Nondeterminism
CSCI 338

DFA vs NFA

Deterministic Finite Automaton (DFA):

Nondeterministic Finite Automaton (NFA):

DFA vs NFA

Deterministic Finite Automaton (DFA):

- Every state has exactly one transition for every character ($e \in \Sigma$).

Nondeterministic Finite Automaton (NFA):

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- Allowed to have multiple (or 0) transitions for each $e \in \Sigma$.

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- Allowed to have multiple (or 0) transitions for each $e \in \Sigma$.
- Allowed to have transitions that happen without input (ε-transition).

DFA vs NFA

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- Every state has exactly one transition for every character ($e \in \Sigma$).
- For the same input, everyone takes the same path to the same final state.

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- Every state has exactly one transition for every character ($e \in \Sigma$).
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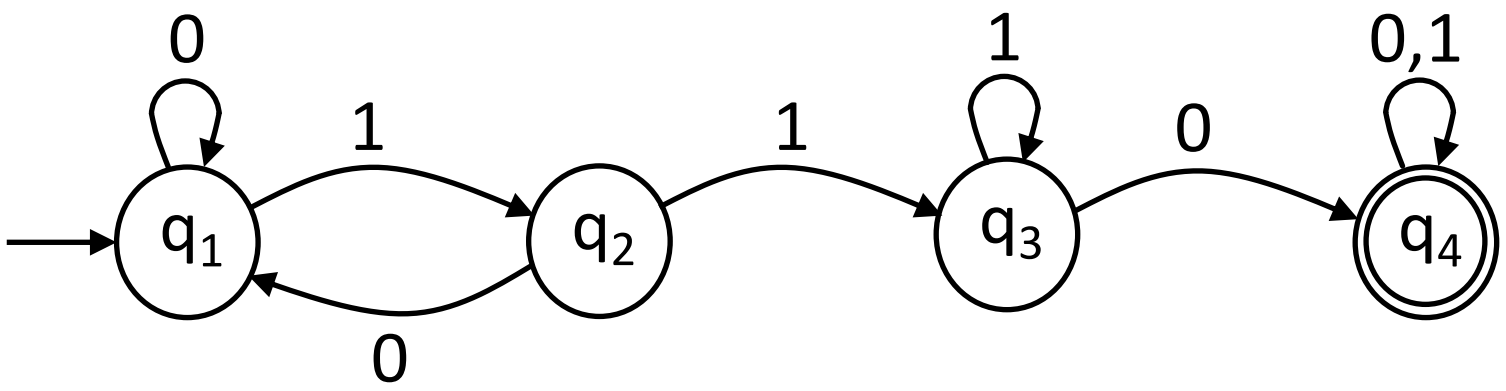
Nondeterministic Finite Automaton (NFA):

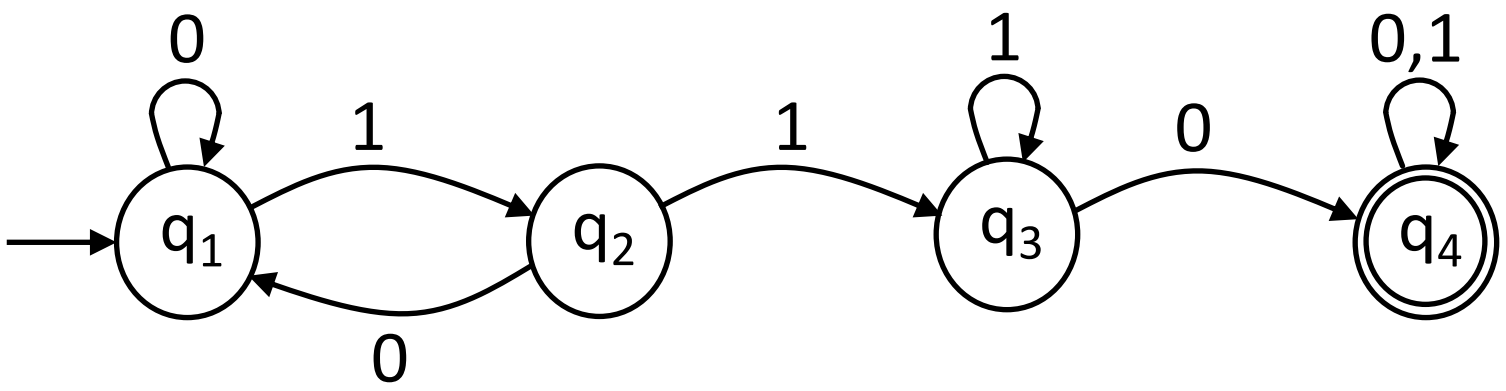
- Allowed to have multiple (or 0) transitions for each $e \in \Sigma$.
- Allowed to have transitions that happen without input (ε-transition).
- Processing strings is different.

DFA Processing



$\{\omega\colon\ \omega$ contains the substring $110\}$

DFA Processing

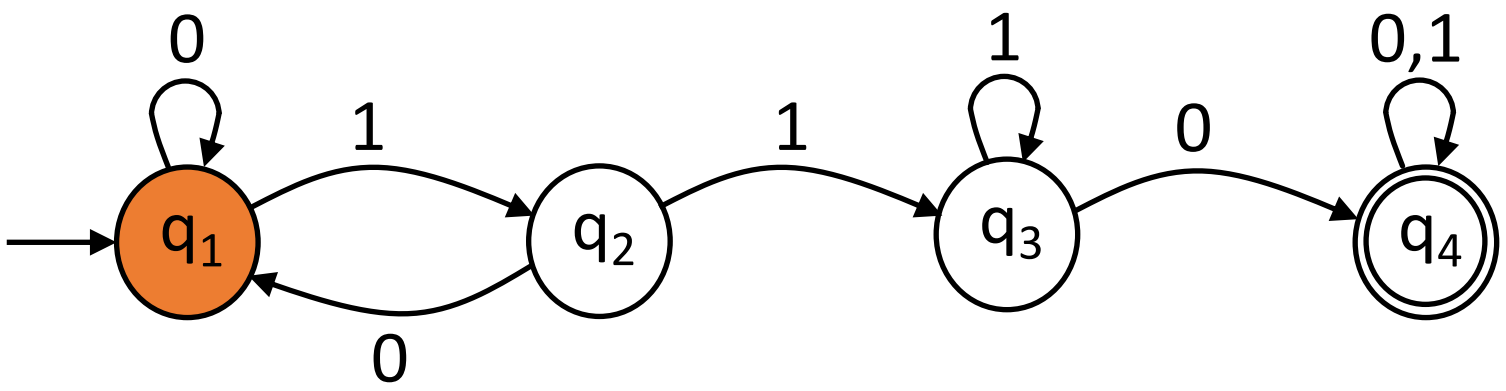


$\{\omega$: ω contains the substring $110\}$

$\omega = 1110$

DFA Processing

q_1



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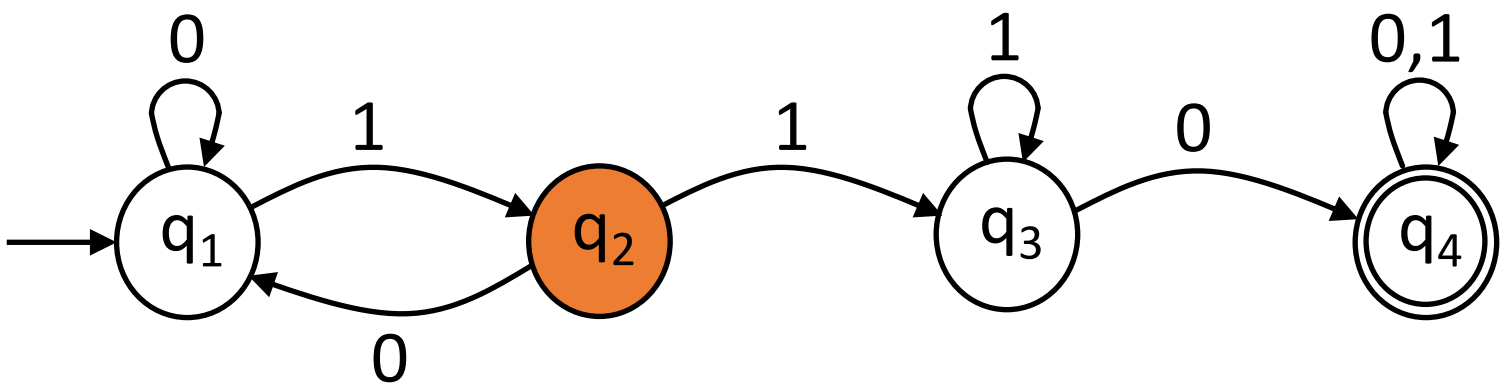
DFA Processing



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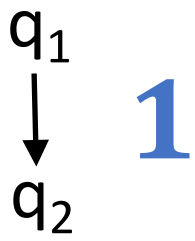
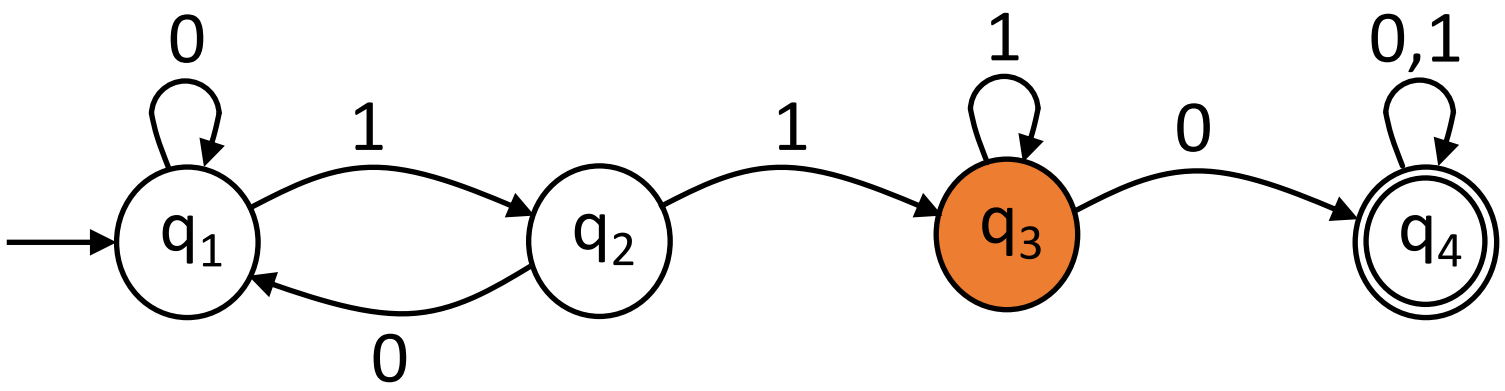


