Finite Automata
CSCI 338
DFA Formal Definition

DFAs consist of:
1. Finite set of states, \( Q \).
2. Finite alphabet, \( \Sigma \).
3. Transition function, \( \delta : Q \times \Sigma \rightarrow Q \).
4. Start state, \( q_0 \in Q \).
5. Set of accept states, \( F \subseteq Q \).

\[
\begin{align*}
Q &= \{q_1, q_2, q_3\} \\
\Sigma &= \{0, 1\} \\
\delta : &\begin{array}{c|cc} 
0 & 1 \\
q_1 & q_1 & q_2 \\
q_2 & q_3 & q_2 \\
q_3 & q_2 & q_2 \\
\end{array} \\
\text{Start state} &= q_1 \\
F &= \{q_2\}
\end{align*}
\]
DFA Practice

Prove that the following languages are regular:
1. Set of all strings over \( \{0,1\} \).

2. Set of all strings with an even number of 0s.

3. Set of all strings that contain the substring: 10.
Prove that the following language is regular: 
\{\omega: \omega \text{ begins with sequence } 10\}.

Proof:
Prove that the following language is regular:
\[\{\omega : \omega \text{ begins with sequence } 10\}\].

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Proof:
Prove that the following language is regular:
\[ \{ \omega : \omega \text{ begins with sequence 10} \} \].

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Prove that the following language is regular:
\{ \omega: \omega \text{ begins with sequence } 10 \}.

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Prove that the following language is regular:
\[ \{ \omega : \omega \text{ begins with sequence } 10 \} . \]

Proof:
Prove that the following language is regular: 
\{\omega: \omega \text{ begins with sequence } 10\}.

Proof:

![Diagram](image)
Prove that the following language is regular:
\[ \{ \omega : \omega \text{ ends with sequence } 10 \} \].

Proof:
Prove that the following language is regular:
\( \{ \omega : \omega \text{ ends with sequence } 10 \} \).

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Prove that the following language is regular:
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Prove that the following language is regular: 
\{\omega: \omega \text{ ends with sequence } 10\}.

Proof:

What about 1010?
Prove that the following language is regular: \( \{ \omega: \omega \text{ ends with sequence 10} \} \).

Proof:
Prove that the following language is regular:
\[ \{ \omega : \omega \text{ ends with sequence } 10 \} \].

Proof:

If you had to name this state, what would you name it?
Prove that the following language is regular: 
\{ \omega : \omega \text{ ends with sequence } 10 \}.

Proof:

If you had to name this state, what would you name it? 
“We just processed a \textbf{1} state”
Prove that the following language is regular:
\( \{ \omega : \omega \text{ ends with sequence 10} \} \).

Proof:
Prove that the following language is regular: \( \{ \omega : \omega \text{ ends with sequence } 10 \} \).

Proof:
Empty string vs. Empty set
Empty string vs. Empty set

\( \varepsilon \) is called the empty string. It is the string that contains no characters.
\[ L = \{ \varepsilon \} \]

Empty string vs. Empty set

\( \varepsilon \) is called the empty string. It is the string that contains no characters.
\(L = \{\varepsilon\}\)

Empty string vs. Empty set

\(\varepsilon\) is called the empty string. It is the string that contains no characters.
$L = \{ \varepsilon \}$

Empty string vs. Empty set

$\varepsilon$ is called the empty string. It is the string that contains no characters.

$\emptyset$ is called the empty set. It is the set that contains no elements.
Empty string vs. Empty set

\( L = \{ \varepsilon \} \) vs. \( L = \emptyset \)

\( \varepsilon \) is called the empty string. It is the string that contains no characters.

\( \emptyset \) is called the empty set. It is the set that contains no elements.
Empty string vs. Empty set

\( \varepsilon \) is called the empty string. It is the string that contains no characters.

\( \emptyset \) is called the empty set. It is the set that contains no elements.
Prove that the following language is regular:
\{ \omega : \omega \text{ starts and ends with a } 0 \}.

Proof:
Prove that the following language is regular:
\{ \omega : \omega \text{ starts and ends with a } 0 \}.

Proof:
Prove that the following language is regular:
\[ \{ \omega : \omega \text{ starts and ends with a } 0 \} \].

Proof:

The string \( \omega = 0 \) starts and ends with a 0 and must be accepted!
Prove that the following language is regular:
\{ \omega : \omega \text{ starts and ends with a } 0 \}.

Proof:
Prove that the following language is regular:
\{ \omega : \omega \text{ starts and ends with a } 0 \}.

Proof:
Prove that the following language is regular: 
\[ \{ \omega: \omega \text{ starts and ends with a } 0 \} \].

Proof:

![DFA Diagram]

1. The DFA starts in the initial state.
2. It reads 0 and moves to the final state.
3. The DFA continues to read 1 and cycles between the two states.
4. It reads 0 and returns to the initial state.
5. The DFA accepts strings that start and end with 0.

This DFA recognizes the language \( \{ \omega: \omega \text{ starts and ends with a } 0 \} \).
Prove that the following language is regular: 
\{ \omega : \omega \text{ starts and ends with a } 0 \}.

Proof:
Prove that the following language is regular:
\{\omega: \omega \text{ starts and ends with a } 0\}.

Proof:
Prove that the following language is regular:
\( \{ \omega: \omega \text{ starts and ends with a } 0 \} \).

Proof:
Prove that the following language is regular:
\[ \{ \omega : \omega \text{ starts and ends with a } 0 \} \].

Proof:

\[ \exists \text{ NFA that recognizes the language} \]
Prove that the following language is regular: 
\{\omega: \omega \text{ starts and ends with a } 0\}.

Proof:

\[ \omega = 0110. \text{ Accept or Reject?} \]
Prove that the following language is regular: \( \{ \omega: \omega \text{ starts and ends with a 0} \} \).

Proof:

\[
\omega = 0110. \text{ Accept or Reject? It rejects but should accept!}
\]
Prove that the following language is regular:
\[ \{ \omega : \omega \text{ starts and ends with a } 0 \} \].

Proof:
Prove that the following language is regular:
\{\omega: \omega\text{ starts and ends with a 0}\}.

Proof:
Prove that the following language is regular: 
\{ \omega: \omega \text{ starts and ends with a } 0 \}. 

Proof:
Prove that the following language is regular:
\[ \{ \omega: \omega \text{ starts and ends with a } 0 \} \].

Proof:
Prove that the following language is regular:
\{ \omega : \omega \text{ starts and ends with a } 0 \}. 

Proof:
Prove that the following language is regular:
\( \{ \omega : \omega \text{ starts and ends with a } 0 \} \).

Proof:
Prove that the following language is regular:
\{ \omega: \omega \text{ consists of some number of } 0\text{s} \text{ followed by the same number of } 1\text{s} \}. \text{ E.g. } 000111

Proof: