Linear Programming
CSCI 432

Maximizing Profit

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2. MSC can sell up to 30 Rippers and 20 Ripper Carbons per day.
3. MSC can manufacture up to 40 skis per day.

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Day's Profit: ?

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Day's Profit: $100x_1 + 300x_2$

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Objective: $100x_1 + 300x_2$

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Objective: $\max 100x_1 + 300x_2$

Subject to: ?

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Objective: $\max 100x_1 + 300x_2$

Subject to: $x_1 \leq 30$

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Objective: $\max 100x_1 + 300x_2$

Subject to: $x_1 \leq 30$

$x_2 \leq 20$

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Objective: $\max 100x_1 + 300x_2$

Subject to: $x_1 \leq 30$

$x_2 \leq 20$

$x_1 + x_2 \leq 40$

Linear Program (LP)

$x_1 =$ # of Rippers sold
$x_2 =$ # of Ripper Carbons

Objective: $\max 100x_1 + 300x_2$
Subject to: $x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$x_1, x_2 \geq 0$

Linear Program (LP)

Decision Variables:

- Real numbers = solvable in polynomial time (called LP).
- Integers = not (yet?) solvable in polynomial time (called integer linear program – ILP).

x_1 = # of Rippers sold
x_2 = # of Ripper Carbons

Objective: $\max 100x_1 + 300x_2$

Subject to:
$x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
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Linear Program (LP)

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Subject to:
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$x_1 + x_2 \leq 40$
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Objective:

- Can be minimization or maximization.
- Must be linear combinations of variables x_i (e.g. $a_1x_1 + \cdots + a_nx_n$ for constants a_i, not $a_ix_1x_2$).

Linear Program (LP)

x_1 = # of Rippers sold
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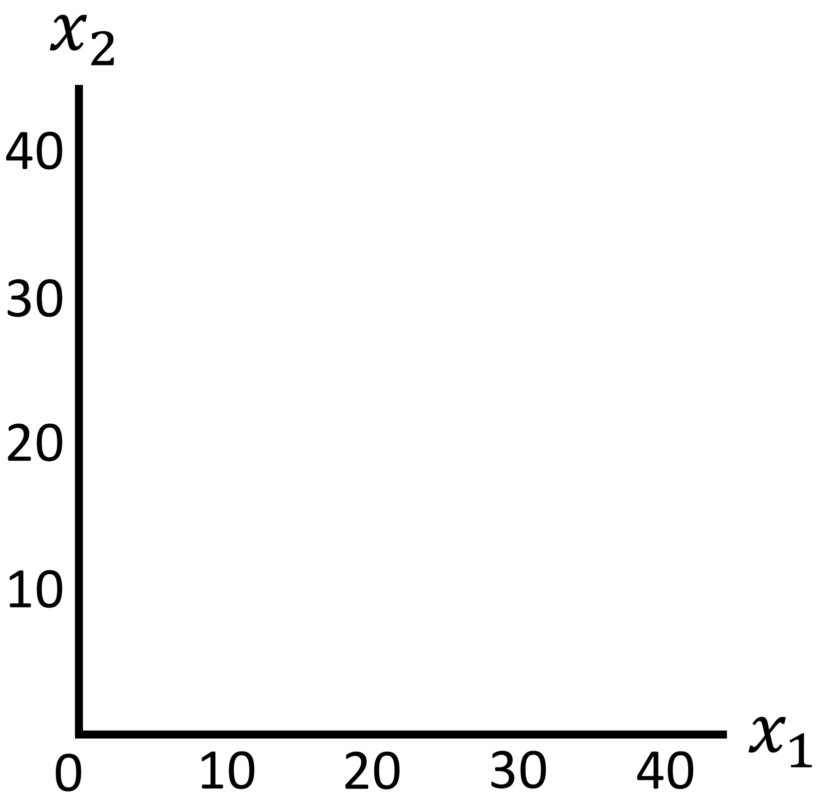
- Must be linear combinations of variables x_i
  (e.g. $a_1x_1 + \cdots + a_nx_n$ for constants a_i, not $a_ix_1x_2$).

Constraints:

- Can be $\leq$, $\geq$, $=$.
- Must be linear combinations of variables.

Maximizing Profit



$x_1 =$ # of Rippers sold in a day
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Objective: $\max 100x_1 + 300x_2$
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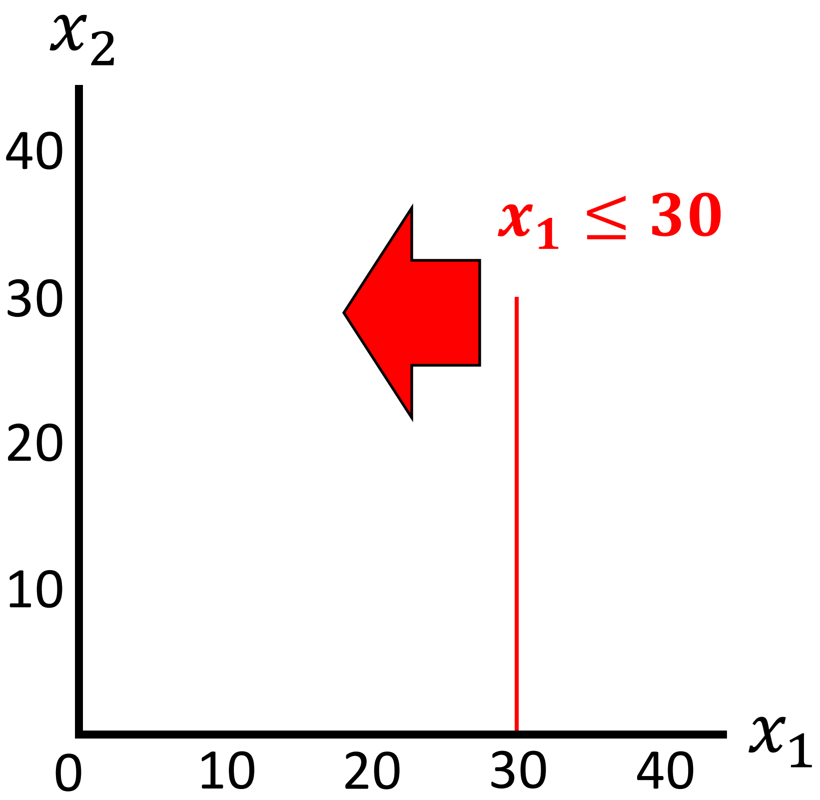

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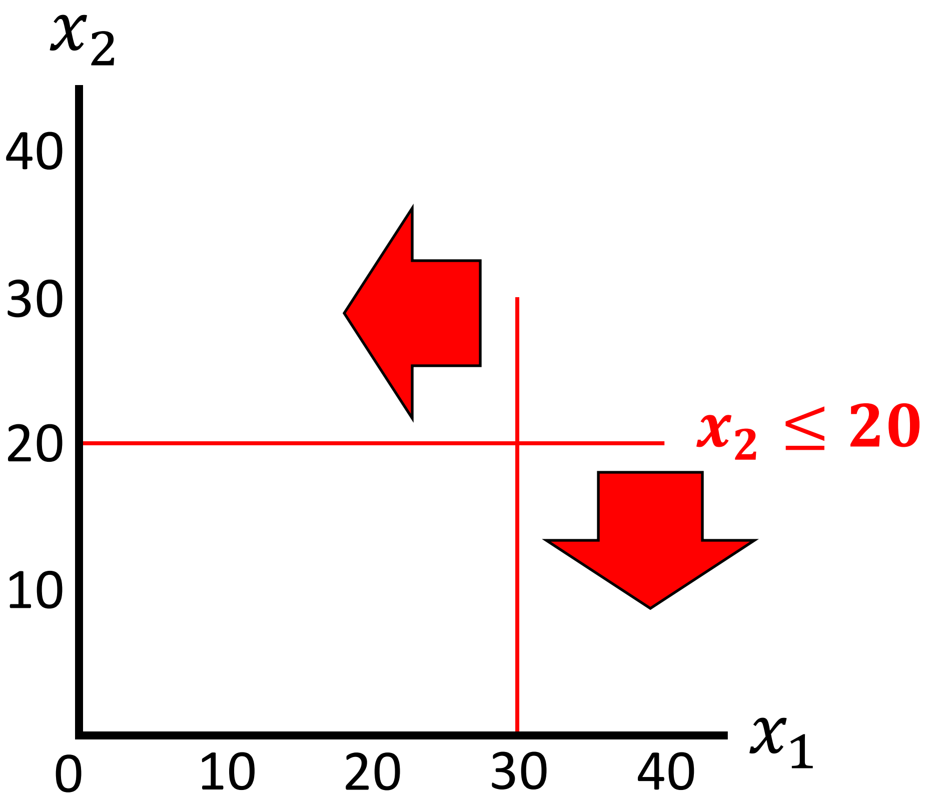
Maximizing Profit



x_1 = # of Rippers sold in a day
x_2 = # of Ripper Carbons sold in a day

Objective: $\max 100x_1 + 300x_2$

Subject to:

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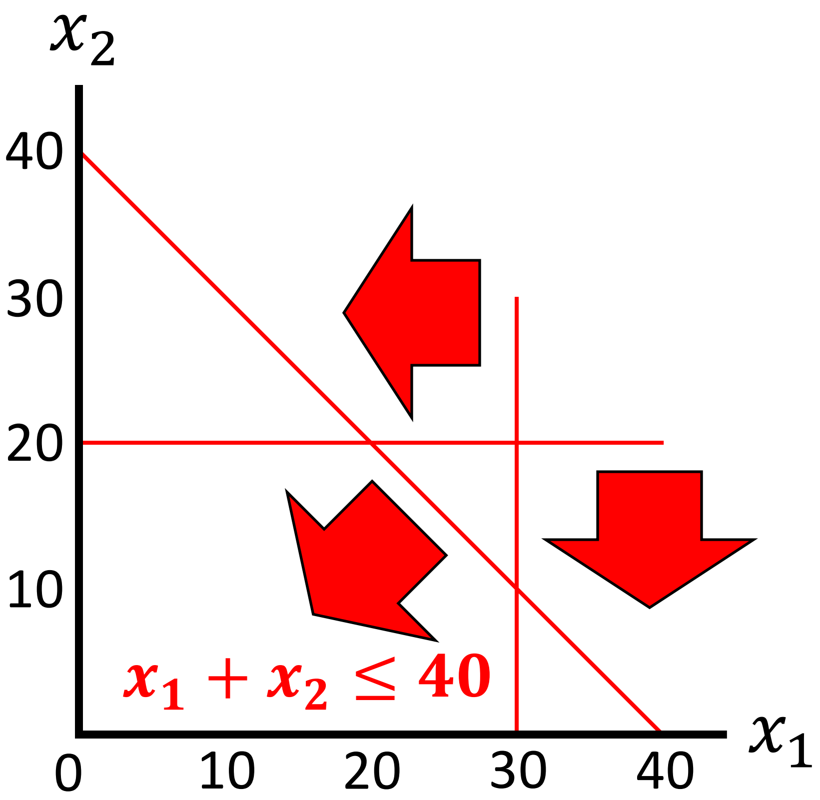
Maximizing Profit