$\omega = \mathbf{1}110$

DFA Processing



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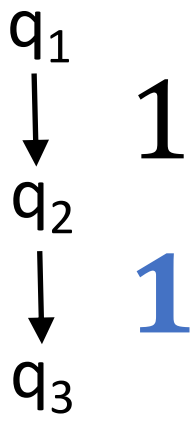

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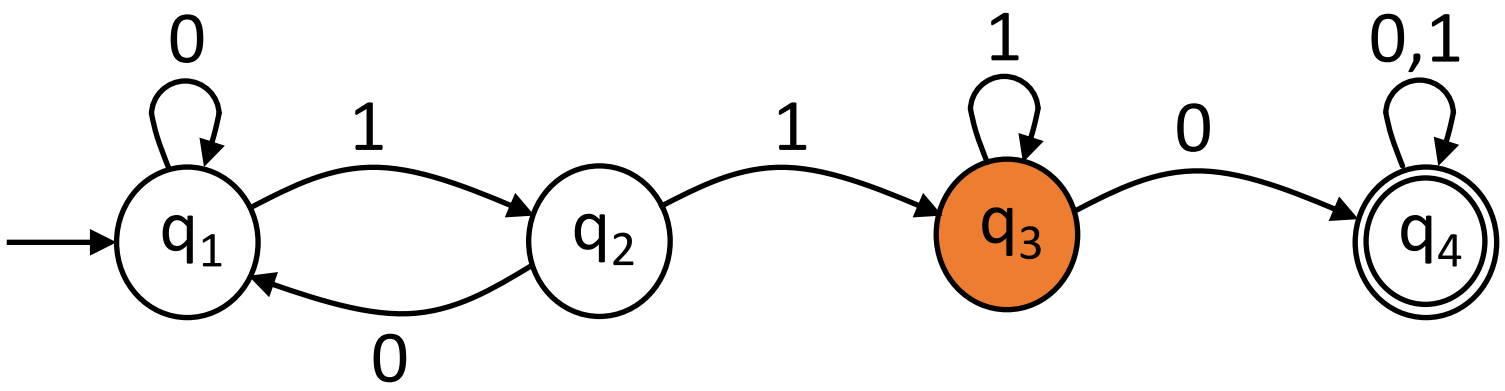
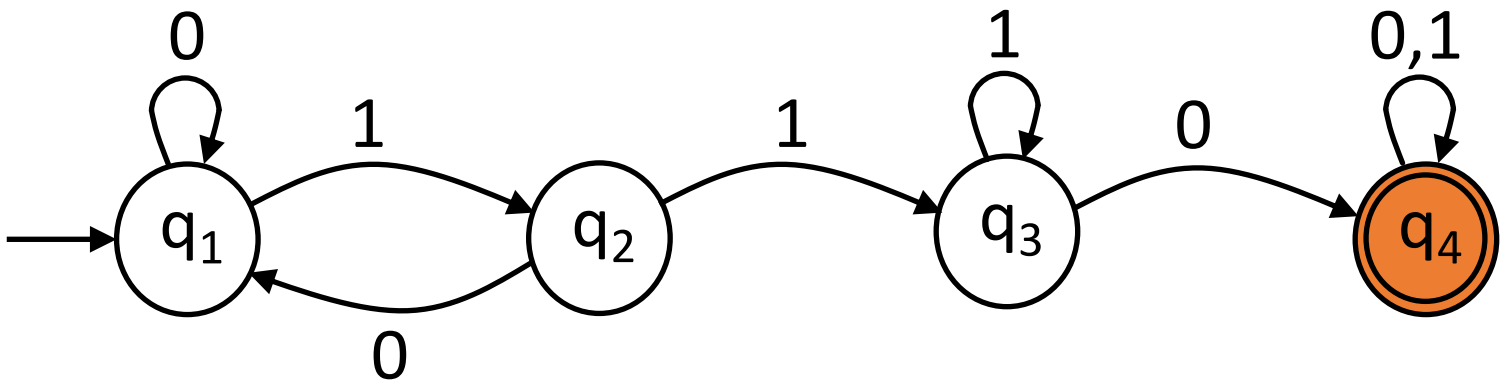


$\{\omega$: ω contains the substring $110\}$

q_1
↓ 1
q_2
↓ 1
q_3
↓ **1**
q_3

$\omega = 11\mathbf{1}0$

DFA Processing



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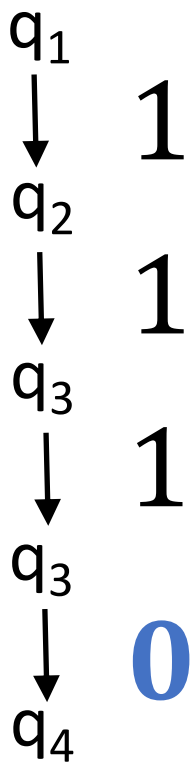

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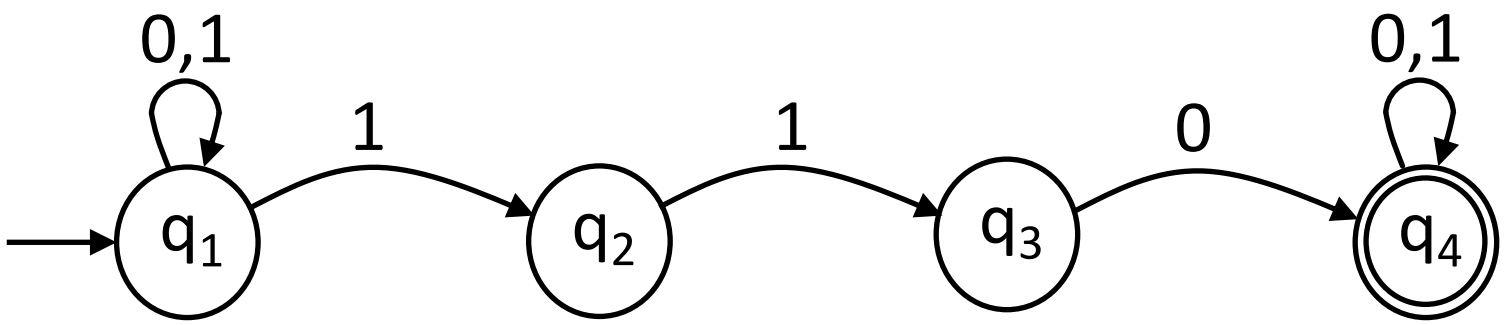
NFA Processing



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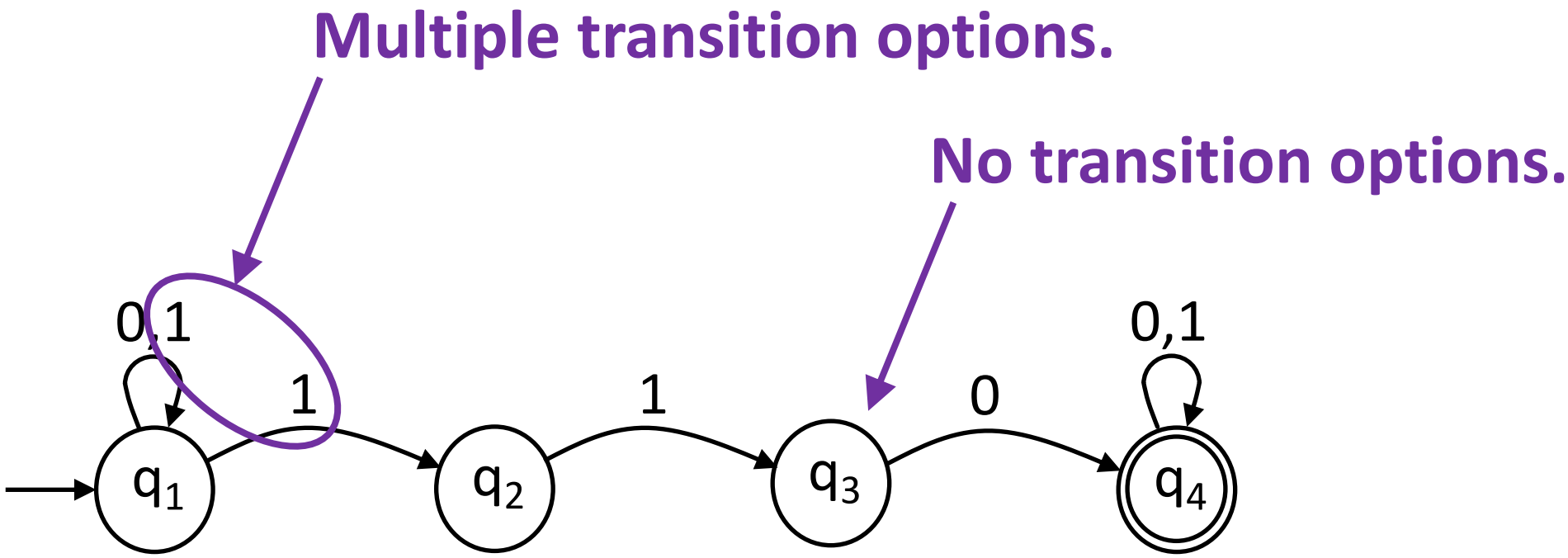

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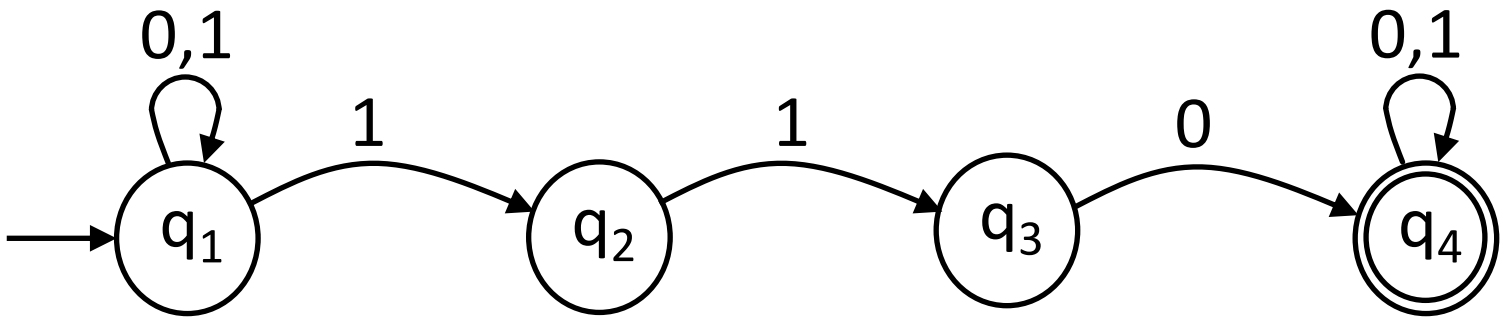

