CLIQUE
CSCI 338

Announcement:

- CIA representatives speaking to ESOF 322 for first 10-20 minutes.
- REID 103 @ 12:00.
- You are invited.

NP-Complete

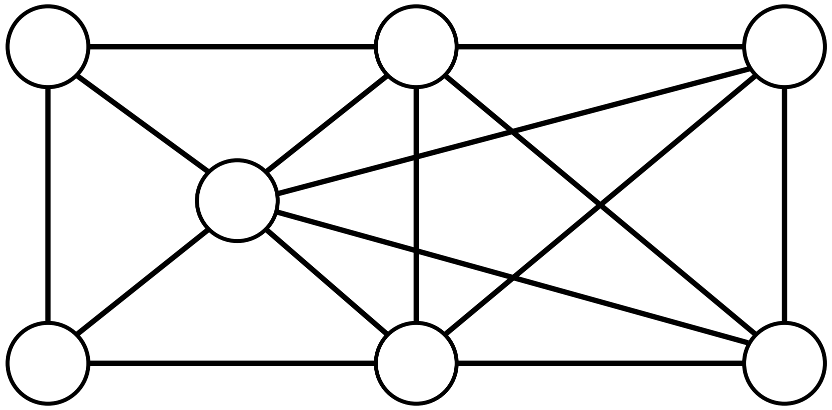
How to show problem B is in NP-Complete:

1. Show B is in NP.
2. Pick some known NP-Complete problem A.
3. Show how generic instances of A can be translated into instances of B.
4. Show that the translation process runs in polynomial time.
5. Show that if the answer to A's instance is 'yes', the answer to B's instance is also 'yes'.
6. Show that if the answer to B's instance is 'yes', the answer to A's instance is also 'yes'.

$CLIQUE$

Clique: a subgraph where every pair of nodes share an edge (i.e. a complete subgraph).

k-Clique: A clique that contains k vertices.



$CLIQUE = \{\langle G, k \rangle : G$ is an undirected graph with a k–clique$\}$

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$CLIQUE$

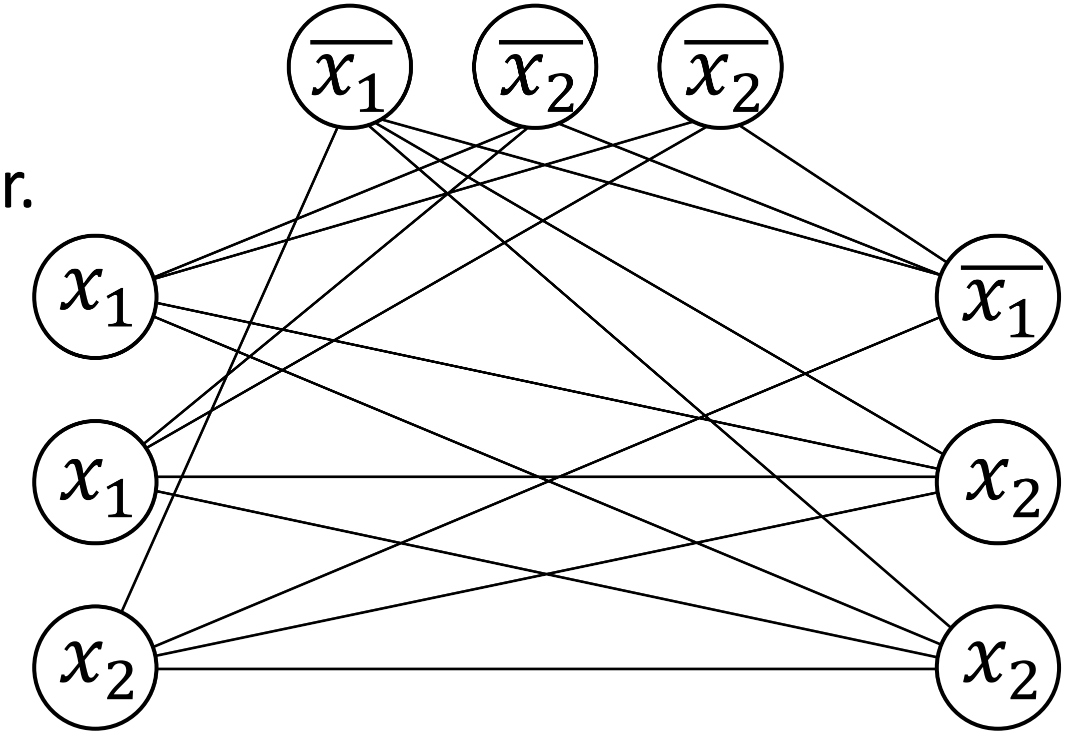
Claim: $3SAT \leq_P CLIQUE$

Proof: Let ϕ be a formula with k clauses. Generate an undirected graph G:

For each **Need to show: $\boldsymbol{\phi}$ is satisfiable $\Leftrightarrow$ $\boldsymbol{G}$ has a $\boldsymbol{k}$-clique.** en every pair of nodes, except:

1. Nodes in the same clause
2. Nodes that are negations of each other.

$$\phi = (x_1 \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee x_2)$$



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Claim: $3SAT \leq_P CLIQUE$

Proof: ϕ is satisfiable $\Leftrightarrow$ G has a k-clique.

For each clause in ϕ, make a node for each literal. Make edge between every pair of nodes, except:

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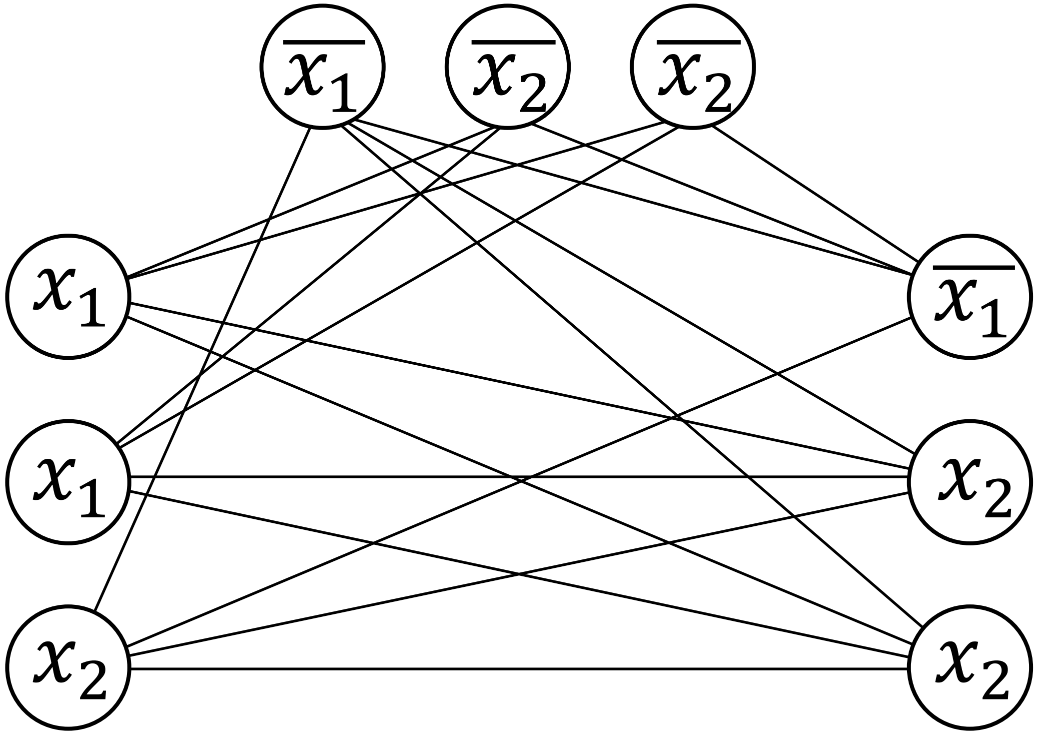

