Regular Expressions
CSCI 338
String Construction

0*: Zero or more 0’s
String Construction

$0^*\colon$ Zero or more 0’s

Accept: $0,0000,\varepsilon$

Reject
String Construction

0*: Zero or more 0’s

Accept: 0, 0000, ε

Reject: 1, 0001, 1000
String Construction

0*: Zero or more 0’s
(01)*: Zero or more 01’s

Accept:
0, 0000, ε

Reject:
1, 0001, 1000
String Construction

$0^* \cdot$ Zero or more 0’s

$0^*$: Zero or more 0’s

$0000$, $\varepsilon$

Accept

$01^* \cdot$ Zero or more 01’s

$(01)^*$: Zero or more 01’s

$01, 010101, \varepsilon$

Reject

$1, 0001, 1000$
String Construction

$0^* : \text{Zero or more } 0\text{'s}$

$0^* : \text{Zero or more } 01\text{'s}$

Accept

$0, 0000, \varepsilon$

$01, 010101, \varepsilon$

Reject

$1, 0001, 1000$

$10, 001, 01010$
## String Construction

<table>
<thead>
<tr>
<th>Expression</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$0^*$</td>
<td>$0, 0000, \varepsilon$</td>
<td>$1, 0001, 1000$</td>
</tr>
<tr>
<td>$(01)^*$</td>
<td>$01, 010101, \varepsilon$</td>
<td>$10, 001, 01010$</td>
</tr>
<tr>
<td>$(0^<em>1)^</em>$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
String Construction

0*: Zero or more 0’s
(01)*: Zero or more 01’s
(0*1)*: ?

<table>
<thead>
<tr>
<th></th>
<th>Accept</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0, 0000, ε</td>
<td>1, 0001, 1000</td>
</tr>
<tr>
<td></td>
<td>01, 010101, ε</td>
<td>10, 001, 01010</td>
</tr>
<tr>
<td>ε?</td>
<td></td>
<td>ε?</td>
</tr>
</tbody>
</table>
String Construction

$0^*:$ Zero or more 0’s

$01^*:$ Zero or more 01’s

$(0^*1)^*:$ ?
# String Construction

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<tbody>
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<td>$0^*$ : Zero or more 0’s</td>
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<td>$1, 0001, 1000$</td>
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<tr>
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<td>$01, 010101, \varepsilon$</td>
<td>$10, 001, 01010$</td>
</tr>
<tr>
<td>$(0^<em>1)^</em>$ : ?</td>
<td>$\varepsilon$</td>
<td>$1?$</td>
</tr>
<tr>
<td>String Construction</td>
<td>Accept</td>
<td>Reject</td>
</tr>
<tr>
<td>---------------------</td>
<td>--------</td>
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<tr>
<td>$0^*$: Zero or more 0’s</td>
<td>$0, 0000, \varepsilon$</td>
<td>$1, 0001, 1000$</td>
</tr>
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<td>$(01)^*$: Zero or more 01’s</td>
<td>$01, 010101, \varepsilon$</td>
<td>$10, 001, 01010$</td>
</tr>
<tr>
<td>$(0^<em>1)^</em>$: ?</td>
<td>$\varepsilon, 1$</td>
<td></td>
</tr>
</tbody>
</table>

1?
String Construction

$0^*$: Zero or more 0’s  
$0, 0000, \varepsilon$  
$1, 0001, 1000$

$(01)^*$: Zero or more 01’s  
$01, 010101, \varepsilon$  
$10, 001, 01010$

$(0^*1)^*$: ?  
$\varepsilon, 1$

$10?$
### String Construction

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<td>$(01)^*$: Zero or more 01’s</td>
<td>01, 010101, $\varepsilon$</td>
<td>10, 001, 01010</td>
</tr>
<tr>
<td>$(0^<em>1)^</em>$: ?</td>
<td>$\varepsilon$, 1</td>
<td>10</td>
</tr>
</tbody>
</table>

$10^?$
<table>
<thead>
<tr>
<th></th>
<th>Accept</th>
<th>Reject</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^* : \text{Zero or more 0's}$</td>
<td>$0,0000, \varepsilon$</td>
<td>$1,0001,1000$</td>
</tr>
<tr>
<td>$(01)^* : \text{Zero or more 01's}$</td>
<td>$01,010101, \varepsilon$</td>
<td>$10,001,01010$</td>
</tr>
<tr>
<td>$(0^<em>1)^</em> : \varepsilon, 1$</td>
<td></td>
<td>$10$</td>
</tr>
</tbody>
</table>

111?
String Construction

$0^*$: Zero or more $0$'s

$(01)^*$: Zero or more $01$'s

$(0^*1)^*$: ?