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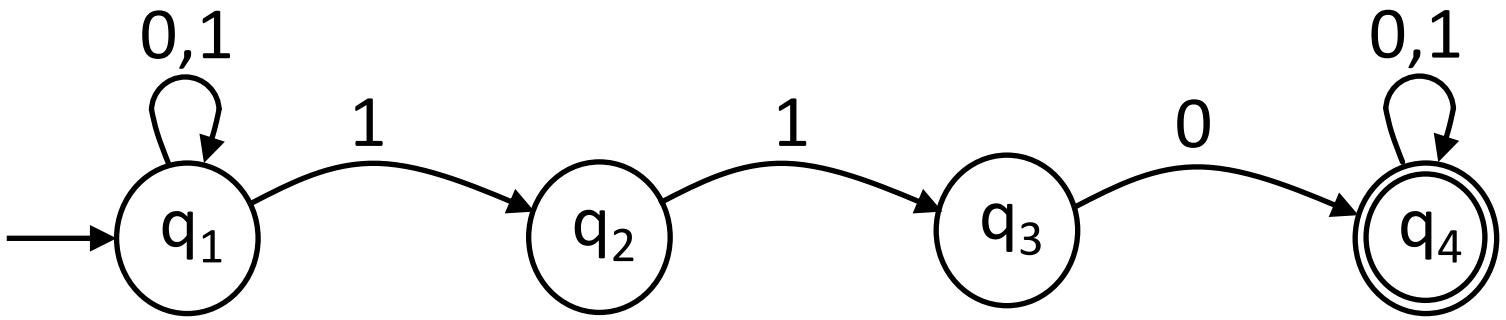

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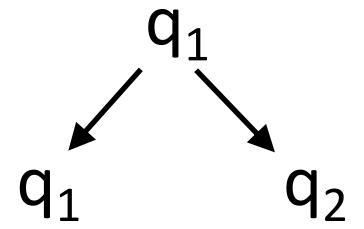
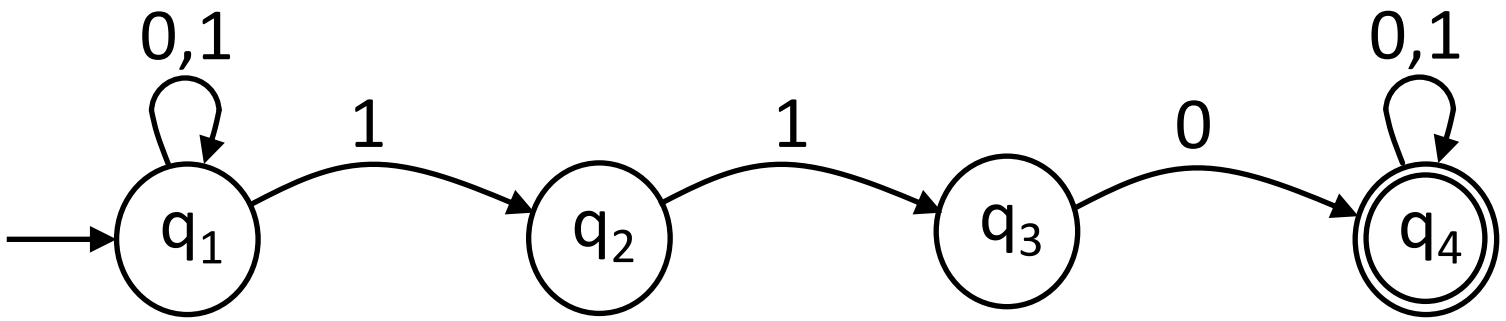


1



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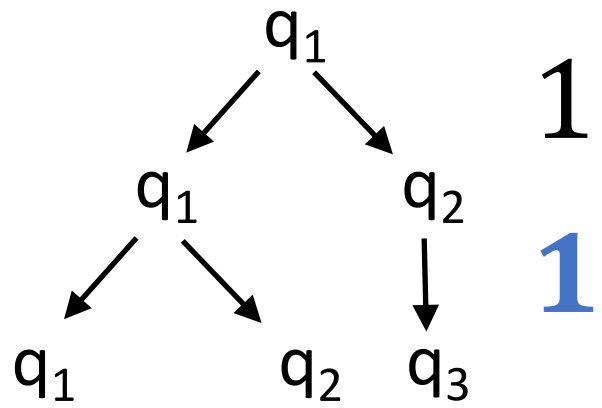

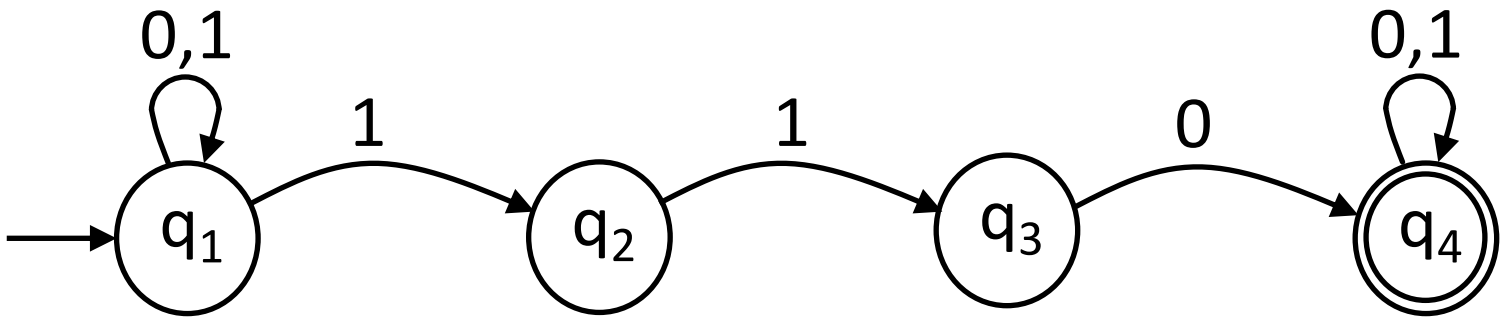

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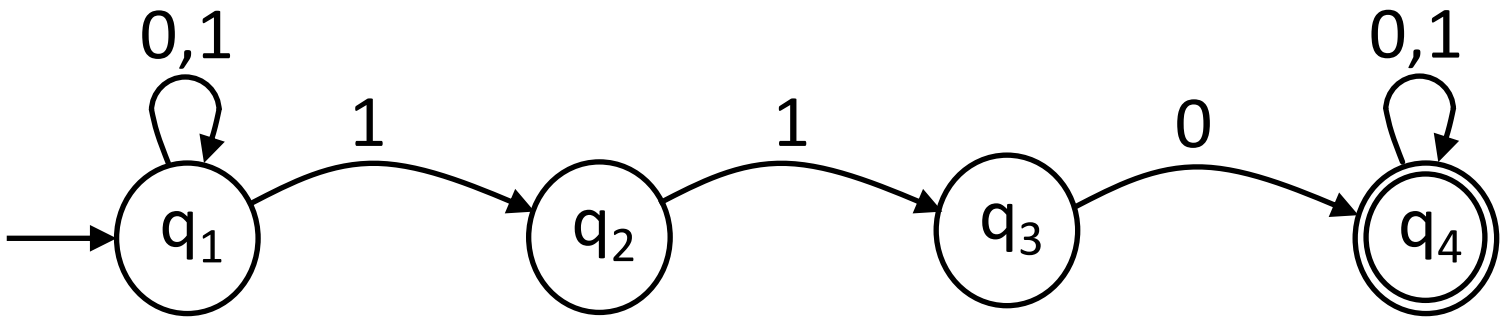

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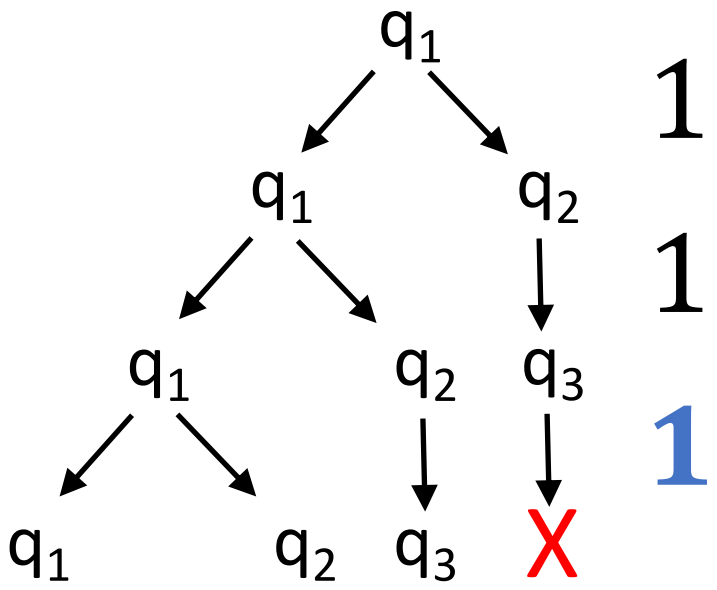

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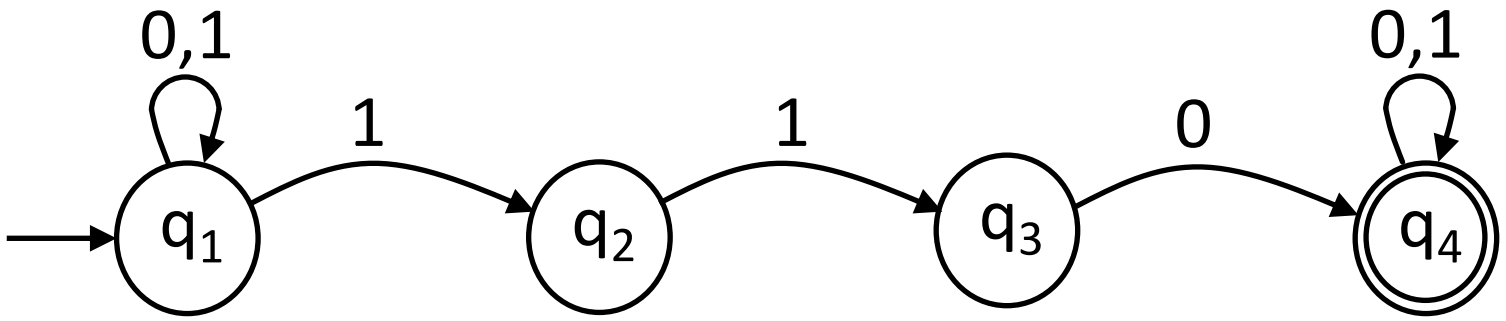

