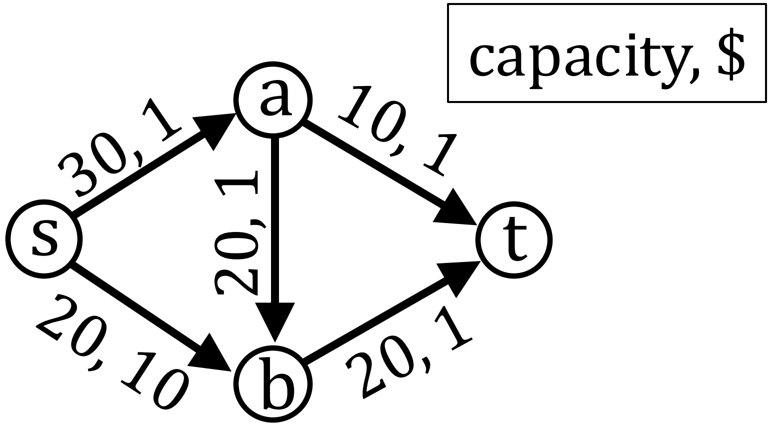
Linear Programming
CSCI 432

Minimum-Cost Flow Problem: Suppose we have a target flow demand d, and a flow network where each edge also has a cost in addition to its capacity. Pushing k flow along edge e incurs the cost $kc(e)$. Find an $s-t$ flow of minimum cost with value d.



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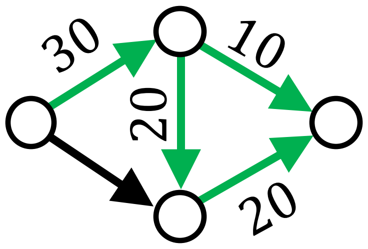
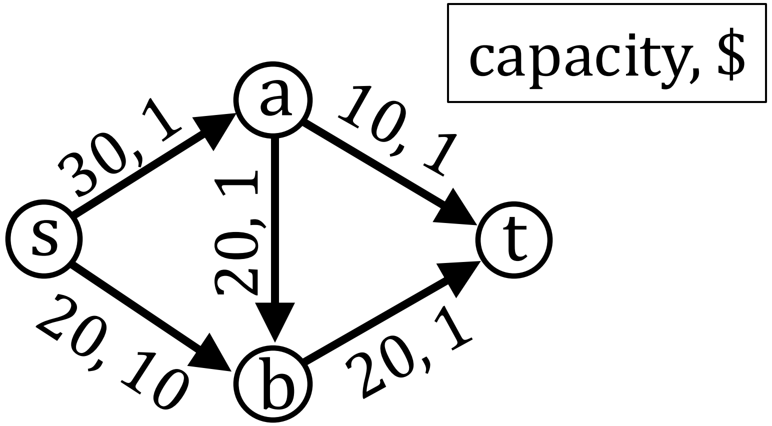
Let $d = 30$



$30 + 20 + 10 + 20 = 80$



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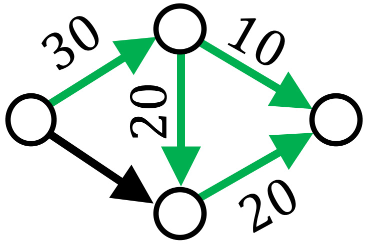

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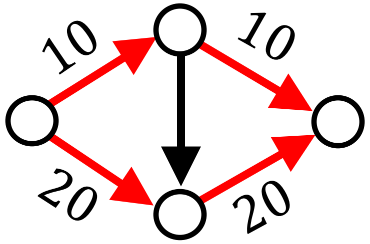
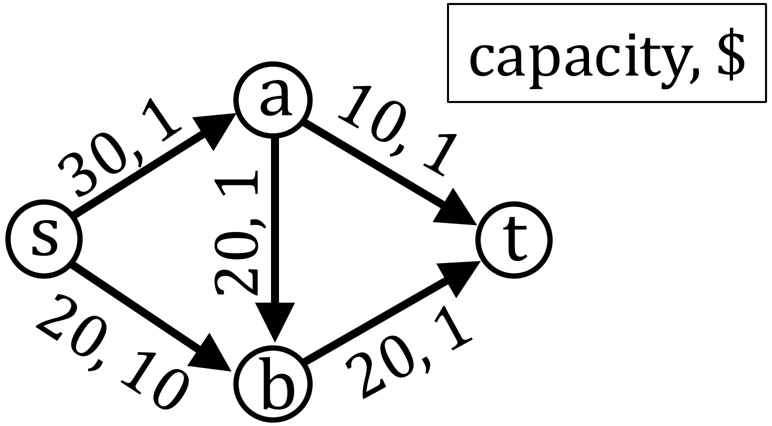


$10 + 10 + 200 + 20 = 240$



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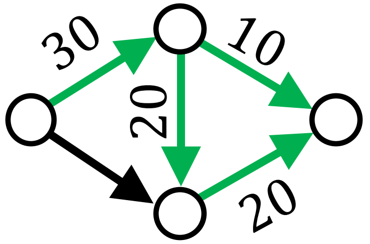

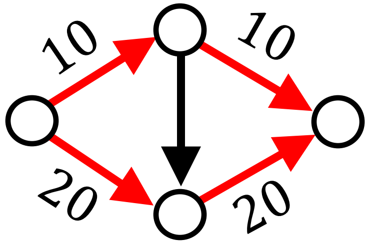

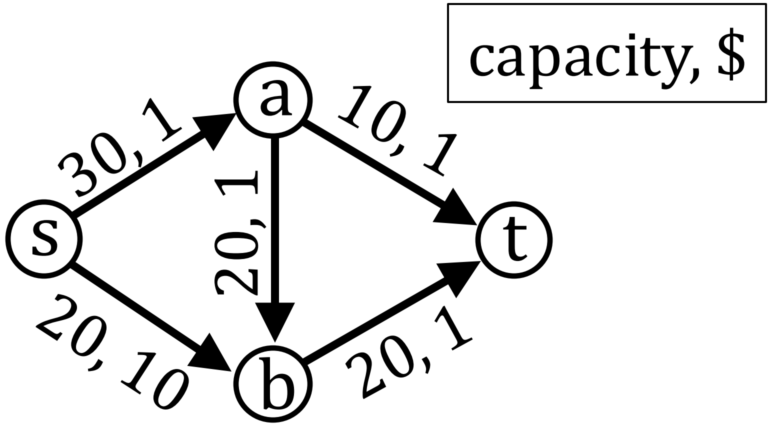

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Linear Program?

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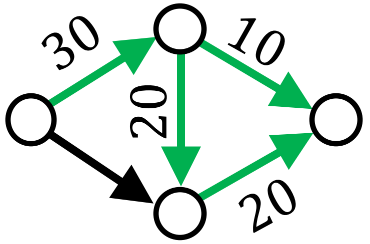

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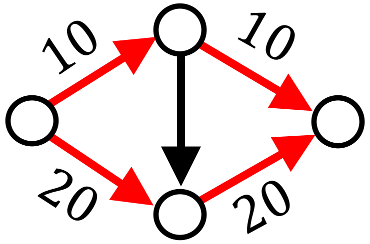

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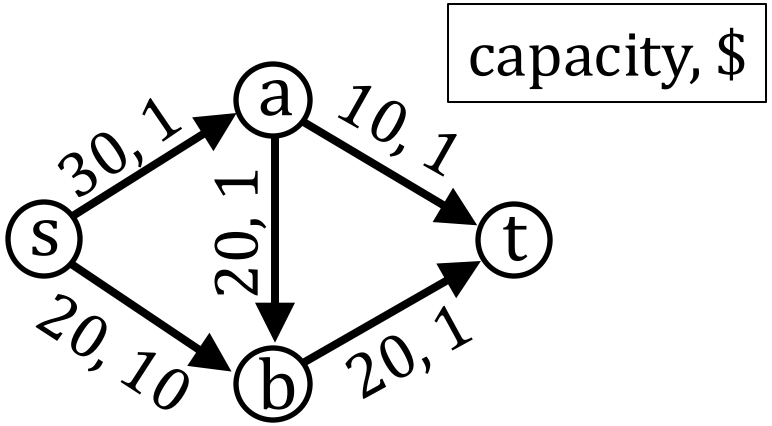

x_e = Amount of flow on edge e.

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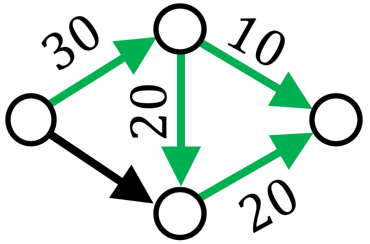

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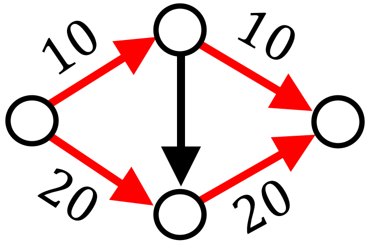

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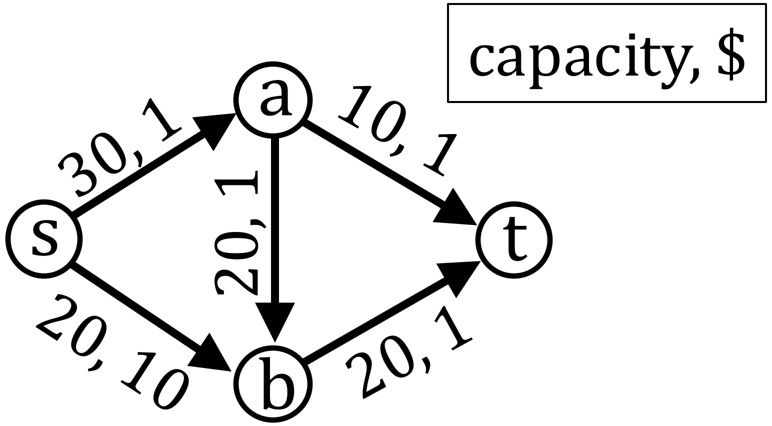

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Objective:

Minimum-Cost Flow Problem: Suppose we have a target flow demand d, and a flow network where each edge also has a cost in addition to its capacity. Pushing k flow along edge e incurs the cost $kc(e)$. Find an $s-t$ flow of minimum cost with value d.

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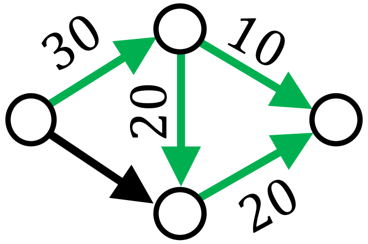

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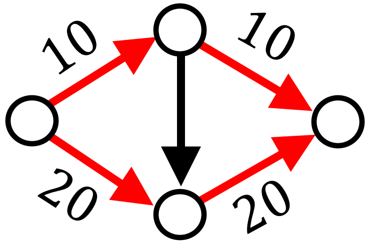

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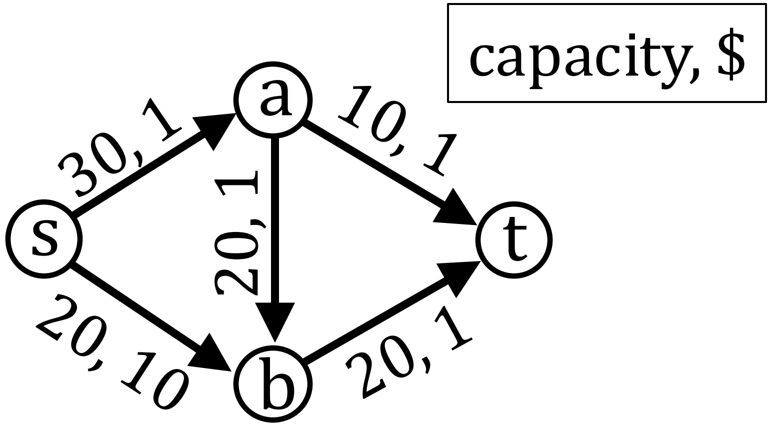

x_e = Amount of flow on edge e.

