Linear Programming
CSCI 432

Linear Program (LP)

$x_1 =$ # of Rippers sold
$x_2 =$ # of Ripper Carbons

Objective: $\max 100x_1 + 300x_2$
Subject to: $x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$x_1, x_2 \geq 0$

Decision Variables:

- Real numbers = solvable in polynomial time (called LP).
- Integers = not (yet?) solvable in polynomial time (called integer linear program – ILP).

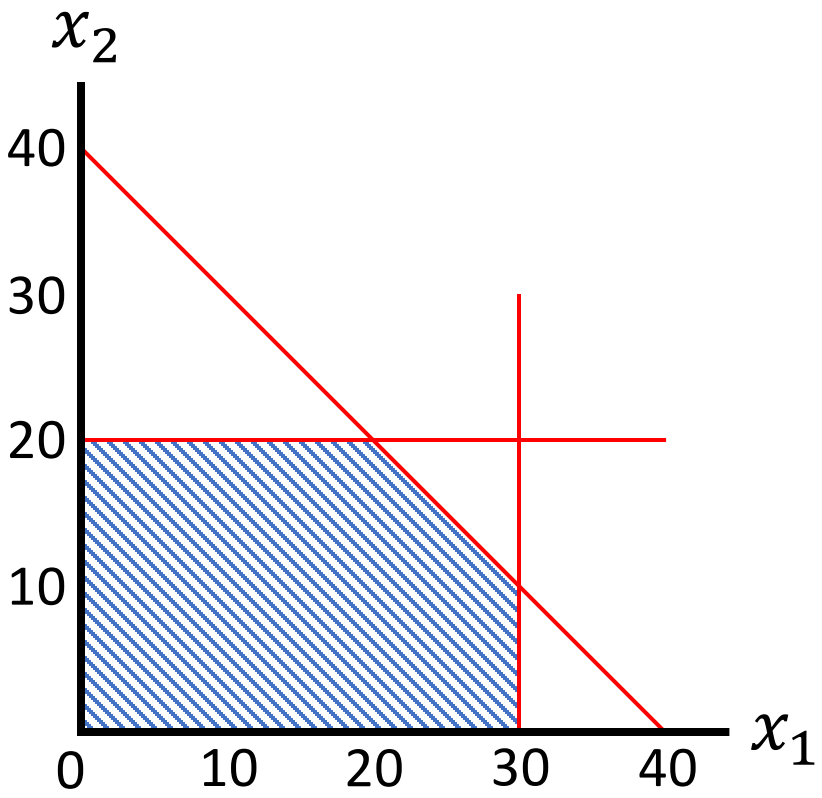
Objective:

- Can be minimization or maximization.
- Must be linear combinations of variables x_i (e.g. $a_1x_1 + \cdots + a_nx_n$ for constants a_i, not $a_ix_1x_2$).

Constraints:

- Can be $\leq$, $\geq$, $=$.
- Must be linear combinations of variables.

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

$c_n(x_1, x_2)$

Properties of optimal solutions:

1. Optimal value occurs at a vertex.
2. Local optimum is global optimum.

Algorithm to find optimal solution:

Test each vertex in order until no neighbors have larger (or smaller) value.

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	**Urban**	**Suburban**	**Rural**
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

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Step 1: Make variables.

"What are the decisions that need to be made?"

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Step 2: Make objective.

"What are we trying to maximize or minimize?"

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https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.linprog.html

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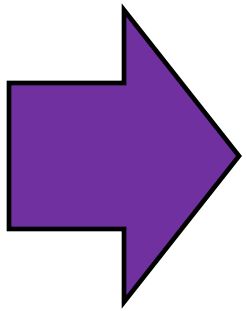

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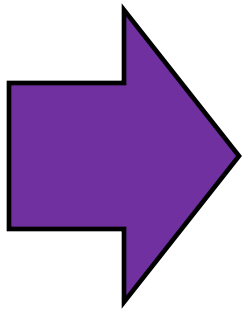

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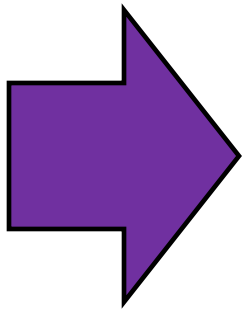

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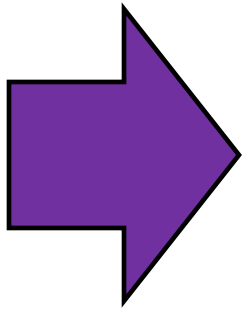

