G, k, V' \rangle, \text{ where } V' \text{ is a subset of } V.\)

\(O(1)\)

1. Test if \(|V'| \geq k\), reject if not.

2. For each pair of vertices \(v_1, v_2\) in \(V'\),
   
   2.1 Test if \((v_1, v_2) \in E\) and reject if it is.

3. accept.
Independent Set (IS)

Claim: IS ∈ \(NP\)

Proof:

Build a polynomial time verifier.

\(|V| = n.\)

\(M = \) on input \(\langle G, k \rangle, V' \rangle\), where \(V'\) is a subset of \(V\).

\(O(1)\) → 1. Test if \(|V'| \geq k\), reject if not.

\(O(?)\) → 2. For each pair of vertices \(v_1, v_2\) in \(V'\),

2.1 Test if \((v_1, v_2) \in E\) and reject if it is.

3. accept.
Independent Set (IS)

Claim: IS $\in \mathcal{NP}$

Proof:

Build a polynomial time verifier.

$|V| = n.$ $M =$ on input $\langle G, k \rangle, V'$, where $V'$ is a subset of $V$.

$O(1) \rightarrow$ 1. Test if $|V'| \geq k$, reject if not.

$O(n^2) \rightarrow$ 2. For each pair of vertices $v_1, v_2$ in $V'$,

2.1 Test if $(v_1, v_2) \in E$ and reject if it is.

3. accept.

Independent Set: Given a graph $G = (V, E)$ and integer $k \leq |V|$, is there $V' \subseteq V$, with $|V'| \geq k$, such that no two vertices $\in V'$ are adjacent?
Independent Set (IS)

Claim: IS ∈ \(NP\)

Proof:

Build a polynomial time verifier.

|\(|V| = n. M = \text{on input } \langle(G, k), V'\rangle, \text{ where } V' \text{ is a subset of } V.\)

\(O(1)\) 1. Test if |\(V'\)| ≥ \(k\), \texttt{reject} if not.

\(O(n^2)\) 2. For each pair of vertices \(v_1, v_2\) in \(V'\),

\(O(\cdot)\) 2.1 Test if \((v_1, v_2) \in E\) and \texttt{reject} if it is.

3. \texttt{accept}.
Independent Set (IS)

Claim: IS ∈ \( NP \)

Proof:

Build a polynomial time verifier.

\[ |V| = n. \]

\[ M = \text{on input } \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

\[ O(1) \rightarrow 1. \text{ Test if } |V'| \geq k, \text{ reject if not.} \]

\[ O(n^2) \rightarrow 2. \text{ For each pair of vertices } v_1, v_2 \text{ in } V', \]

\[ O(n^2) \rightarrow 2.1 \text{ Test if } (v_1, v_2) \in E \text{ and reject if it is.} \]

\[ 3. \text{ accept.} \]
Independent Set (IS)

Claim: IS ∈ NP

Proof:
Build a polynomial time verifier.

\[ |V| = n. \]

\[ M = \text{on input } \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

\[ O(1) \rightarrow 1. \text{ Test if } |V'| \geq k, \text{ reject if not.} \]

\[ O(n^2) \rightarrow 2. \text{ For each pair of vertices } v_1, v_2 \text{ in } V', \]

\[ O(n^2) \rightarrow 2.1 \text{ Test if } (v_1, v_2) \in E \text{ and reject if it is.} \]

\[ O(?) \rightarrow 3. \text{ accept.} \]
Independent Set (IS)

Claim: IS $\in \mathcal{NP}$

Proof:

Build a polynomial time verifier.

$|V| = n$. $M = on input \langle \langle G, k \rangle, V' \rangle$, where $V'$ is a subset of $V$.

$O(1) \rightarrow 1. \) Test if $|V'| \geq k$, reject if not.

$O(n^2) \rightarrow 2. \) For each pair of vertices $v_1, v_2$ in $V'$,

$O(n^2) \rightarrow 2.1 \) Test if $(v_1, v_2) \in E$ and reject if it is.

$O(1) \rightarrow 3. \) accept.
Independent Set (IS)

Claim: IS ∈ NP

Proof:

Build a polynomial time verifier.

\[ |V| = n. \]

\[ M = \text{on input } \langle \langle G, k \rangle, V' \rangle, \text{ where } V' \text{ is a subset of } V. \]

1. Test if \(|V'| \geq k\), reject if not.

2. For each pair of vertices \(v_1, v_2\) in \(V'\),

2.1 Test if \((v_1, v_2) \in E\) and reject if it is.

3. accept.

For \(|V| = n\), M runs in \(O(n^4)\) time, therefore IS ∈ NP.