Objective: $\min \sum_{e \in E} \text{cost}_e x_e$

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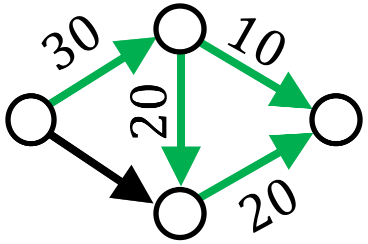

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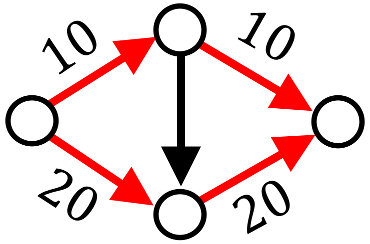

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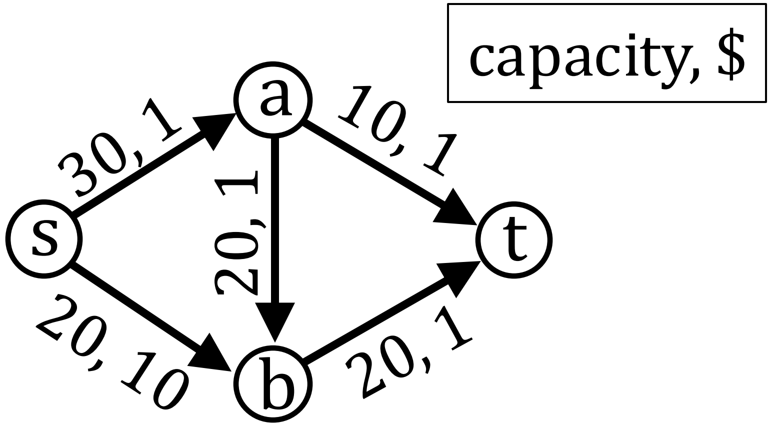

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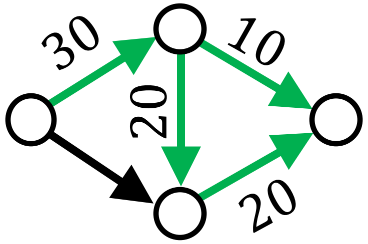

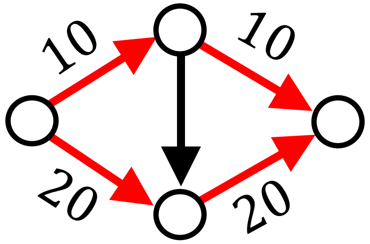

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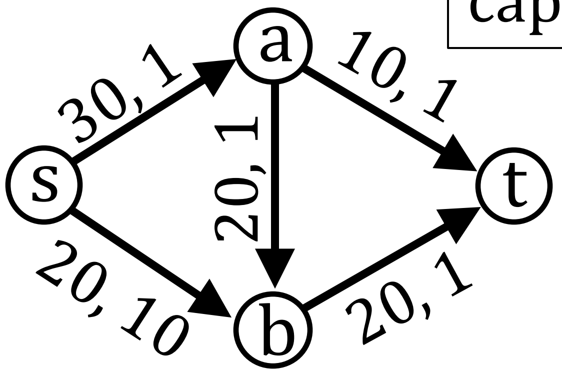

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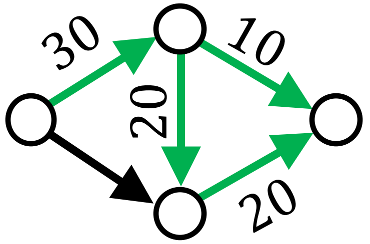

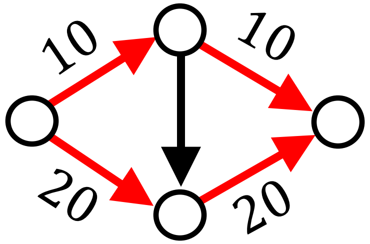

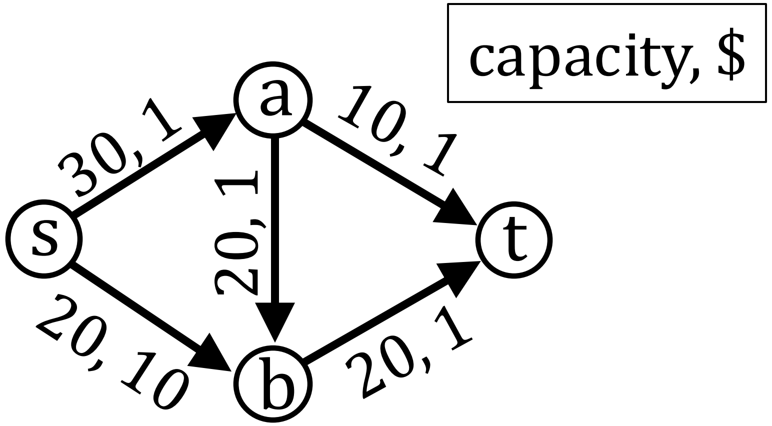

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$\sum_{e \in \text{input}(v)} x_e - \sum_{e \in \text{output}(v)} x_e = 0, \forall v \in V \setminus \{s, t\}$

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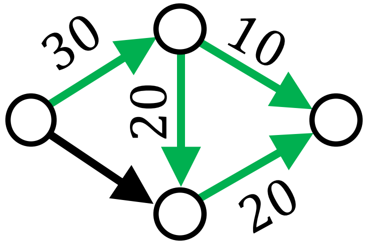

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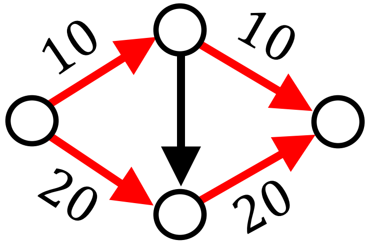

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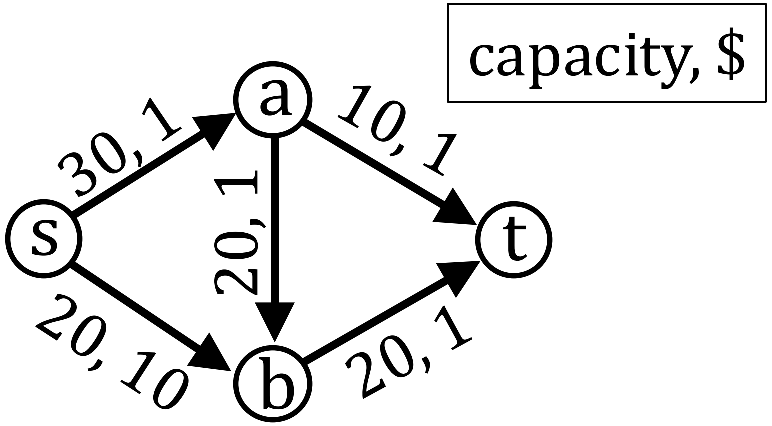

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$\sum_{e \in \text{output}(s)} x_e \geq d$

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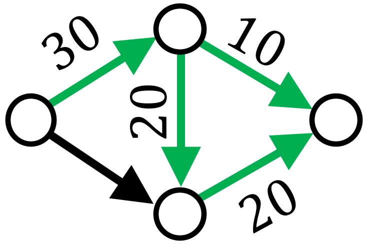

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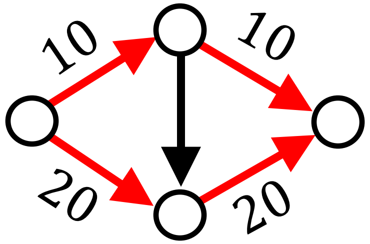

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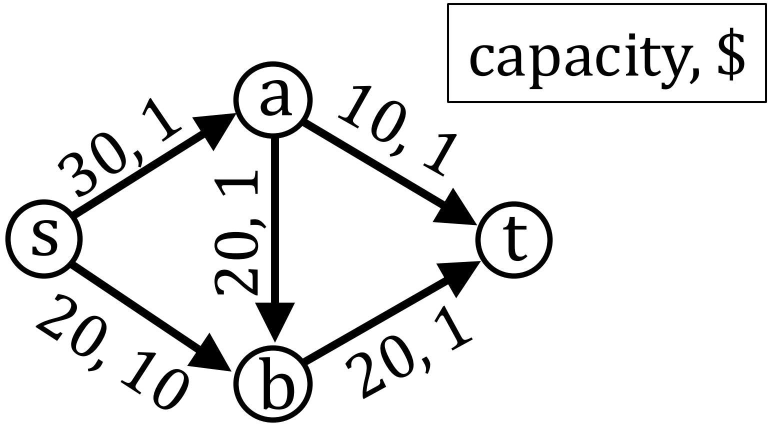

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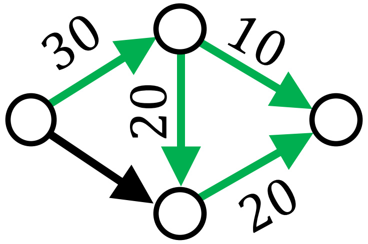

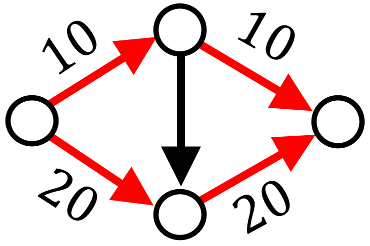

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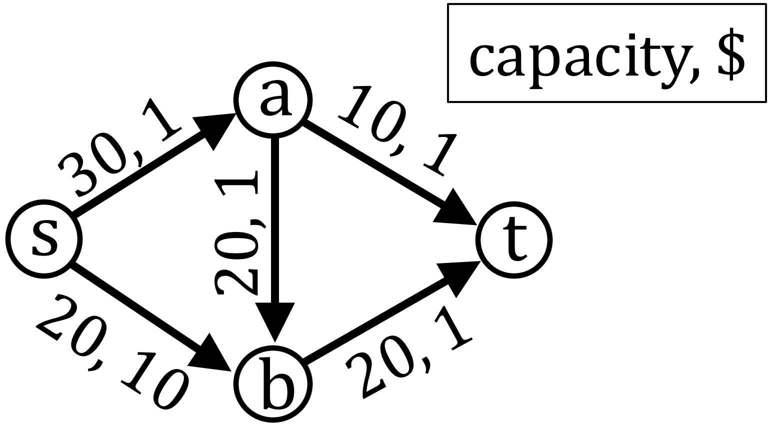

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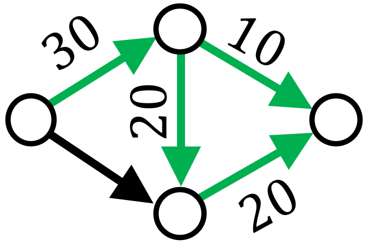

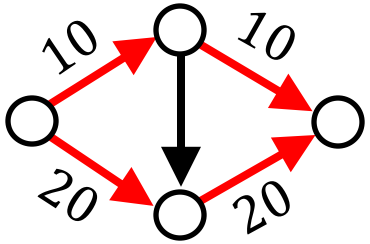

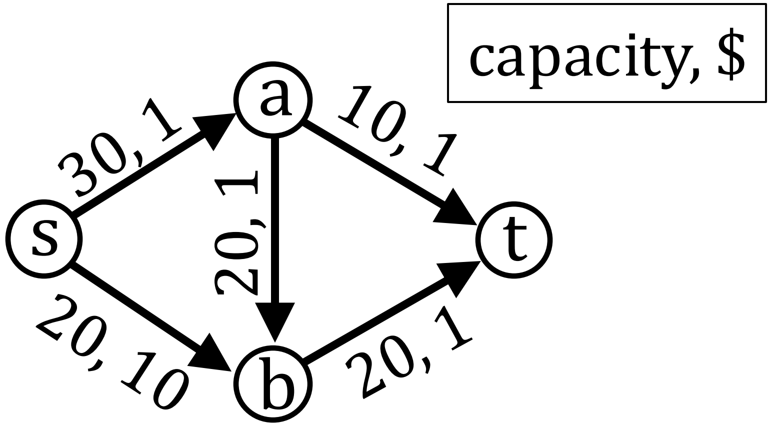

$30 + 20 + 10 + 20 = 80$ $\quad$ $10 + 10 + 200 + 20 = 240$

What if we get rid of the capacity constraint?

x_e = Amount of flow on edge e.

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Subject to: $\cancel{x_e \le \text{capacity}_e, \forall e \in E}$

$\sum_{e \in \text{input}(v)} x_e - \sum_{e \in \text{output}(v)} x_e = 0, \forall v \in V \setminus \{s, t\}$

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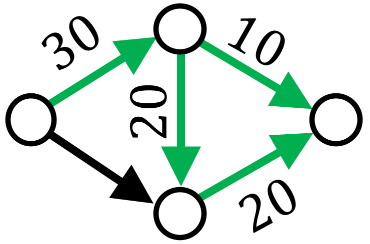

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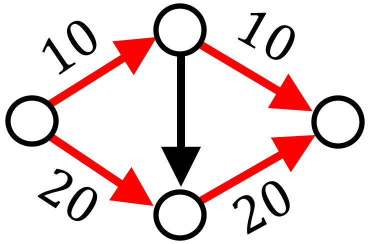

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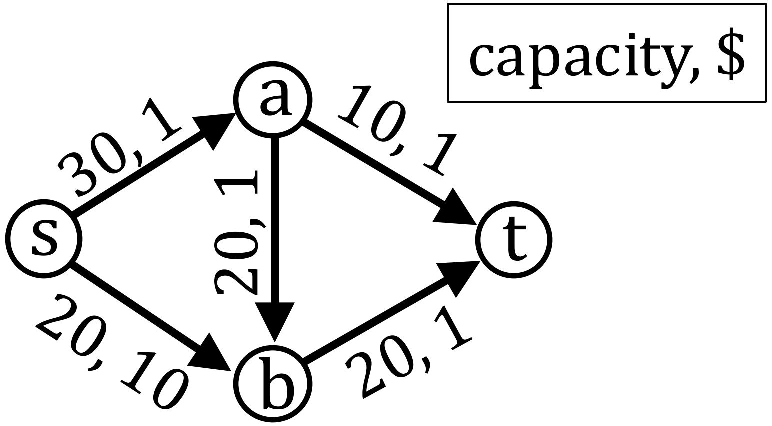

What if we get rid of the capacity constraint?

Shortest Path!