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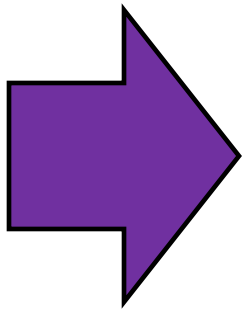

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$f(x) \geq a$ if and only if $-f(x) \leq -a$

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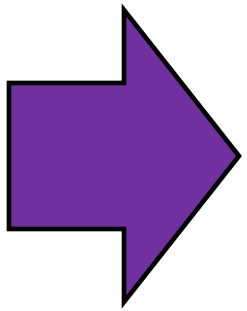

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$$-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$$

$$\Rightarrow \quad 2x_1 - 8x_2 - 10x_4 \leq -50{,}000$$

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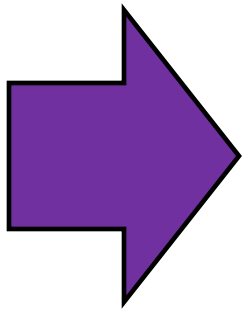

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Code…

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Do we need non-negativity constraints?

$x_1 =$ \$ spent on infrastructure.
$x_2 =$ \$ spent on gun control.
$x_3 =$ \$ spent on farm subsidies.
$x_4 =$ \$ spent on gasoline tax.

Objective: $\min x_1 + x_2 + x_3 + x_4$

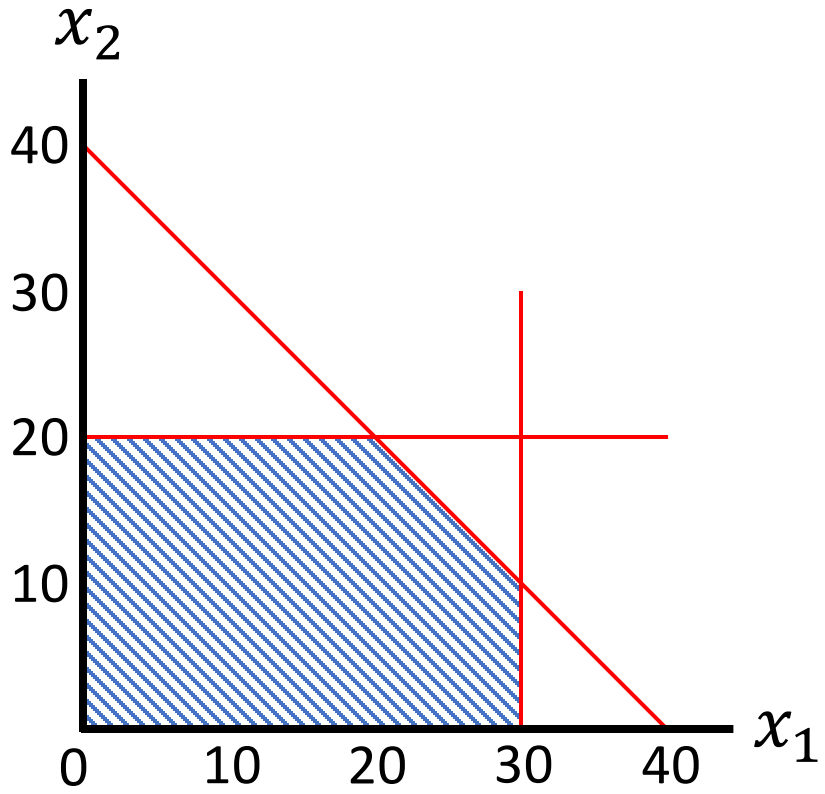
Subject to: $-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$