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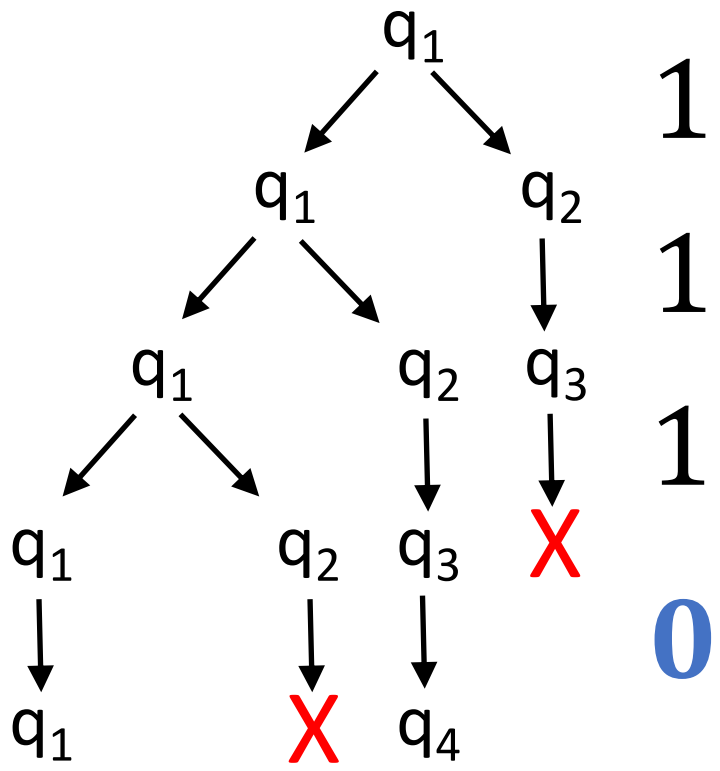

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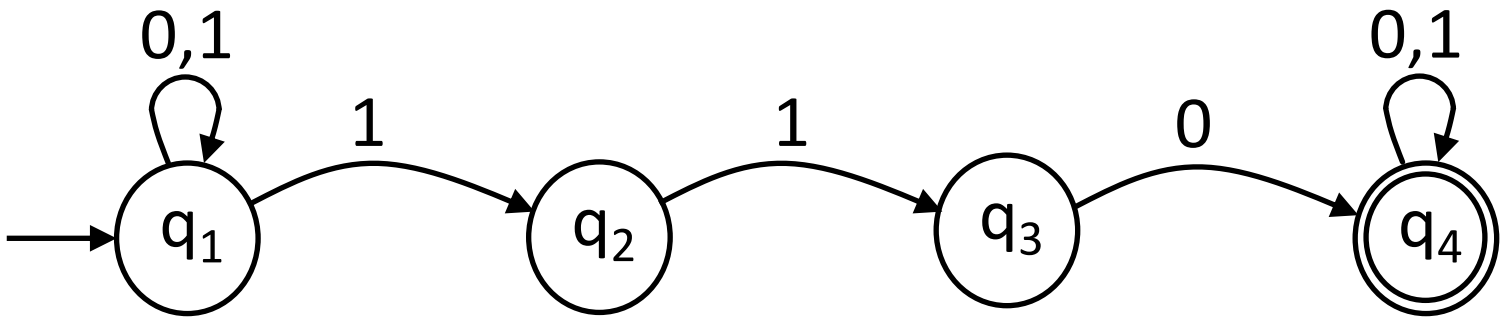

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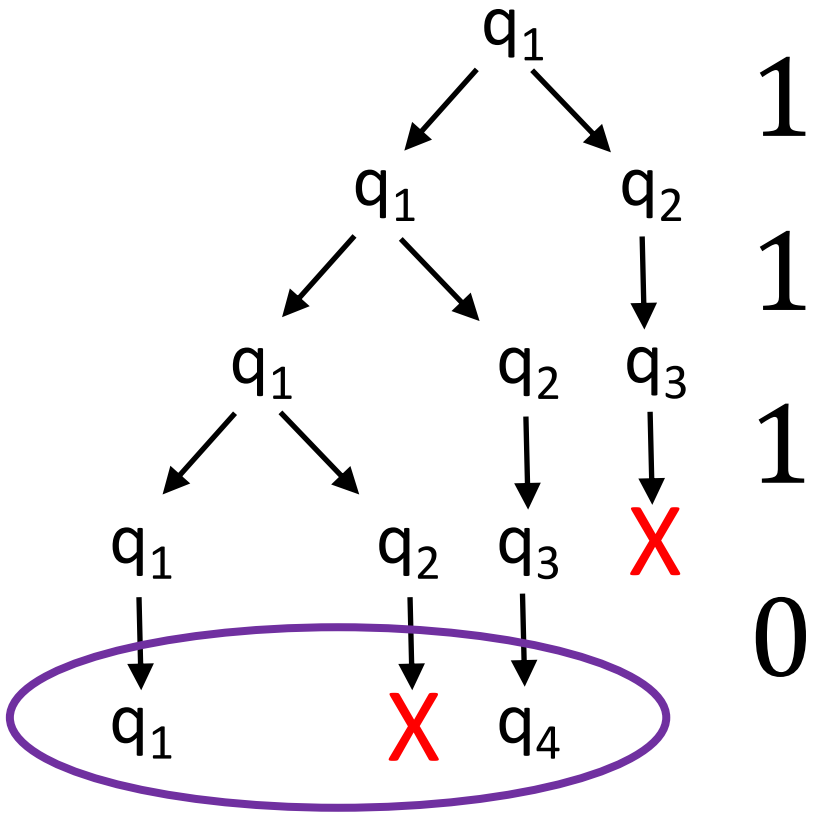
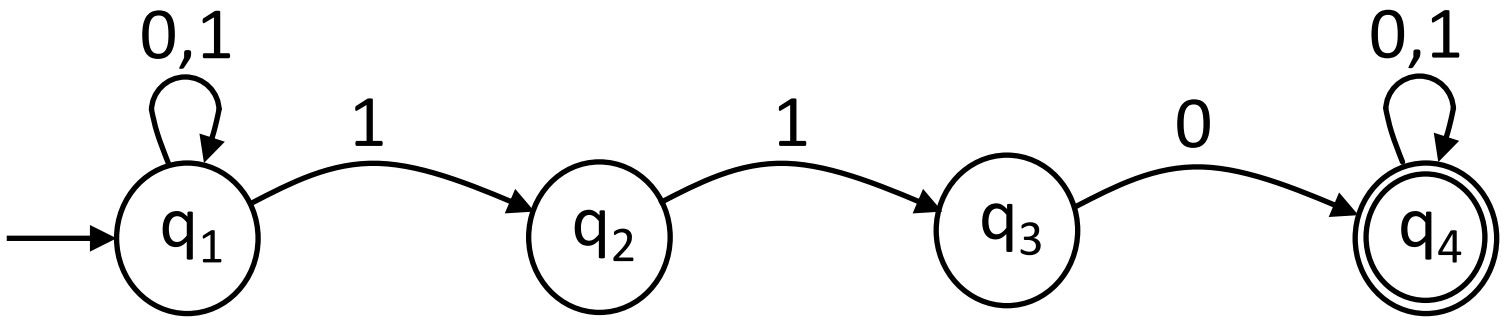


$\omega = 1110$

NFA Processing

1. If a "decision" is encountered, split
   ar
2. If
   ou
3. If
   accept. If not, reject.

Alternate approach: If it is possible to end on an accept state, accept.



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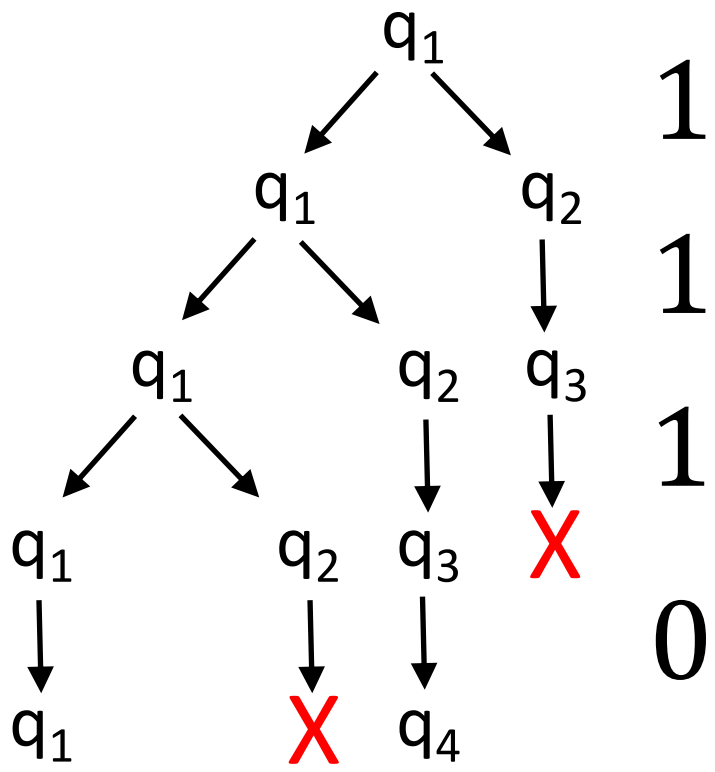

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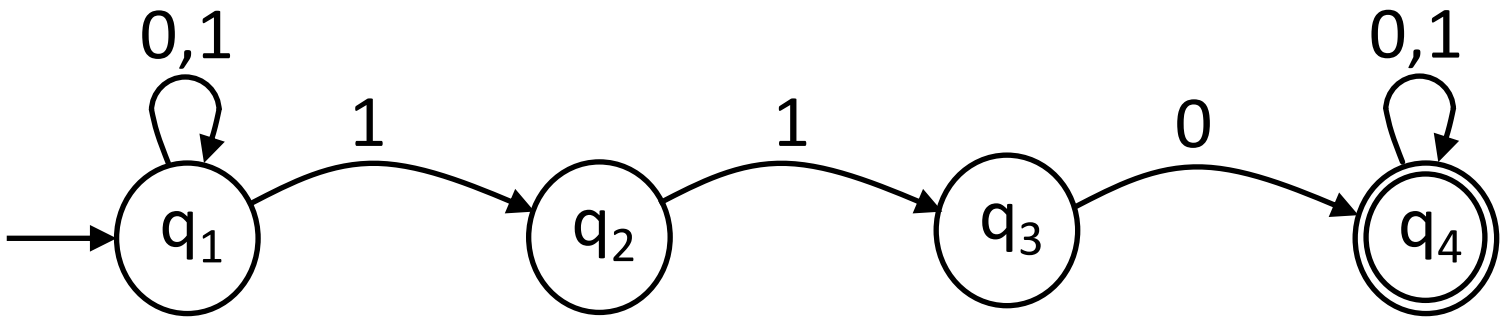
?