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Subject to: $\cancel{x_e \leq \text{capacity}_e, \forall e \in E}$

$\sum_{e \in \text{input}(v)} x_e - \sum_{e \in \text{output}(v)} x_e = 0, \forall v \in V \setminus \{s, t\}$

$\sum_{e \in \text{output}(s)} x_e = d$

$x_e \geq 0, \forall e \in E$

<u>Minimum Fixed-Cost Flow Problem</u>: Suppose we have a target flow demand d, and a flow network where each edge also has a cost in addition to its capacity. Pushing k flow along edge e incurs the cost $\cancel{\boldsymbol{k}}\boldsymbol{c}(\boldsymbol{e})$. Find an $s - t$ flow of minimum cost with value d.

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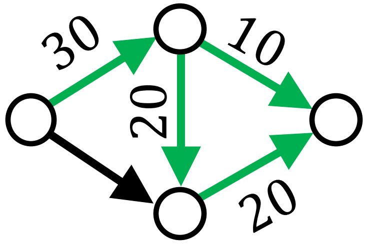

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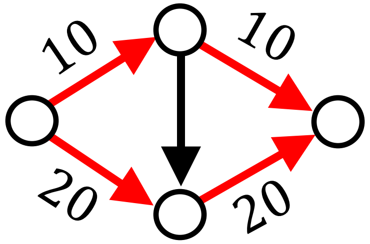

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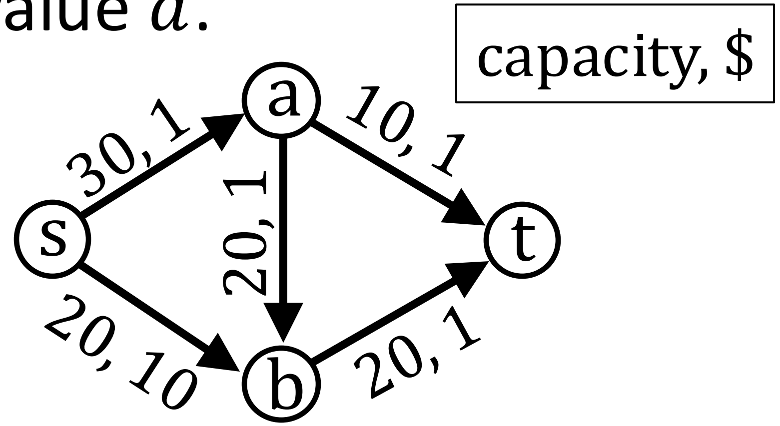

What if edge charges are fixed if edge is used?

x_e = Amount of flow on edge e.

Objective: $\min \sum_{e \in E} \text{cost}_e x_e$

Subject to: $x_e \leq \text{capacity}_e, \forall e \in E$

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$x_e \geq 0, \forall e \in E$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	**# Required**
7	276
9	100
12	250

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

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Linear Program?

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x_1 = {7’, 7’} cut (6’ waste).
x_2 = {9’, 7’} cut (4’ waste).
x_3 = {9’, 9’} cut (2’ waste).
x_4 = {12’, 7’} cut (1’ waste).

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Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

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Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$
Subject to: $2x_1 + x_2 + x_4 \geq 276$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	# Required
7	276
9	100
12	250

x_1 = {7’, 7’} cut (6’ waste).
x_2 = {9’, 7’} cut (4’ waste).
x_3 = {9’, 9’} cut (2’ waste).
x_4 = {12’, 7’} cut (1’ waste).

Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$

Subject to: $2x_1 + x_2 + x_4 \geq 276$

$x_2 + 2x_3 \geq 100$

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Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$

Subject to:
$$2x_1 + x_2 + x_4 \geq 276$$
$$x_2 + 2x_3 \geq 100$$
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Subject to:
$$2x_1 + x_2 + x_4 \geq 276$$
$$x_2 + 2x_3 \geq 100$$
$$x_4 \geq 250$$
$$x_1, x_2, x_3 \geq 0$$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	# Required
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9	100
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Does solution need to be integer?

x_1 = {7’, 7’} cut (6’ waste).
x_2 = {9’, 7’} cut (4’ waste).
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Does solution need to be integer?

Is solution guaranteed to be integer?

x_1 = {7’, 7’} cut (6’ waste).
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Does solution need to be integer?

Is solution guaranteed to be integer?

If feasible region is defined by integer vertices, the solution will be integer.

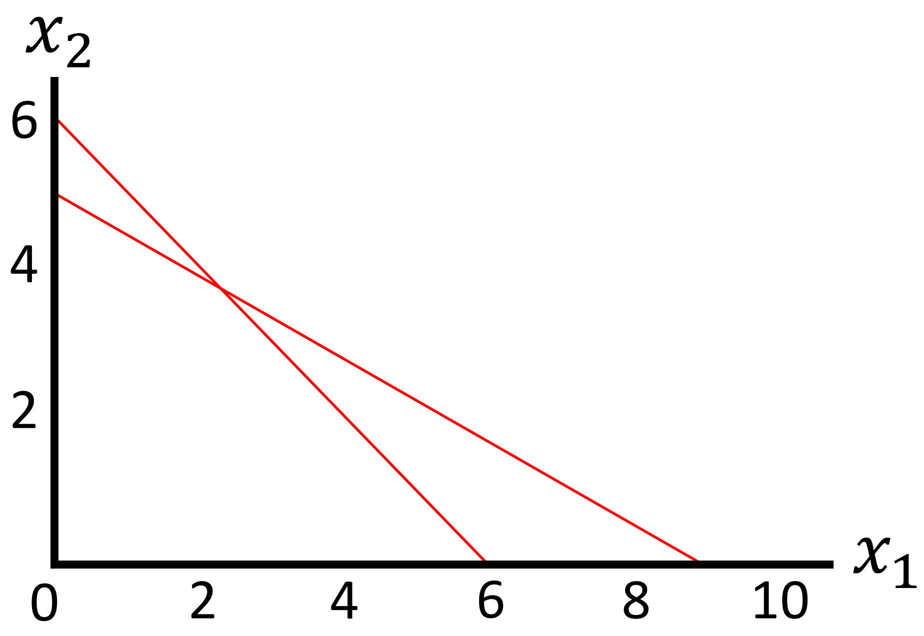
$x_1, x_2 \in \mathbb{R}$

Objective: $\max 5x_1 + 8x_2$

Subject to: $x_1 + x_2 \leq 6$

$5x_1 + 9x_2 \leq 45$

$x_1, x_2 \geq 0$



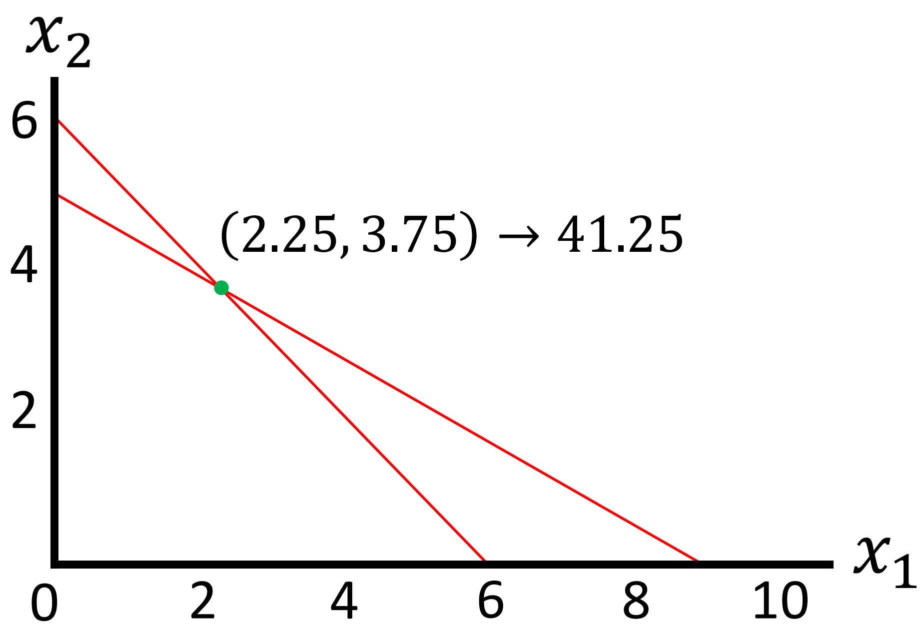
$x_1, x_2 \in \mathbb{R}$

Objective: $\max 5x_1 + 8x_2$

Subject to: $x_1 + x_2 \leq 6$

$5x_1 + 9x_2 \leq 45$

$x_1, x_2 \geq 0$



$x_1, x_2 \in \mathbb{R}$

Objective: $\max 5x_1 + 8x_2$

Subject to: $x_1 + x_2 \leq 6$

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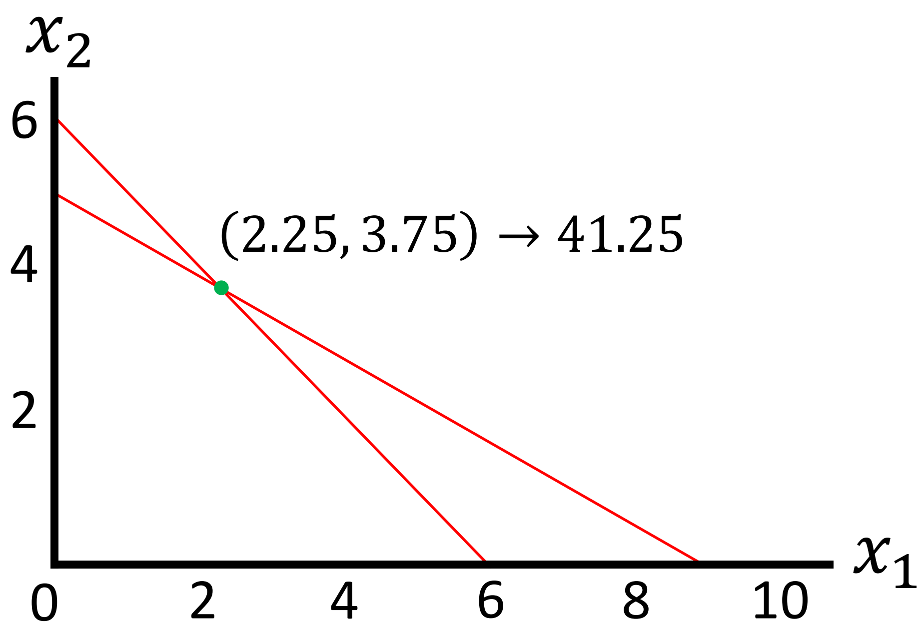


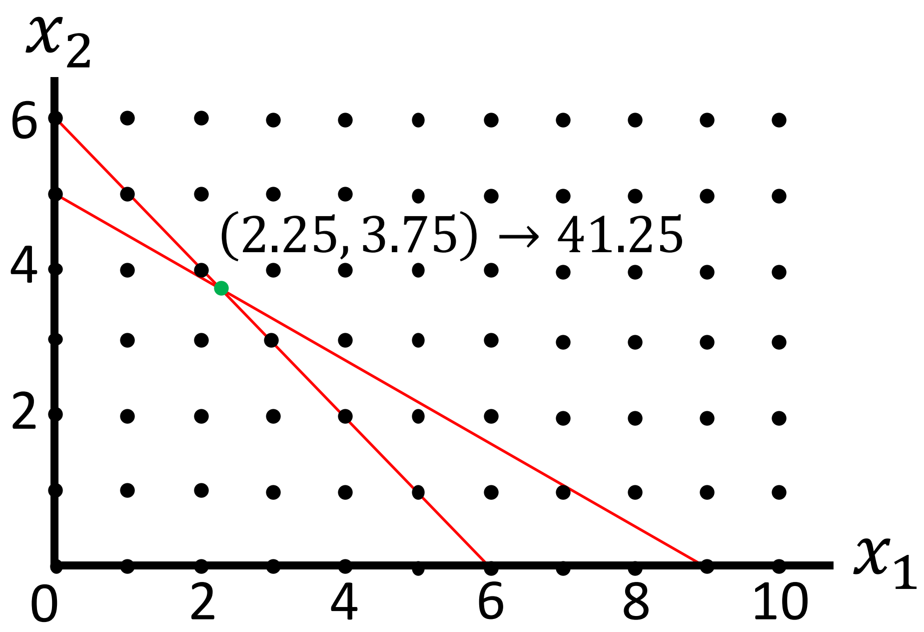
$x_1, x_2 \in \mathbb{N}$

Objective: $\max 5x_1 + 8x_2$

Subject to: $x_1 + x_2 \leq 6$

$5x_1 + 9x_2 \leq 45$

$x_1, x_2 \geq 0$

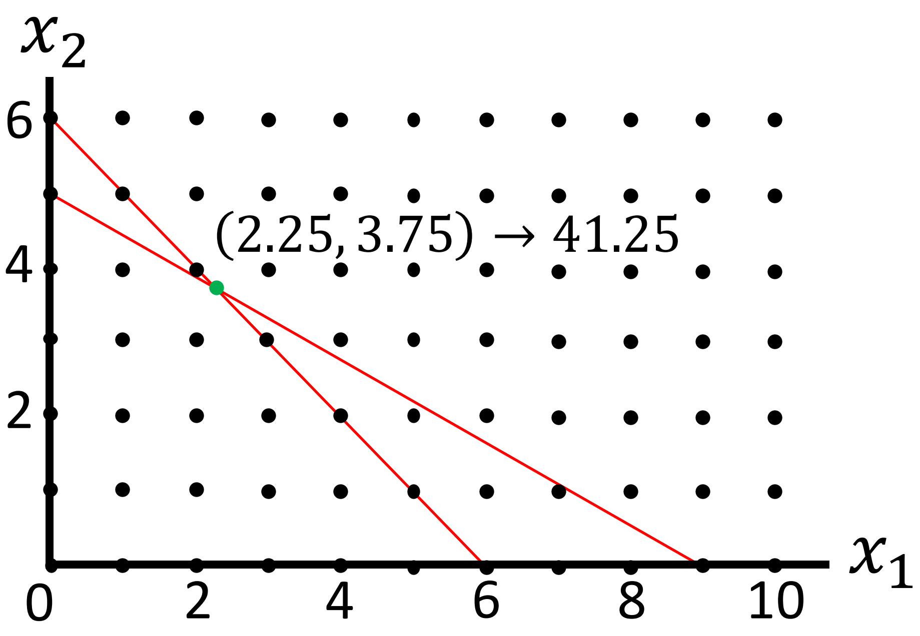


$x_1, x_2 \in \mathbb{N}$

Objective: $\max 5x_1 + 8x_2$

Subject to:

$$x_1 + x_2 \leq 6$$
$$5x_1 + 9x_2 \leq 45$$
$$x_1, x_2 \geq 0$$



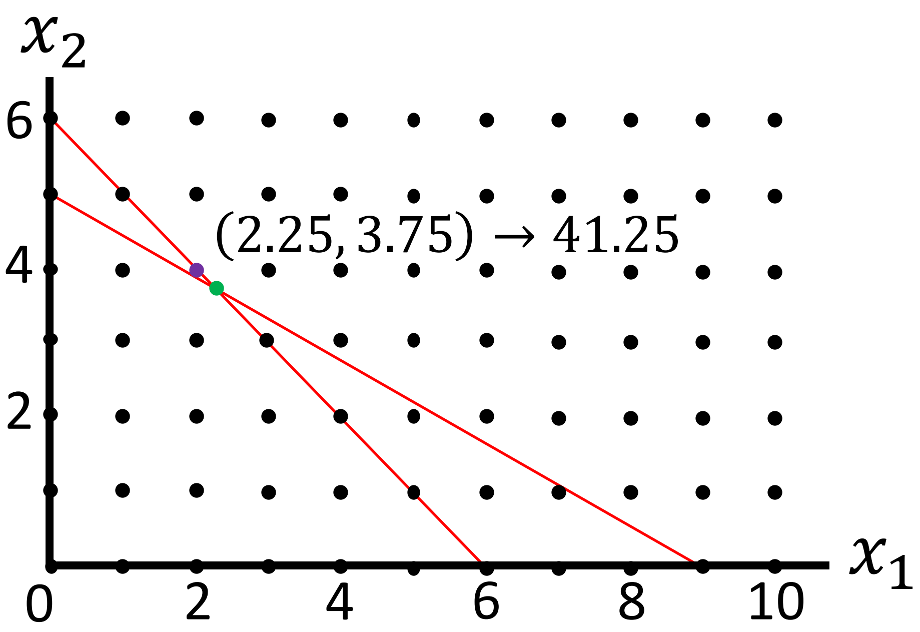
$x_1, x_2 \in \mathbb{N}$

Objective: $\max 5x_1 + 8x_2$

Subject to:
$$x_1 + x_2 \leq 6$$
$$5x_1 + 9x_2 \leq 45$$
$$x_1, x_2 \geq 0$$

Optimal continuous solution → optimal integer solution?

- Closest integer solution?
- Closest feasible integer solution?
- Closest feasible integer solution on feasible region boundary?



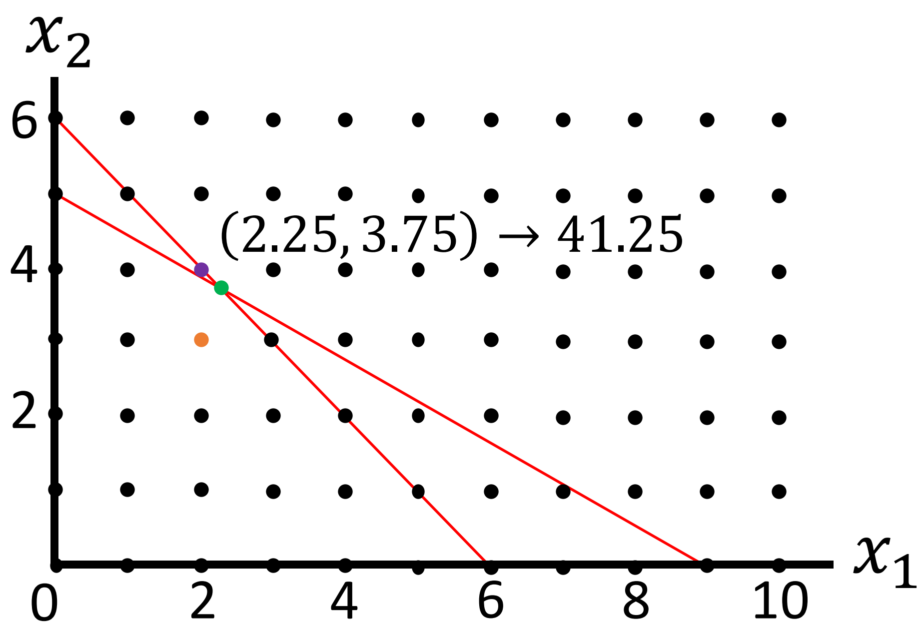
$x_1, x_2 \in \mathbb{N}$

Objective: $\max 5x_1 + 8x_2$

Subject to:
$$x_1 + x_2 \leq 6$$
$$5x_1 + 9x_2 \leq 45$$
$$x_1, x_2 \geq 0$$

Optimal continuous solution → optimal integer solution?

- Closest integer solution? – Not feasible
- Closest feasible integer solution?
- Closest feasible integer solution on feasible region boundary?



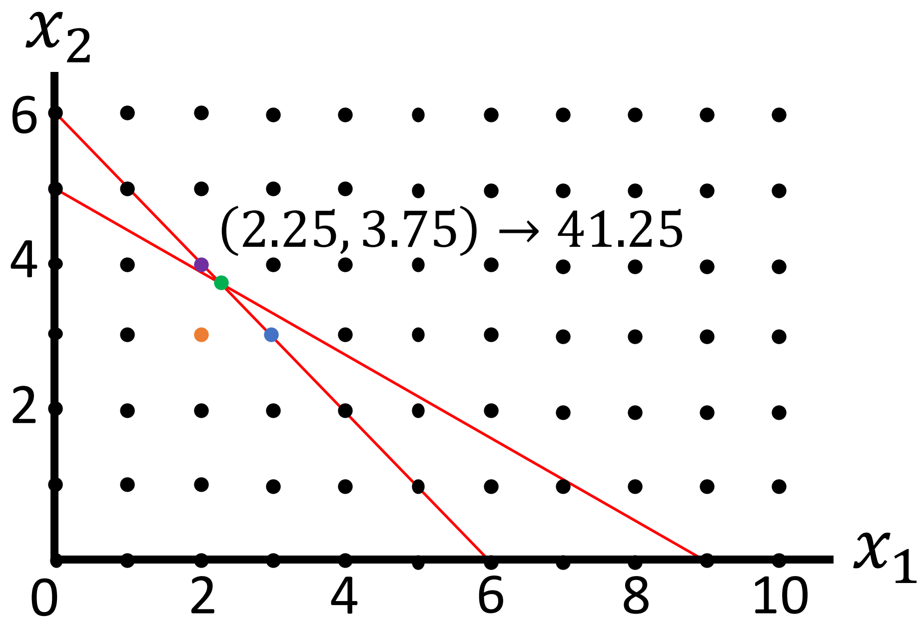
$x_1, x_2 \in \mathbb{N}$

Objective: $\max 5x_1 + 8x_2$

Subject to:
$x_1 + x_2 \leq 6$
$5x_1 + 9x_2 \leq 45$
$x_1, x_2 \geq 0$

Optimal continuous solution → optimal integer solution?

- Closest integer solution? – Not feasible
- Closest feasible integer solution? – Obj = 34
- Closest feasible integer solution on feasible region boundary?



$x_1, x_2 \in \mathbb{N}$

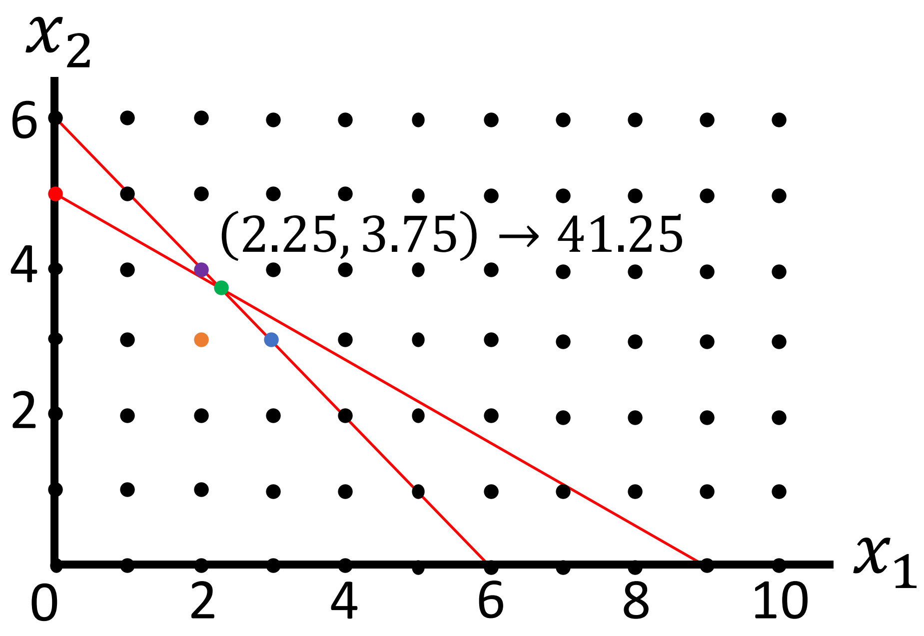
Objective: $\max 5x_1 + 8x_2$

Subject to:

$$x_1 + x_2 \leq 6$$
$$5x_1 + 9x_2 \leq 45$$
$$x_1, x_2 \geq 0$$

Optimal continuous solution → optimal integer solution?

- Closest integer solution? – Not feasible
- Closest feasible integer solution? – Obj = 34
- Closest feasible integer solution on feasible region boundary? – Obj = 39

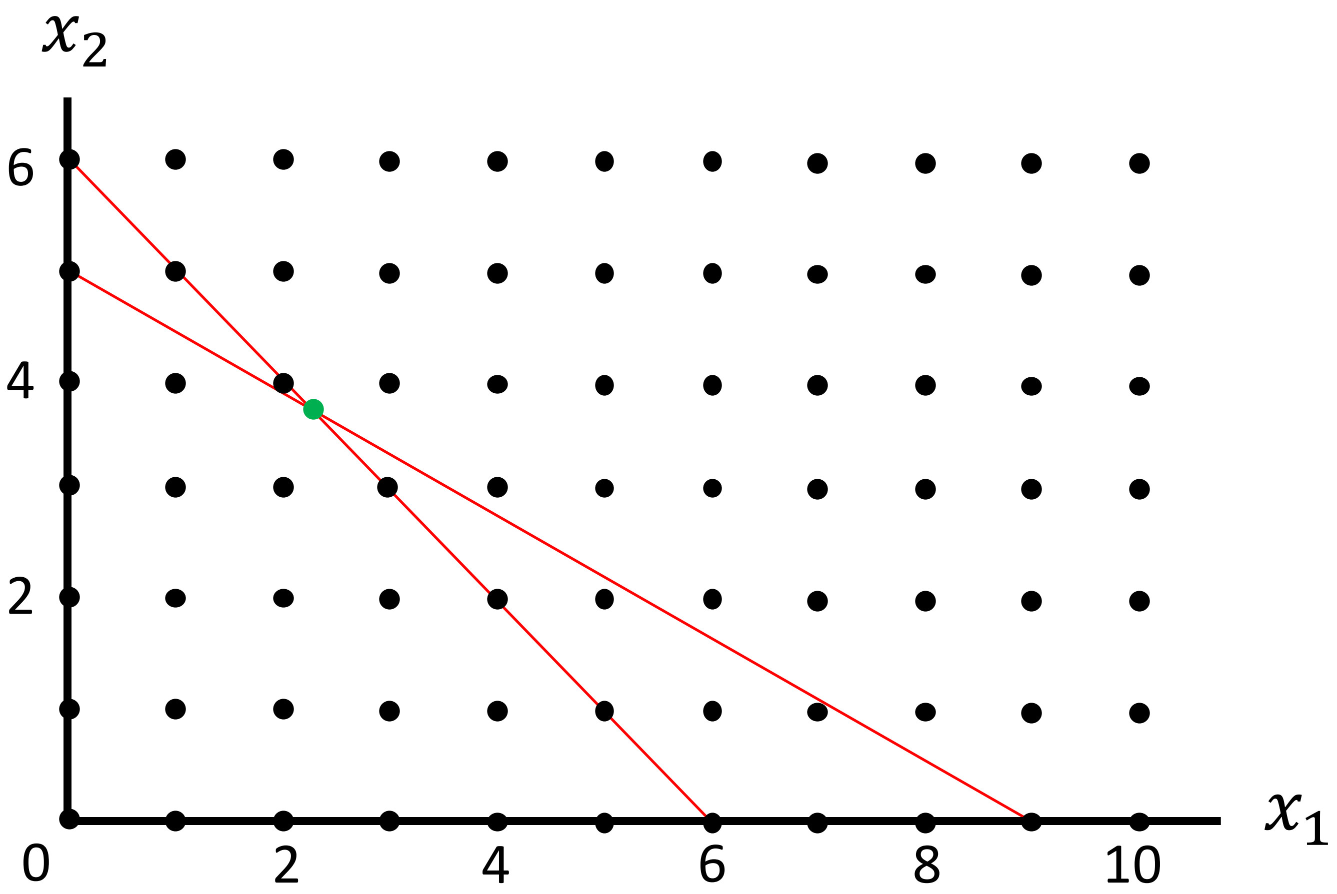


$x_1, x_2 \in \mathbb{N}$

Objective:	$\max 5x_1 + 8x_2$
Subject to:	$x_1 + x_2 \leq 6$
	$5x_1 + 9x_2 \leq 45$
	$x_1, x_2 \geq 0$

Optimal continuous solution $\to$ optimal integer solution?