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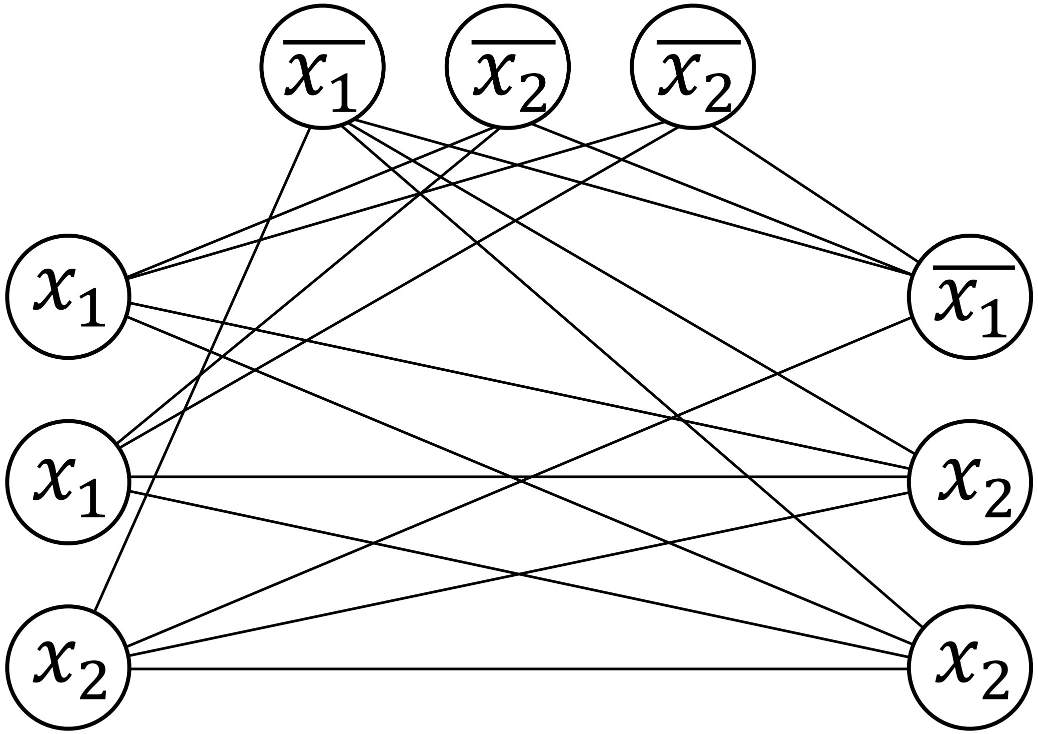

$CLIQUE$

Claim: $3SAT \leq_P CLIQUE$

Proof: ϕ is satisfiable $\Leftrightarrow$ G has a k-clique.

$\Rightarrow$ Suppose ϕ is satisfiable. Then at least one literal is true in each clause.

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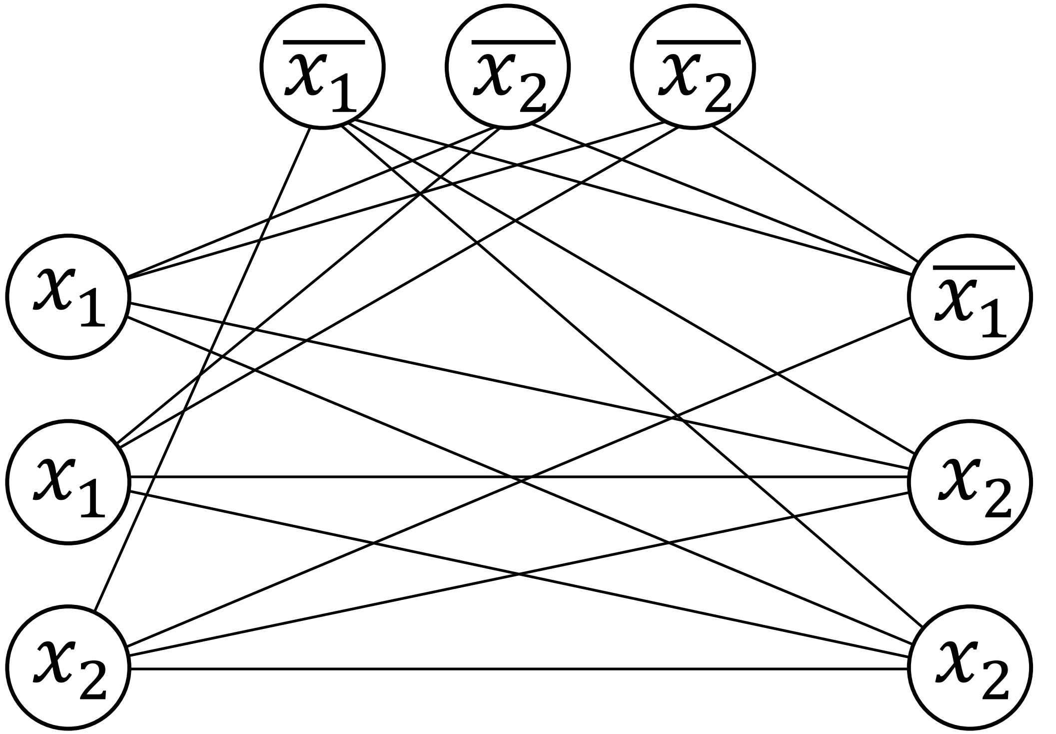

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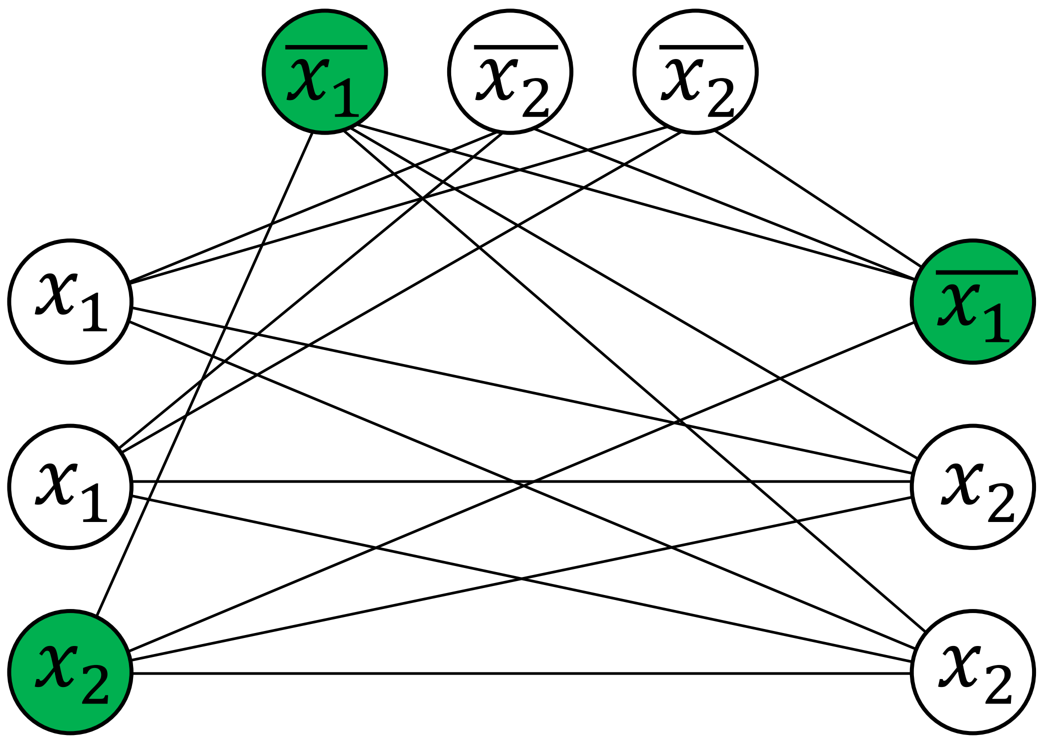

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Proof: ϕ is satisfiable $\Leftrightarrow$ G has a k-clique.