<table>
<thead>
<tr>
<th>Accept</th>
<th>Reject</th>
</tr>
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<tbody>
<tr>
<td>$0, 0000, \varepsilon$</td>
<td>$1, 0001, 1000$</td>
</tr>
<tr>
<td>$01, 010101, \varepsilon$</td>
<td>$10, 001, 01010$</td>
</tr>
<tr>
<td>$\varepsilon, 1, 111$</td>
<td>$10$</td>
</tr>
</tbody>
</table>
String Construction

$0^*$: Zero or more 0’s

Accept

$0, 0000, \varepsilon$

Reject

$1, 0001, 1000$

$(01)^*$: Zero or more 01’s

$01, 010101, \varepsilon$

$10, 001, 01010$

$(0^*1)^*$: ?

$\varepsilon, 1, 111$

$10$

000?
String Construction

0*: Zero or more 0’s
(01)*: Zero or more 01’s
(0*1)*: ?

Accept

0, 0000, ε
01, 010101, ε
ε, 1, 111

Reject

1, 0001, 1000
10, 001, 01010
10, 000

000?
String Construction

$0^*$: Zero or more 0’s

Accept: 0, 0000, $\varepsilon$

Reject: 1, 0001, 1000

$(01)^*$: Zero or more 01’s

Accept: 01, 010101, $\varepsilon$

Reject: 10, 001, 01010

$(0^*1)^*$: ?

Accept: $\varepsilon$, 1, 111

Reject: 10, 000

01001?

String Construction

0*: Zero or more 0’s
(01)*: Zero or more 01’s
(0*1)*: ?

Accept

0, 0000, $\varepsilon$
01, 010101, $\varepsilon$
$\varepsilon$, 1, 111, 01001

Reject

1, 0001, 1000
10, 001, 01010
10, 000

01001?
String Construction

$0^*$: Zero or more 0’s  
$01^*$: Zero or more 01’s  
$(0^*1)^*$: ?

Accept:  
$0, 0000, \varepsilon$  
$01, 010101, \varepsilon$  
$\varepsilon, 1, 111, 01001$  

Reject:  
$1, 0001, 1000$  
$10, 001, 01010$  
$10, 000$

101?
String Construction

$0^*$: Zero or more 0’s
$(01)^*$: Zero or more 01’s
$(0^*1)^*$: ?

Accept

$0, 0000, \varepsilon$
$01, 010101, \varepsilon$
$\varepsilon, 1, 111, 01001, 101$

Reject

$1, 0001, 1000$
$10, 001, 01010$
$10, 000$

101?
<table>
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<tr>
<th>Expression</th>
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<tr>
<td>$0^* : \text{Zero or more 0's}$</td>
<td>0, 0000, $\varepsilon$</td>
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<td>01, 010101, $\varepsilon$</td>
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</tr>
<tr>
<td>$(0^<em>1)^</em> : ?$</td>
<td>$\varepsilon, 1, 111, 01001, 101$</td>
<td>10, 000</td>
</tr>
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</table>

$0001110110001$
String Construction

- $0^*$: Zero or more 0’s
- $(01)^*$: Zero or more 01’s
- $(0^*1)^*$: ?

<table>
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<tbody>
<tr>
<td>$0, 0000, \varepsilon$</td>
<td>$1, 0001, 1000$</td>
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<tr>
<td>$01, 010101, \varepsilon$</td>
<td>$10, 001, 01010$</td>
</tr>
<tr>
<td>$\varepsilon, 1, 111, 01001, 101$</td>
<td>$10, 000$</td>
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</tbody>
</table>

Accept if string can be broken into sequences of $0^*1$
String Construction

0*: Zero or more 0’s
(01)*: Zero or more 01’s
(0*1)*: ?

