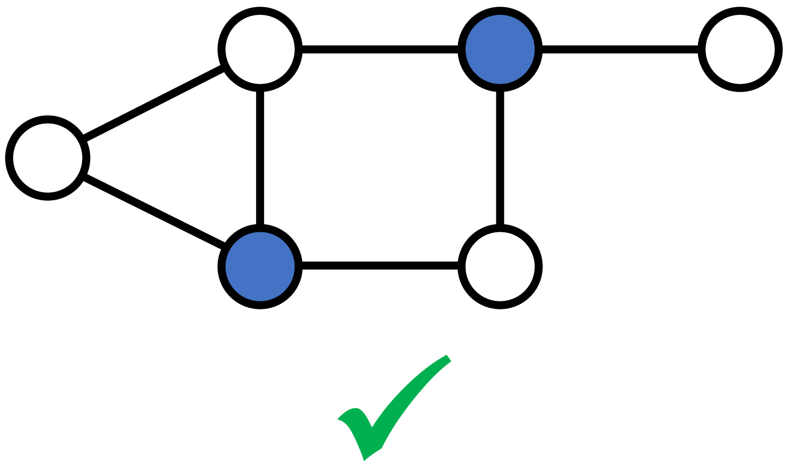
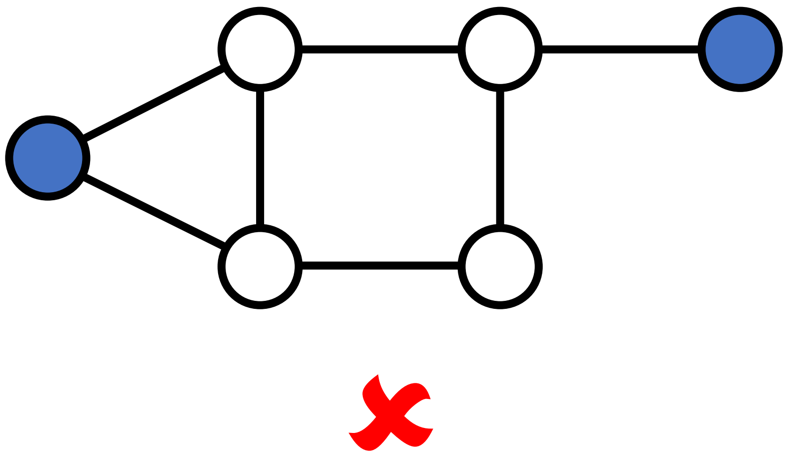
Dominating Set
CSCI 338

Dominating Set

Dominating Set: Given a graph $G = (V, E)$ and integer $k \leq |V|$, is there a subset V' of size $\leq k$, such that every vertex $\in V \backslash V'$ shares an edge with a vertex $\in V'$?





Dominating Set

Dominating Set: For $G = (V, E)$ and $k \leq |V|$, $\exists\, V' \subseteq V$, $|V'| \leq k$, s.t. each $v \in V \setminus V'$ shares an edge with a $u \in V'$?

Claim: Dominating Set $\in NP - C$

Proof:

???

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Claim: Dominating Set $\in NP - C$

Proof:

1. Show Dominating Set $\in NP$.

2. Show $A \leq_P$ Dominating Set, for some $A \in NP - C$.

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Claim: Dominating Set $\in NP - C$

Proof:

1. Show Dominating Set $\in NP$.

   Given $G = (V, E)$, k and a subset V' of V, confirm that $|V| \geq k$ and that for each vertex $v \in V$, $v \in V'$ or there is some edge $(v, u) \in E$ such that $u \in V'$.

   $O(n^3)$ running time $\Rightarrow$ Dominating Set $\in NP$.

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SAT?

3SAT?

Clique?

Vertex Cover?

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Vertex Cover: For $G = (V, E)$ and $k \leq |V|$, $\exists\, V' \subseteq V$, $|V'| \leq k$, s.t. each edge contains an endpoint from V'?