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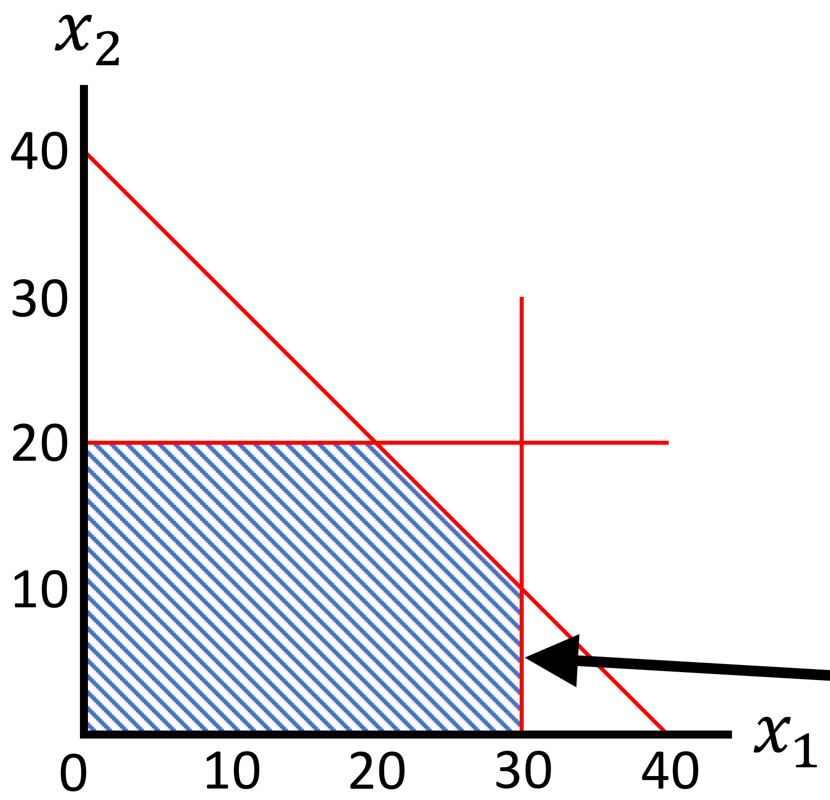
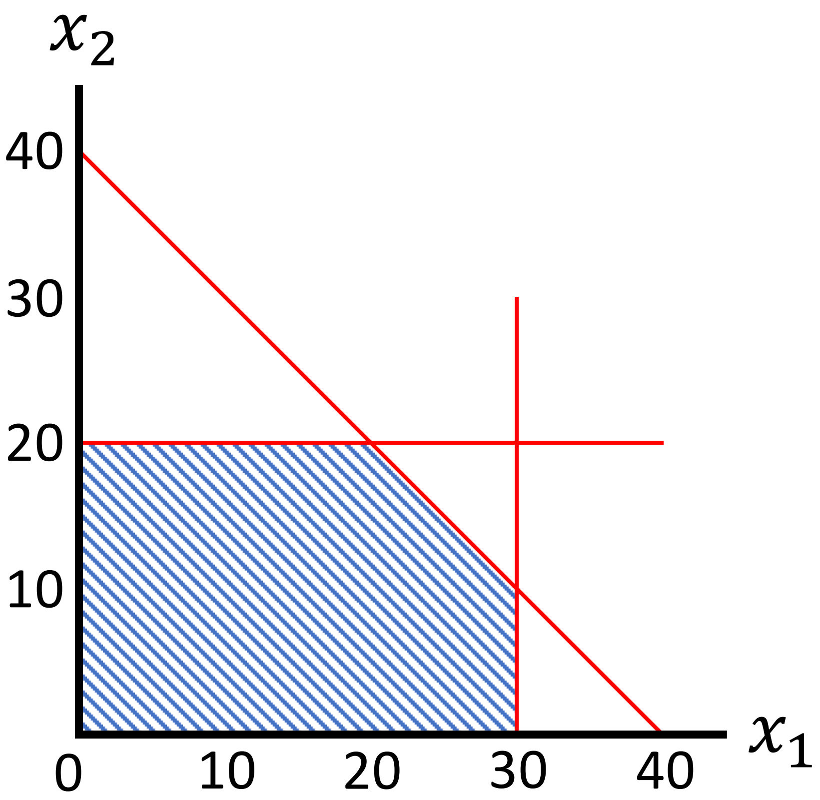


Feasible Region
(area where *all* valid solutions lie)

Maximizing Profit



x_1 = # of Rippers sold in a day
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Objective: $\max 100x_1 + 300x_2$

Subject to:
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What is the optimal value?

Maximizing Profit

x_1 = # of Rippers sold in a day
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Objective: $\max 100x_1 + 300x_2$
Subject to: $x_1 \leq 30$
$x_2 \leq 20$
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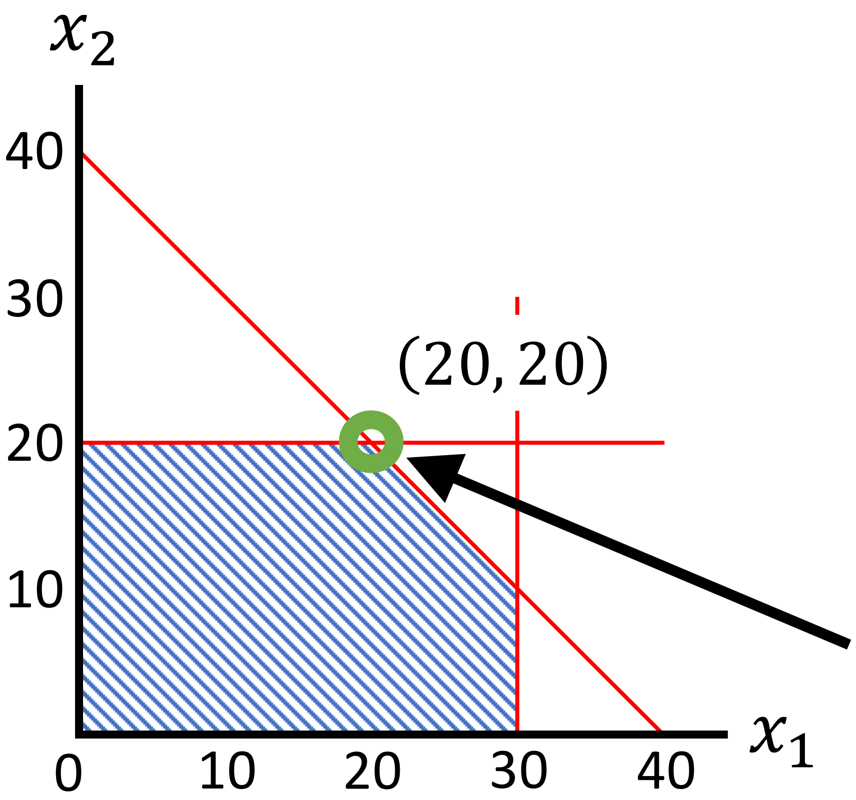
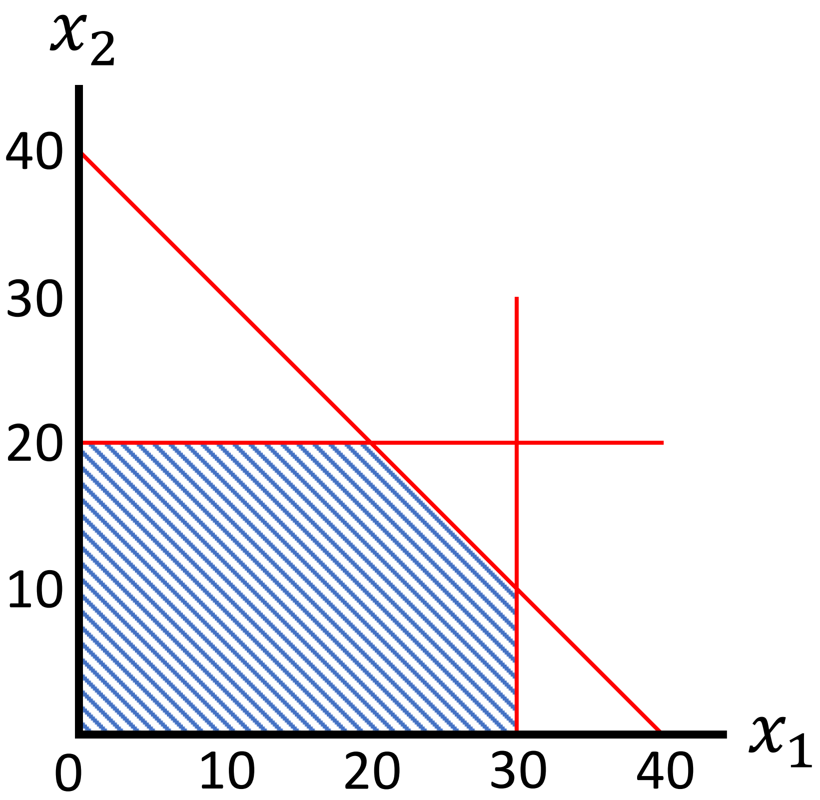


$\text{obj} = 100 * 20 + 300 * 20 = 8000$

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

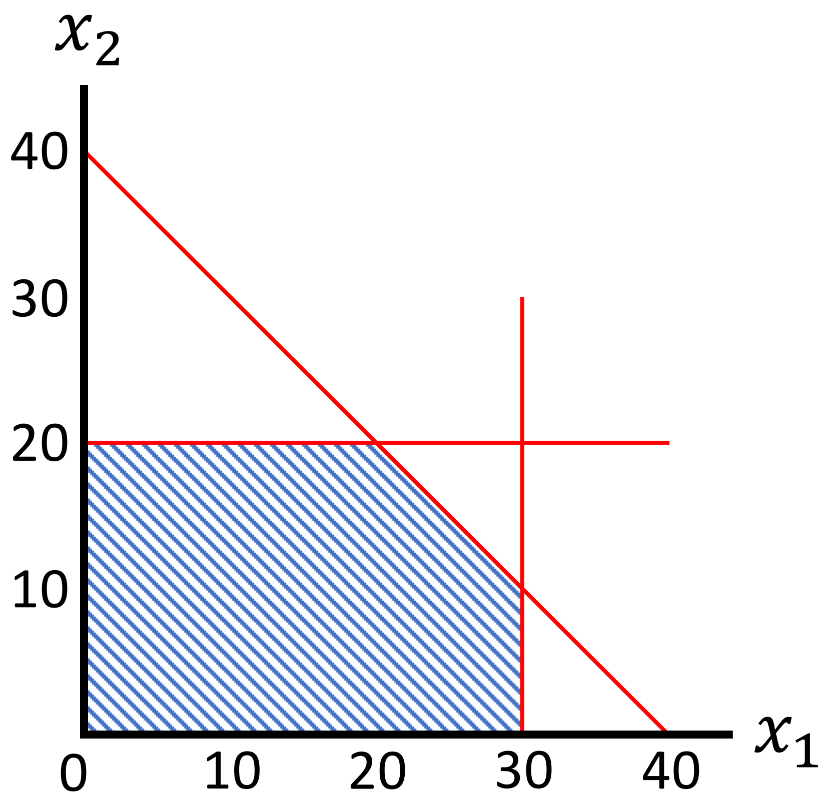
$c_2(x_1, x_2)$

$\vdots$

$c_n(x_1, x_2)$

How can we efficiently find optimal solutions?

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

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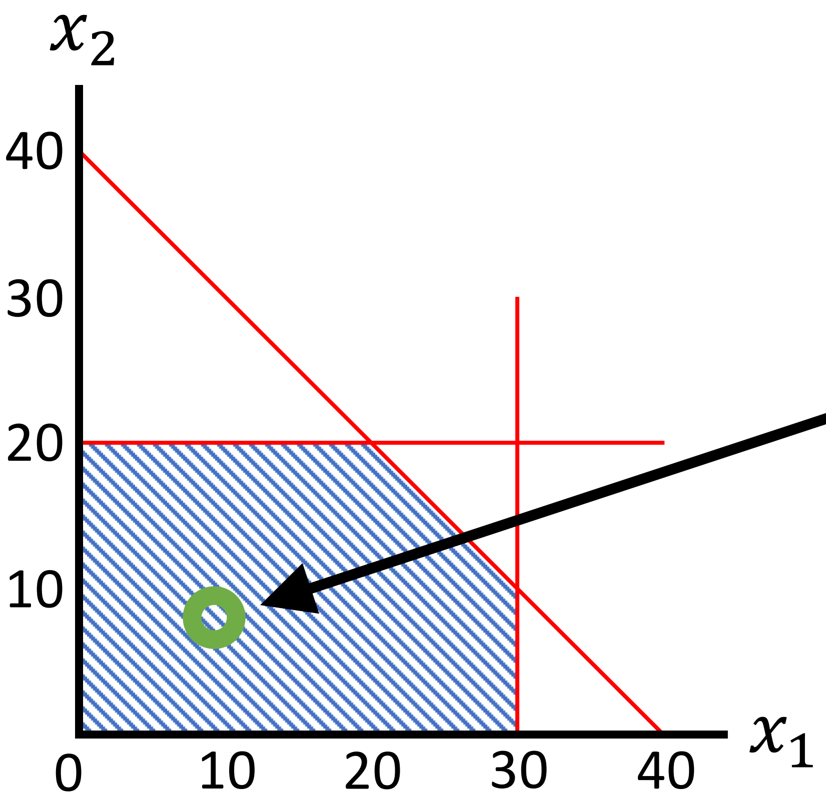
$c_n(x_1, x_2)$

How can we efficiently find optimal solutions?

Identify two key properties of optimal solutions:

1. ?
2. ?