$5x_1 + 2x_2 \geq 100{,}000$

$3x_1 - 5x_2 + 10x_3 - 2x_4 \geq 25{,}000$

$x_1, x_2, x_3, x_4 \geq 0$

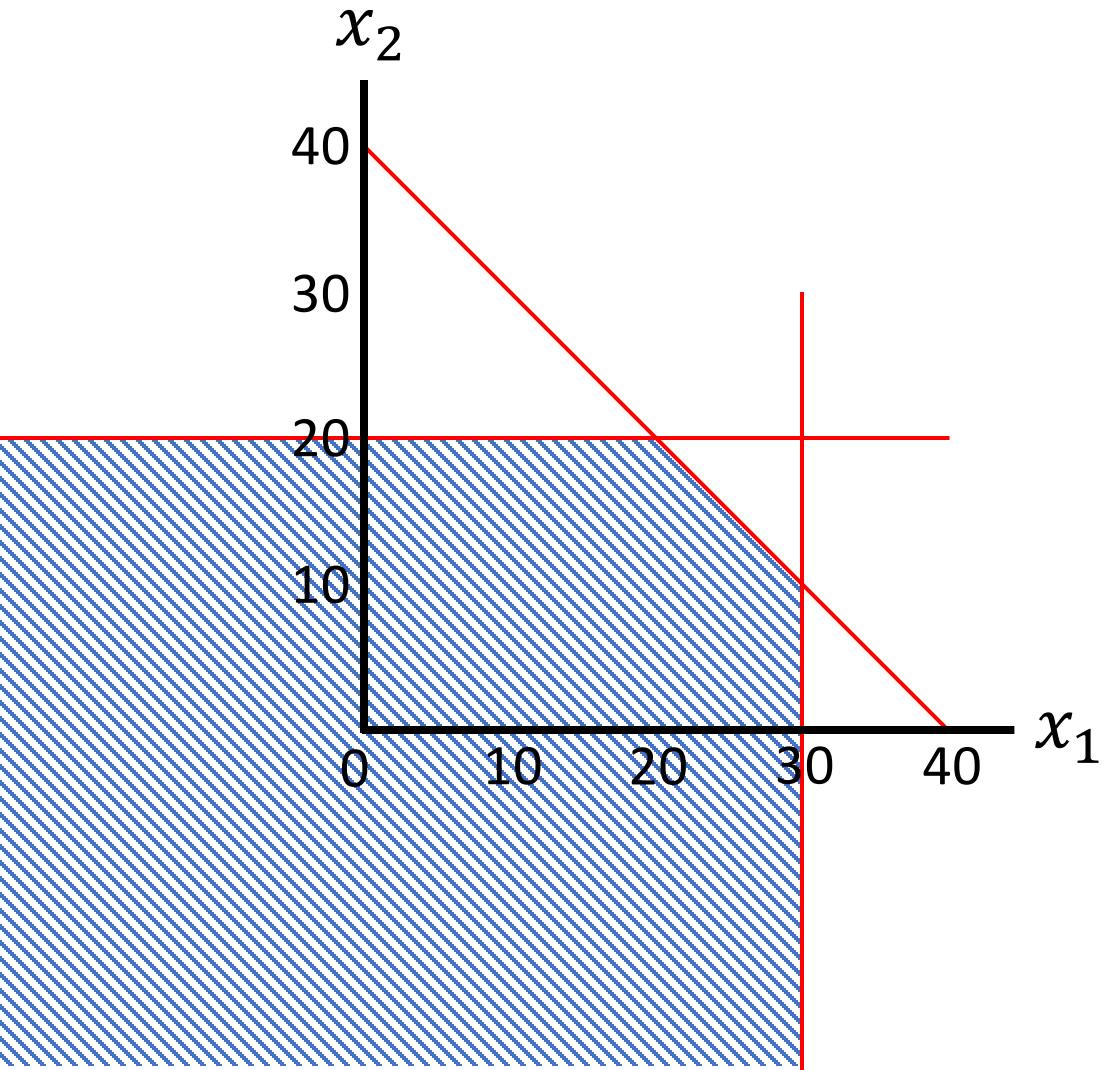
Non-negativity Constraints



Objective: $\max 100x_1 + 300x_2$

Subject to:
$x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$x_1, x_2 \geq 0$

Non-negativity Constraints



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~~$x_1, x_2 \geq 0$~~

Non-negativity Constraints

Objective: $\min x$
Subject to: $x \geq 0$

Optimal Value: ?

Non-negativity Constraints

Objective: $\min x$
Subject to: $x \geq 0$

Optimal Value: $x = 0$

Non-negativity Constraints

Objective: $\min x$

Subject to: ~~$x \geq 0$~~

Optimal Value: ?

Non-negativity Constraints

Objective: $\min x$

Subject to: ~~$x \geq 0$~~

Optimal Value: $x = -\infty$

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	Urban	Suburban	Rural
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

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Do we need non-negativity constraints?

Check and see!

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Infrastructure	-2	+5	
Gun Control	+8	[illegible]	
Farm Subsidies	+0	[illegible]	
Gasoline Tax	[illegible]	[illegible]	

Lesson: If the problem implicitly calls for non-negativity constraints, put them in!

Do we need non-negativity constraints?

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