$\Rightarrow$ Suppose ϕ is satisfiable. Then at least one literal is true in each clause. For each clause, select a node in G for one of the true literals. This forms a ***k*-clique**, since…

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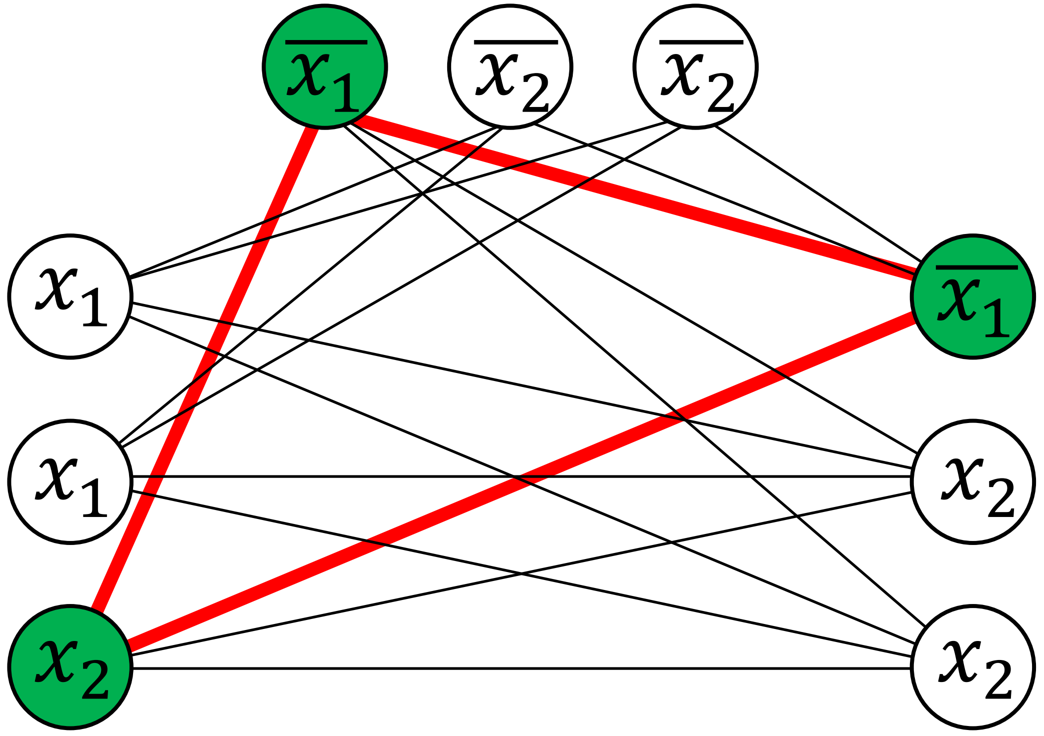

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Claim: $3SAT \leq_P CLIQUE$

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$\Rightarrow$ Suppose ϕ is satisfiable. Then at least one literal is true in each clause. For each clause, select a node in G for one of the true literals. This forms a **k-clique**, since **k nodes** are selected and each is joined by an edge.

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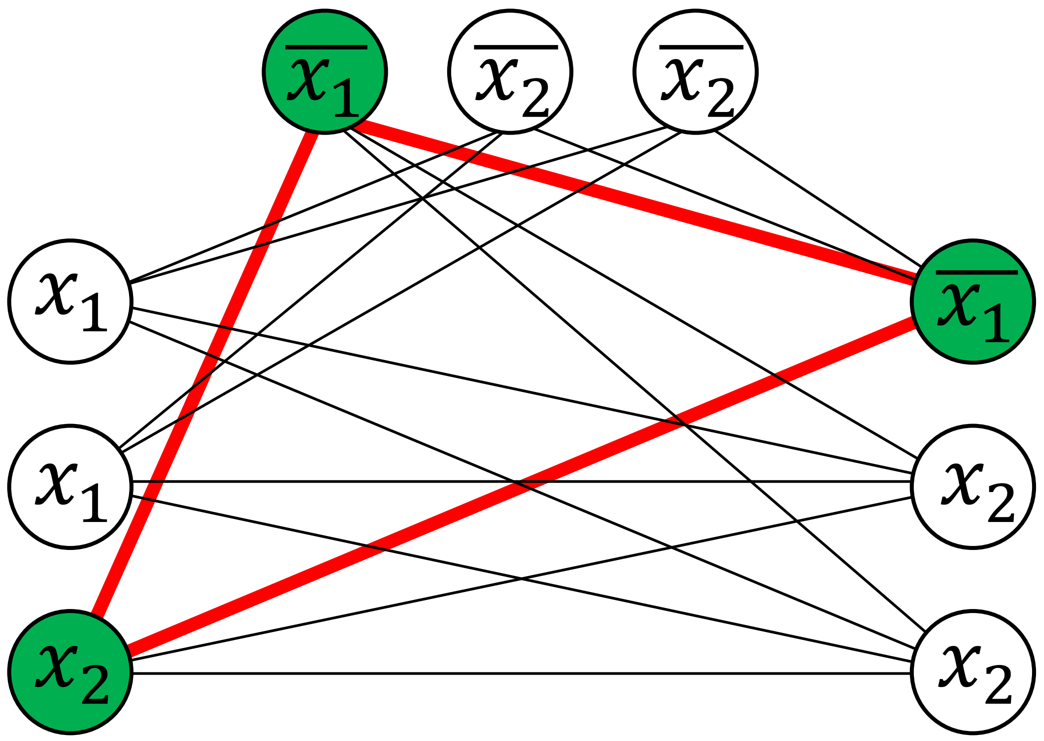

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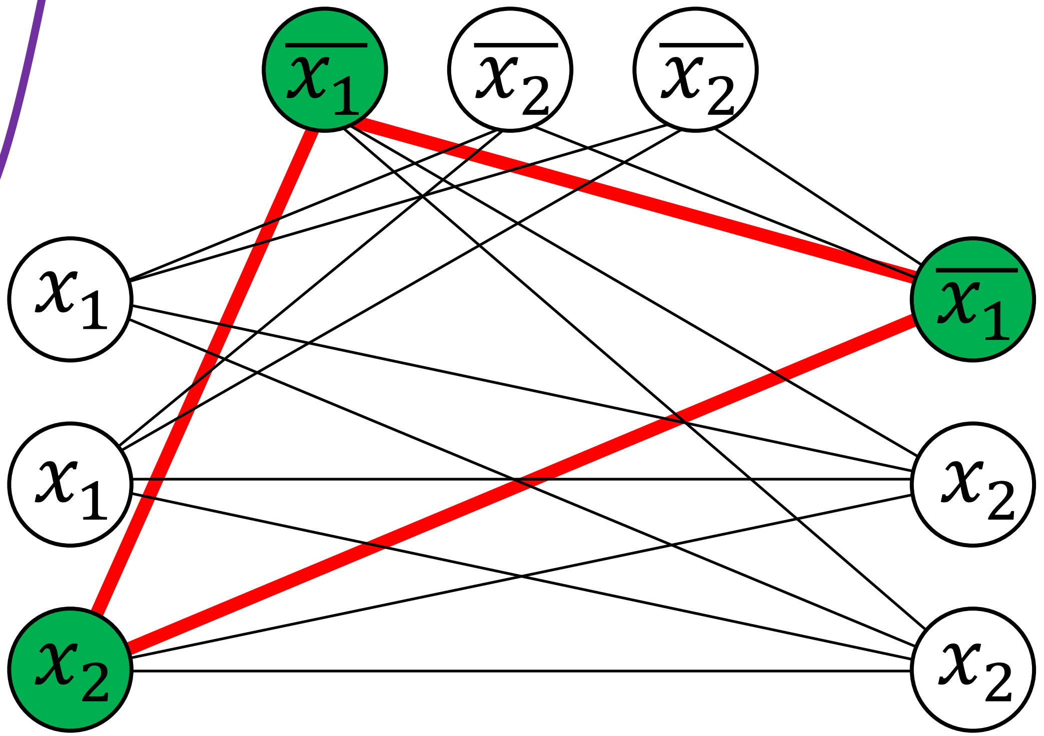