$\{\omega\colon\ \omega$ contains the substring $110\}$

$\omega = 101$

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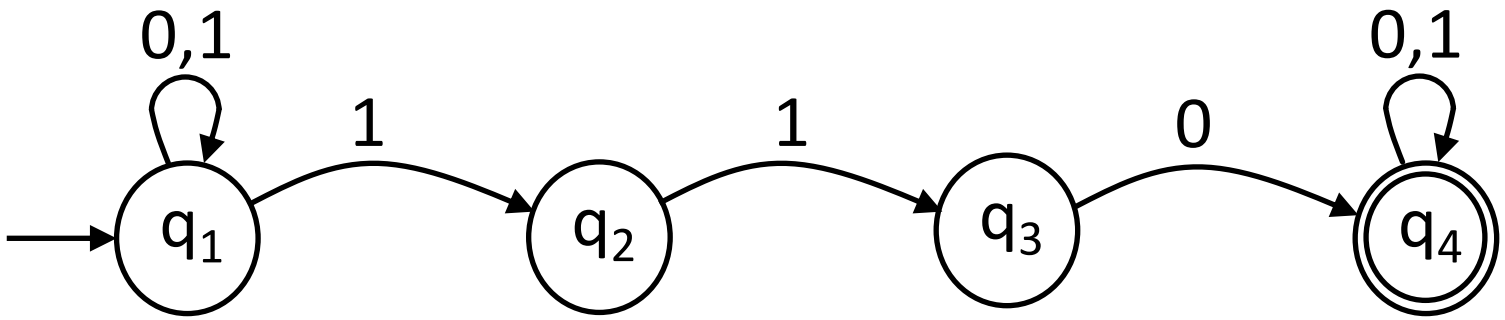

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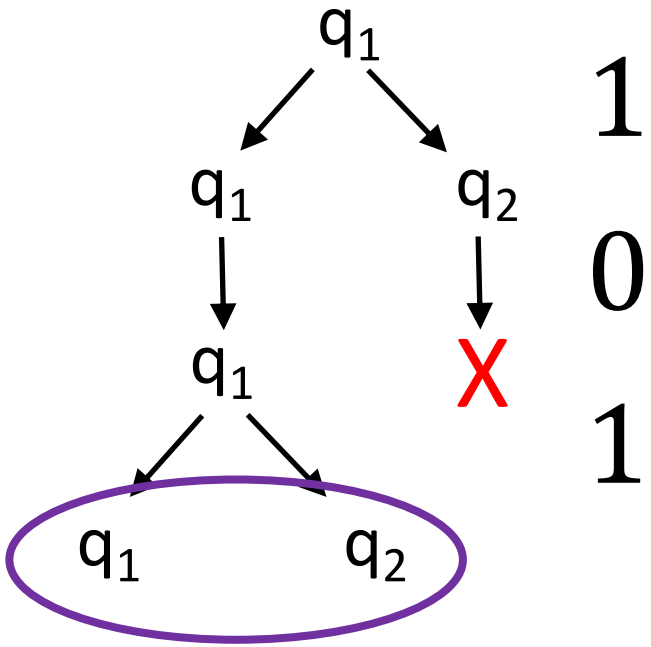
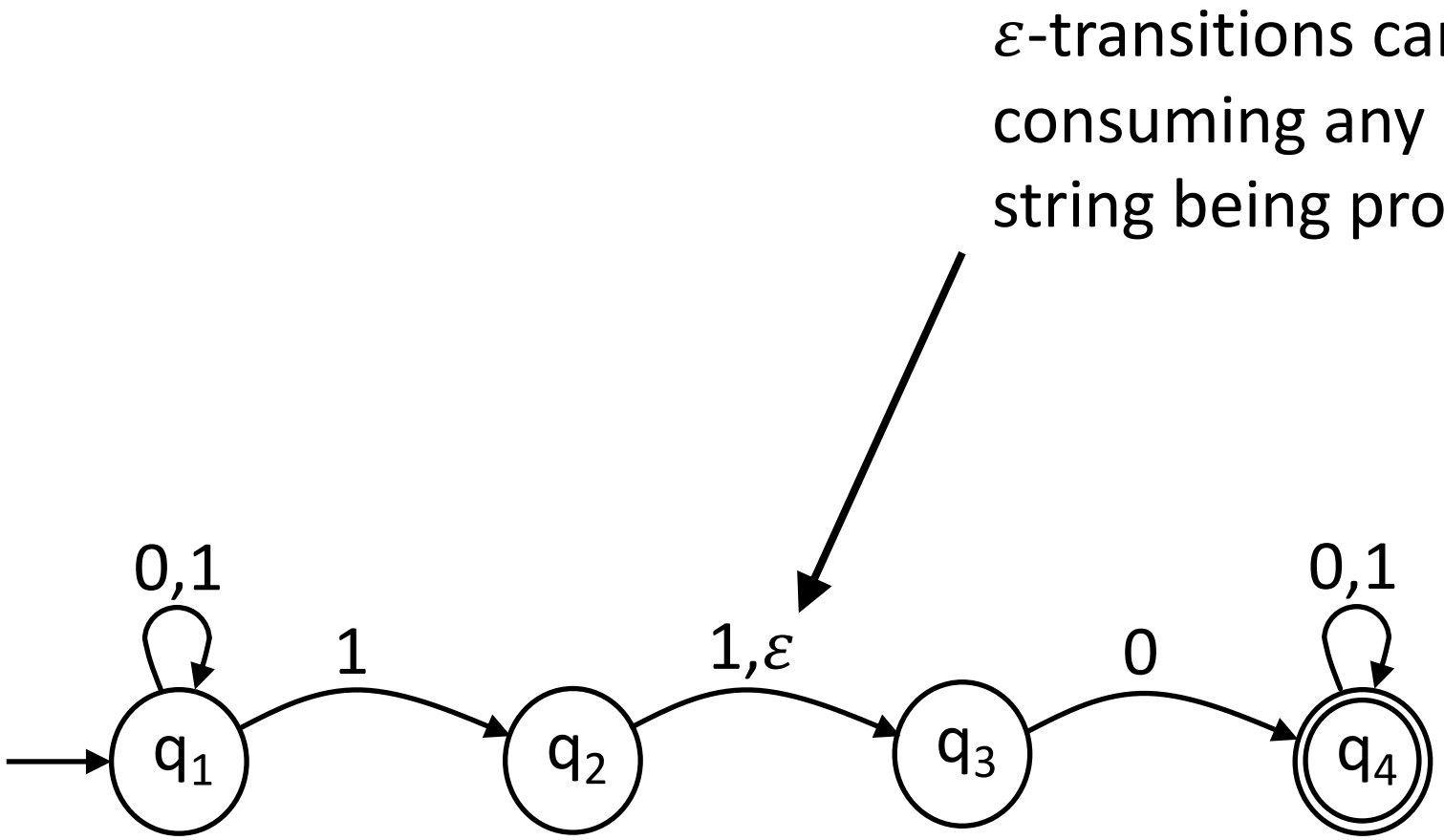


$\omega = 101$

ε-Transitions

ε-transitions can be taken without consuming any characters from the string being processed.



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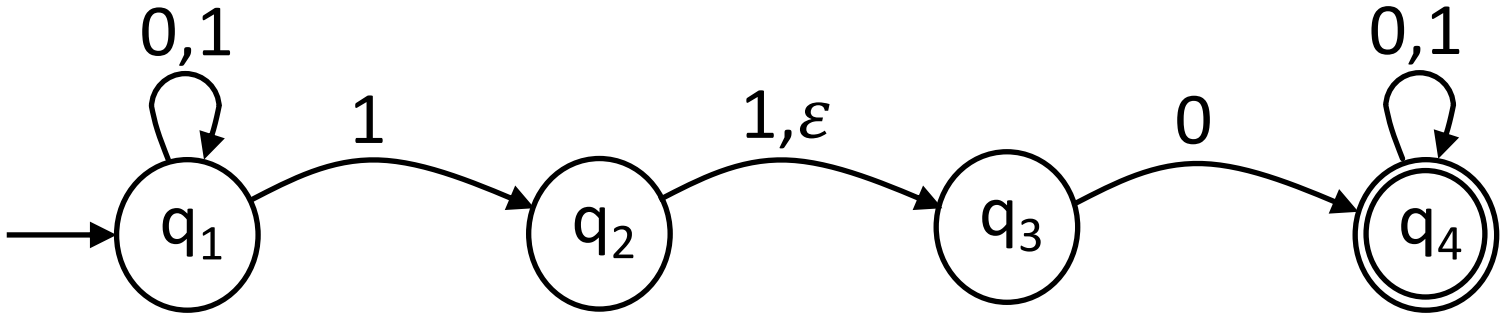

ε-Transitions

q_1

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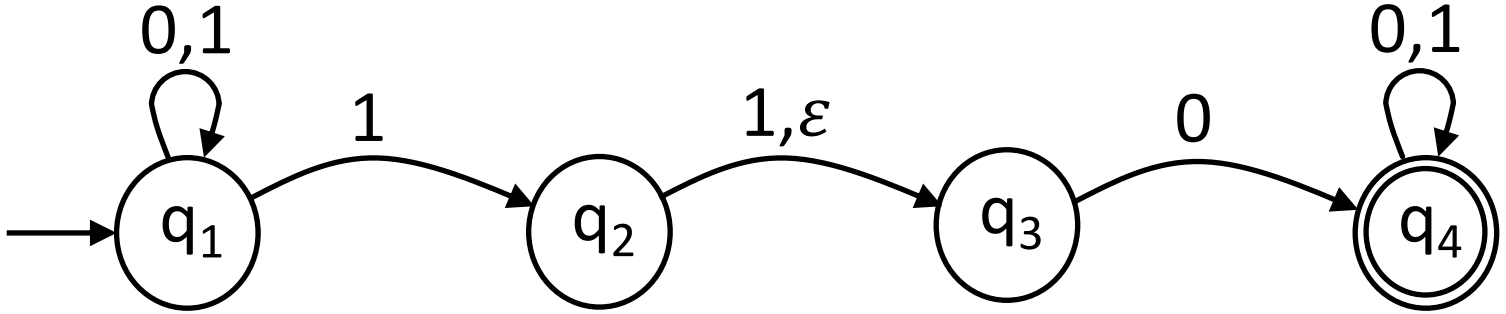