Claim: Vertex Cover $\leq_P$ Dominating Set

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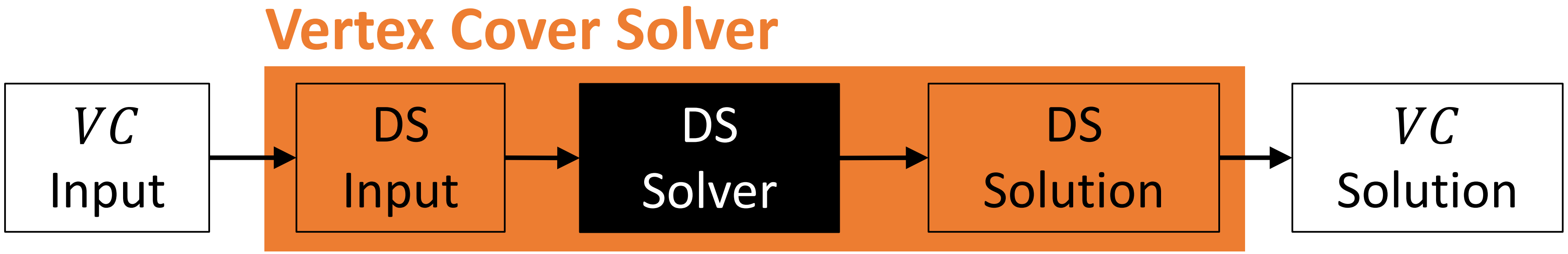

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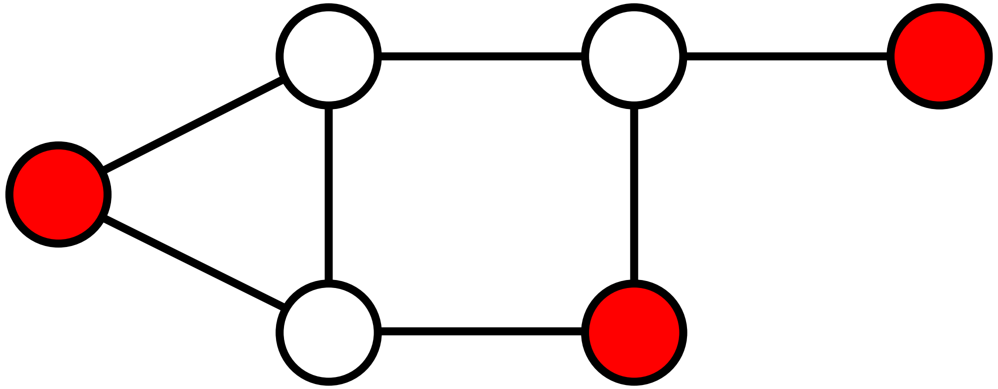

Vertex Cover ⇨ Dominating Set

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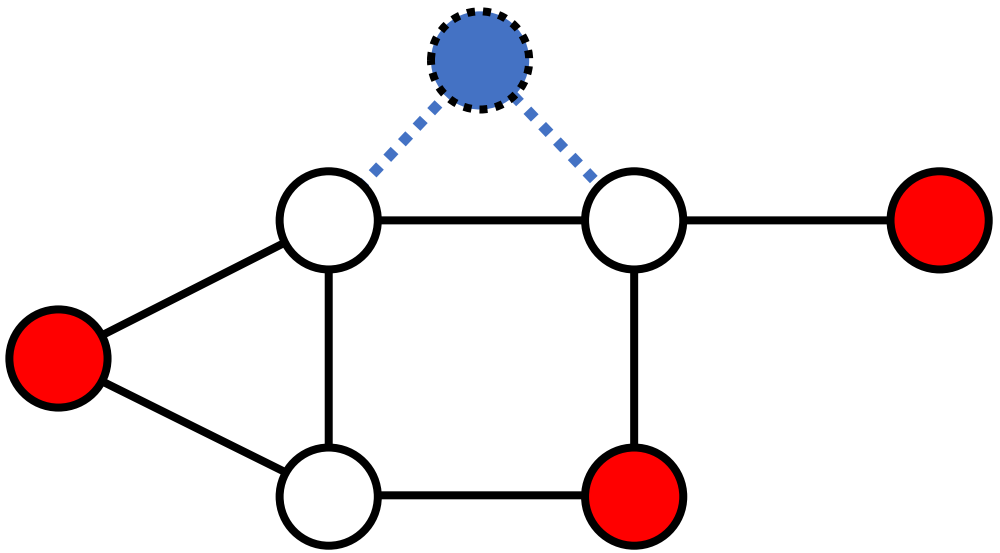

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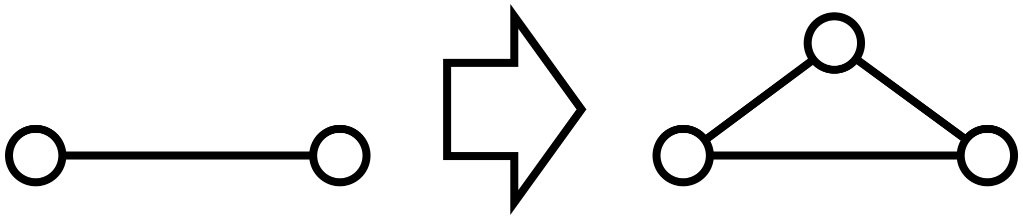