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

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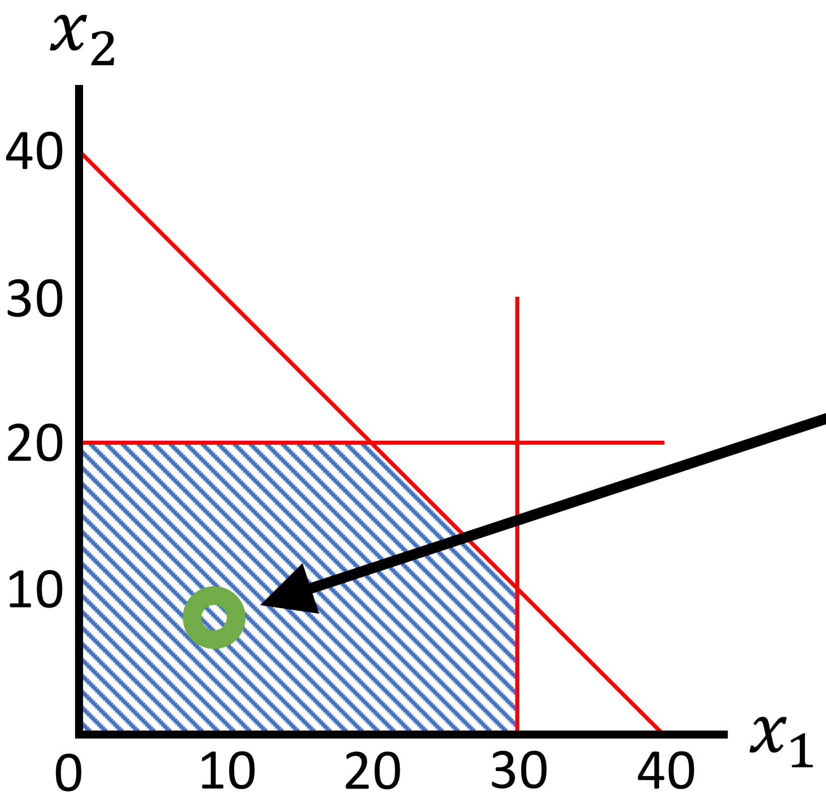
$c_n(x_1, x_2)$

Could this ever be a maximum value of the objective function?

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

$c_n(x_1, x_2)$

Could this ever be a maximum value of the objective function?

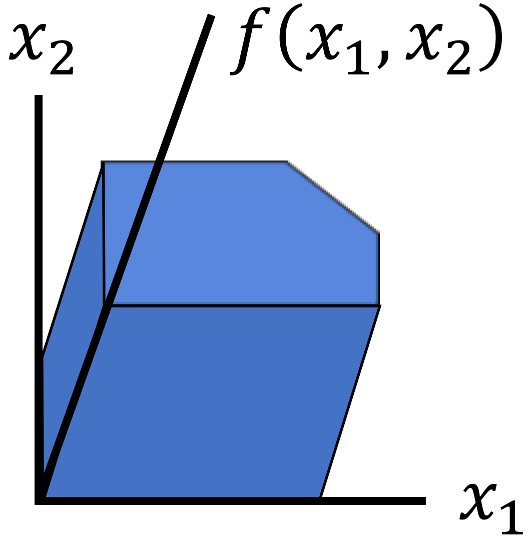
Yes, if $f(x_1, x_2) =$ constant.

Objective: $\max 5$

Subject to: $c_1(x_1, x_2)$

$\vdots$

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

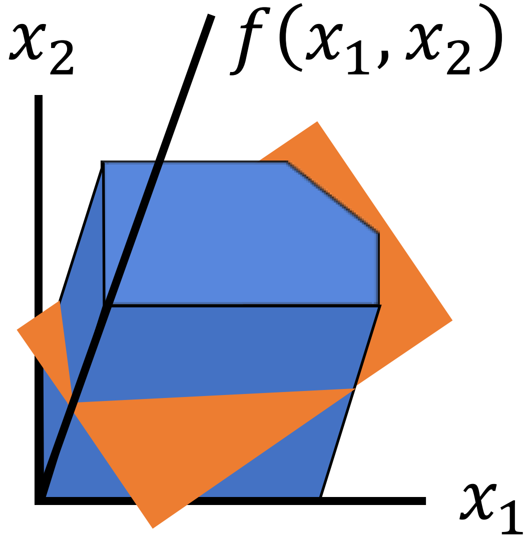
$\vdots$

$c_n(x_1, x_2)$

Could this ever be a maximum value of the objective function?

Yes, if $f(x_1, x_2) = \text{constant}$.

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

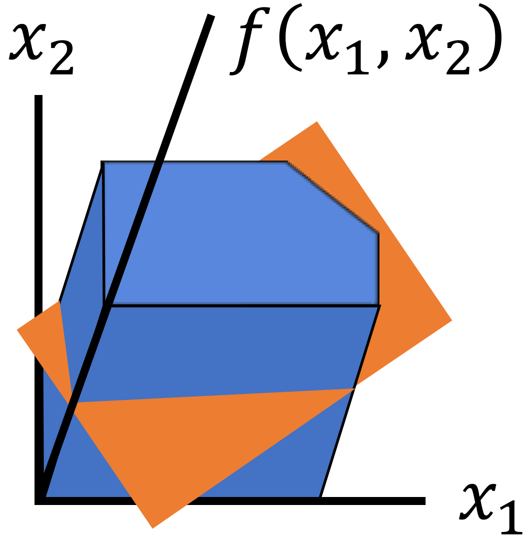
$c_n(x_1, x_2)$

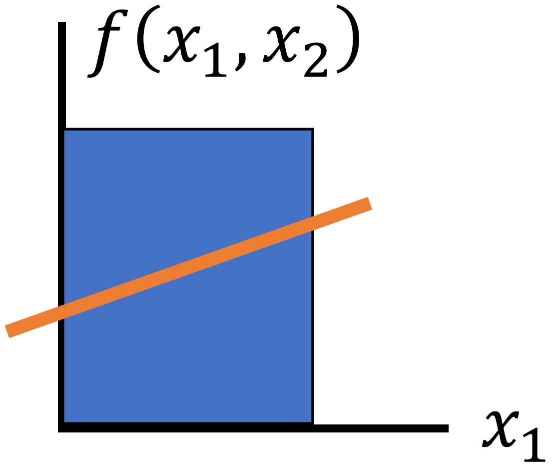
Could this ever be a maximum value of the objective function?

Yes, if $f(x_1, x_2) = \text{constant}$.

$f(x_1, x_2)$ is a plane

Optimal Value





Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

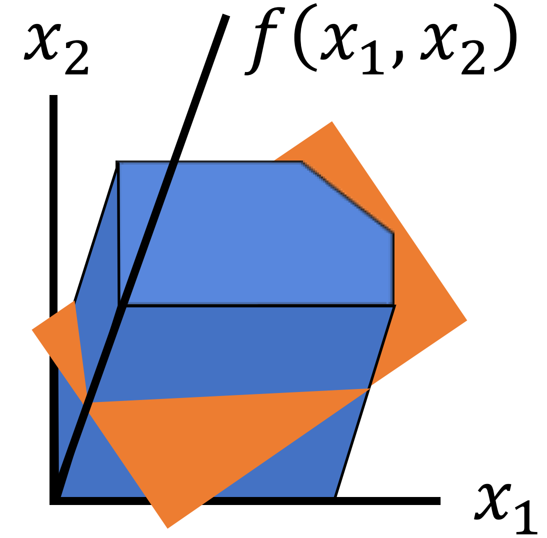
$c_n(x_1, x_2)$

Could this ever be a maximum value of the objective function?

Yes, if $f(x_1, x_2) = \text{constant}$.

$f(x_1, x_2)$ is a plane $\Longrightarrow$ a max/min of $f(x_1, x_2)$ occurs on the boundary of the feasible region.

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

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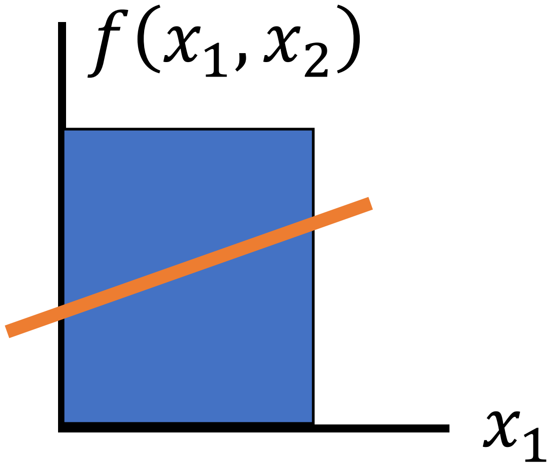
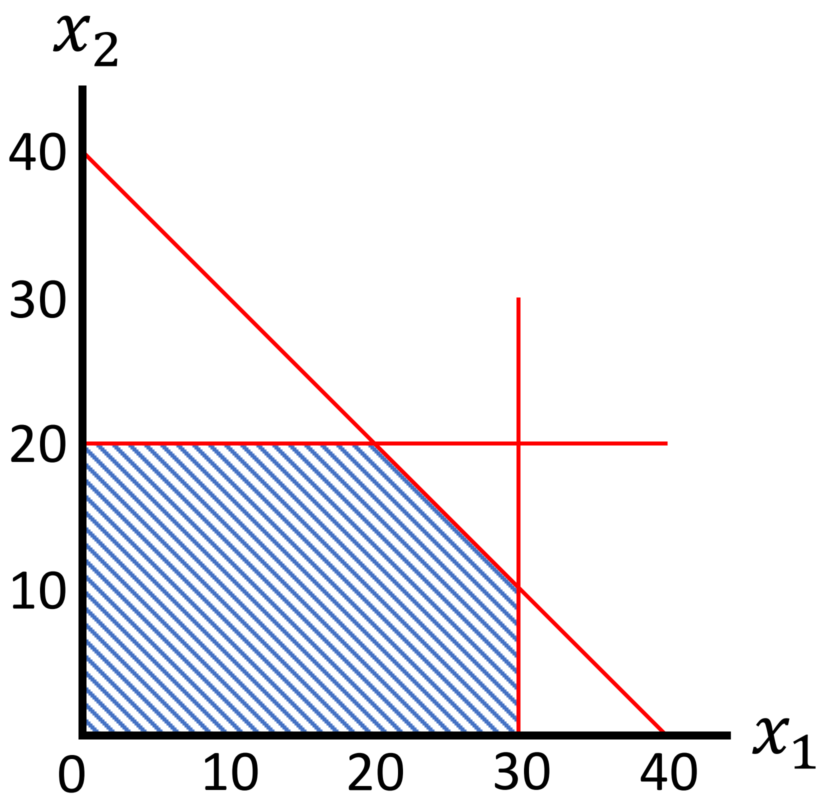
Could this ever be a maximum value of the objective function?



Yes, if $f(x_1, x_2) = \text{constant}$.

$f(x_1, x_2)$ is a plane $\Longrightarrow$ a max/min of $f(x_1, x_2)$ occurs on the boundary of the feasible region. Since feasible region has linear boundaries, max/min must occur at a vertex in the feasible region.

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

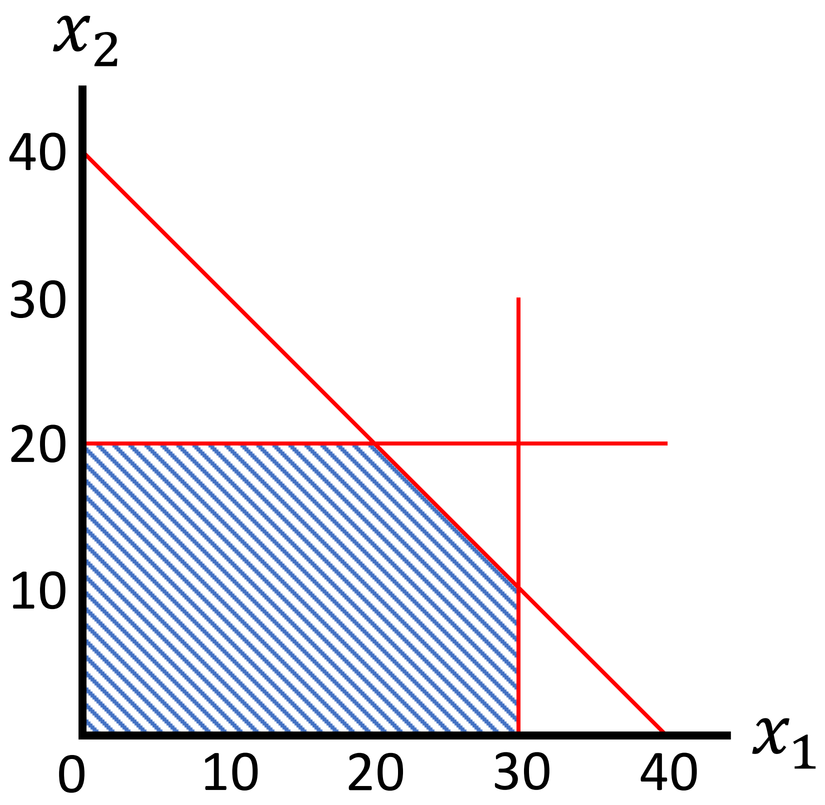
$c_n(x_1, x_2)$

How can we efficiently find optimal solutions?