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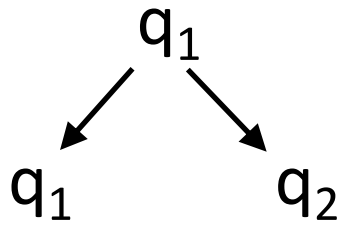

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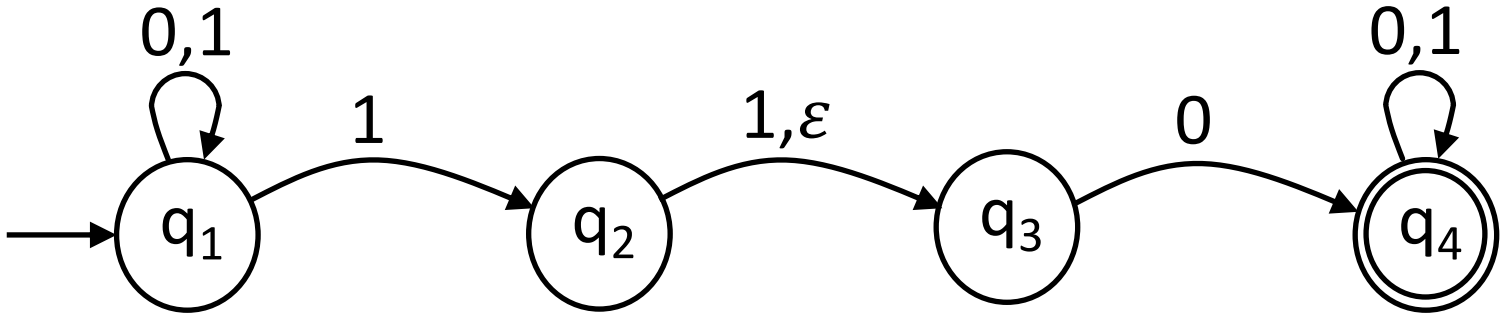

$\omega = \mathbf{1}01$

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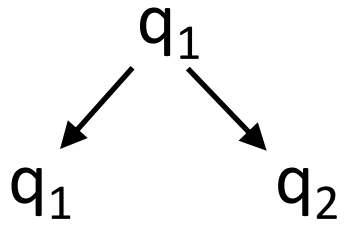

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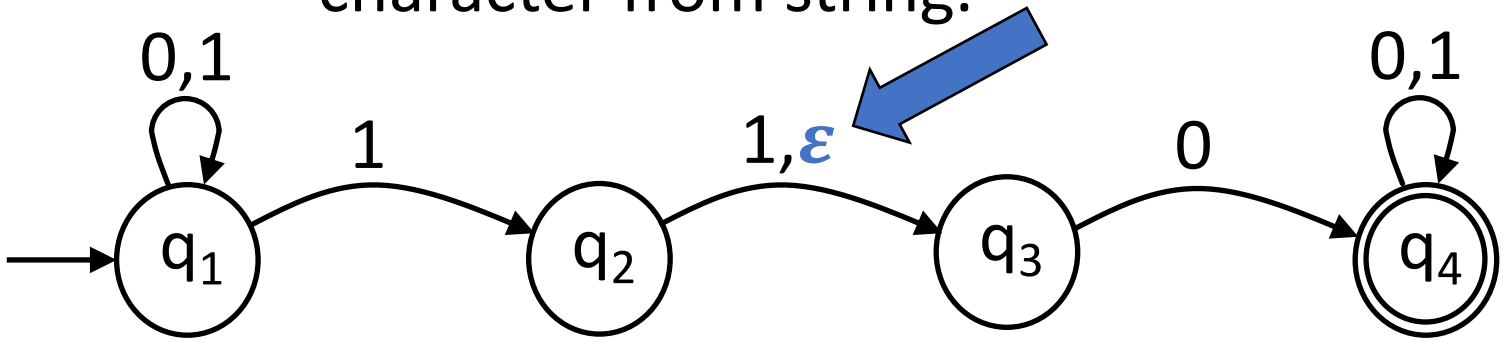

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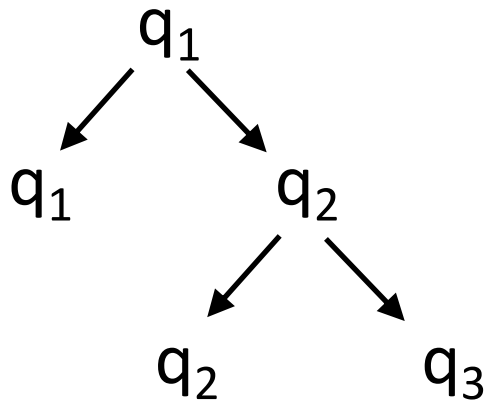
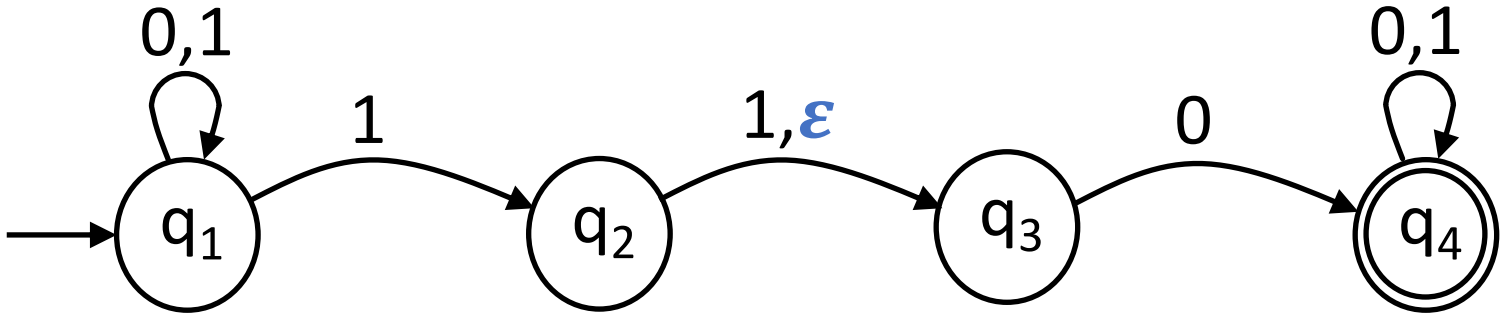


1

ε



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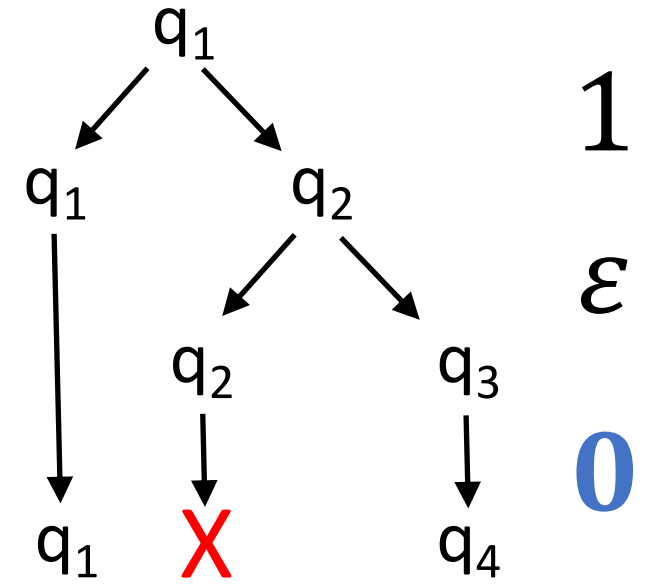

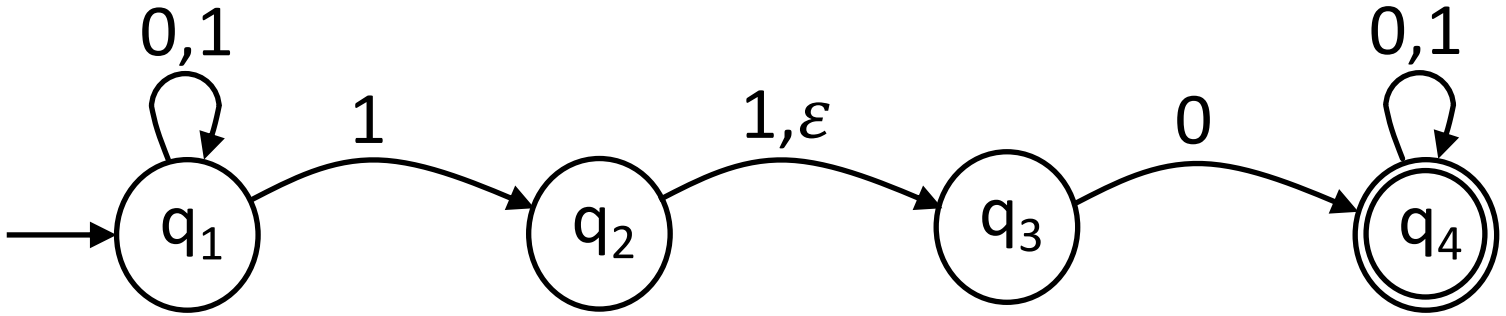

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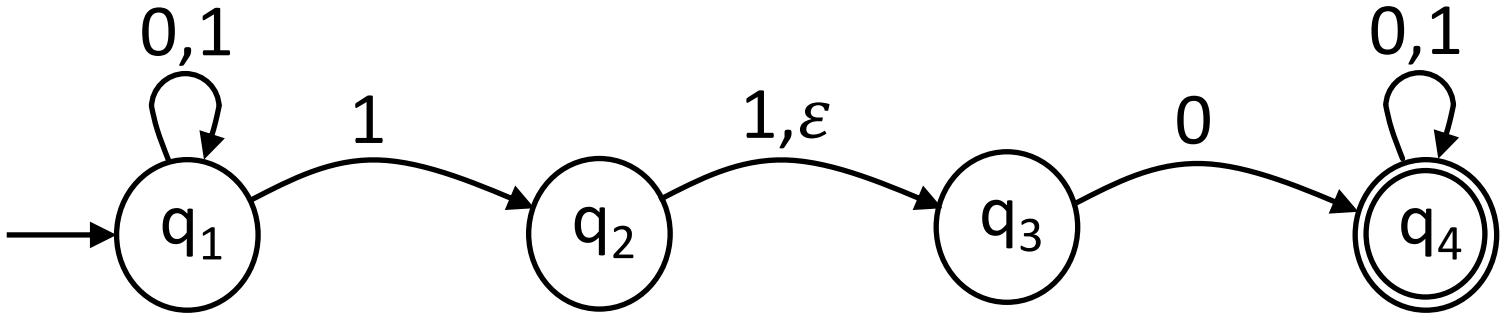