Accept
0, 0000, \(\varepsilon\)
01, 010101, \(\varepsilon\)
\(\varepsilon\), 1, 111, 01001, 101

Reject
1, 0001, 1000
10, 001, 01010
10, 000

Accept if string can be broken into sequences of 0*1
## String Construction

<table>
<thead>
<tr>
<th>Expression</th>
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<tr>
<td>$0^*$: Zero or more 0’s</td>
<td>$0, 0000, \varepsilon$</td>
<td>$1, 0001, 1000$</td>
</tr>
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<td>$(01)^*$: Zero or more 01’s</td>
<td>$01, 010101, \varepsilon$</td>
<td>$10, 001, 01010$</td>
</tr>
<tr>
<td>$(0^* 1)^*$: ?</td>
<td>$\varepsilon, 1, 111, 01001, 101$</td>
<td>$10, 000$</td>
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</tbody>
</table>

Accept if string can be broken into sequences of $0^* 1$
### String Construction

<table>
<thead>
<tr>
<th>Description</th>
<th>Accept</th>
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<tbody>
<tr>
<td>0*: Zero or more 0’s</td>
<td>0, 0000, (\varepsilon)</td>
<td>1, 0001, 1000</td>
</tr>
<tr>
<td>(01)*: Zero or more 01’s</td>
<td>01, 010101, (\varepsilon)</td>
<td>10, 001, 01010</td>
</tr>
<tr>
<td>(0<em>1)</em>: Doesn’t end with 0</td>
<td>(\varepsilon), 1, 111, 01001, 101</td>
<td>10, 000</td>
</tr>
</tbody>
</table>

Accept if string can be broken into sequences of 0*1
### String Construction

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<td>$(01)^*$: Zero or more 01’s</td>
<td>01, 010101, $\varepsilon$</td>
<td>10, 001, 01010</td>
</tr>
<tr>
<td>$(0^<em>1)^</em>$: Doesn’t end with 0</td>
<td>$\varepsilon$, 1, 111, 01001, 101</td>
<td>10, 000</td>
</tr>
<tr>
<td>$0^+$: One or more 0’s</td>
<td>0, 0000</td>
<td>$\varepsilon$, 1</td>
</tr>
</tbody>
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### String Construction

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<td>$0^+$: One or more 0’s</td>
<td>0, 0000</td>
<td>$\varepsilon$, 1</td>
</tr>
<tr>
<td>$(001^+)^*$</td>
<td>001, 0011, 0010011, $\varepsilon$</td>
<td>1, 00</td>
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<td>$(01)^*$: Zero or more 01’s</td>
<td>01, 010101, ε</td>
<td>10, 001, 01010</td>
</tr>
<tr>
<td>$(0^<em>1)^</em>$: Doesn’t end with 0</td>
<td>ε, 1, 111, 01001, 101</td>
<td>10, 000</td>
</tr>
<tr>
<td>$0^+$: One or more 0’s</td>
<td>0, 0000</td>
<td>ε, 1</td>
</tr>
<tr>
<td>$(001^+)^*$</td>
<td>001, 0011, 0010011, ε</td>
<td>1, 00</td>
</tr>
<tr>
<td>$1^<em>(001^+)^</em>$</td>
<td>ε, 1, 1001, 10010011, 001</td>
<td>00, 101</td>
</tr>
</tbody>
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<td>$(0^<em>1)^</em>$: Doesn’t end with 0</td>
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<tr>
<td>$(001^+)^*$</td>
<td>001, 0011, 0010011, $\varepsilon$</td>
<td>1, 00</td>
</tr>
<tr>
<td>$1^<em>(001^+)^</em>$</td>
<td>$\varepsilon$, 1, 1001, 10010011, 001</td>
<td>00, 101</td>
</tr>
<tr>
<td>$(0 \cup 1)$: A single 0 or 1</td>
<td>1, 0</td>
<td>$\varepsilon$, 00, 101</td>
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## String Construction

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<td>0*: Zero or more 0’s</td>
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<td>1, 0001, 1000</td>
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<tr>
<td>(01)*: Zero or more 01’s</td>
<td>01, 010101, ε</td>
<td>10, 001, 01010</td>
</tr>
<tr>
<td>(0<em>1)</em>: Doesn’t end with 0</td>
<td>ε, 1, 111, 01001, 101</td>
<td>10, 000</td>
</tr>
<tr>
<td>0+: One or more 0’s</td>
<td>0, 0000</td>
<td>ε, 1</td>
</tr>
<tr>
<td>(001+)</td>
<td>001, 0011, 0010011, ε</td>
<td>1, 00</td>
</tr>
<tr>
<td>1*(001+)</td>
<td>ε, 1, 1001, 10010011, 001</td>
<td>00, 101</td>
</tr>
<tr>
<td>(0 ∪ 1): A single 0 or 1</td>
<td>1, 0</td>
<td>ε, 00, 101</td>
</tr>
<tr>
<td>(0 ∪ 1)0*: A 0 or 1 followed by zero or more 0s.</td>
<td></td>
<td></td>
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## String Construction