Identify two key properties of optimal solutions:

1. Optimal value occurs at a vertex.
2. ?

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

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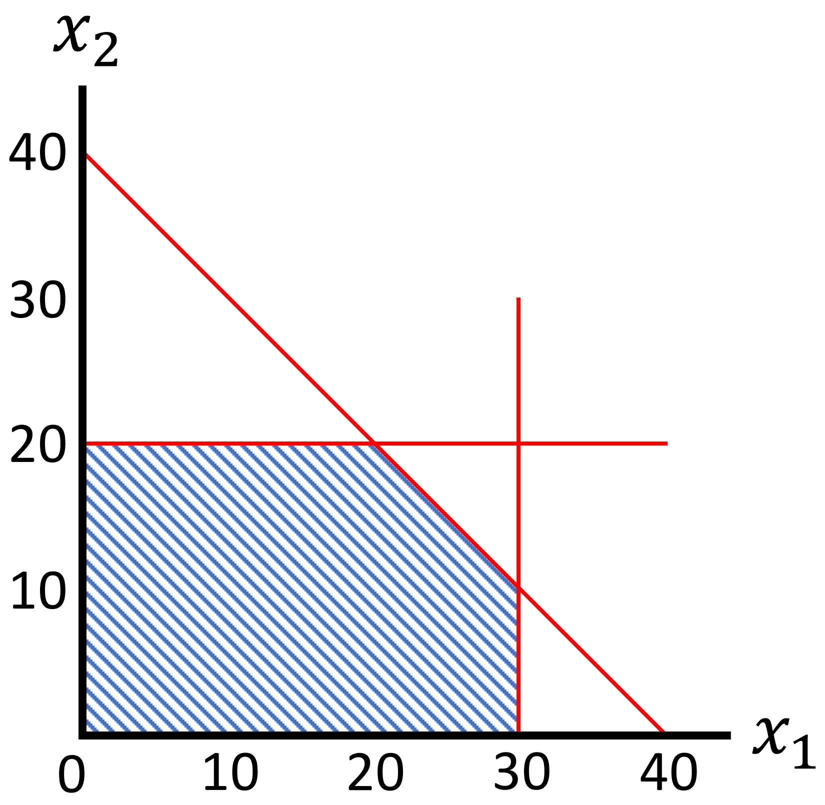
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

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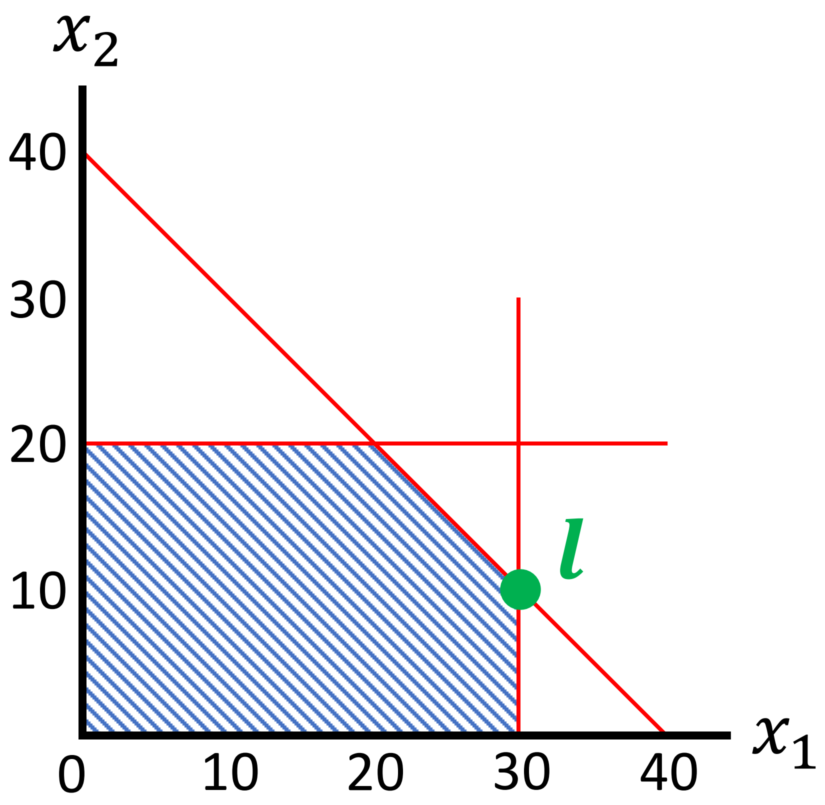
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min? local max/min = global max/min.

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

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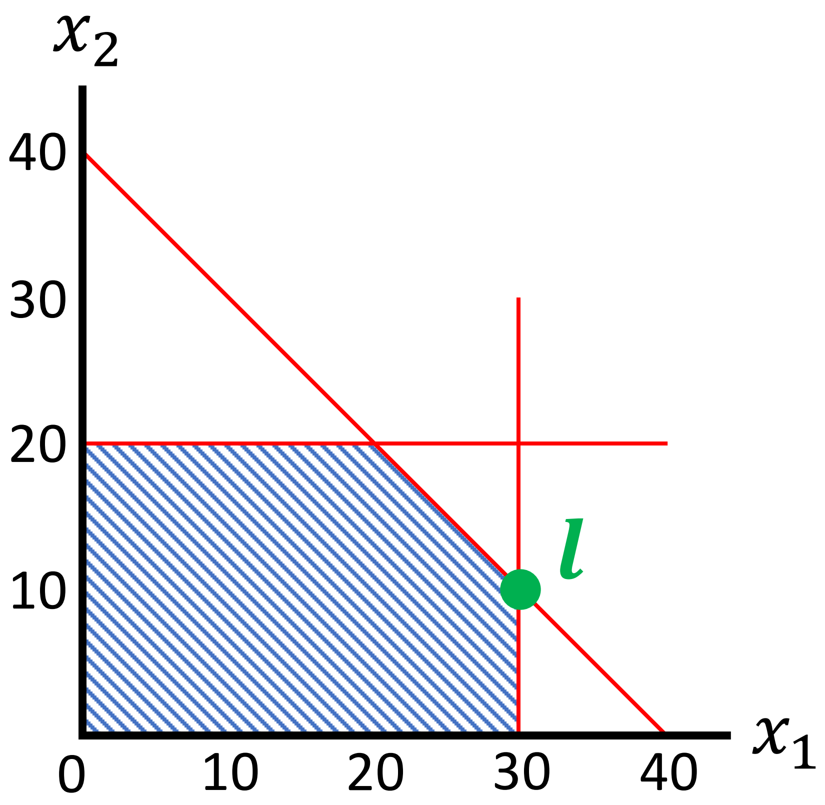
Is there a relationship between a local max/min and a global max/min? local max/min = global max/min.

local max ⇒ ?

Optimal Value



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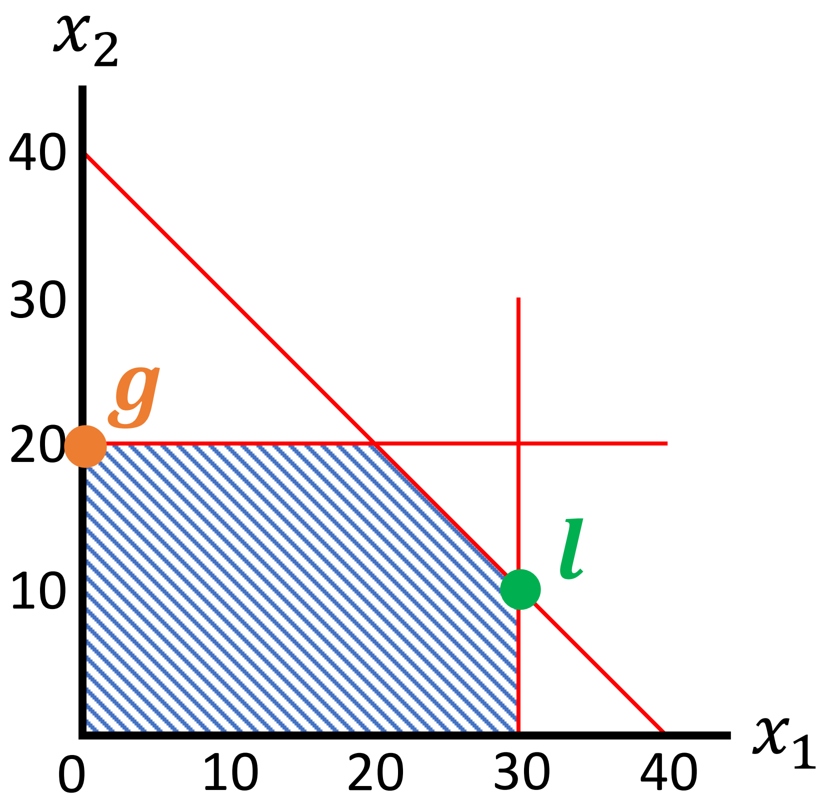
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min? local max/min = global max/min.

local max ⇒ all points in ε-neighborhood of l have lower objective values.

Optimal Value



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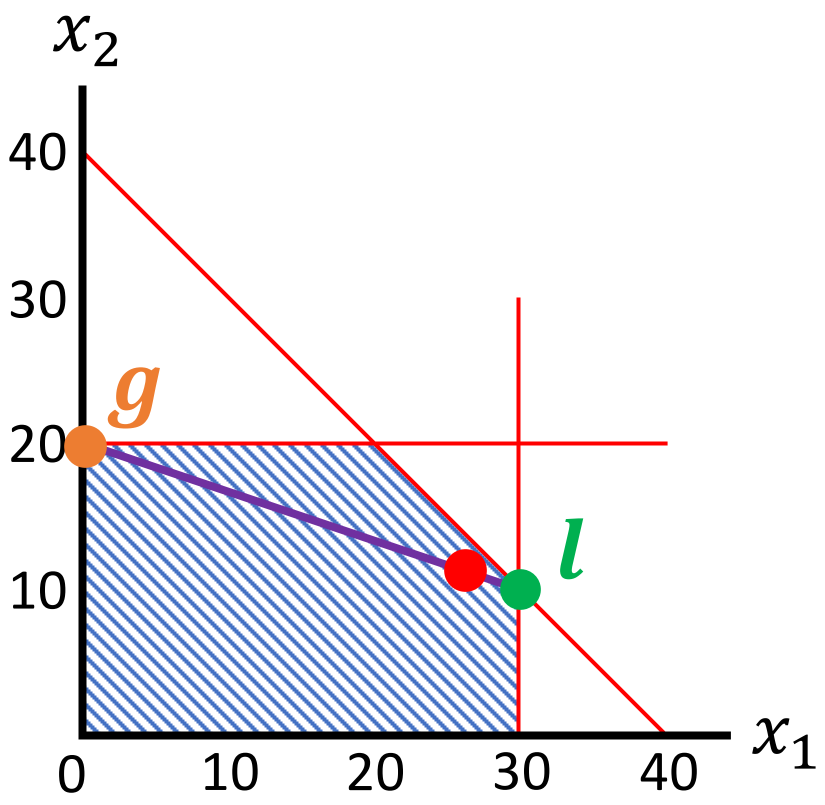
Subject to: $c_1(x_1, x_2)$
$c_2(x_1, x_2)$
$\vdots$
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min? local max/min = global max/min.

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.
Let g be global max.

Optimal Value

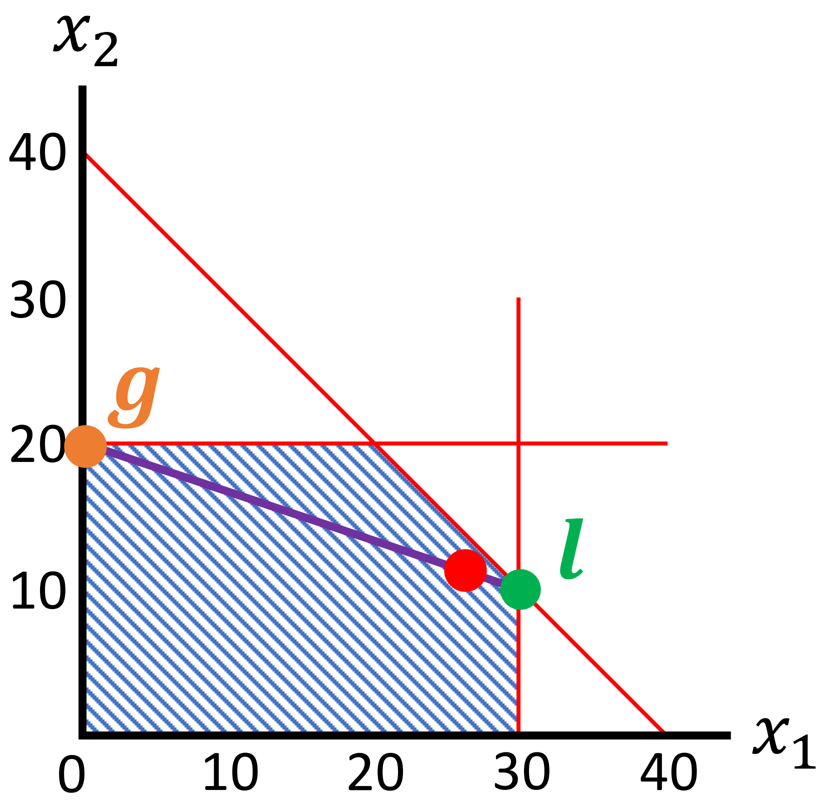


Objective:	$\max f(x_1, x_2)$
Subject to:	$c_1(x_1, x_2)$
	$c_2(x_1, x_2)$
	$\vdots$
	$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min? local max/min = global max/min.