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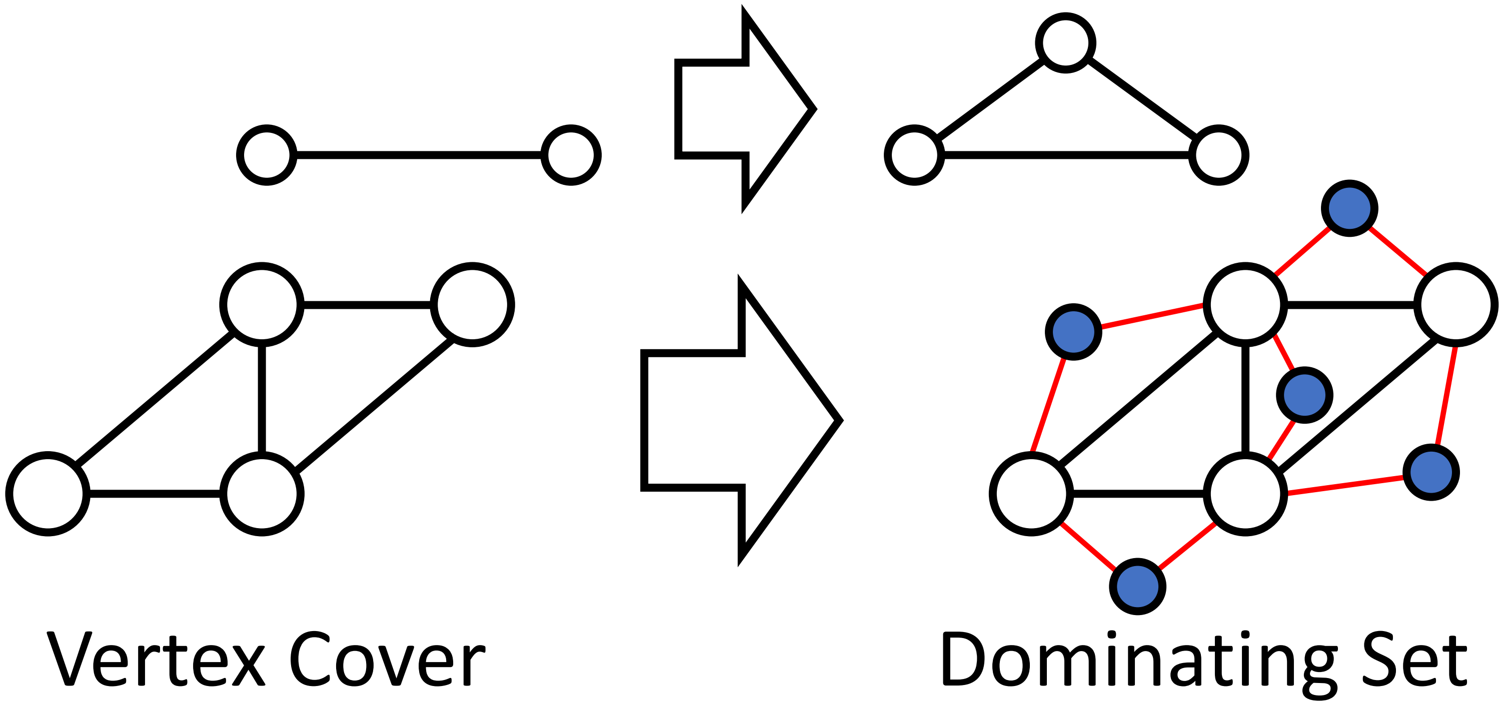

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Proof: Let $G = (V, E)$, k be input to the vertex cover problem, where $|V| = n$.

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Proof: Let $G = (V, E)$, k be input to the vertex cover problem, where $|V| = n$. Create $G' = (V', E')$ as follows:

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Proof: Let $G = (V, E)$, k be input to the vertex cover problem, where $|V| = n$. Create $G' = (V', E')$ as follows: For each $e = (u, v) \in E$, add u and v to V' and add e to E'.