- Closest integer solution? – Not feasible
- Closest feasible integer solution? – Obj = 34
- Closest feasible integer solution on feasible region boundary? – Obj = 39
- **Actual optimal – Obj = 40**



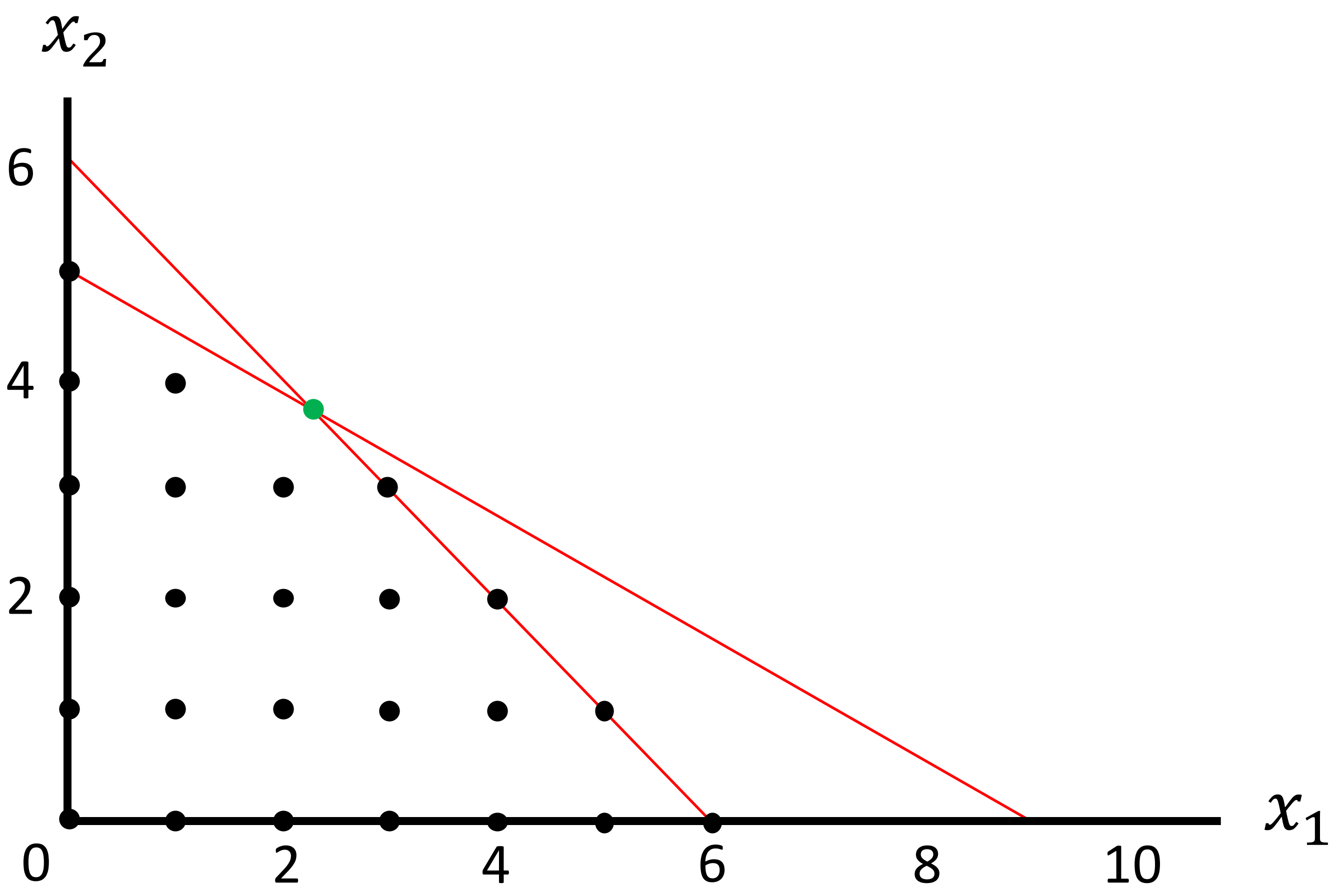
$x_1, x_2 \in \mathbb{N}$

Objective: $\max 5x_1 + 8x_2$

Subject to: $x_1 + x_2 \leq 6$

$5x_1 + 9x_2 \leq 45$

$x_1, x_2 \geq 0$



$x_1, x_2 \in \mathbb{N}$

Objective: $\max 5x_1 + 8x_2$

Subject to: $x_1 + x_2 \leq 6$

$5x_1 + 9x_2 \leq 45$

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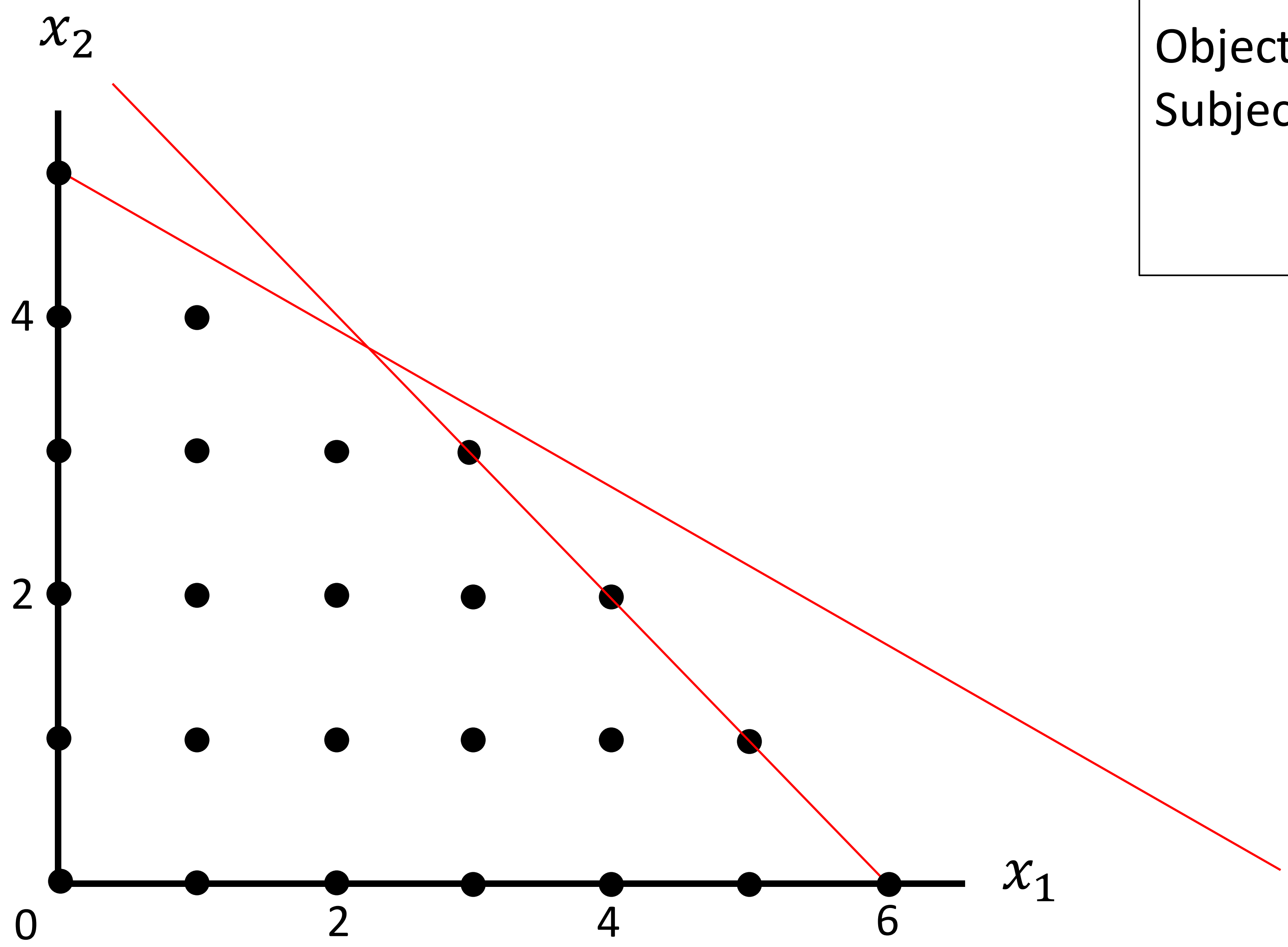

$x_1, x_2 \in \mathbb{N}$

Objective: $\max 5x_1 + 8x_2$

Subject to:

$$x_1 + x_2 \leq 6$$
$$5x_1 + 9x_2 \leq 45$$
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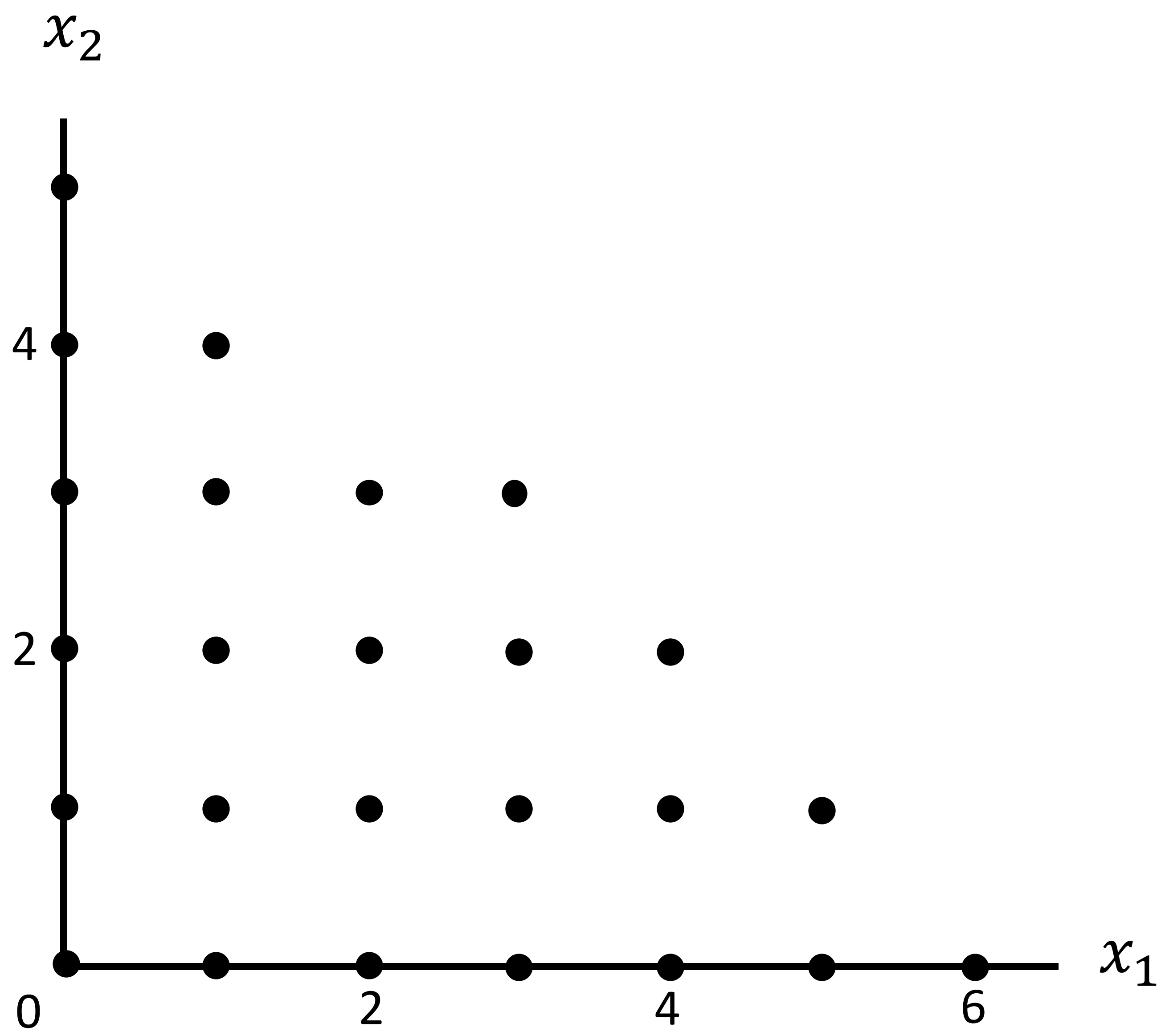

