Regular Expressions

CSCI 338

Regular Operations

If A and B are regular languages, so are:

- Complement

  $\bar{A} = \{\omega : \omega \notin A\}$
- Union

  $A \cup B = \{\omega :\ \omega \in A \text{ or } \omega \in B\}$
- Intersection

  $A \cap B = \{\omega :\ \omega \in A \text{ and } \omega \in B\}$
- Concatenation

  $A \circ B = \{xy : x \in A, y \in B\}$
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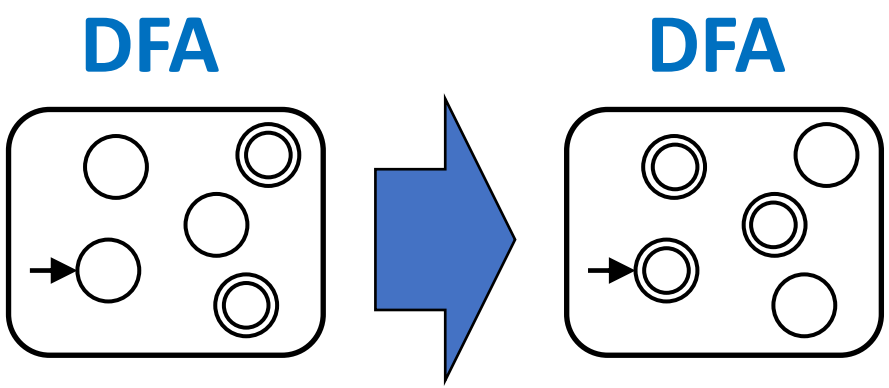

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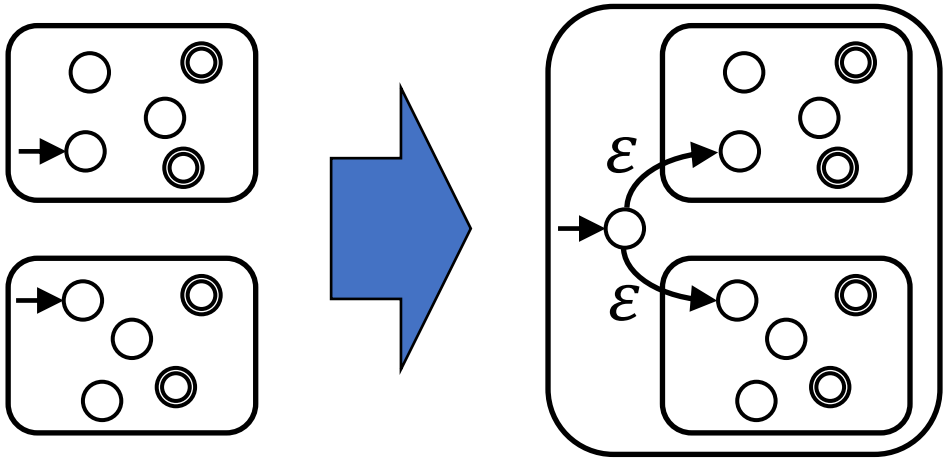

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  $$A \cap B = \overline{\bar{A} \cup \bar{B}}$$

  (De Morgan's Law)

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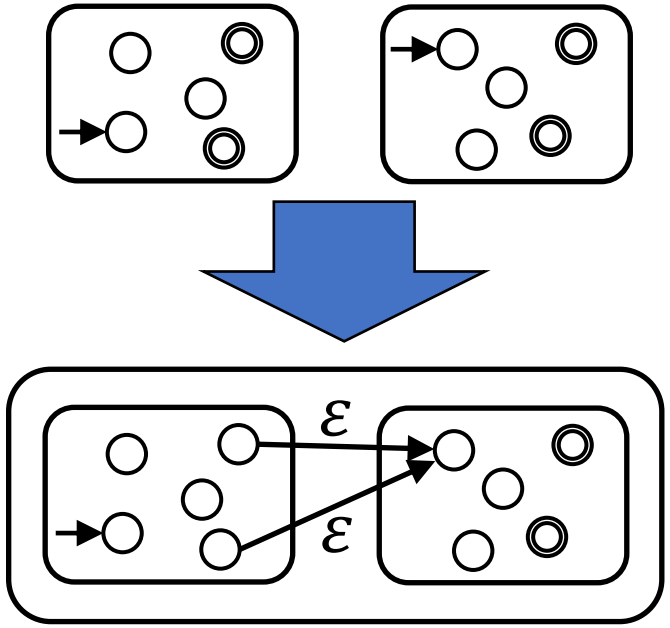

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String Construction

	Yes	No
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$$0001110110001$$

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Accept if string can be broken into sequences of 0^*1

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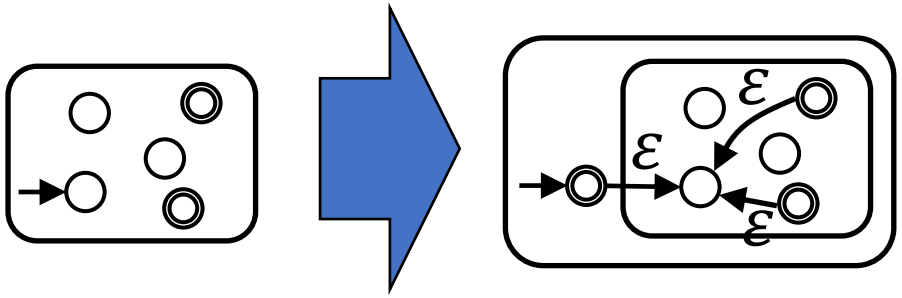

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$(0 \cup 1)^*$: A string with any number of 0s and 1s.

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Regular Expressions:

- **Algebraic formula that specifies a pattern of strings (i.e. a language).**
- **Used for text matching/searching**