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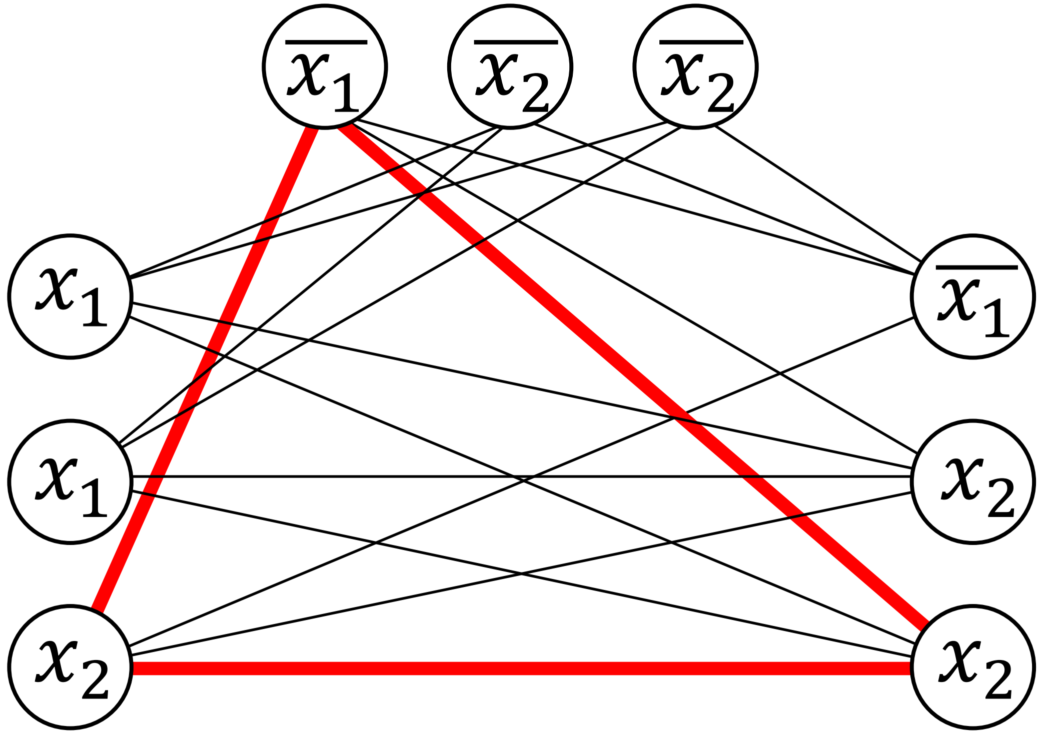

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$\Rightarrow$ Suppose ϕ is satisfiable. Then at least one literal is true in each clause. For each clause, select a node in G for one of the true literals. This forms a k-clique, since k nodes are selected and each is joined by an edge.

$\Leftarrow$ Suppose G has a ***k*-clique**. Then there is a non-contradictory **node** from the k-clique in each clause.

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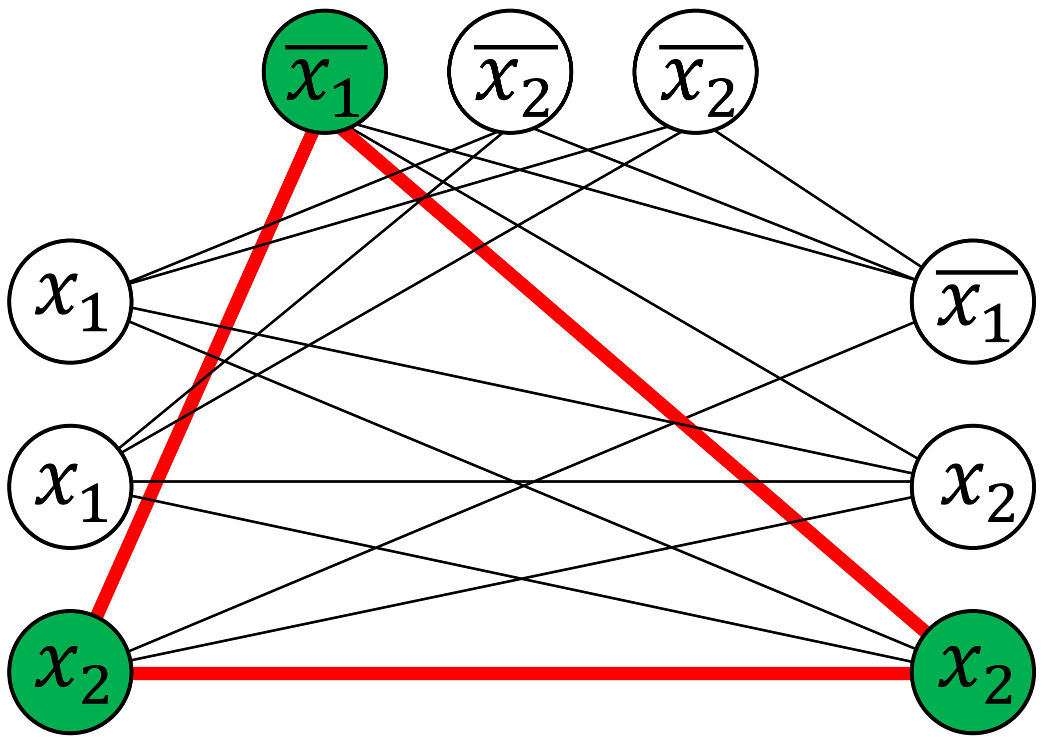

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$\Leftarrow$ Suppose G has a ***k*-clique**. Then there is a **non-contradictory node** from the k-clique in each clause. **(nodes in the same clause can't share an edge!)**

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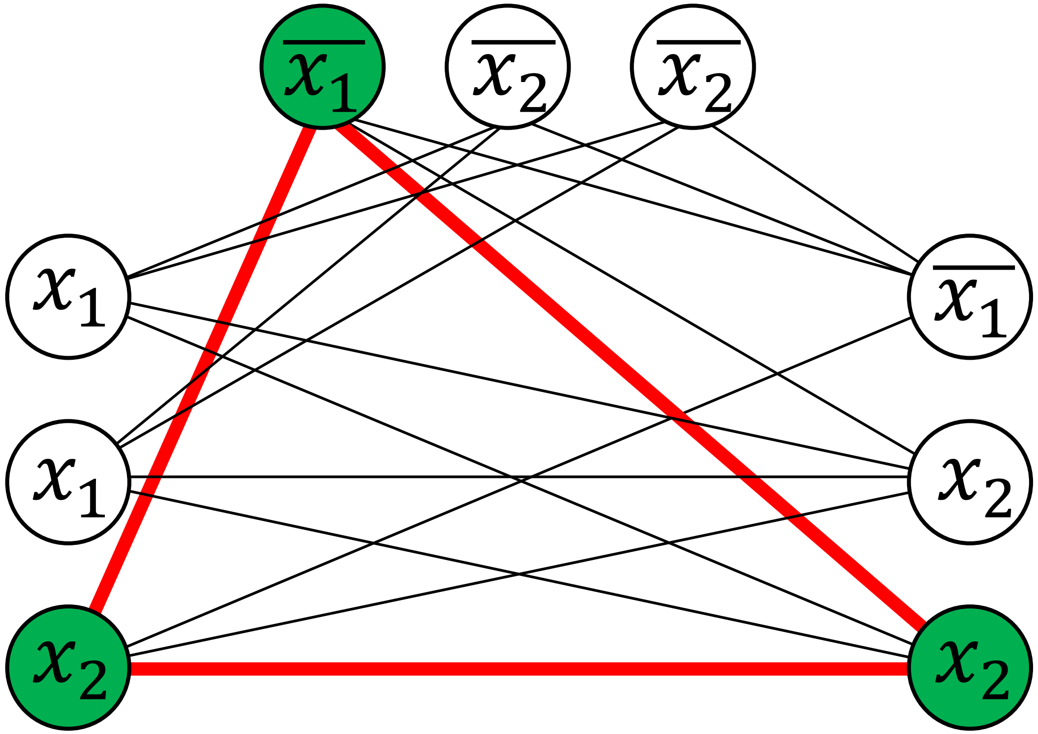