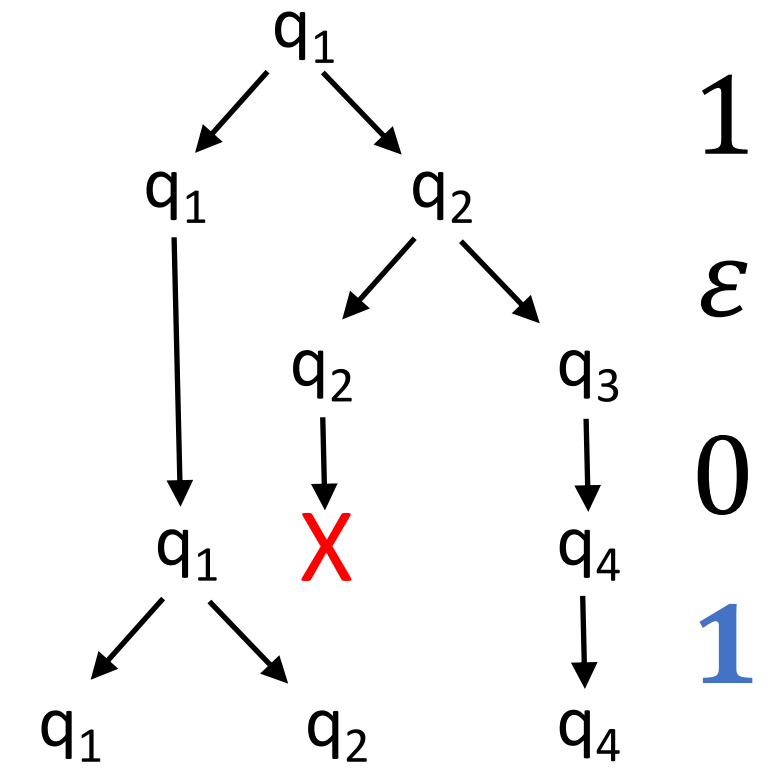

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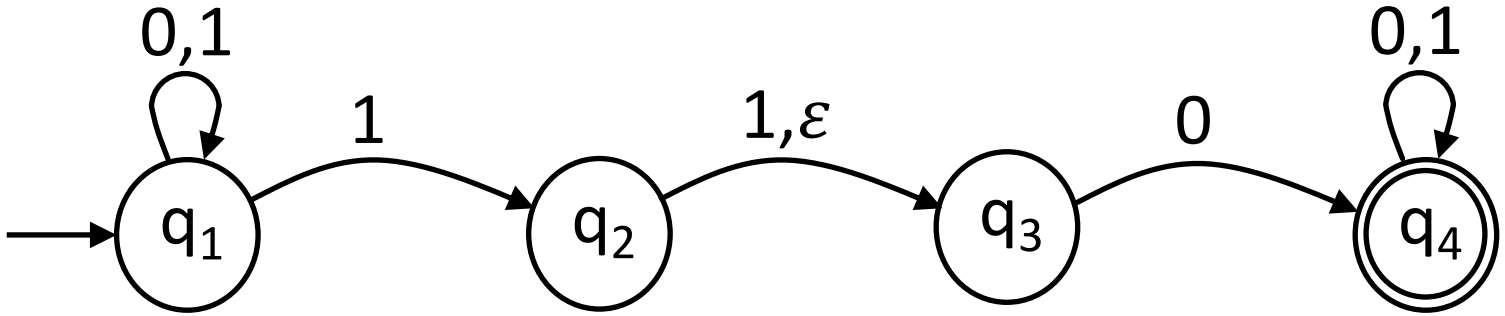

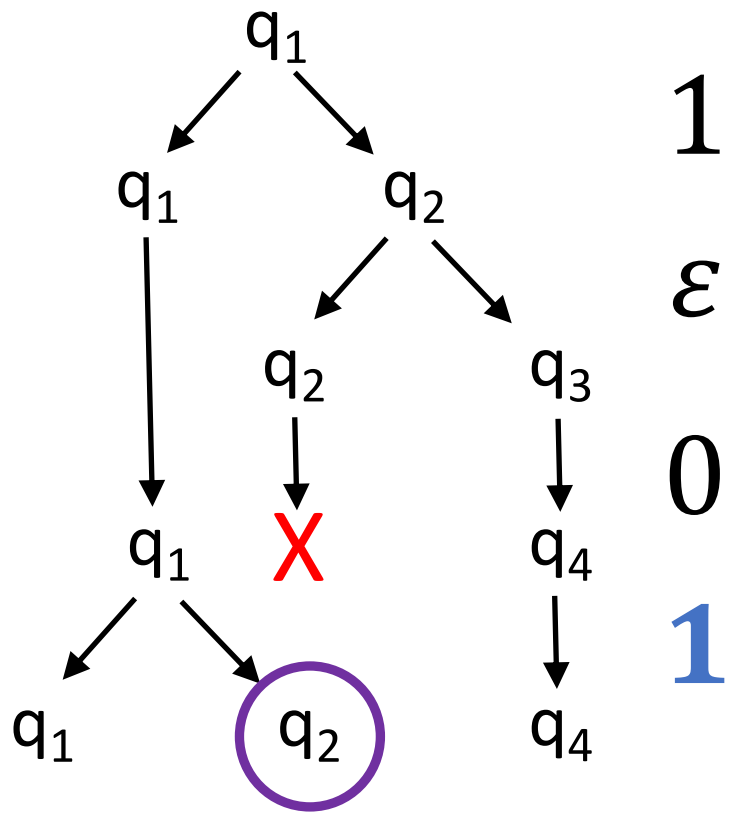

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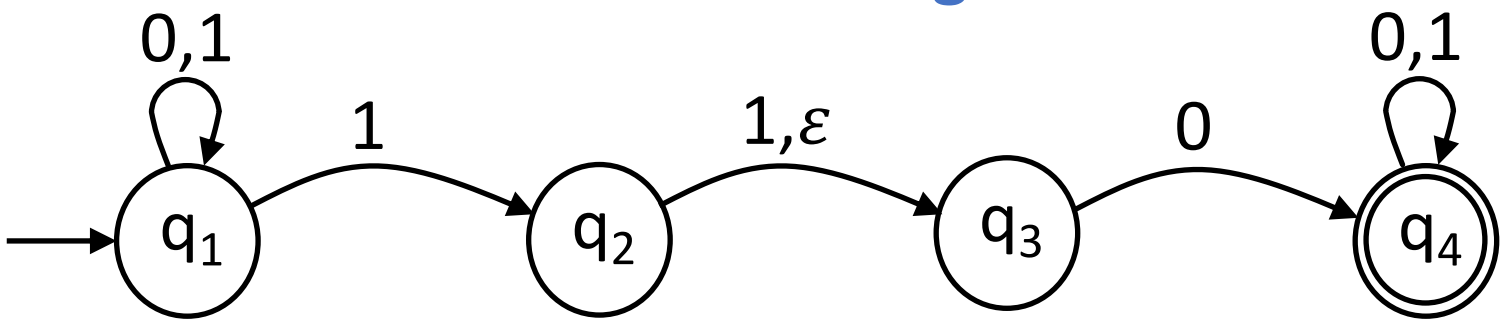
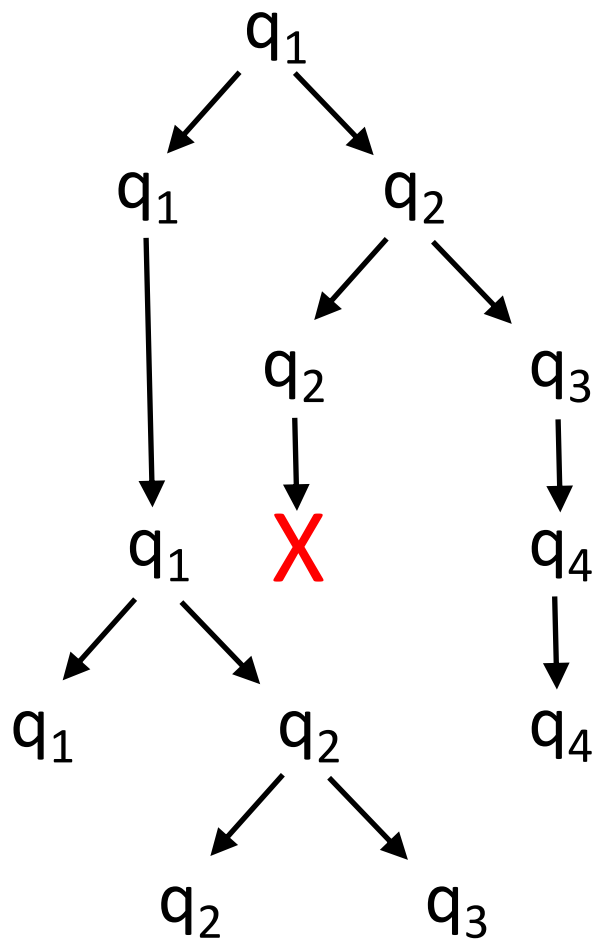
3. If any branch ends in an accept state, accept. If not, reject.
4. **If ε is encountered, split and take all options without consuming a character from string.**