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Objective: $\max 5x_1 + 8x_2$

Subject to:
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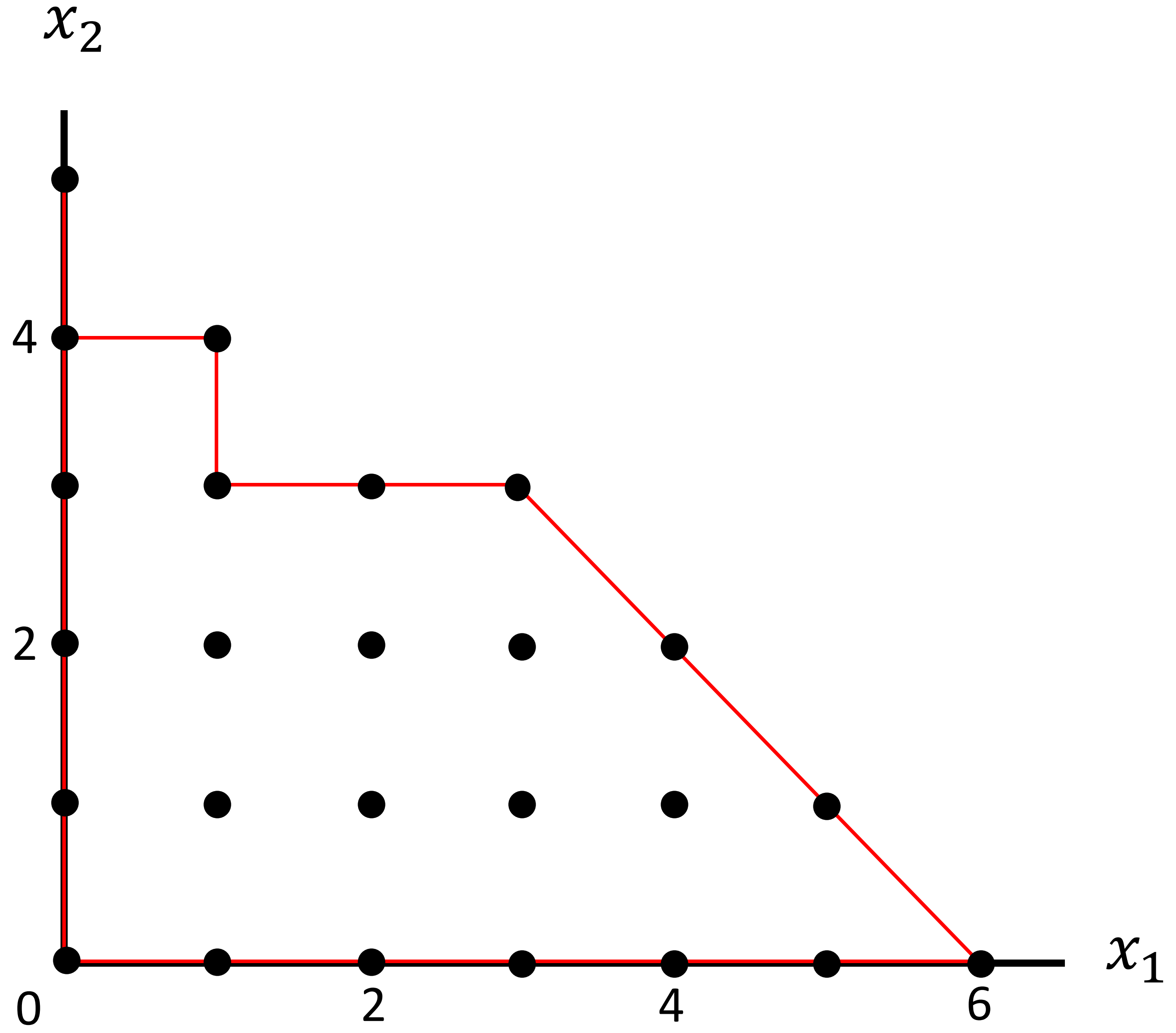
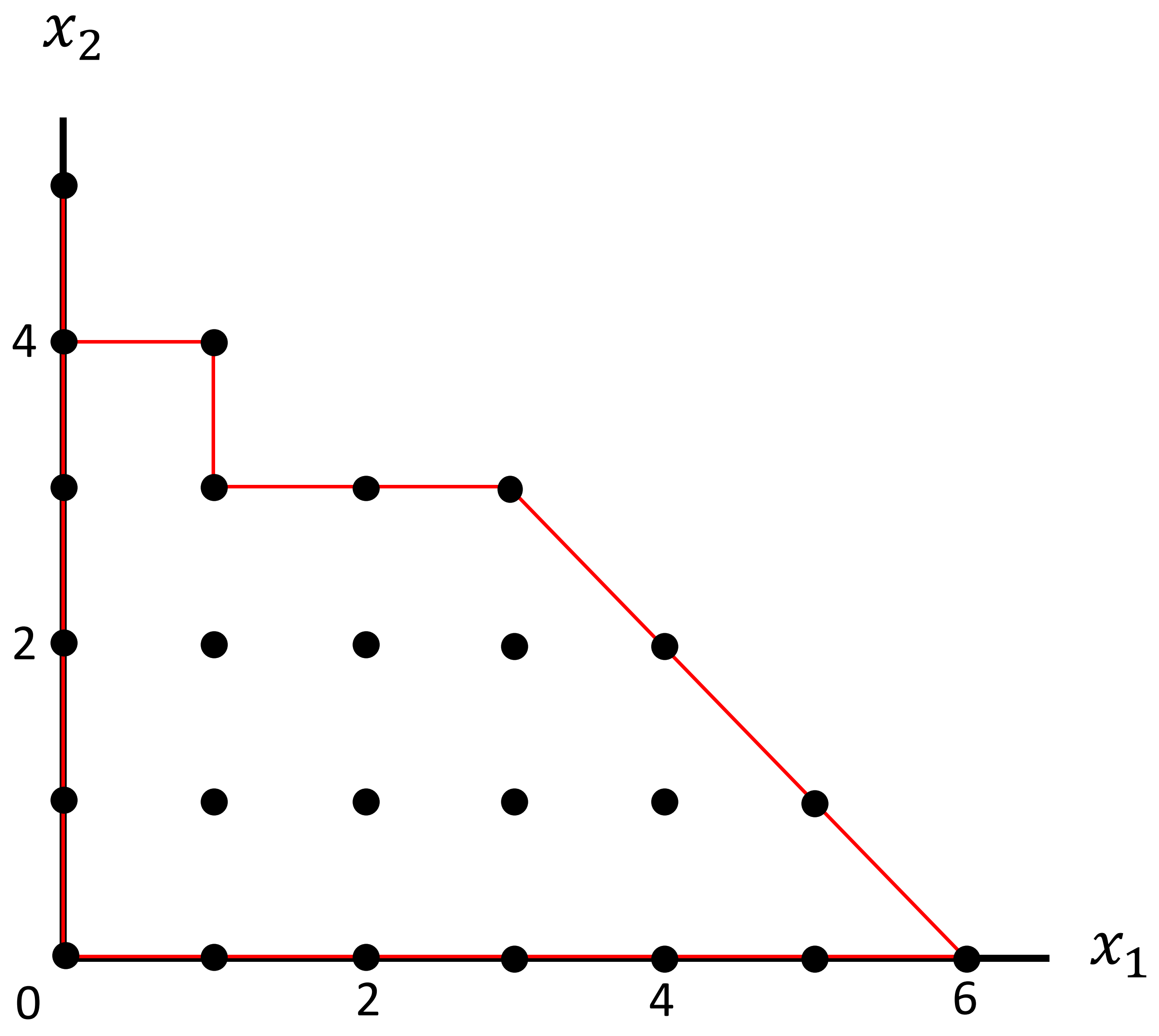


Integer feasible region:



$x_1, x_2 \in \mathbb{N}$

Objective:	$\max 5x_1 + 8x_2$
Subject to:	$x_1 + x_2 \leq 6$
	$5x_1 + 9x_2 \leq 45$
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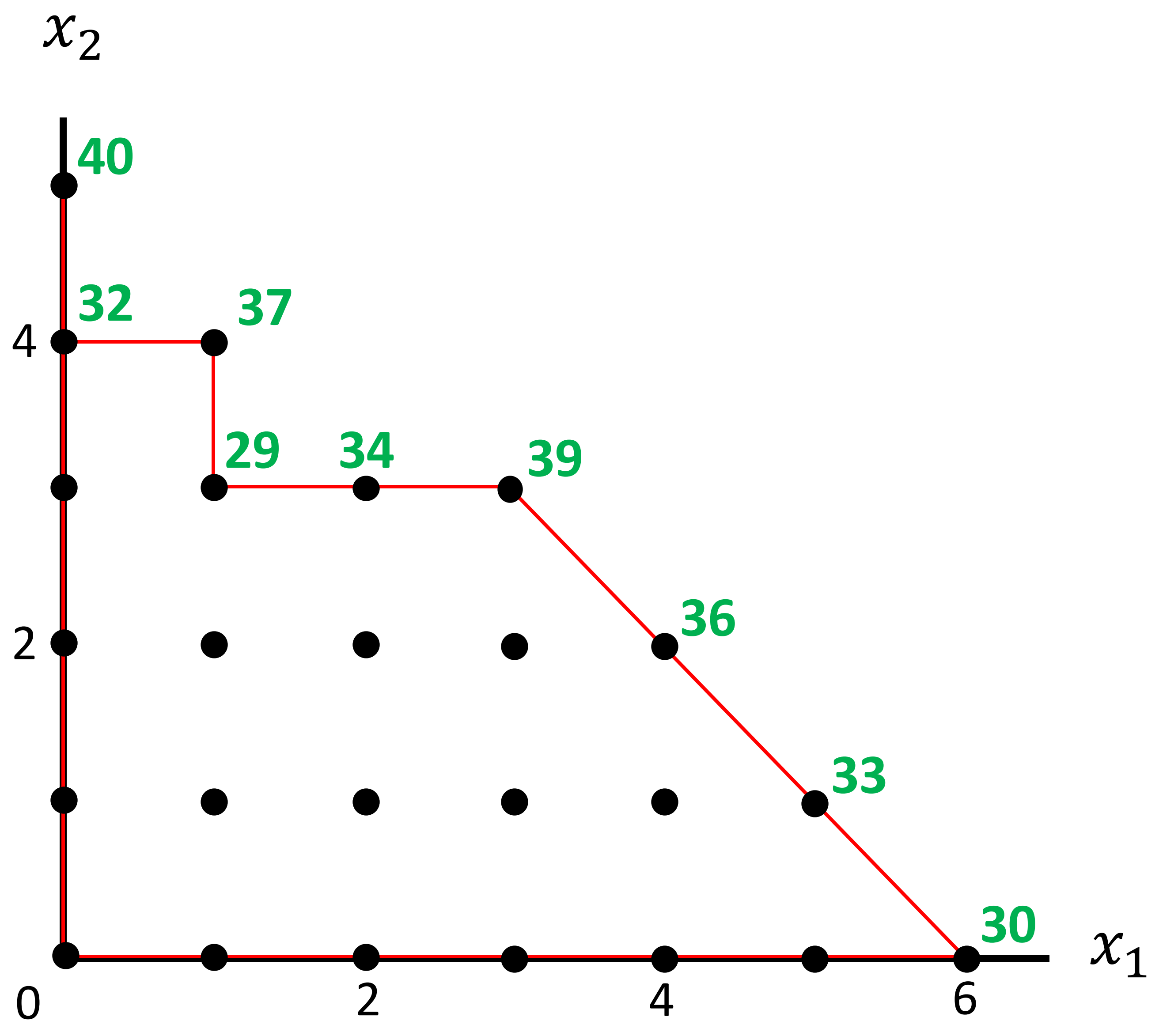
Integer feasible region:

- Not convex.