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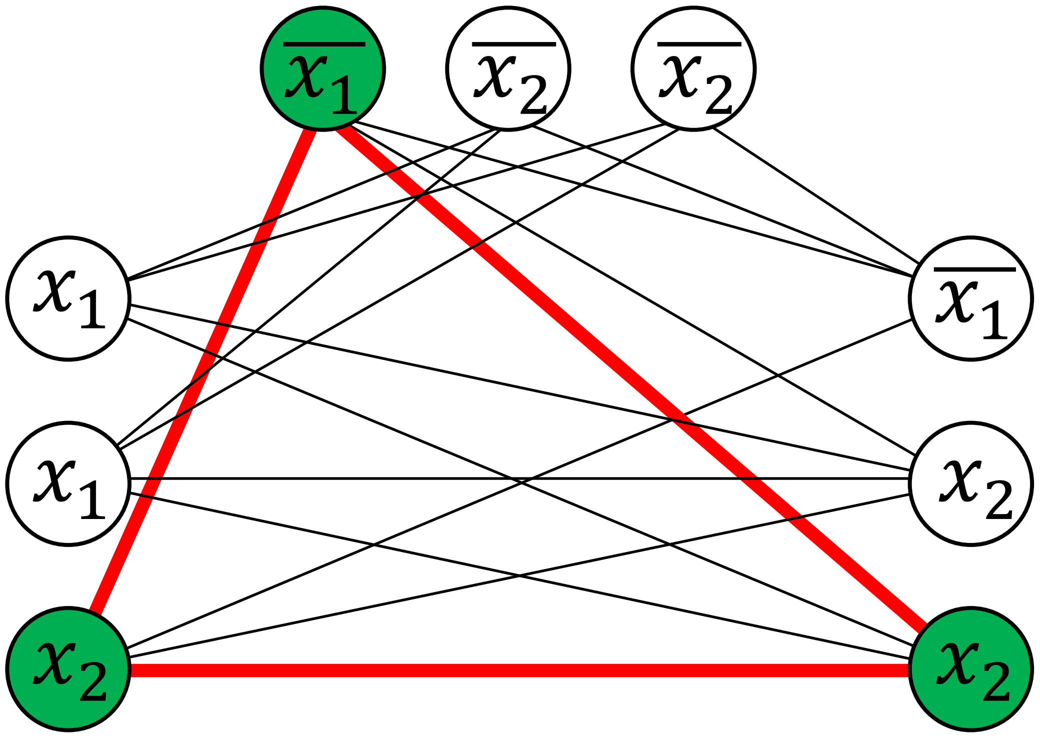

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$\Leftarrow$ Suppose G has a ***k*-clique**. Then there is a non-contradictory **node** from the k-clique in each clause. Making each node in the k-clique **true** results in…

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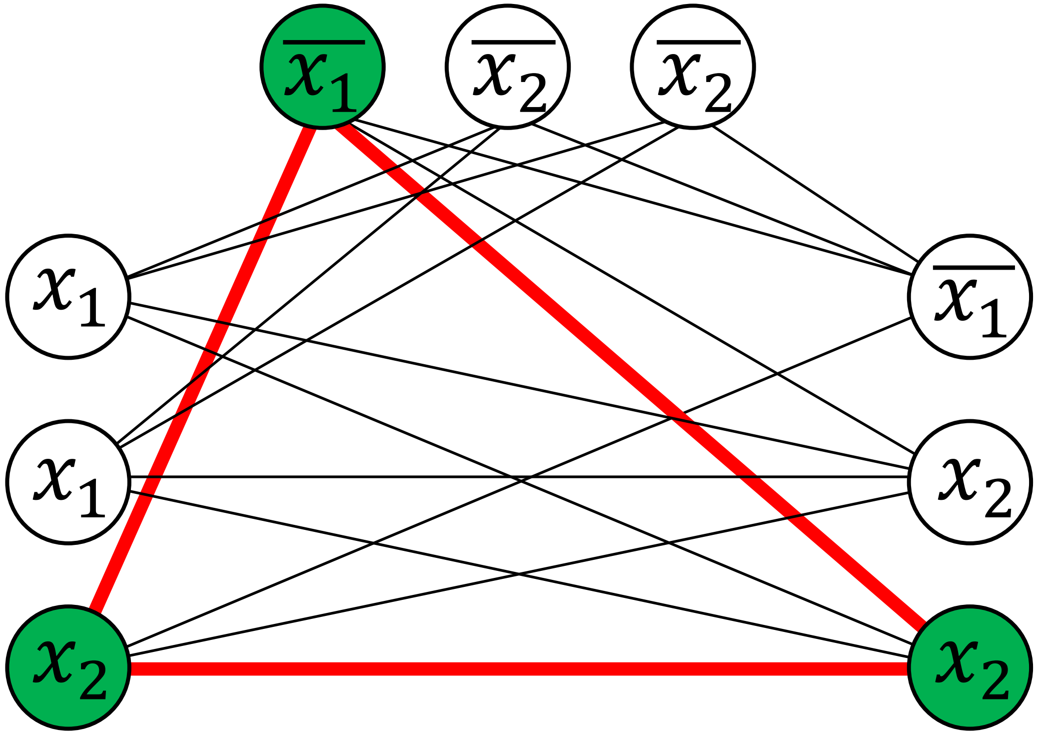

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$\Leftarrow$ Suppose G has a ***k*-clique**. Then there is a non-contradictory **node** from the k-clique in each clause. Making each node in the k-clique **true** results in ϕ being true.