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.
Let g be global max. Some point in ε-nbhd lies on the line between l and g and …

Optimal Value



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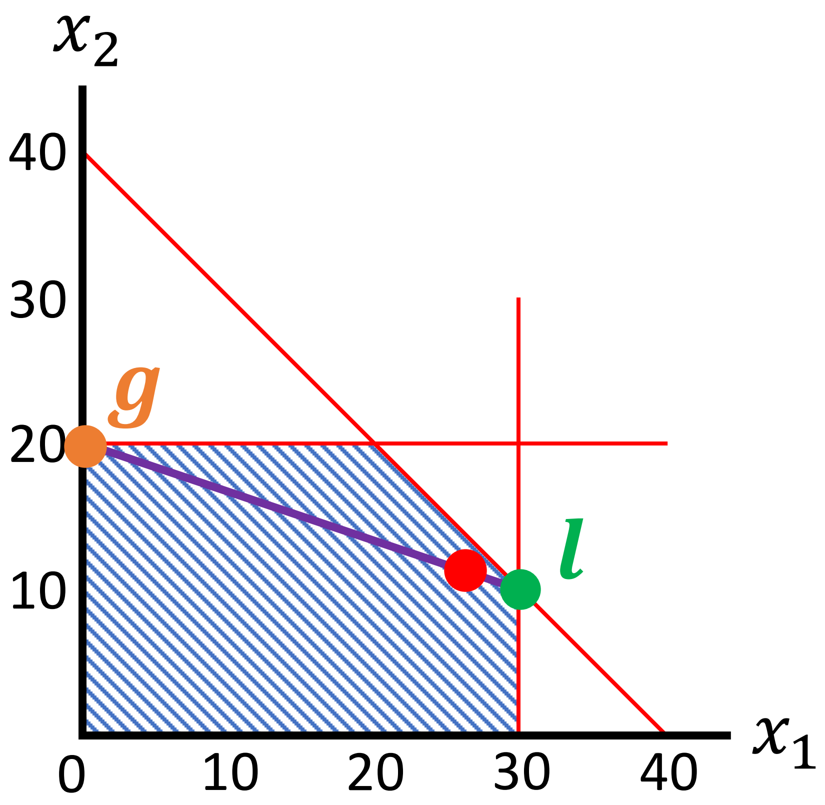
Is there a relationship between a local max/min and a global max/min? local max/min = global max/min.

local max ⇒ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (?).

Optimal Value



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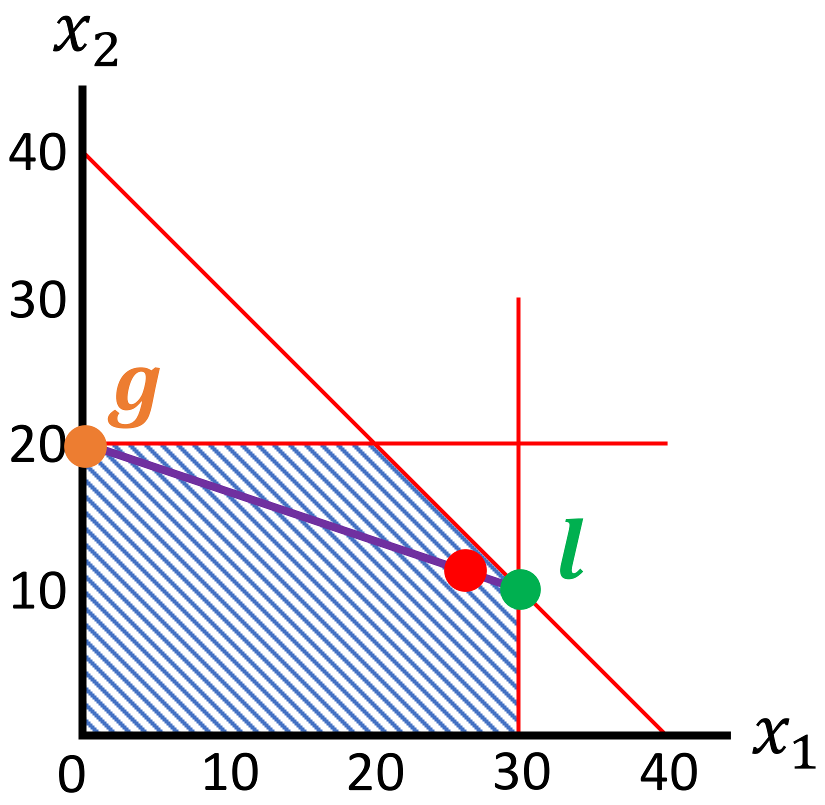
Is there a relationship between a local max/min and a global max/min? local max/min = global max/min.

local max ⇒ all points in ε-neighborhood of l have lower objective values.
Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region).

Optimal Value



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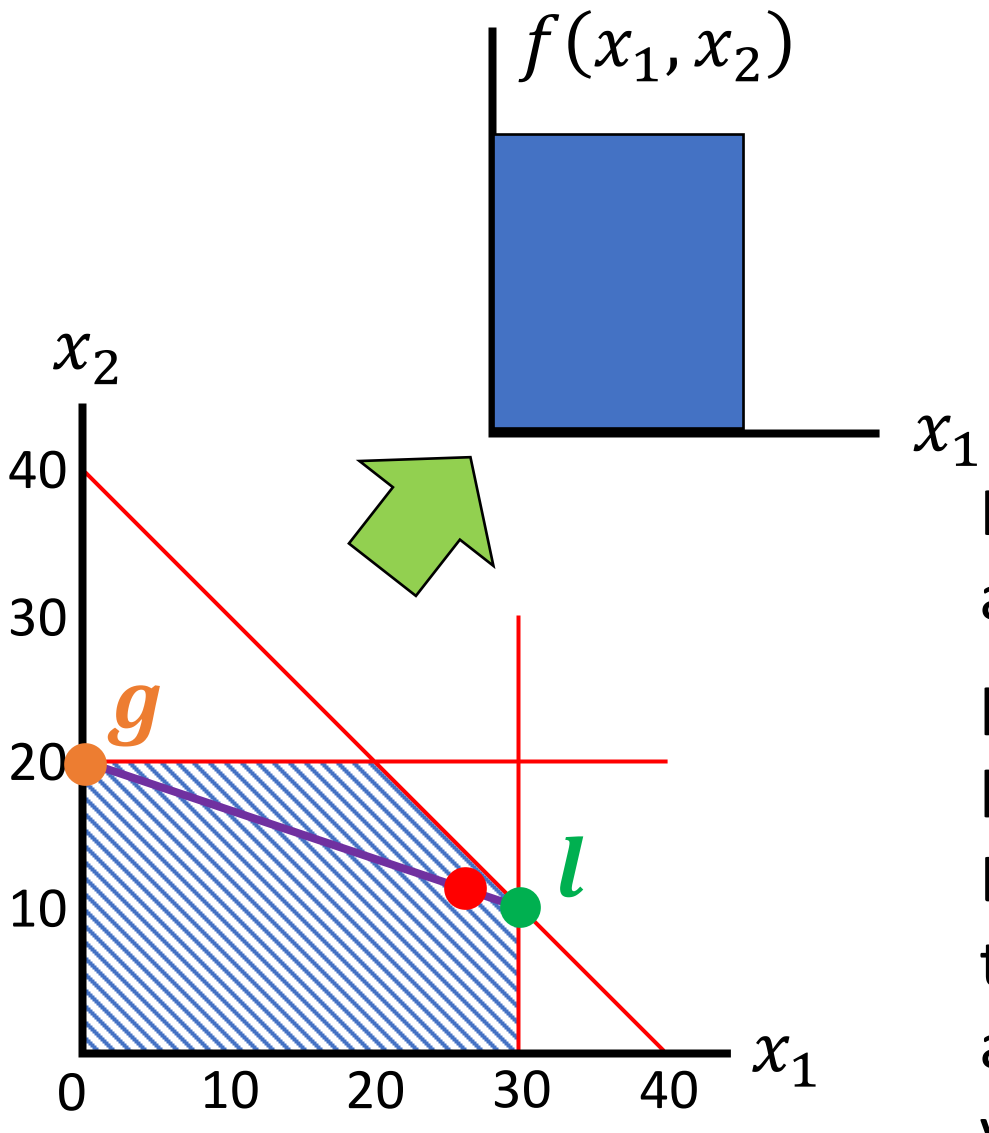
Optimal Value



Objective: $\max f(x_1, x_2)$

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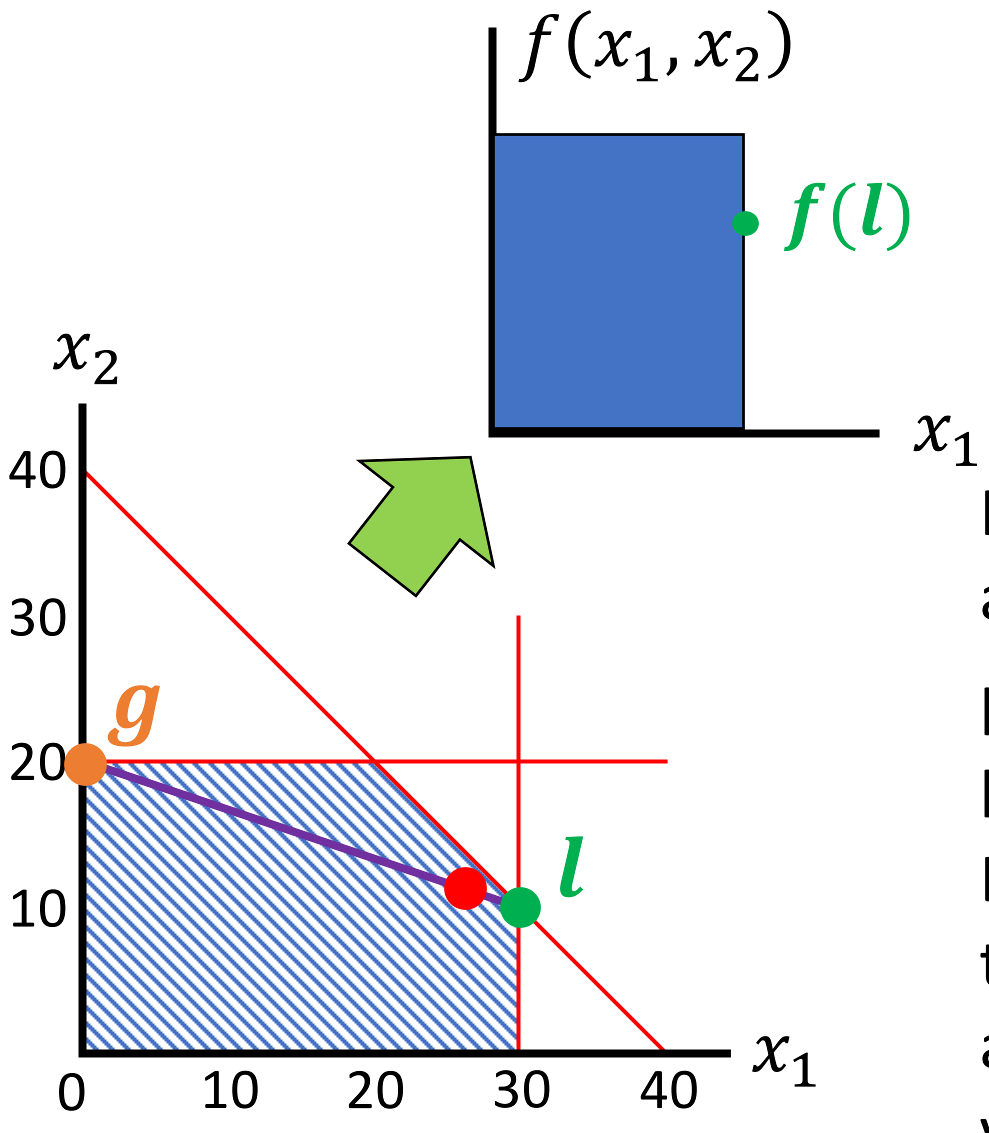