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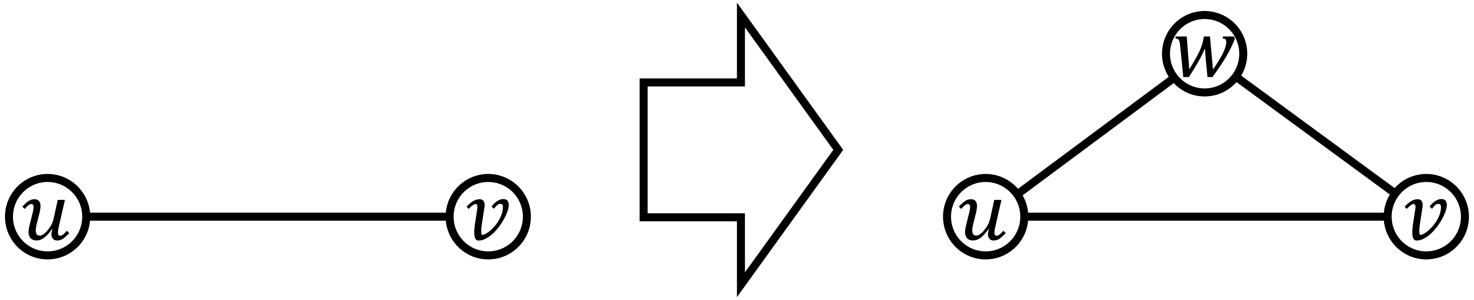

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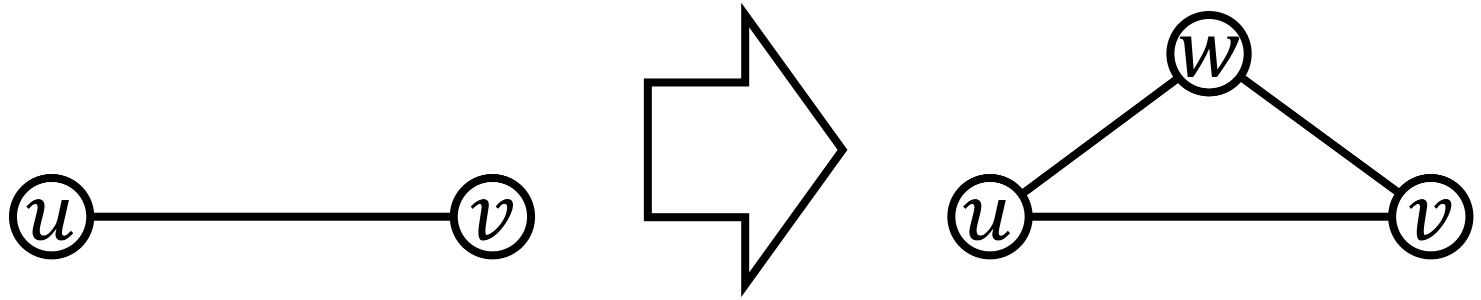

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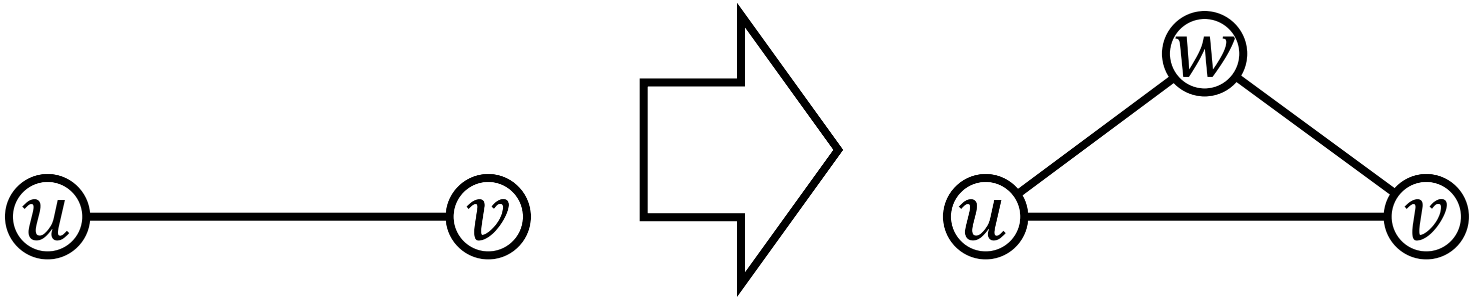
Claim: Vertex Cover $\leq_P$ Dominating Set

Proof: Turn $G = (V, E)$ into $G' = (V', E')$ as follows: For each $e = (u, v) \in E$, add u and v to V' and add e to E'. Also, add a new vertex w to V' and add edges (u, w) and (v, w) to E'. $O(n + n^2 + n^2 + 2n^2) = O(n^2)$ time.



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G has a k-VC $\Leftrightarrow$ G' has a k-DS.