## Finite Automata CSCI 338

## Deterministic Finite Automaton (DFA)



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DFA string processing:

1. Start at start state.
2. Select first character in string.
3. Update state by following transition that corresponds to character.
4. Select next character in string.
5. Repeat step 3 and 4 until no
 characters remain.
6. If final state is accept state, accept. Otherwise, reject.

DFAs either accept or reject strings.
Given string $\omega=01101$, does this DFA accept or reject?

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1. Finite set of states, $Q$.
2. Finite alphabet, $\Sigma$.
$\Sigma$ consists of the transition characters (i.e. characters in the strings the DFA processes).


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Not allowed! $q_{3}$
needs to handle the value 1 somehow!

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Exactly one start state needed.
4. Start state, $q_{0} \in Q$.


## DFA Formal Definition

DFAs consist of:

1. Finite set of states, $Q$.
2. Finite alphabet, $\Sigma$.
3. Transition function, $\delta: Q \times \Sigma \rightarrow Q$.
$F$ is allowed to equal $Q$ or be empty.
4. Start state, $q_{0} \in Q$.
5. Set of accept states, $F \subseteq Q$.


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$$
\left\{\begin{array}{l}
Q=\left\{q_{1}, q_{2}, q_{3}\right\} \\
\Sigma=\{0,1\} \\
\delta: \\
\\
\\
\hline
\end{array} \left\lvert\, \begin{array}{l|l} 
\\
\hline \mathrm{q}_{1} & \mathrm{q}_{1} \\
\mathrm{q}_{2} & \mathrm{q}_{2} \\
& \mathrm{q}_{2} \\
\mathrm{q}_{3} & \mathrm{q}_{2} \\
& \mathrm{q}_{3} \\
\mathrm{q}_{2} & \mathrm{q}_{2}
\end{array}\right.\right.
$$

Start state $=q_{1}$
$F=\left\{q_{2}\right\}$

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Definitions:
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$M$ recognizes $A$.

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$L(M)=\begin{aligned} & \{\omega: \omega \text { contains at least one } 1 \text { and an } \\ & \text { even number of } 0 \mathrm{~s} \text { following the final } 1\}\end{aligned}$

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Make a DFA that recognizes it.

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How do you prove a language is regular? Make a DFA that recognizes it.

Set of regular languages are "things we can do" with DFAs.

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Prove that the following languages are regular:

1. Set of all strings over $\{0,1\}$.

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The set of all strings $A$ that a DFA $M$ accepts is called its language, $L(M)=A$.


DFA Language Rules:

1. If the DFA accepts it, it is in the language.
2. If it is in the language, the DFA must accept it.

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3. Set of all strings that contain the substring: 10.

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