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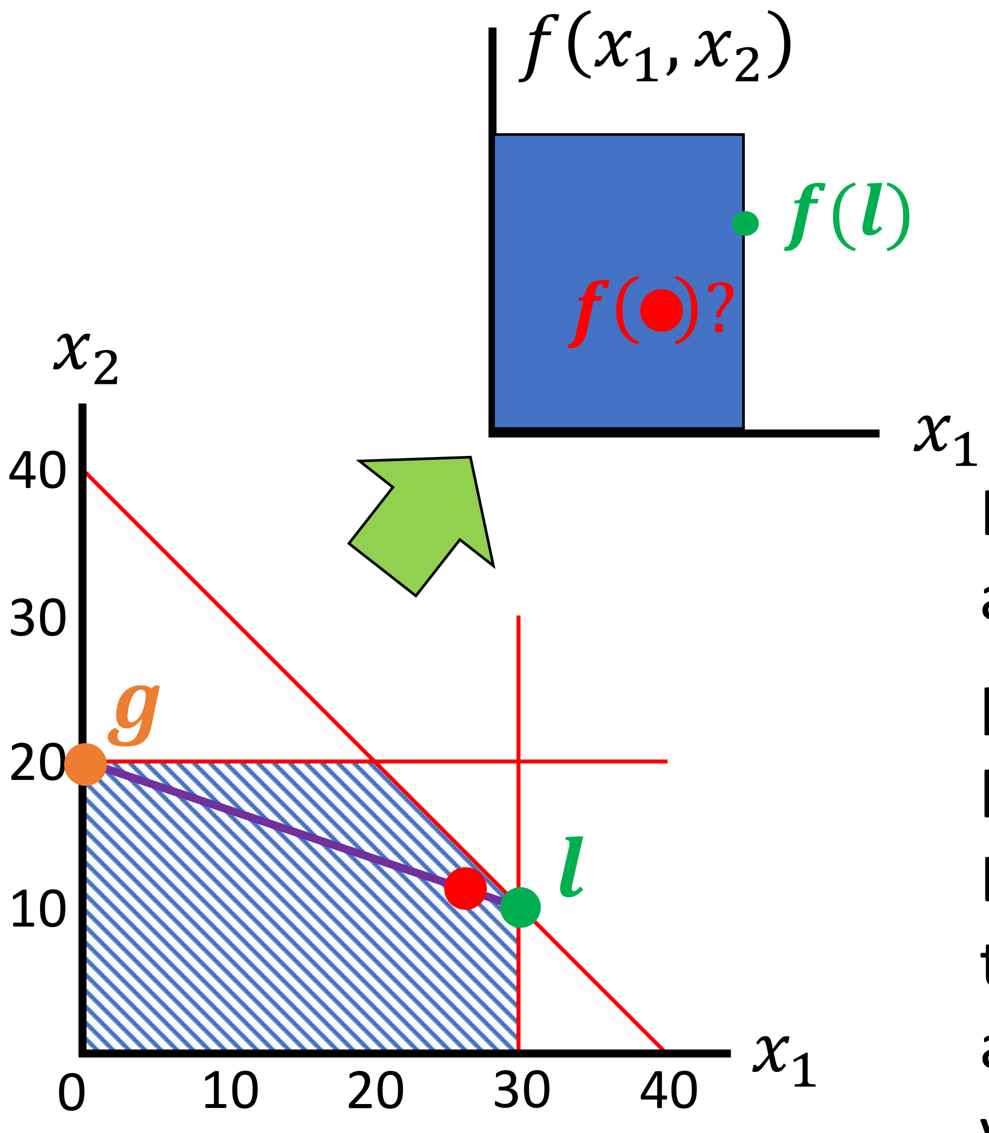

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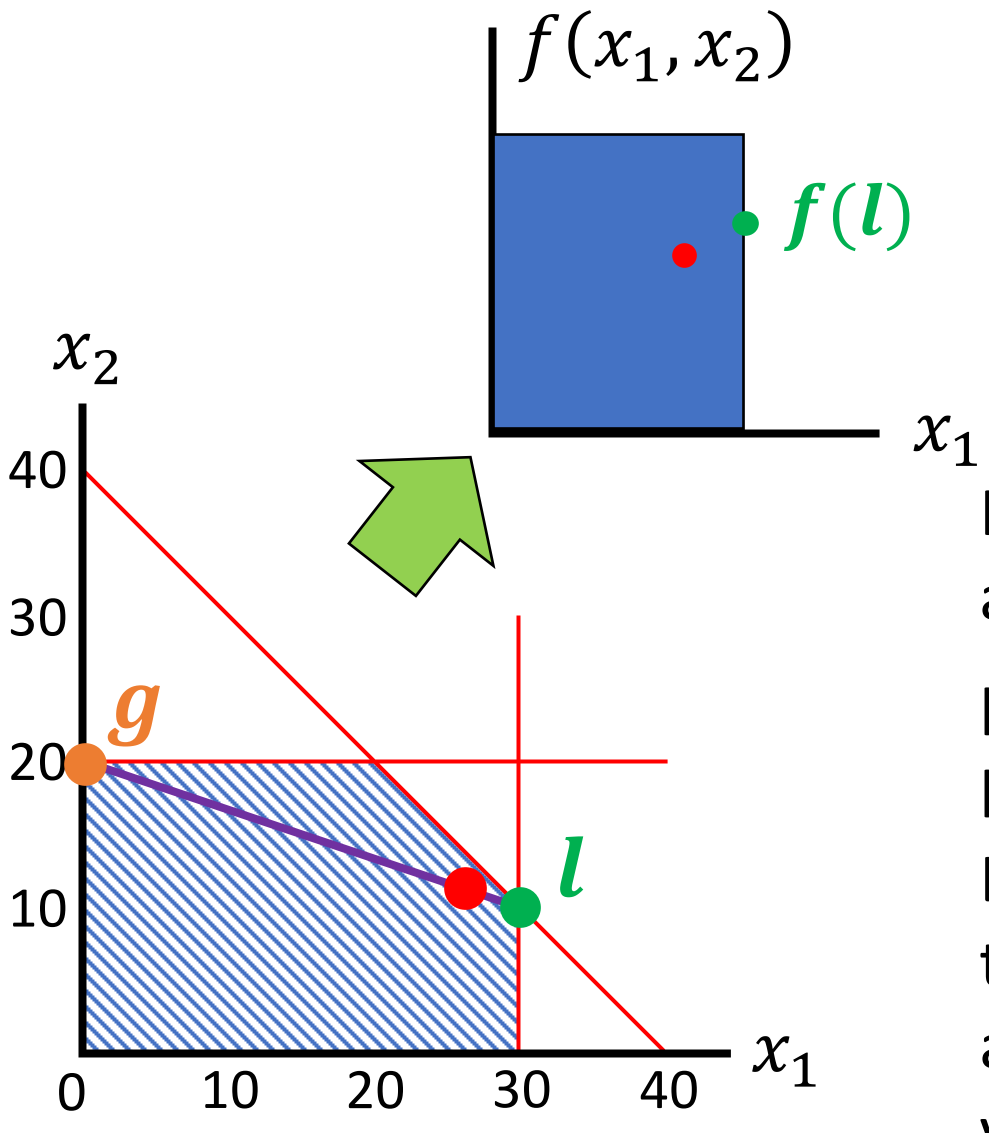

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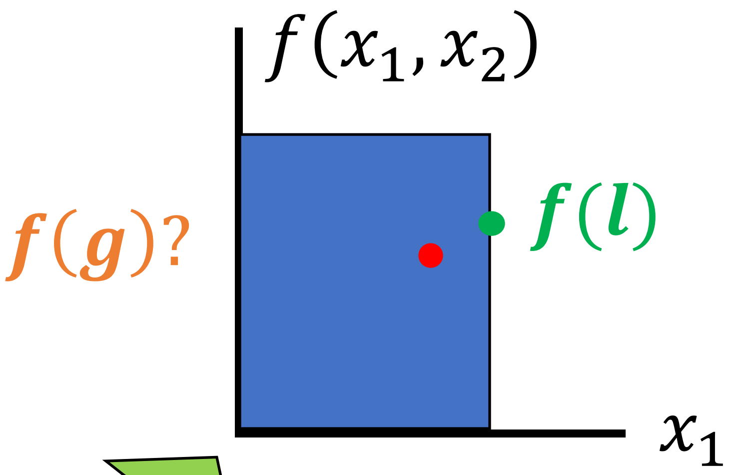

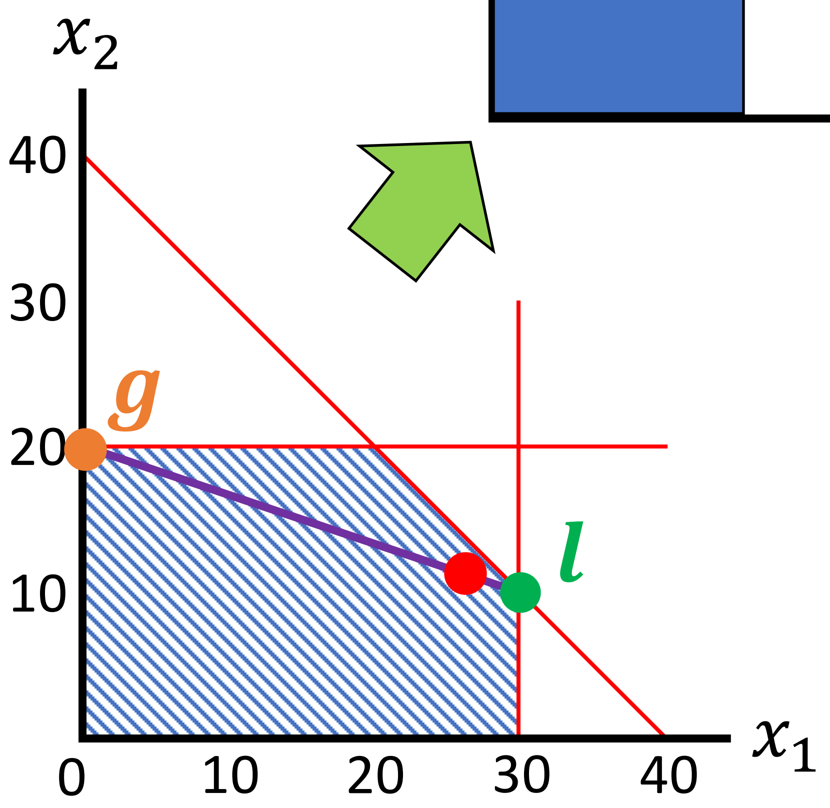

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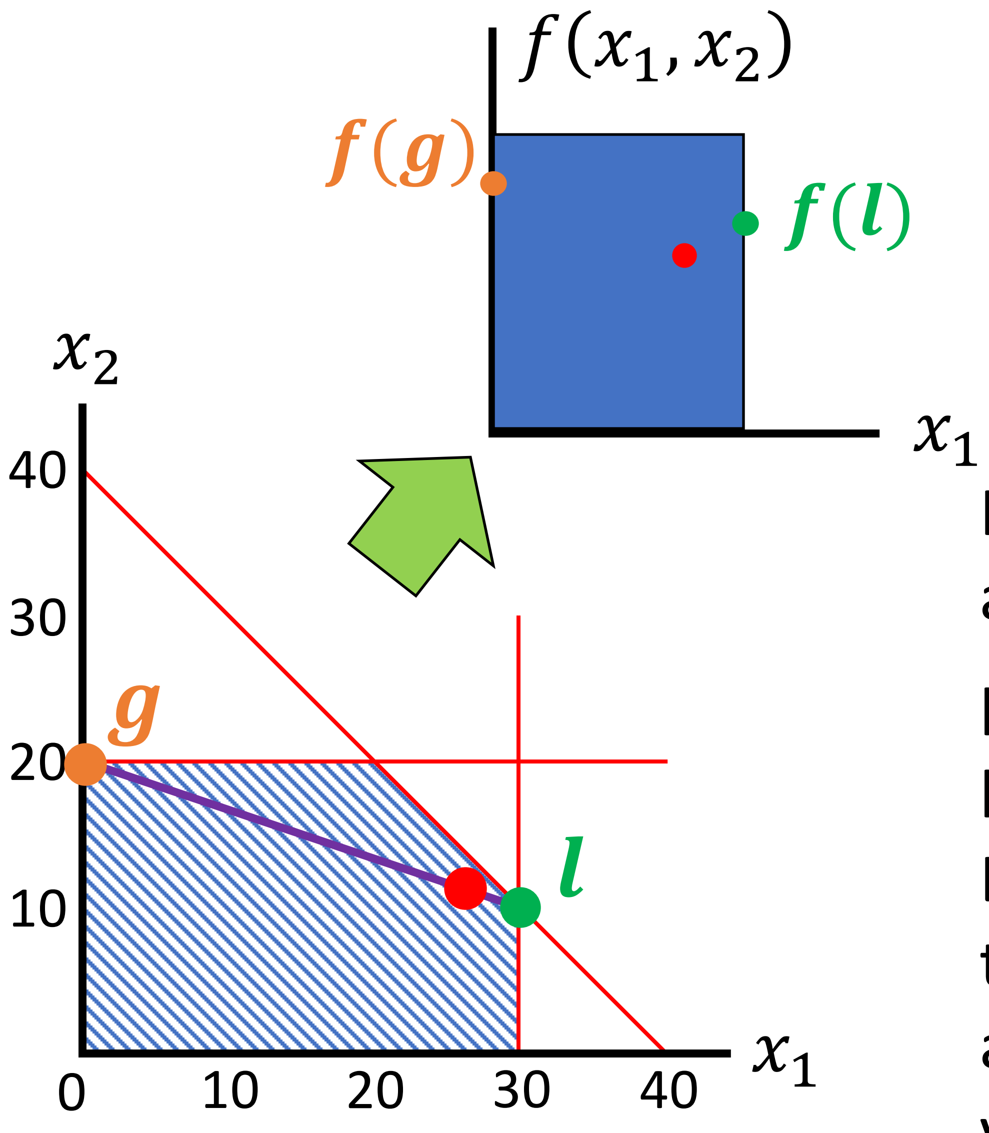