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<td>((01)^*): Zero or more 01’s</td>
<td>01, 010101, (\varepsilon)</td>
<td>10, 001, 01010</td>
</tr>
<tr>
<td>((0^<em>1)^</em>): Doesn’t end with 0</td>
<td>(\varepsilon), 1, 111, 01001, 101</td>
<td>10, 000</td>
</tr>
<tr>
<td>(0^+): One or more 0’s</td>
<td>0, 0000</td>
<td>(\varepsilon), 1</td>
</tr>
<tr>
<td>((001^+)^*)</td>
<td>001, 0011, 0010011, (\varepsilon)</td>
<td>1, 00</td>
</tr>
<tr>
<td>(1^<em>(001^+)^</em>)</td>
<td>(\varepsilon), 1, 1001, 10010011, 001</td>
<td>00, 101</td>
</tr>
<tr>
<td>((0 \cup 1)): A single 0 or 1</td>
<td>1, 0</td>
<td>(\varepsilon), 00, 101</td>
</tr>
<tr>
<td>((0 \cup 1)^0^*): A 0 or 1 followed by zero or more 0s.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((0 \cup 1)^*): A string with any number of 0s and 1s.</td>
<td></td>
<td></td>
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</table>
String Construction

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<thead>
<tr>
<th>Regular Expressions:</th>
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<td>0*: Zero or more 0’s</td>
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<td>(01)*: Zero or more 01’s</td>
<td>01, 010101, ε</td>
<td>10, 001, 01010</td>
</tr>
<tr>
<td>(0<em>1)</em>: One or more 01’s</td>
<td>ε, 1</td>
<td>1, 00</td>
</tr>
<tr>
<td>0+: One or more 0’s</td>
<td>0000, 000</td>
<td>1, 00</td>
</tr>
<tr>
<td>(001+)*: A string with at least one 001</td>
<td>ε, 1</td>
<td>1, 110</td>
</tr>
<tr>
<td>1*(001+): A string with at least one 001 followed by any number of 1’s</td>
<td>0, 101</td>
<td>1, 101</td>
</tr>
<tr>
<td>(0 U 1): A 0 or 1</td>
<td>00, 101</td>
<td>100, 101</td>
</tr>
<tr>
<td>(0 U 1)0*: A 0 or 1 followed by zero or more 0’s</td>
<td>00, 101</td>
<td>100, 101</td>
</tr>
<tr>
<td>(0 U 1)*: A string with any number of 0s and 1s</td>
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</table>
Regular Expressions

Rules for building regular expressions (regex):
1. Each $e \in \Sigma$ is a regex
Regular Expressions

Rules for building regular expressions (regex):
1. Each $e \in \Sigma$ is a regex
2. $\{\varepsilon\}$ is a regex Language with one string: The empty string.
Regular Expressions

Rules for building regular expressions (regex):
1. Each $e \in \Sigma$ is a regex
2. $\{\varepsilon\}$ is a regex \quad \text{Language with one string: The empty string.}
3. $\emptyset$ is a regex \quad \text{Language with no strings.}
Regular Expressions

Rules for building regular expressions (regex):
1. Each $\varepsilon \in \Sigma$ is a regex
2. $\{\varepsilon\}$ is a regex
3. $\emptyset$ is a regex

Language with one string: The empty string.
Language with no strings.
Regular Expressions

Rules for building regular expressions (regex):
1. Each $e \in \Sigma$ is a regex
2. $\{\varepsilon\}$ is a regex \textcolor{green}{Language with one string: The empty string.}
3. $\emptyset$ is a regex \textcolor{blue}{Language with no strings.}
4. $(R_1 \cup R_2)$ is a regex
   \[ R_1 \text{ and } R_2 \text{ are regexs} \]
Regular Expressions