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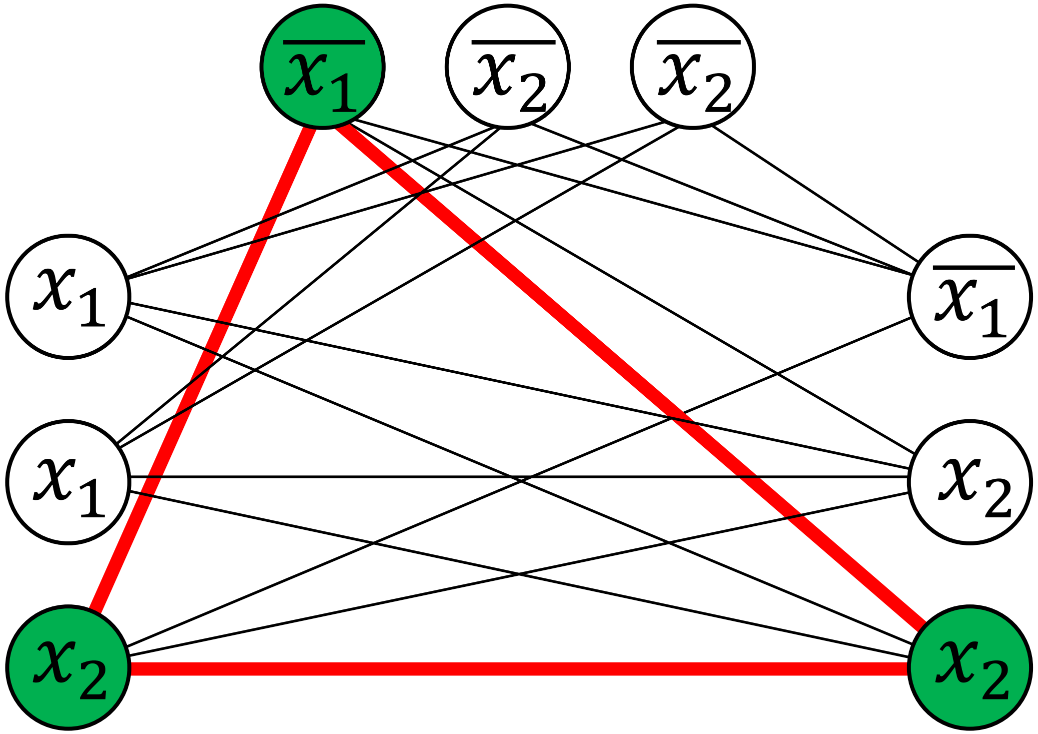

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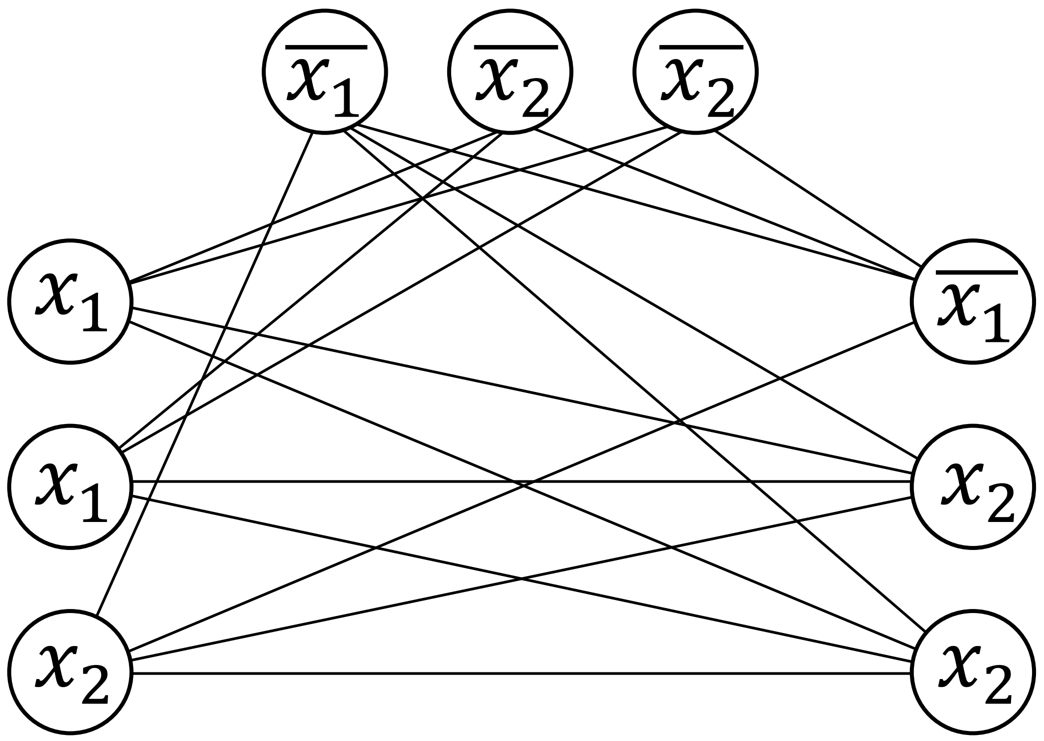


$CLIQUE$

Claim: $CLIQUE \in NP$-Complete

Proof:

