Dominating Set
CSCI 338
Dominating Set

Dominating Set: Given a graph $G = (V, E)$ and integer $k \leq |V|$, is there a subset $V'$ of size $\leq k$, such that every vertex $\in V \setminus V'$ shares an edge with a vertex $\in V'$?
Dominating Set

Claim: Dominating Set $\in NP - C$

Proof:

???
Dominating Set

Claim: Dominating Set $\in NP - C$

Proof:
1. Show Dominating Set $\in NP$.

2. Show $A \leq_p$ Dominating Set, for some $A \in NP - C$. 

Dominating Set: For $G = (V, E)$ and $k \leq |V|$, $\exists V' \subseteq V$, $|V'| \leq k$, s.t. each $v \in V \setminus V'$ shares an edge with a $u \in V'$?
Dominating Set

Claim: Dominating Set $\in \text{NP} - \text{C}$

Proof:
1. Show Dominating Set $\in \text{NP}$. 

Dominating Set: For $G = (V, E)$ and $k \leq |V|$, $\exists V' \subseteq V$, $|V'| \leq k$, s.t. each $v \in V \setminus V'$ shares an edge with a $u \in V'$?
Dominating Set

Claim: Dominating Set $\in \mathcal{NP} - \mathcal{C}$

Proof:
1. Show Dominating Set $\in \mathcal{NP}$.

Given $G = (V, E)$, $k$ and a subset $V'$ of $V$, confirm that $|V| \geq k$ and that for each vertex $v \in V, v \in V'$ or there is some edge $(v, u) \in E$ such that $u \in V'$.

$O(n^3)$ running time $\Rightarrow$ Dominating Set $\in \mathcal{NP}$. 

Dominating Set: For $G = (V, E)$ and $k \leq |V|, \exists V' \subseteq V, |V'| \leq k$, s.t. each $v \in V \setminus V'$ shares an edge with a $u \in V'$?
Dominating Set

Claim: Dominating Set $\in NP - C$

Proof:
2. Show $A \leq_p$ Dominating Set, for some $A \in NP - C$. 

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Dominating Set

Claim: Dominating Set $\in NP - C$

Proof:
2. Show $A \leq_p$ Dominating Set, for some $A \in NP - C$.

SAT?
3SAT?
Clique?
Vertex Cover?
Dominating Set

Claim: Vertex Cover \(\leq_p\) Dominating Set

Proof:

Vertex Cover Solver

Dominating Set: For \(G = (V, E)\) and \(k \leq |V|\), \(\exists V' \subseteq V, |V'| \leq k\), s.t. each \(v \in V \setminus V'\) shares an edge with a \(u \in V'\)?

Vertex Cover: For \(G = (V, E)\) and \(k \leq |V|\), \(\exists V' \subseteq V, |V'| \leq k\), s.t. each edge contains an endpoint from \(V'\)?
Dominating Set

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Dominating Set: For $G = (V, E)$ and $k \leq |V|$, $\exists V' \subseteq V$, $|V'| \leq k$, s.t. each $v \in V \setminus V'$ shares an edge with a $u \in V'$.

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Dominating Set: For \(G = (V, E)\) and \(k \leq |V|\), \(\exists V' \subseteq V\), \(|V'| \leq k\), s.t. each vertex \(v \in V \setminus V'\) shares an edge with a vertex \(u \in V'\).

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**Dominating Set:** For $G = (V, E)$ and $k \leq |V|$, $\exists V' \subseteq V$, $|V'| \leq k$, s.t. each $v \in V \setminus V'$ shares an edge with a $u \in V'$?

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Dominating Set: For $G = (V, E)$ and $k \leq |V|$, $\exists V' \subseteq V$, $|V'| \leq k$, s.t. each $v \in V \setminus V'$ shares an edge with a $u \in V$.

Vertex Cover: For $G = (V, E)$ and $k \leq |V|$, $\exists V' \subseteq V$, $|V'| \leq k$, s.t. each edge contains an endpoint from $V'$.
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Claim: Vertex Cover $\leq_p$ Dominating Set

Proof: Let $G = (V, E)$, $k$ be input to the vertex cover problem, where $|V| = n$. 
Dominating Set

Claim: Vertex Cover \(\leq_p\) Dominating Set

Proof: Let \(G = (V, E)\), \(k\) be input to the vertex cover problem, where \(|V| = n\). Create \(G' = (V', E')\) as follows:
Claim: Vertex Cover $\leq_p$ Dominating Set

Proof: Let $G = (V, E)$, $k$ be input to the vertex cover problem, where $|V| = n$. Create $G' = (V', E')$ as follows: For each $e = (u, v) \in E$, add $u$ and $v$ to $V'$ and add $e$ to $E'$. For each $u \in V'$, add $u$ to $V' \subseteq V$, $|V'| \leq k$, s.t. each $v \in V \setminus V'$ shares an edge with a $u \in V'$. Dominating Set: For $G = (V, E)$ and $k \leq |V|$, $\exists V' \subseteq V, |V'| \leq k$, s.t. each $v \in V \setminus V'$ shares an edge with a $u \in V'$.
Claim: Vertex Cover $\leq_p$ Dominating Set

Proof: Let $G = (V, E)$, $k$ be input to the vertex cover problem, where $|V| = n$. Create $G' = (V', E')$ as follows: For each $e = (u, v) \in E$, add $u$ and $v$ to $V'$ and add $e$ to $E'$. Also, add a new vertex $w$ to $V'$ and add edges $(u, w)$ and $(v, w)$ to $E'$.

Dominating Set: For $G = (V, E)$ and $k \leq |V|$, $\exists V' \subseteq V$, $|V'| \leq k$, s.t. each $v \in V \setminus V'$ shares an edge with a $u \in V'$?
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Proof: Let $G = (V, E)$, $k$ be input to the vertex cover problem, where $|V| = n$. Create $G' = (V', E')$ as follows: For each $e = (u, v) \in E$, add $u$ and $v$ to $V'$ and add $e$ to $E'$. Also, add a new vertex $w$ to $V'$ and add edges $(u, w)$ and $(v, w)$ to $E'$. $O(n + n^2 + n^2 + 2n^2) = O(n^2)$ time.
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Claim: Vertex Cover $\leq_p$ Dominating Set

Proof: Turn $G = (V, E)$ into $G' = (V', E')$ as follows: For each $e = (u, v) \in E$, add $u$ and $v$ to $V'$ and add $e$ to $E'$. Also, add a new vertex $w$ to $V'$ and add edges $(u, w)$ and $(v, w)$ to $E'$. $O(n + n^2 + n^2 + 2n^2) = O(n^2)$ time.
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$G$ has a $k$-VC $\iff$ $G'$ has a $k$-DS.