Rules for building regular expressions (regex):
1. Each $e \in \Sigma$ is a regex
2. $\{\varepsilon\}$ is a regex  \(\rightarrow\) Language with one string: The empty string.
3. $\emptyset$ is a regex  \(\rightarrow\) Language with no strings.
4. $(R_1 \cup R_2)$ is a regex
5. $(R_1 \circ R_2)$ is a regex  \(\rightarrow\) $R_1$ and $R_2$ are regexes
Regular Expressions

Rules for building regular expressions (regex):
1. Each $e \in \Sigma$ is a regex
2. $\{\varepsilon\}$ is a regex $\Rightarrow$ Language with one string: The empty string.
3. $\emptyset$ is a regex $\Rightarrow$ Language with no strings.
4. $(R_1 \cup R_2)$ is a regex
5. $(R_1 \circ R_2)$ is a regex $\Rightarrow$ $R_1$ and $R_2$ are regexes
6. $R_1^*$ is a regex
Regular Expressions

Regular Expression notation:

- \( R^* \) (i.e. zero or more strings from \( R \))
  
  e.g. \( 1^* \) includes: 1, 11111111, \( \varepsilon \)

- \( RR = R \circ R \) (i.e. two strings from \( R \) concatenated)
  
  e.g. \( 1^*0 \) includes: 10, 111111110, 0

- \( R^+ = RR^* \) (i.e. at least one string from \( R \))
  
  e.g. \( 1^+ \) includes: 1, 11111111, but not \( \varepsilon \)

Order of operations:

- Parentheses, star (and plus), concatenation, union.
Regular Expression Practice

Suppose that $\Sigma = \{0, 1\}$.

- $1^* 0^* 1 = ?$
Regular Expression Practice

Suppose that \( \Sigma = \{0,1\} \).

- \( 1^*0^*1 = \{w : w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\} \)
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^*0^*1 = \{w : w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^*1 = ?$
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^* 0^* 1 = \{w: w \text{ contains } \geq 0 \ 1\text{s, then } \geq 0 \ 0\text{s, then a } 1\}$
- $(1 \cup 0)^* 1 = \{w: w \text{ ends in } 1\}$
Regular Expression Practice

Suppose that $\Sigma = \{0,1\}$.

- $1^* 0^* 1 = \{w: w \text{ contains } \geq 0 \text{ 1s, then } \geq 0 \text{ 0s, then a 1}\}$
- $(1 \cup 0)^* 1 = \{w: w \text{ ends in } 1\}$
- $\{w: w \text{ contains a single 1}\} = ?$
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- $\{w: w \text{ contains at least one 1}\} = \Sigma^*1\Sigma^* \text{ or } (0 \cup 1)^*1(0 \cup 1)^*$
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- $\{w: \text{ every 0 is followed by at least one 1} \} =$?
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- \( \{w: \text{ every 0 is followed by at least one 1} \} = 1^*(01^+)^* \)
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- $(\Sigma \Sigma)^* = \{w: w \text{ has even length}\}$
- $\{w: \text{every } 0 \text{ is followed by at least one } 1\} = 1^* (0 1^+)^*$
- $1^* \emptyset = \text{?}$

By definition, $A \circ B = \{xy: x \in A, y \in B\}$
Suppose that $\Sigma = \{0,1\}$.

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- $1^*\emptyset = \emptyset$

By definition, $A \circ B = \{xy: x \in A, y \in B\}$

Since there is no element in $\emptyset$, there cannot be any $xy$ such that $y \in \emptyset$. 
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- $1^*\varepsilon = ?$
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By definition, $A \circ B = \{xy: x \in A, y \in B\}$
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By definition, $A^* = \{x_1 x_2 \ldots x_k: k \geq 0, x_i \in A\}$
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- $1^*\emptyset = \emptyset$
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By definition, $A^* = \{x_1x_2 \ldots x_k: k \geq 0, x_i \in A\}$

Thus, it can append 0 elements of $\emptyset$ and get the empty string $\varepsilon$. 

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- $\{w : \text{every 0 is followed by at least one 1}\} = 1^*(01^+)^*$
- $1^*\emptyset = \emptyset$
- $1^*\varepsilon = 1^*$
- $\emptyset^* = \varepsilon$  \(\emptyset^+ = ?\)
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- $\emptyset^* = \varepsilon$    $\emptyset^+ = \emptyset$