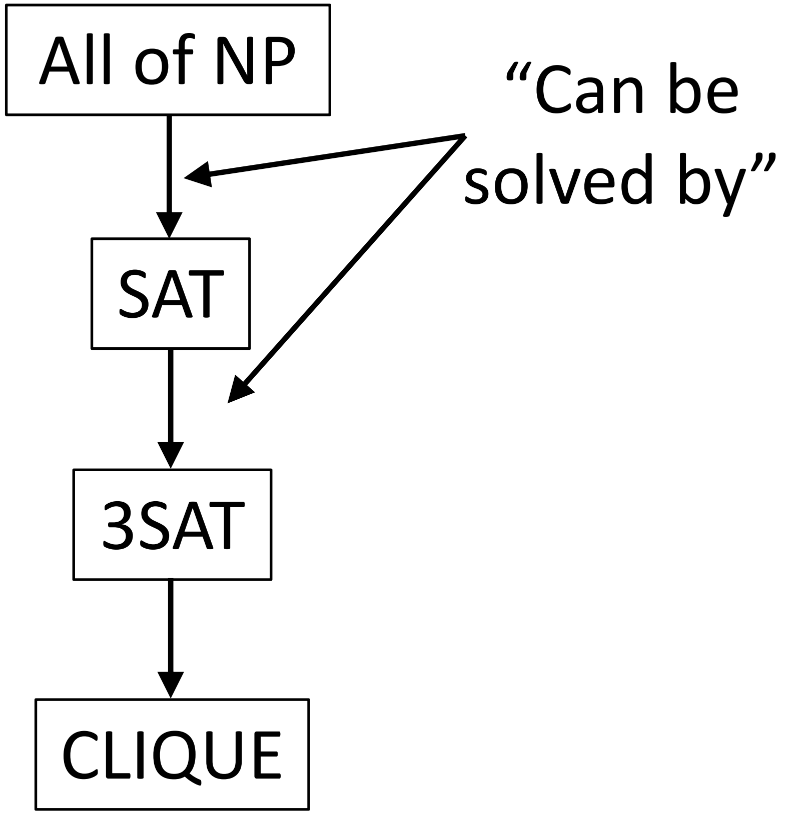
1. $CLIQUE \in NP$

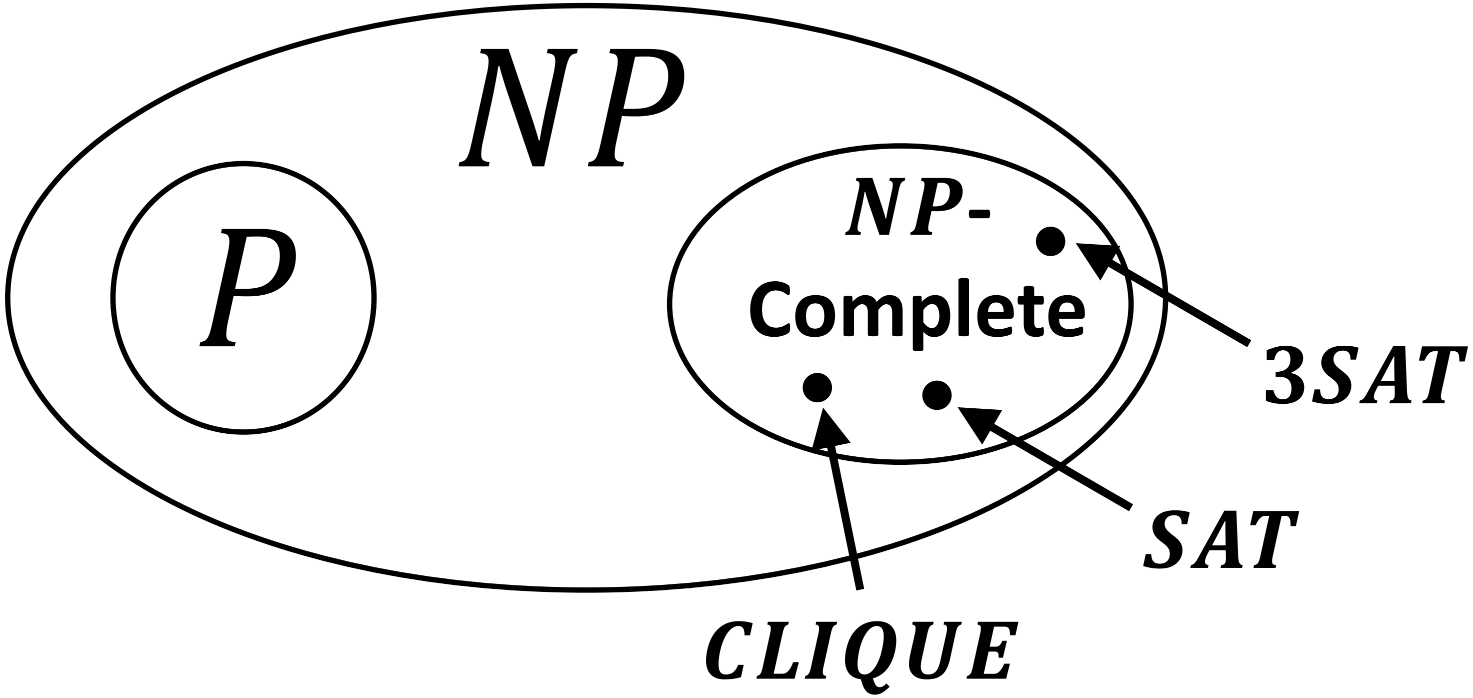
   Given a graph $G = (V, E)$, where $|V| = n$, and a subset $S \subseteq V$, where $|S| \geq k$, check if all pairs of vertices in S are in E. Running time: $O(n^2)$.

2. $3SAT \leq_P CLIQUE$

$$\therefore CLIQUE \in NP - C$$

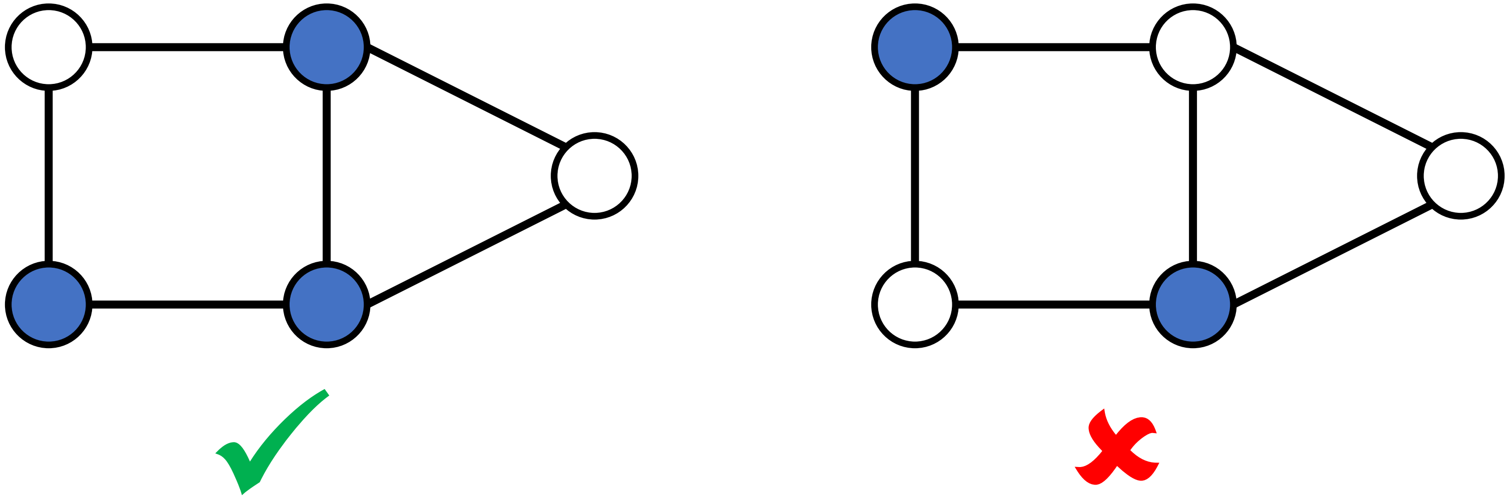
$NP - C$


All of NP
SAT
3SAT
CLIQUE
"Can be solved by"


NP
P
NP-Complete
3SAT
SAT
CLIQUE

Vertex Cover (VC)

Vertex Cover: Given graph $G = (V, E)$ and integer $k \leq |V|$, is there $V' \subseteq V$, with $|V'| \leq k$, such that each edge in E contains an end point in V'?



Vertex Cover (VC)

Claim: VC $\in NP$-Complete

Proof:

1. VC $\in NP$

2. ??? $\leq_P$ VC

Vertex Cover (VC)

Claim: VC ∈ NP-Complete

Proof:

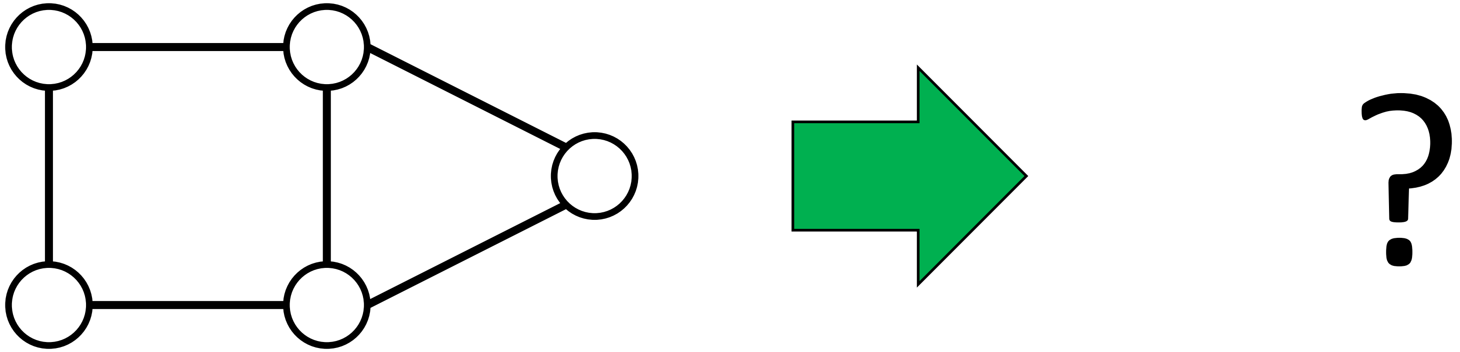
1. VC ∈ NP

2. $CLIQUE \leq_P$ VC

Vertex Cover (VC)

Claim: $CLIQUE \leq_P$ VC

Proof:



Clique

Vertex Cover

Vertex Cover (VC)