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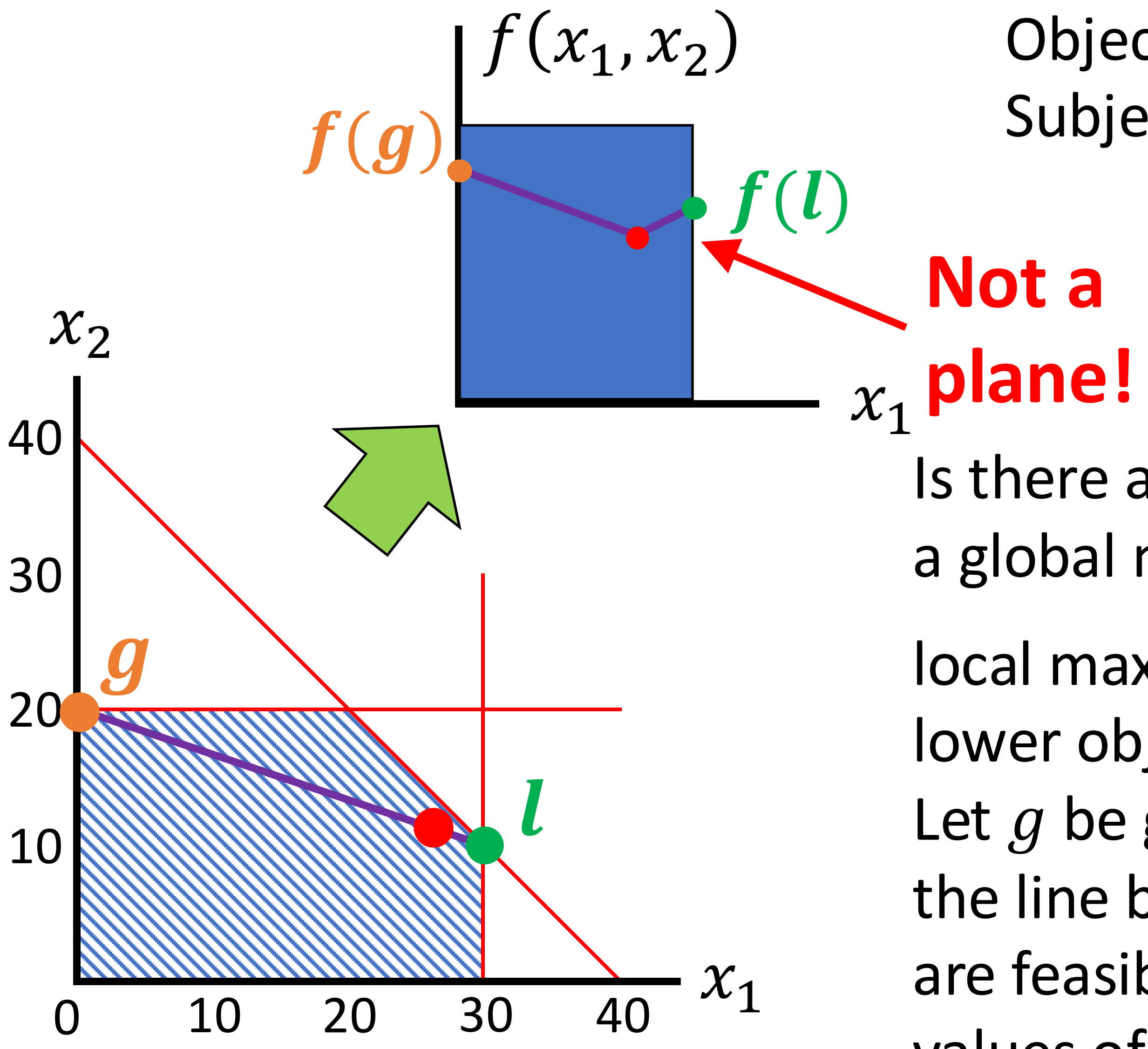

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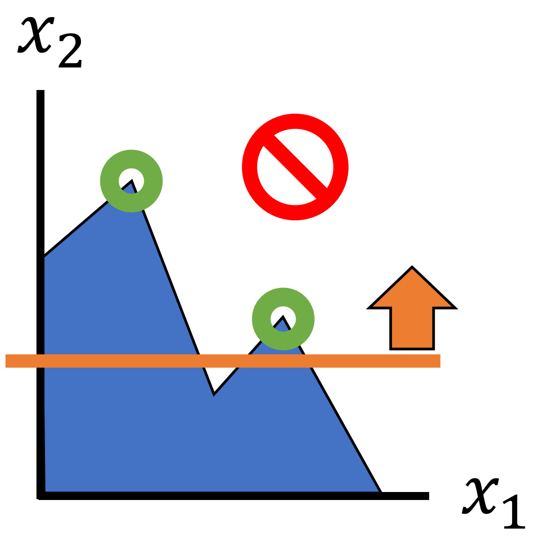

The only way for local optimum ≠ global optimum ***and*** objective be linear is for feasible region to not be convex.

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Subject to: $c_1(x_1, x_2)$
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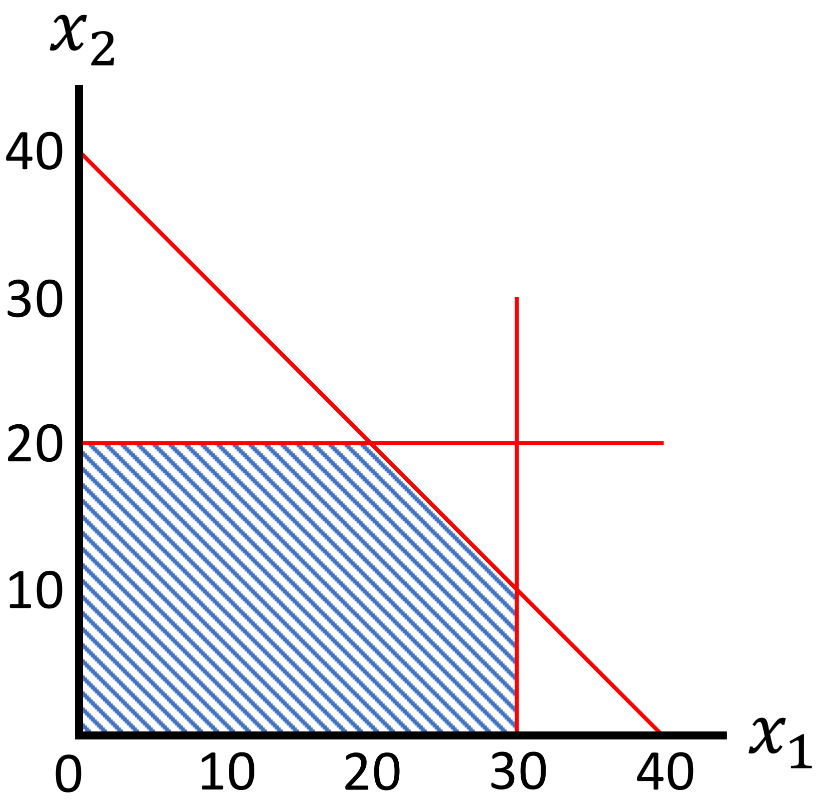
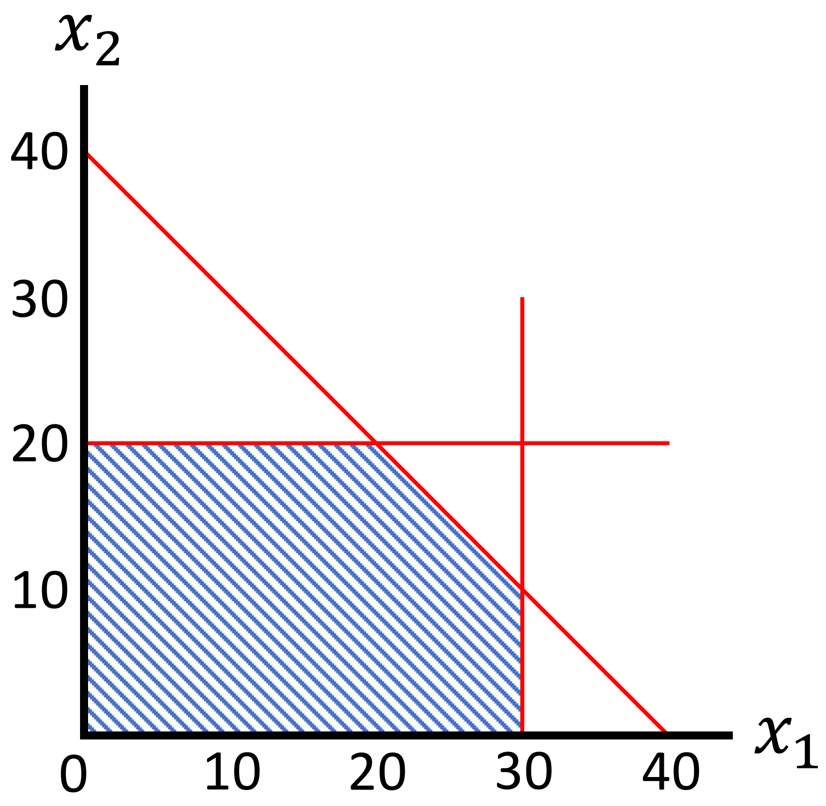


How can we efficiently find optimal solutions?

Identify two key properties of optimal solutions:

1. Optimal value occurs at a vertex.
2. Local optimum is global optimum.

Optimal Value



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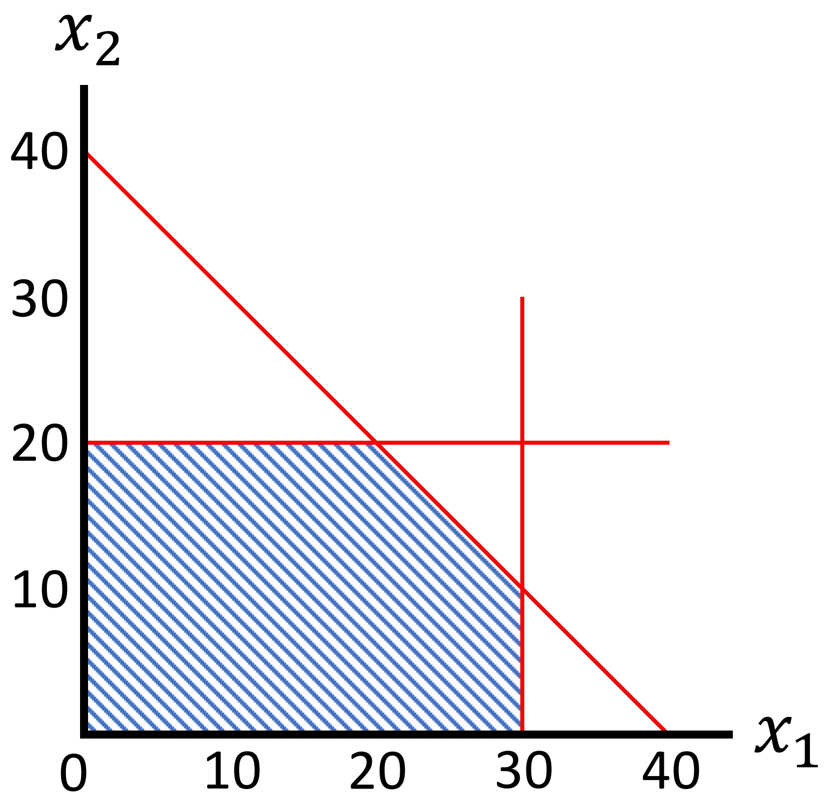
Algorithm to find optimal solution:

?