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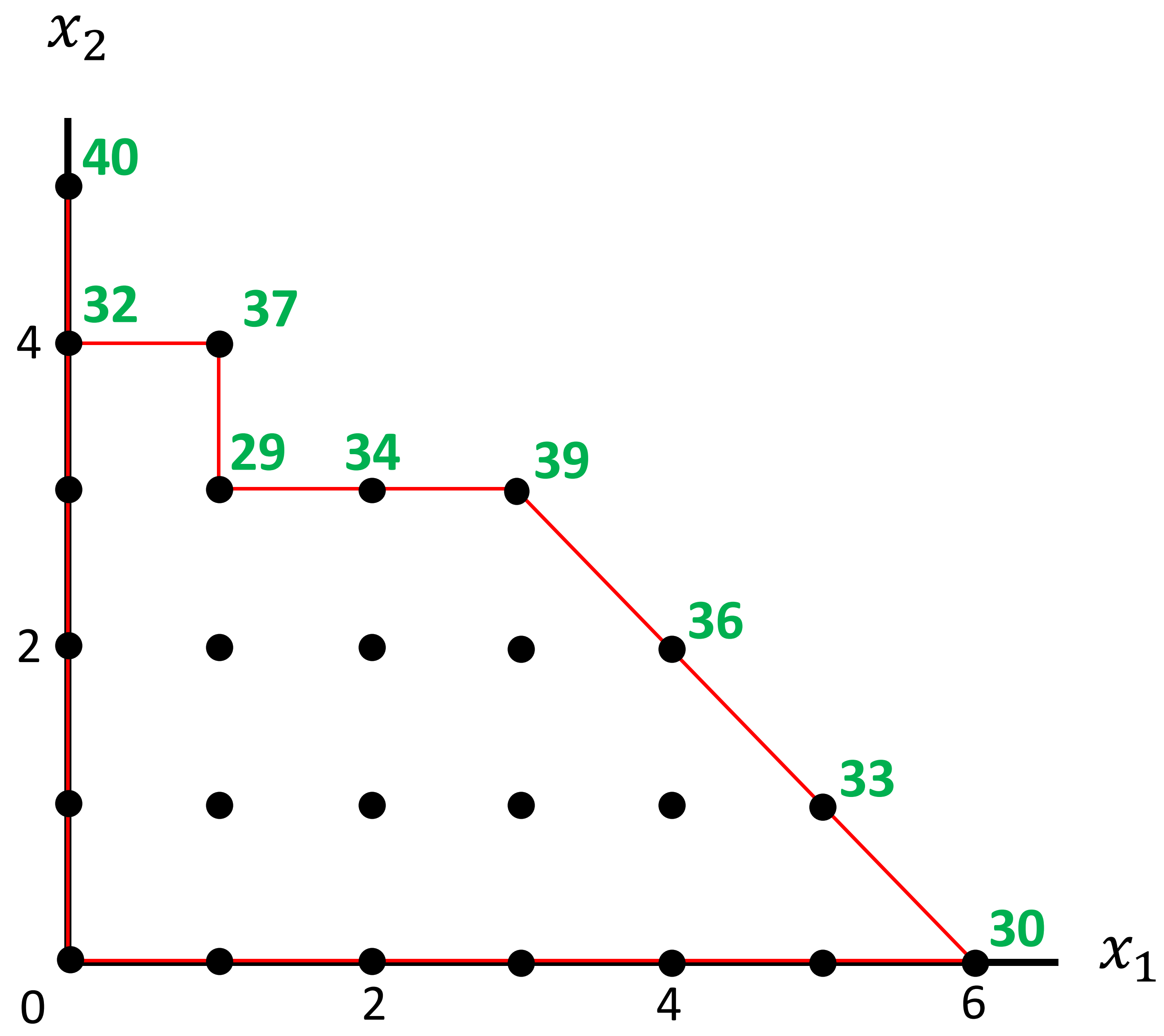
Integer feasible region:

- Not convex.



$x_1, x_2 \in \mathbb{N}$

Objective: $\max 5x_1 + 8x_2$

Subject to:
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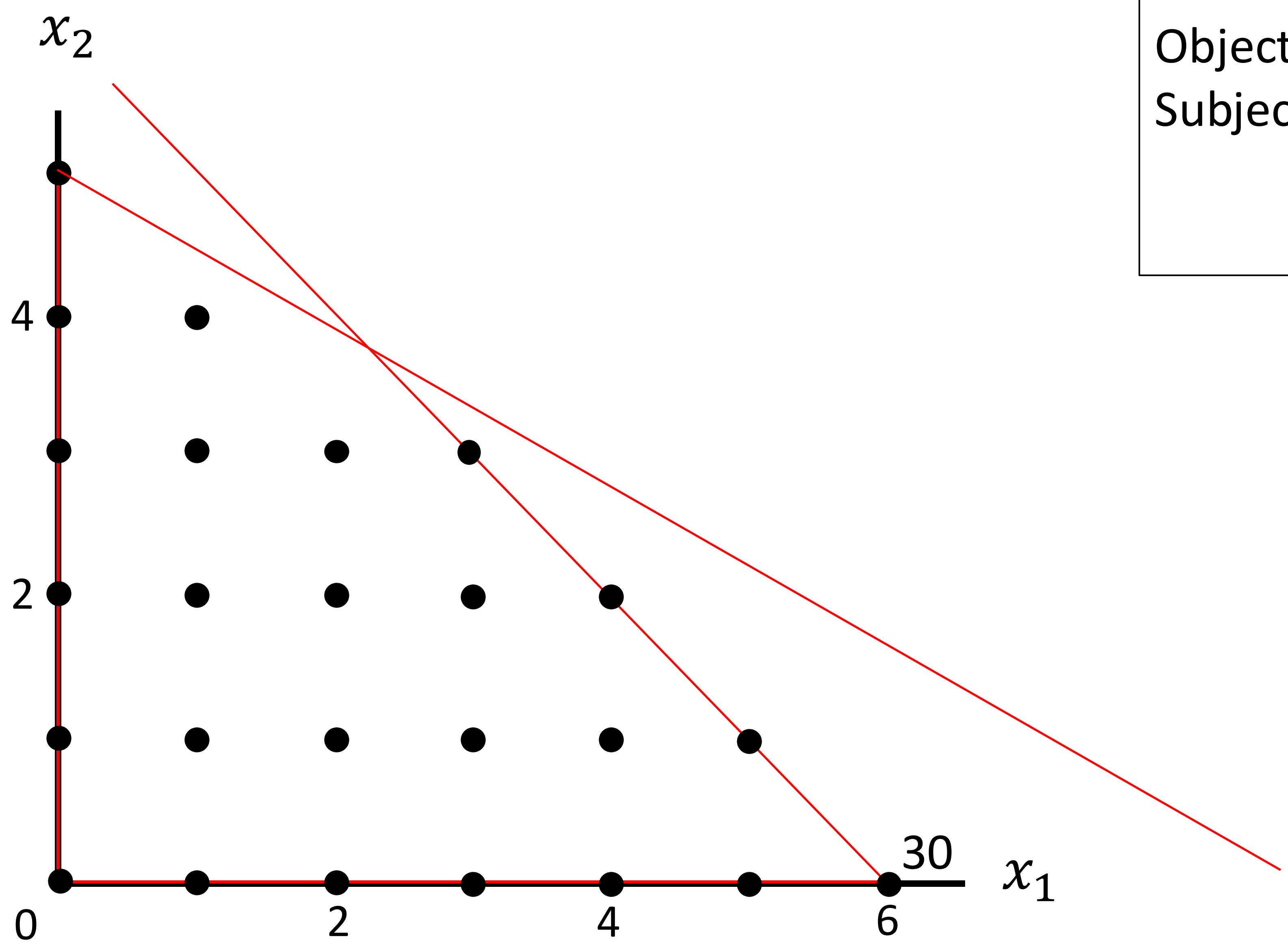
$$x_1, x_2 \geq 0$$

Integer feasible region:

- Not convex.
- local optimum $\neq$ global optimum.



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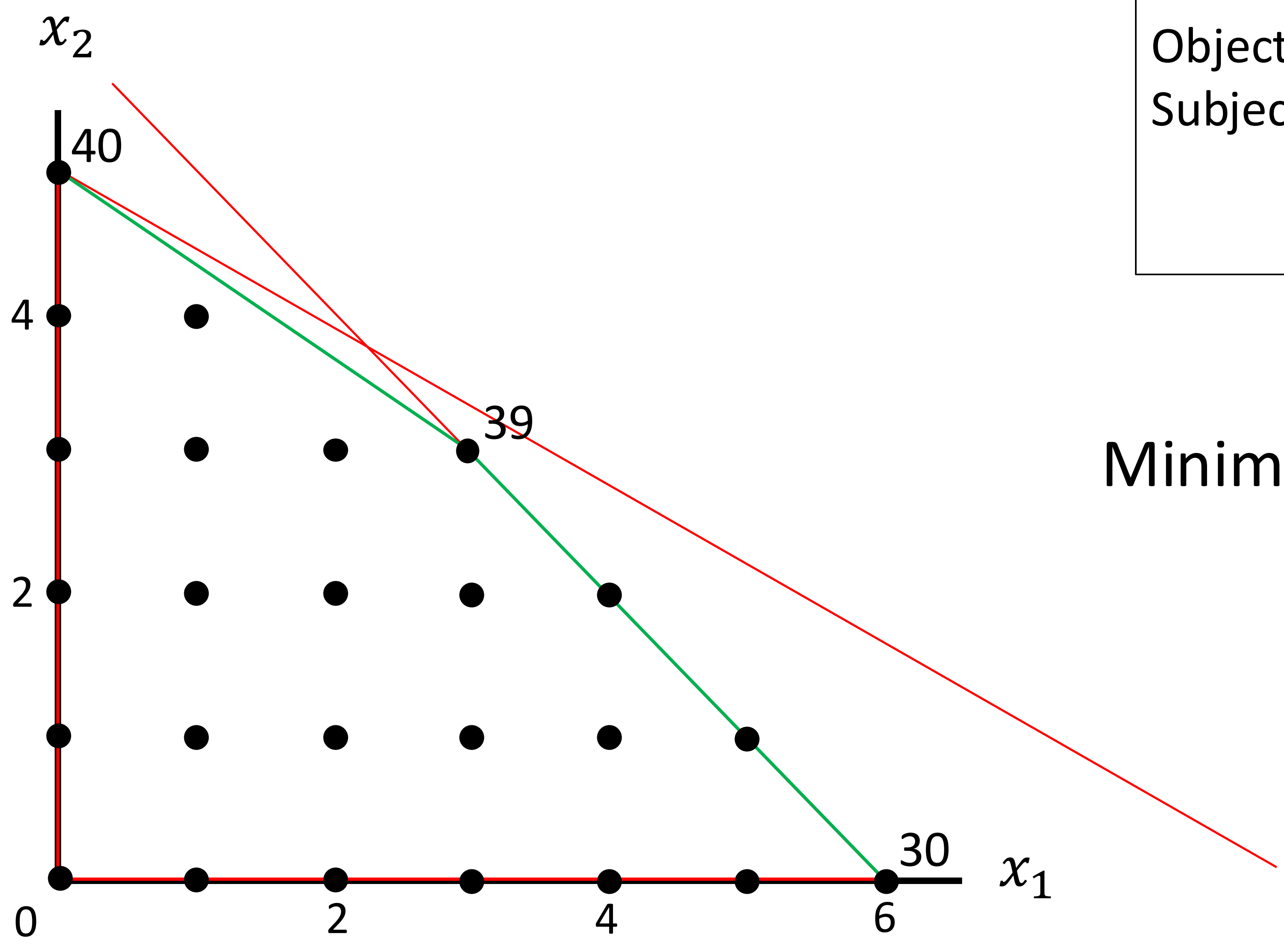
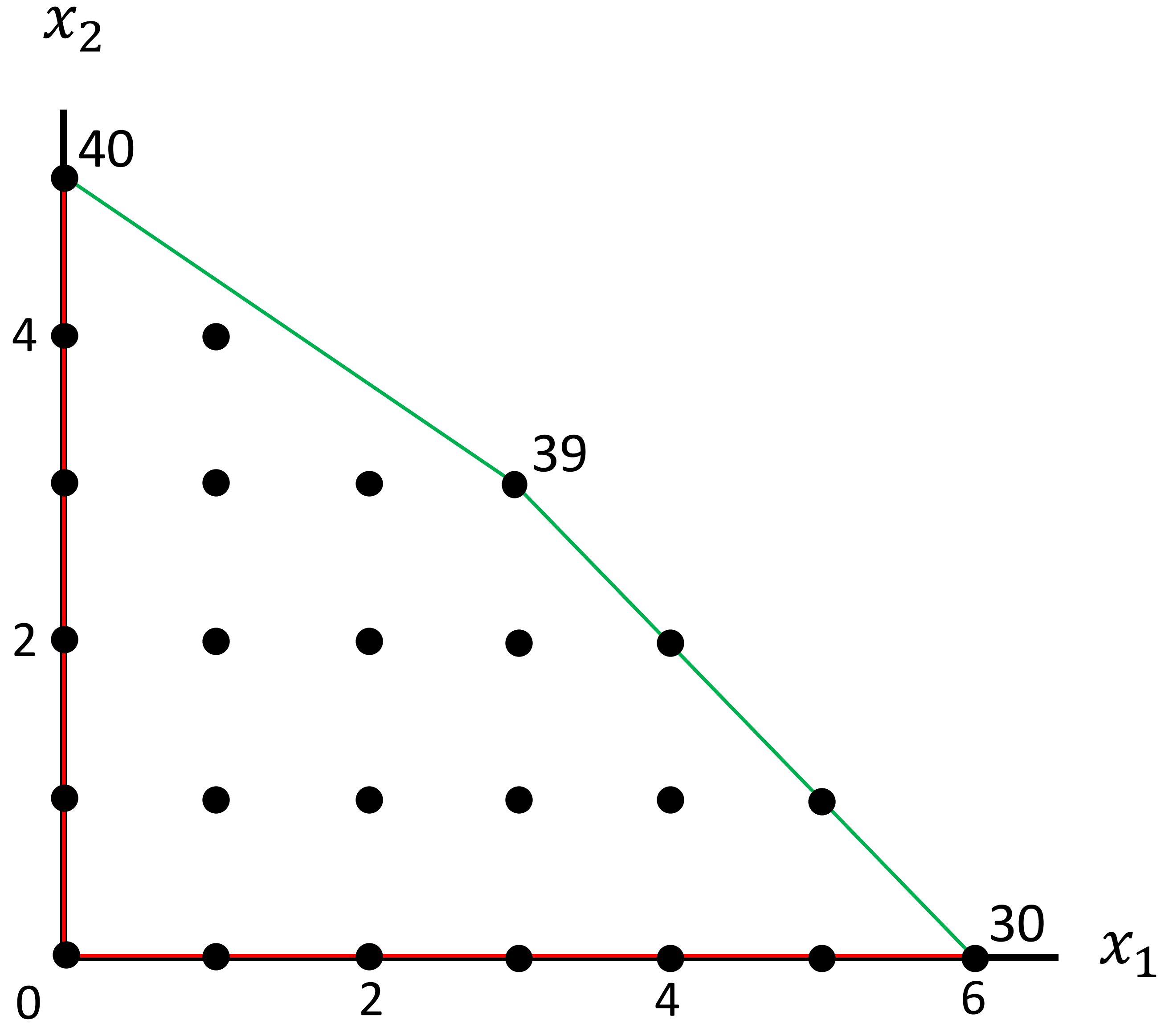