Claim: $CLIQUE \leq_P$ VC

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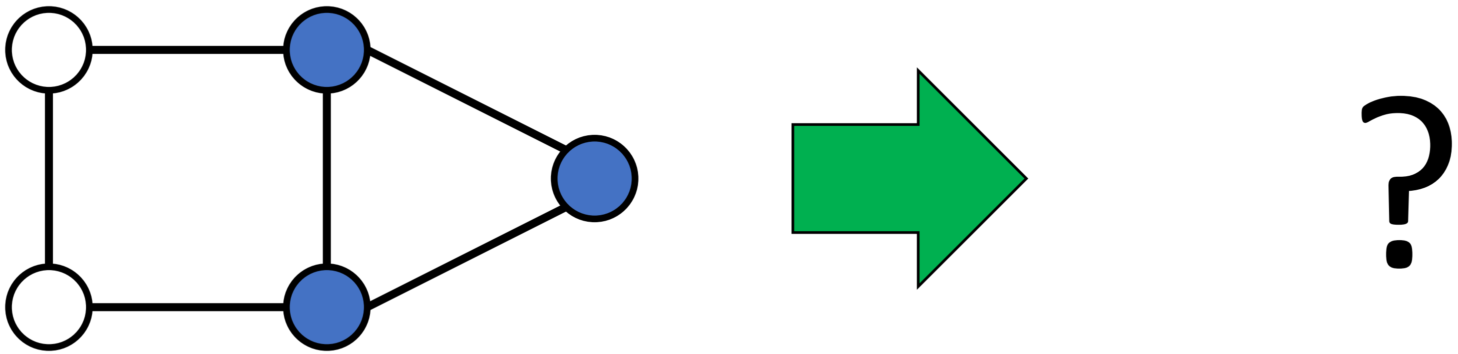

Clique　　　　　Vertex Cover

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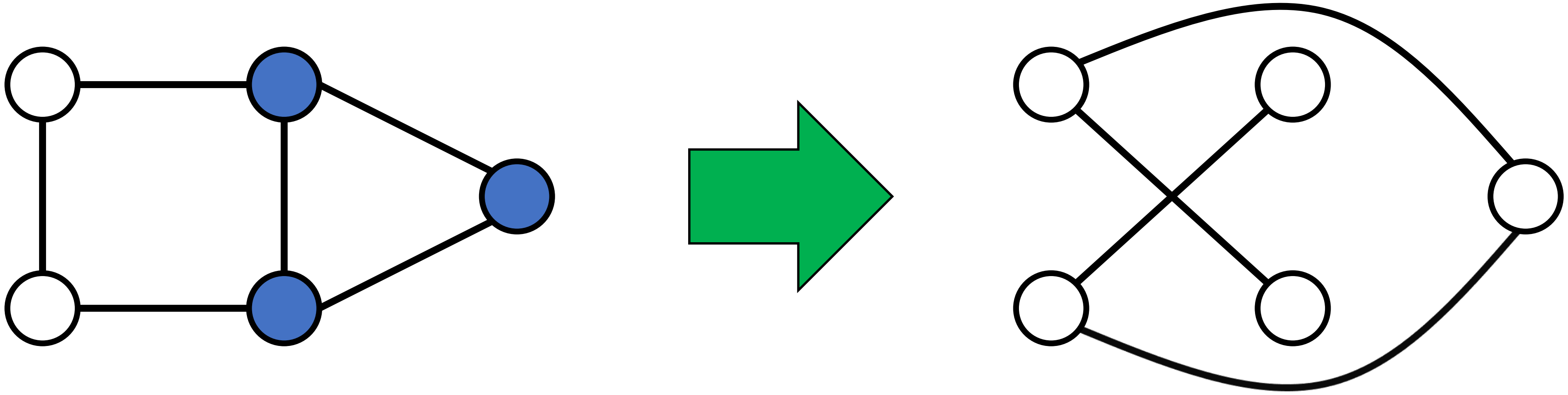

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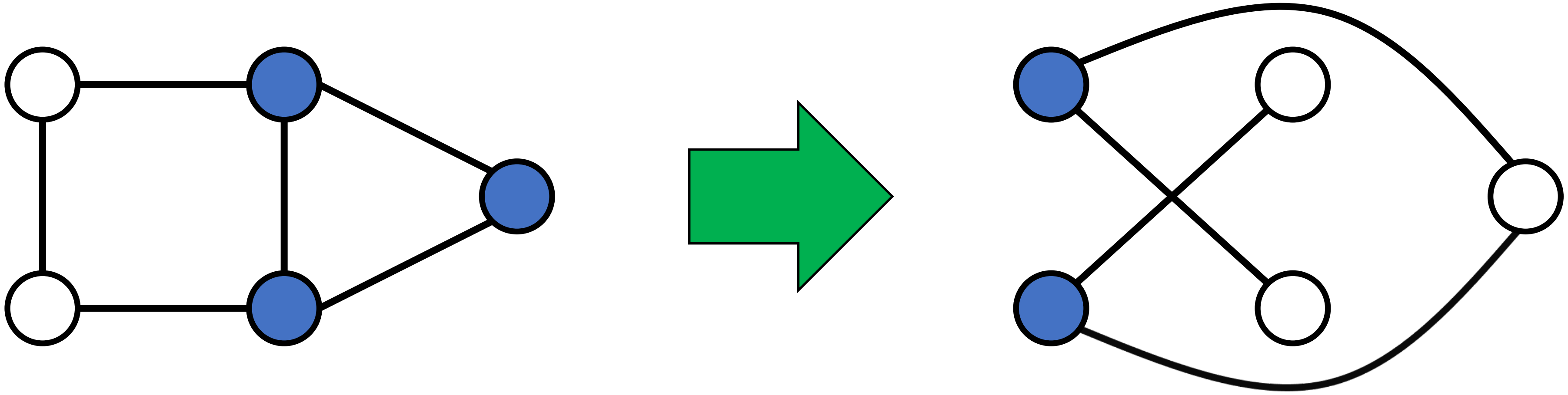

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Claim: $CLIQUE \leq_P \text{VC}$

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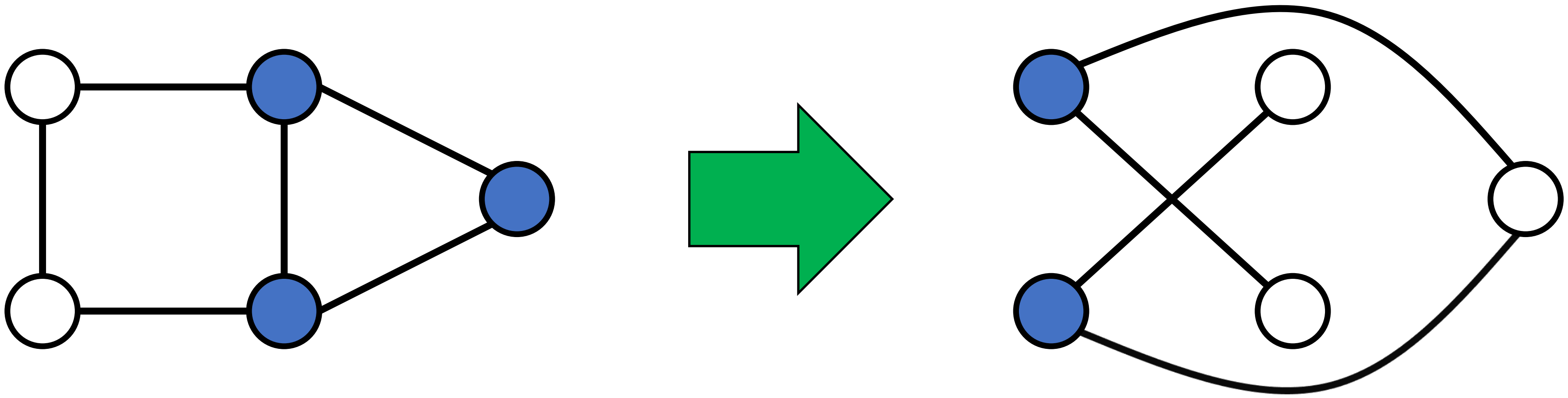

$\exists\ k$ - Clique $\iff$ $\exists\ (n-k)$ - Vertex Cover