Optimal Value



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Algorithm to find optimal solution:

Test each vertex in order until no neighbors have larger (or smaller) value.

Max Flow

Given a directed, edge-weighted graph, where each edge has an associated capacity, make a linear program that will determine the maximum flow on the network.

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Decision Variables?
Objective?
Constraint?

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$$x_e = \text{Amount of flow on edge } e.$$

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$x_e =$ Amount of flow on edge e.
$c_e =$ Capacity of edge e?

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~~c_e = Capacity of edge e?~~

Not a decision variable!!

I.e., the solver is not allowed to modify this to influence the solution

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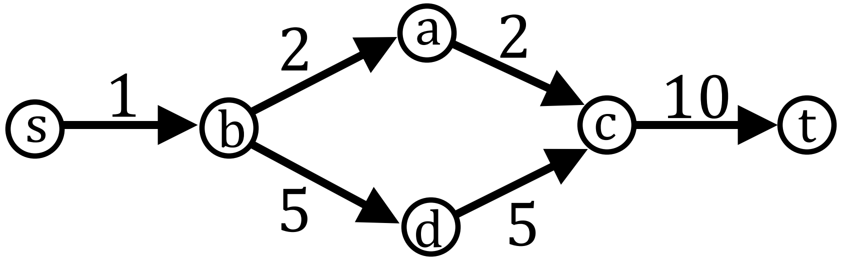

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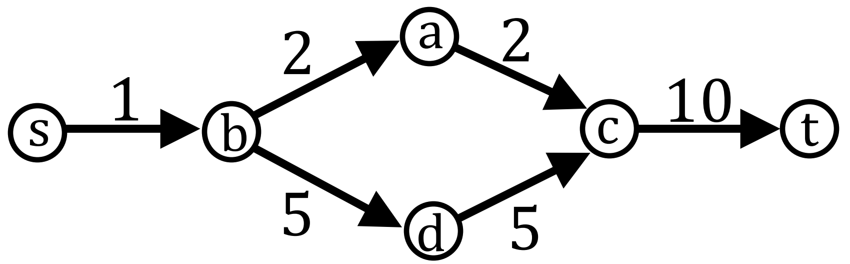

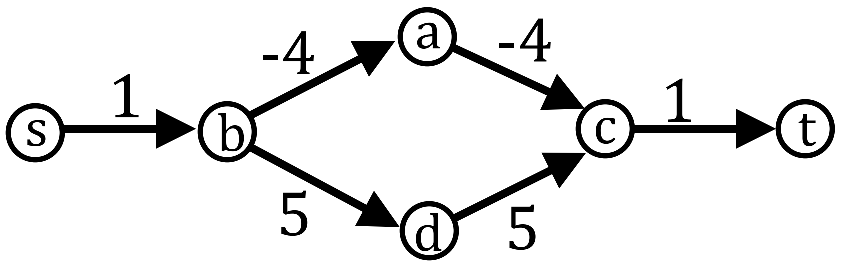

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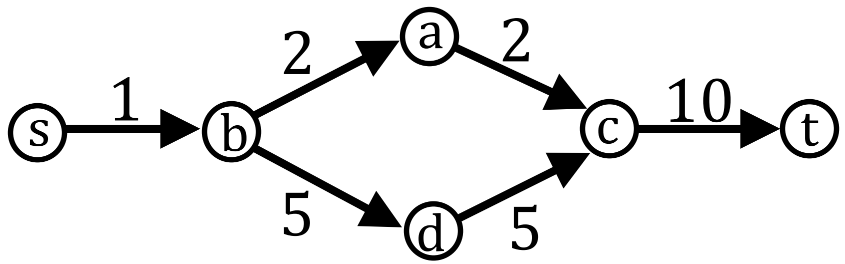

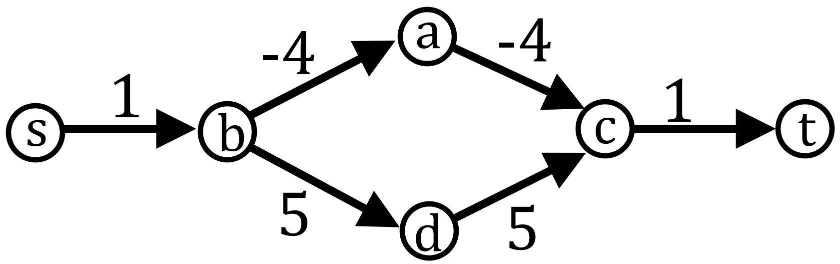

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Step 1: Make variables.

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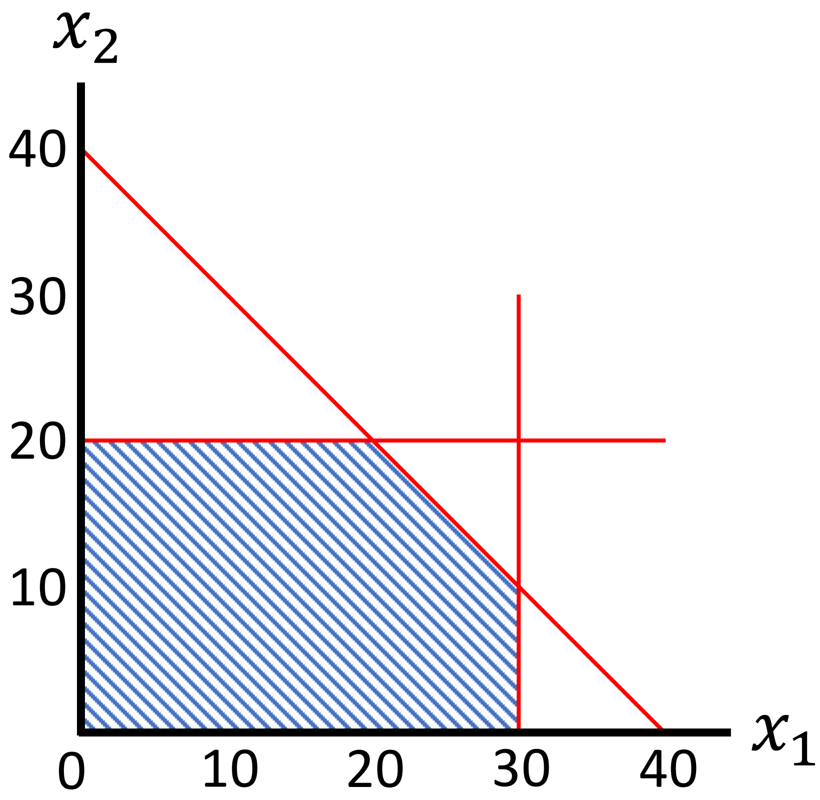
Do we need non-negativity constraints?

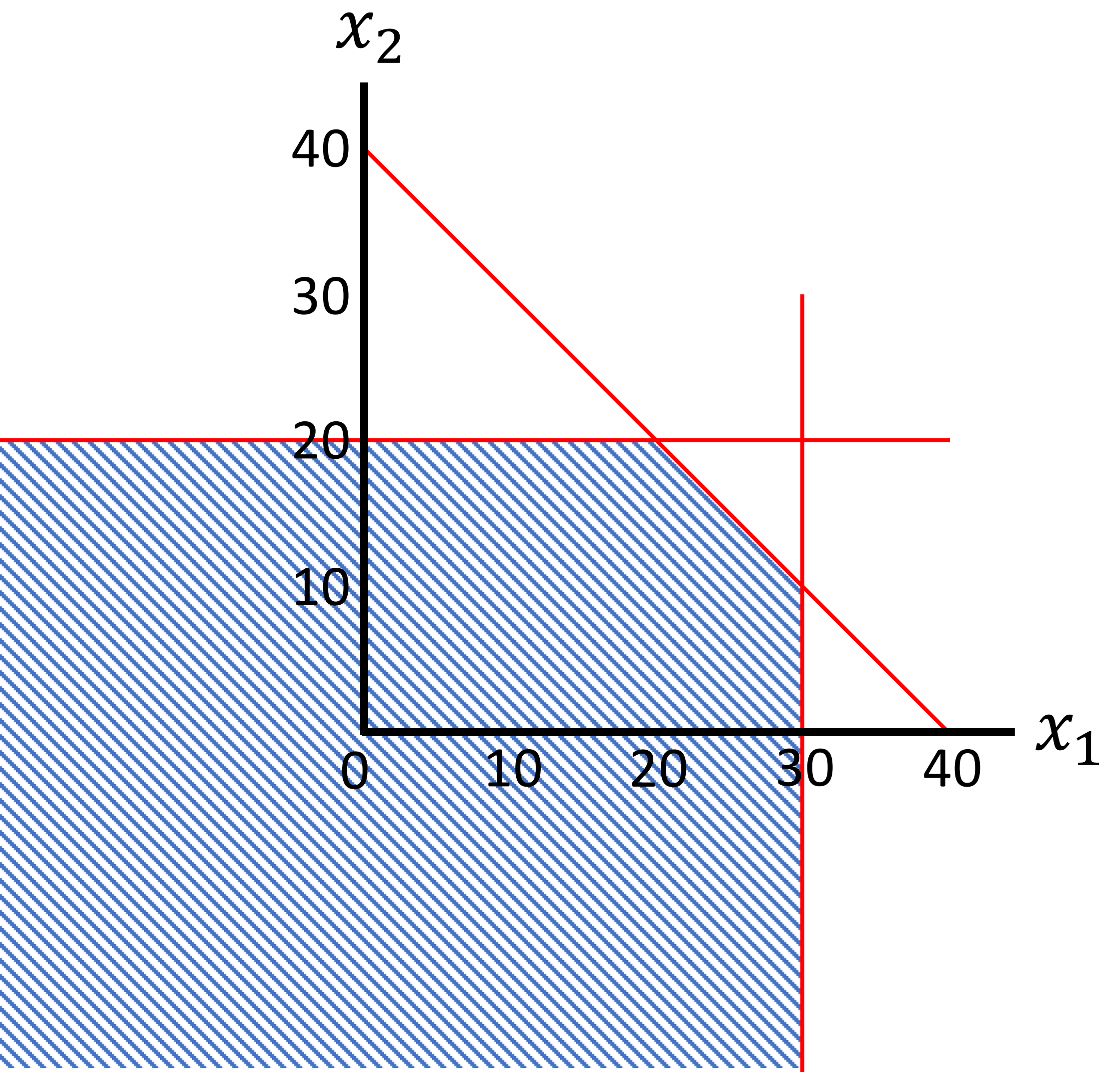
Non-negativity Constraints



Objective: $\max 100x_1 + 300x_2$

Subject to:
$$x_1 \leq 30$$
$$x_2 \leq 20$$
$$x_1 + x_2 \leq 40$$
$$x_1, x_2 \geq 0$$

Non-negativity Constraints



Objective: $\max 100x_1 + 300x_2$

Subject to: $x_1 \leq 30$

$x_2 \leq 20$

$x_1 + x_2 \leq 40$

~~$x_1, x_2 \geq 0$~~

Non-negativity Constraints

Objective: $\min x$

Subject to: $x \geq 0$

Optimal Value: ?

Non-negativity Constraints

Objective: $\min x$

Subject to: $x \geq 0$

Optimal Value: $x = 0$

Non-negativity Constraints

Objective: $\min x$

Subject to: $\cancel{x \geq 0}$

Optimal Value: ?

Non-negativity Constraints

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Subject to: $\cancel{x \geq 0}$

Optimal Value: $x = -\infty$

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$$x_1, x_2, x_3, x_4 \geq 0$$

Do we need non-negativity constraints?

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estim[illegible]e number of voters gained or lost based on each \$1 spent advertisi[illegible] campaign aims to minimize advertising expenses[illegible]

Issue	Urban	Suburban	Rural
Infrastructure	-2	+5	+3
Gun Control	+8	+2	[illegible]
Farm Subsidies	+0	[illegible]	[illegible]
Gasoline Tax	[illegible]	[illegible]	[illegible]

Lesson: If the problem calls for implicitly calls for non-negativity constraints, put them in!

[illegible]rastructure.

[illegible] on gun control.

[illegible] \$ spent on farm subsidies.

x_4 = \$ spent on gasoline tax.

Objective: $\min x_1 + x_2 + x_3 + x_4$

Subject to: $-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$

$5x_1 + 2x_2 \geq 100{,}000$

$3x_1 - 5x_2 + 10x_3 - 2x_4 \geq 25{,}000$

$x_1, x_2, x_3, x_4 \geq 0$

Do we [illegible] non-negativity constraints?