Minimal convex hull:



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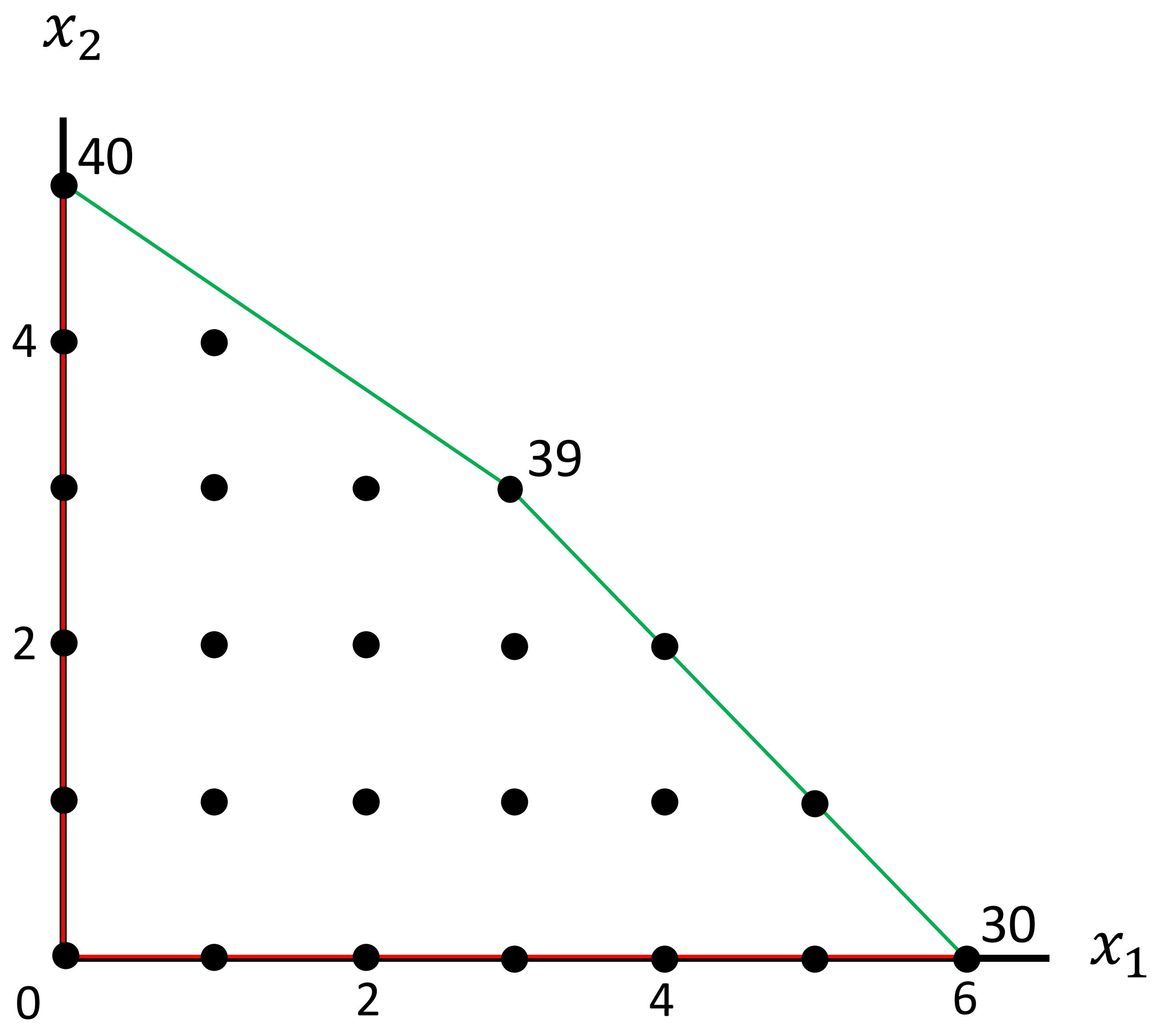
Minimal convex hull:

- Convex.



$x_1, x_2 \in \mathbb{N}$

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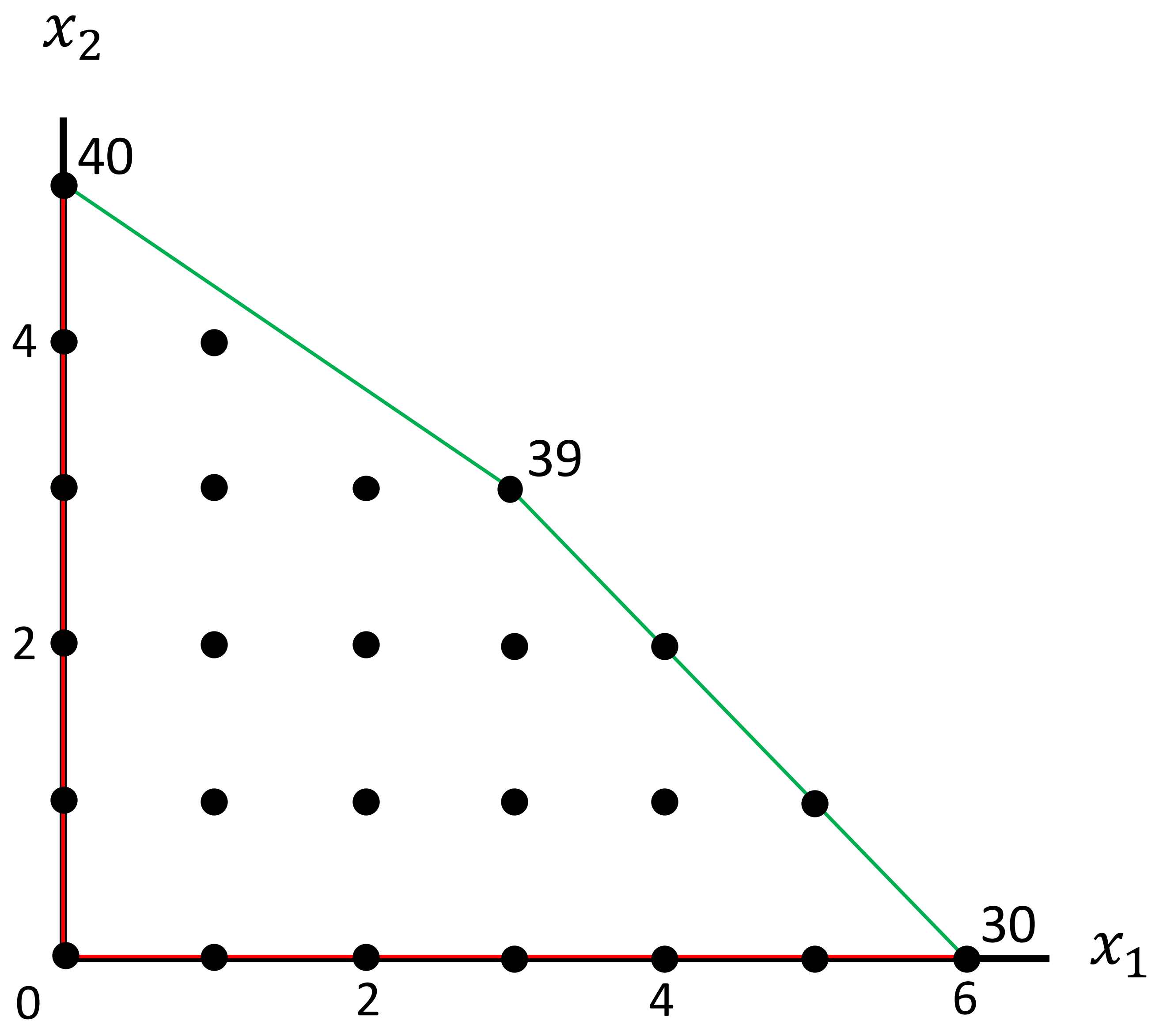
Minimal convex hull:

- Convex.
- local optimum = global optimum.



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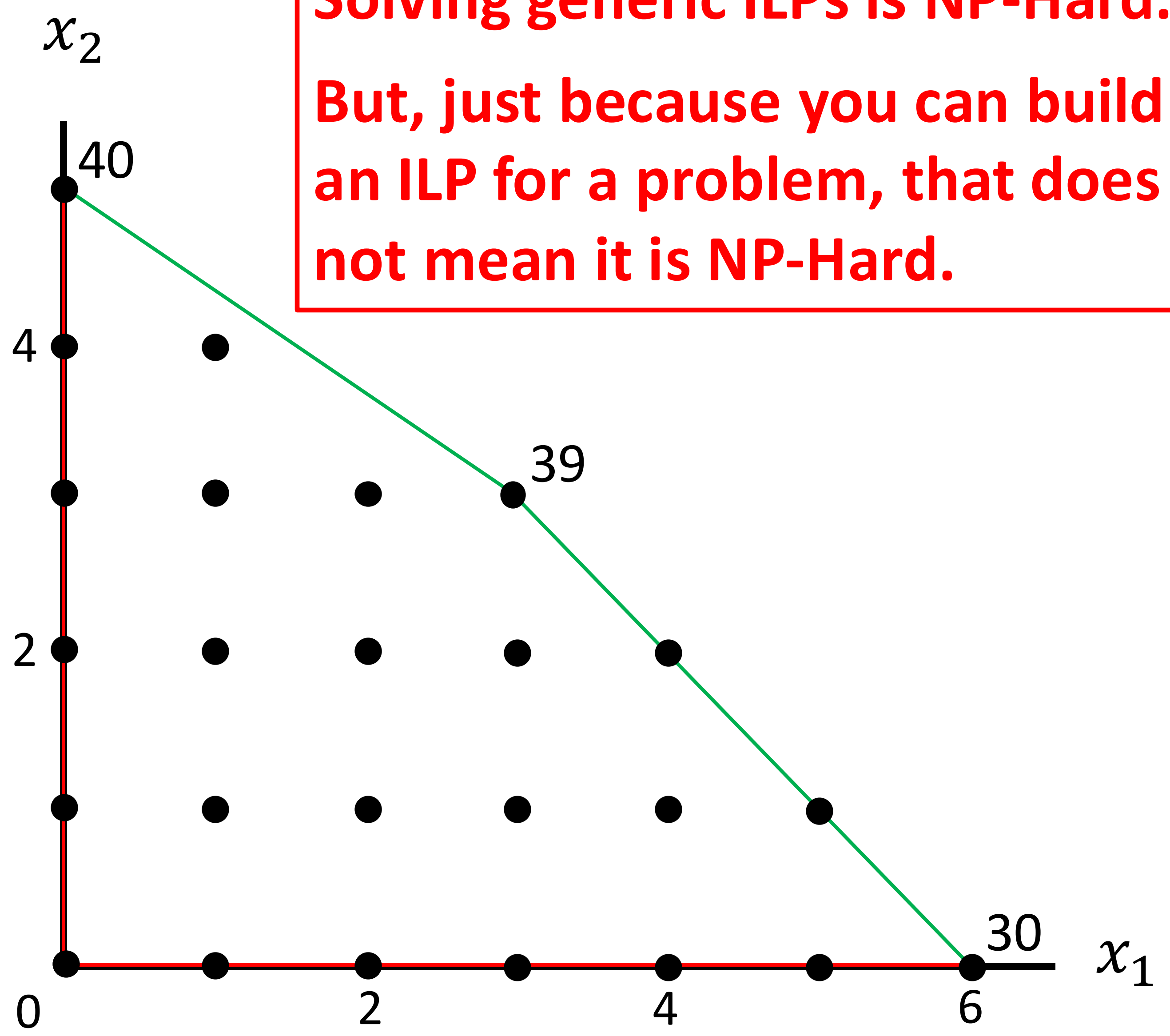
Minimal convex hull:

- Convex.
- local optimum = global optimum.
- $O\left(n^{\lfloor d/2 \rfloor}\right)$ faces, $n = \#$ points and $d = \#$ dimensions



Solving generic ILPs is NP-Hard.

But, just because you can build an ILP for a problem, that does not mean it is NP-Hard.

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