$\omega = 101$

ε-Transitions

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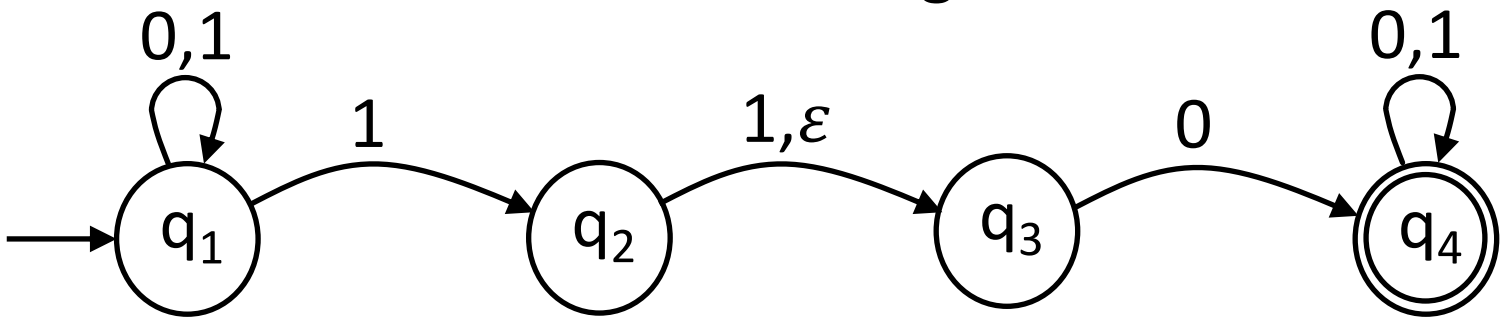
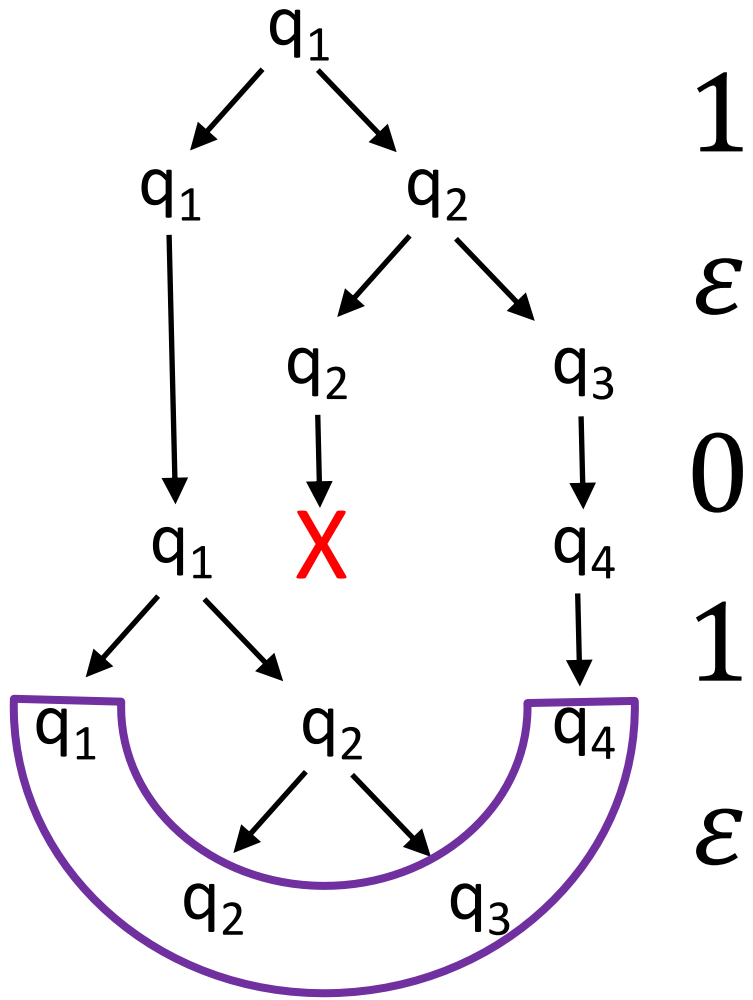


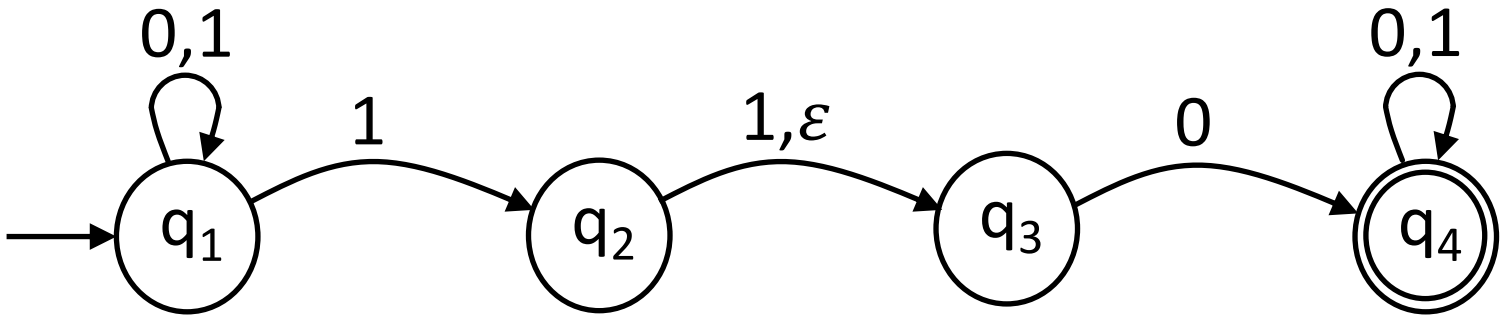


$\omega = 101$

NFA Formal Definition

NFAs consist of:

1. Finite set of states, Q.
2. Finite alphabet, Σ.
3. Transition function, $\delta: Q \times (\Sigma \cup \{\varepsilon\}) \rightarrow \mathcal{P}(Q)$.
4. Start state, $q_0 \in Q$.
5. Set of accept states, $F \subseteq Q$.



NFA Formal Definition

Power set of Q.

I.e. set of all subsets.

E.g. $Q = \{q_1, q_2\}$

$\Rightarrow \mathcal{P}(Q) = \{\emptyset, \{q_1\}, \{q_2\}, \{q_1, q_2\}\}$

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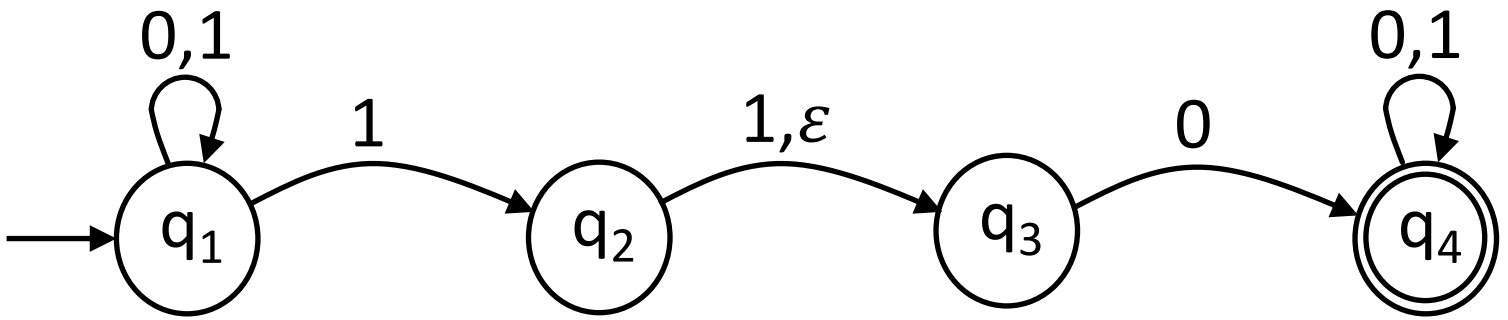

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I.e. $\exists$ 0 or more transitions for each $e \in \Sigma \cup \{\varepsilon\}$ at each state

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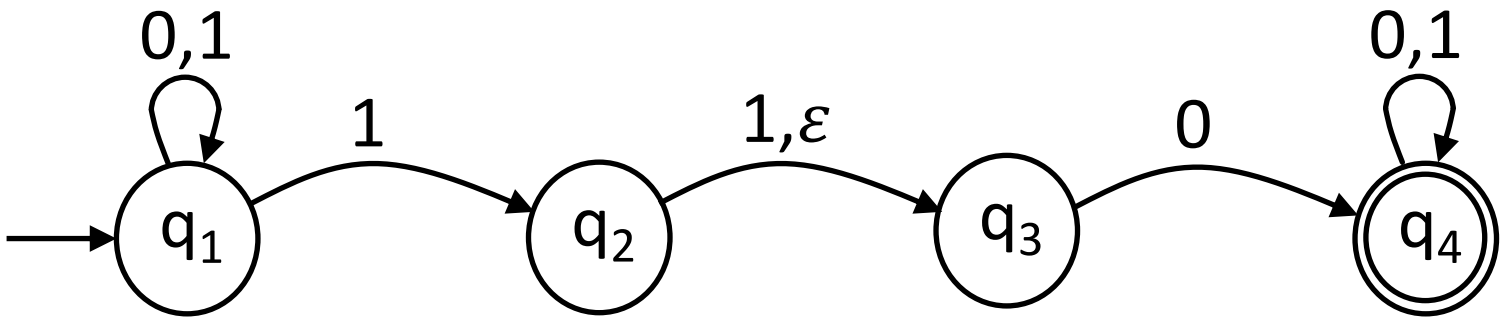

NFA Practice

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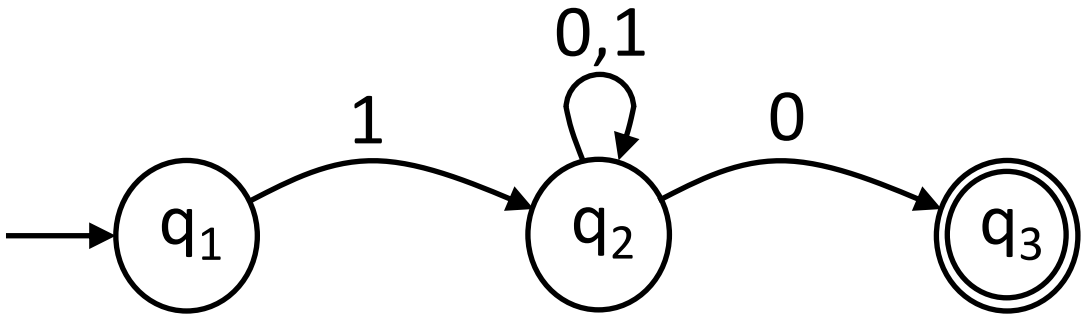

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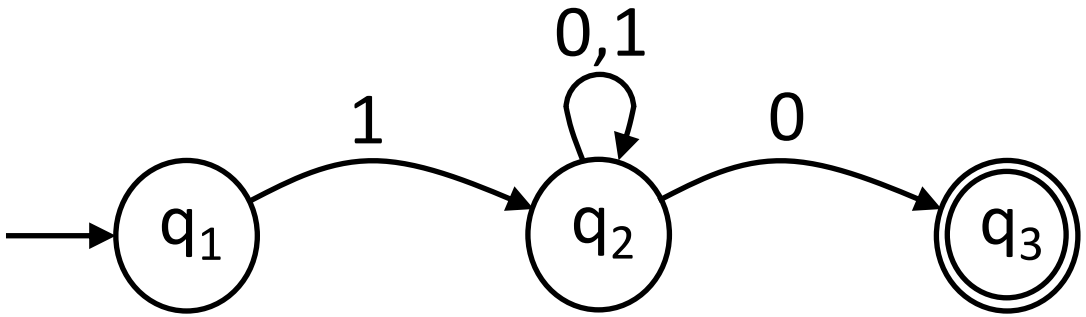

Only ω's that start with 1 get to q_2.
Any string that gets to q_2, can get to q_3 and terminate, if it ends with 0.