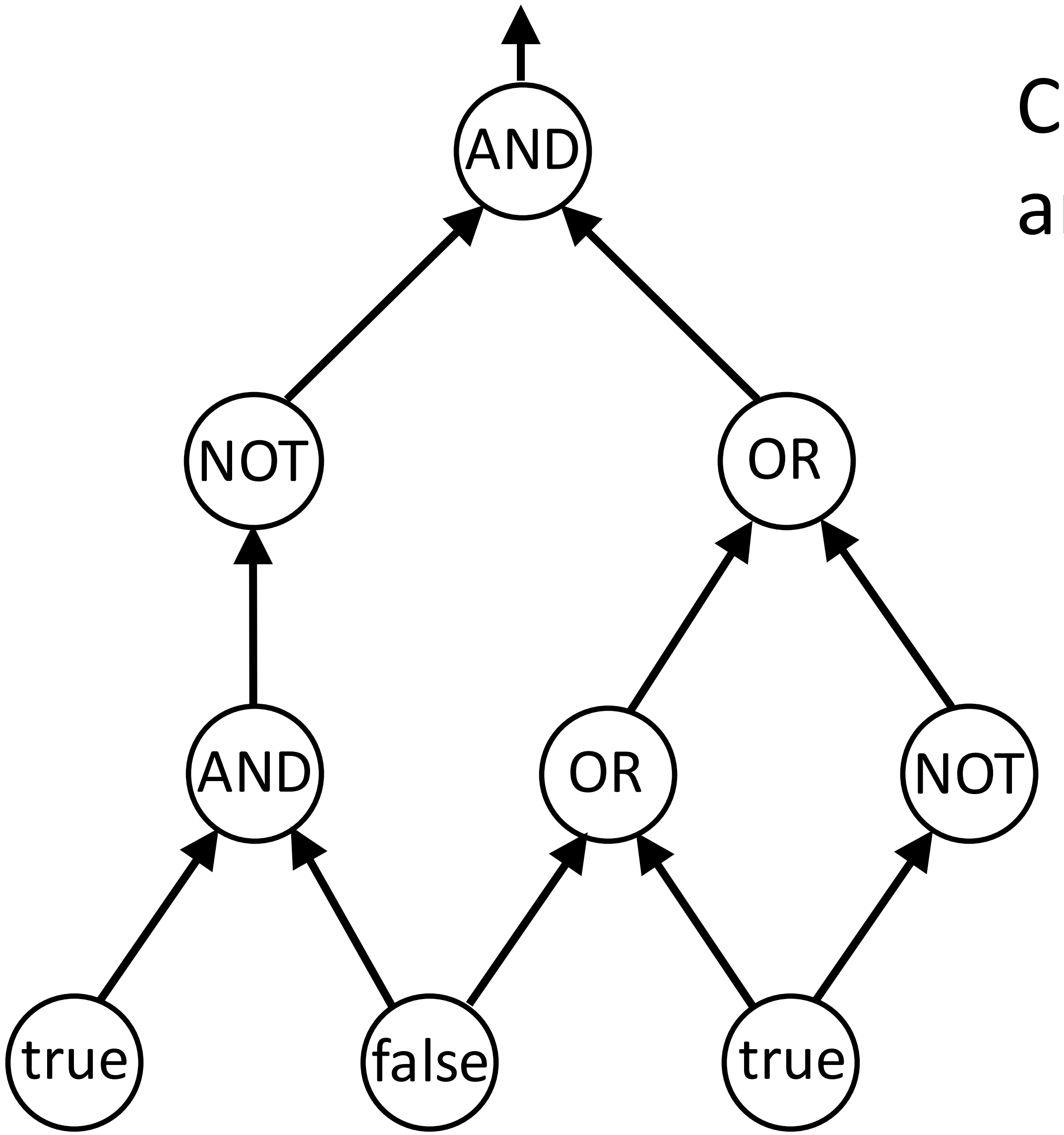
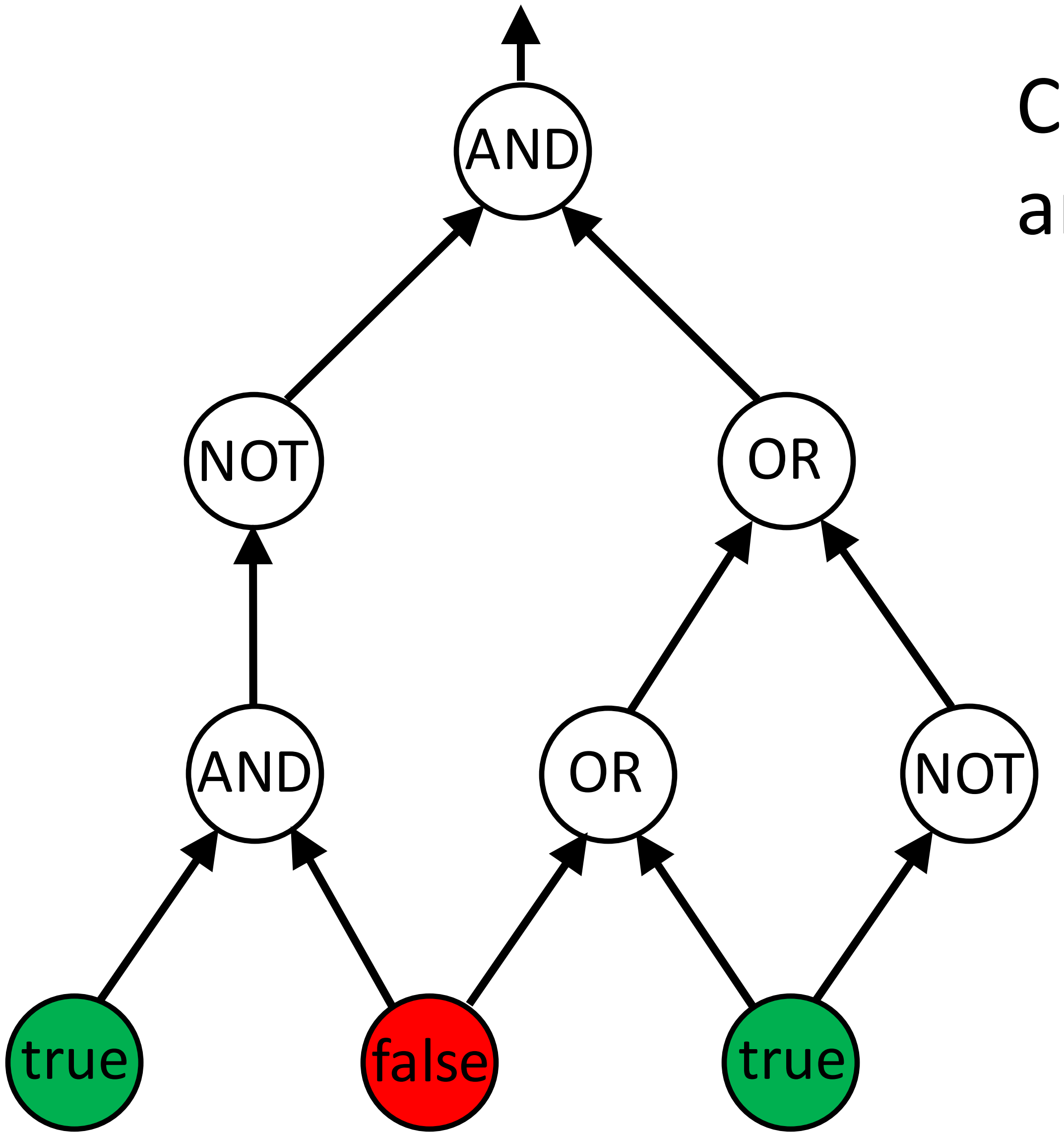
Linear Programming
CSCI 432



Circuit Value Problem: When the gates are evaluated, is the output true?

Given a circuit schematic, what is the answer?



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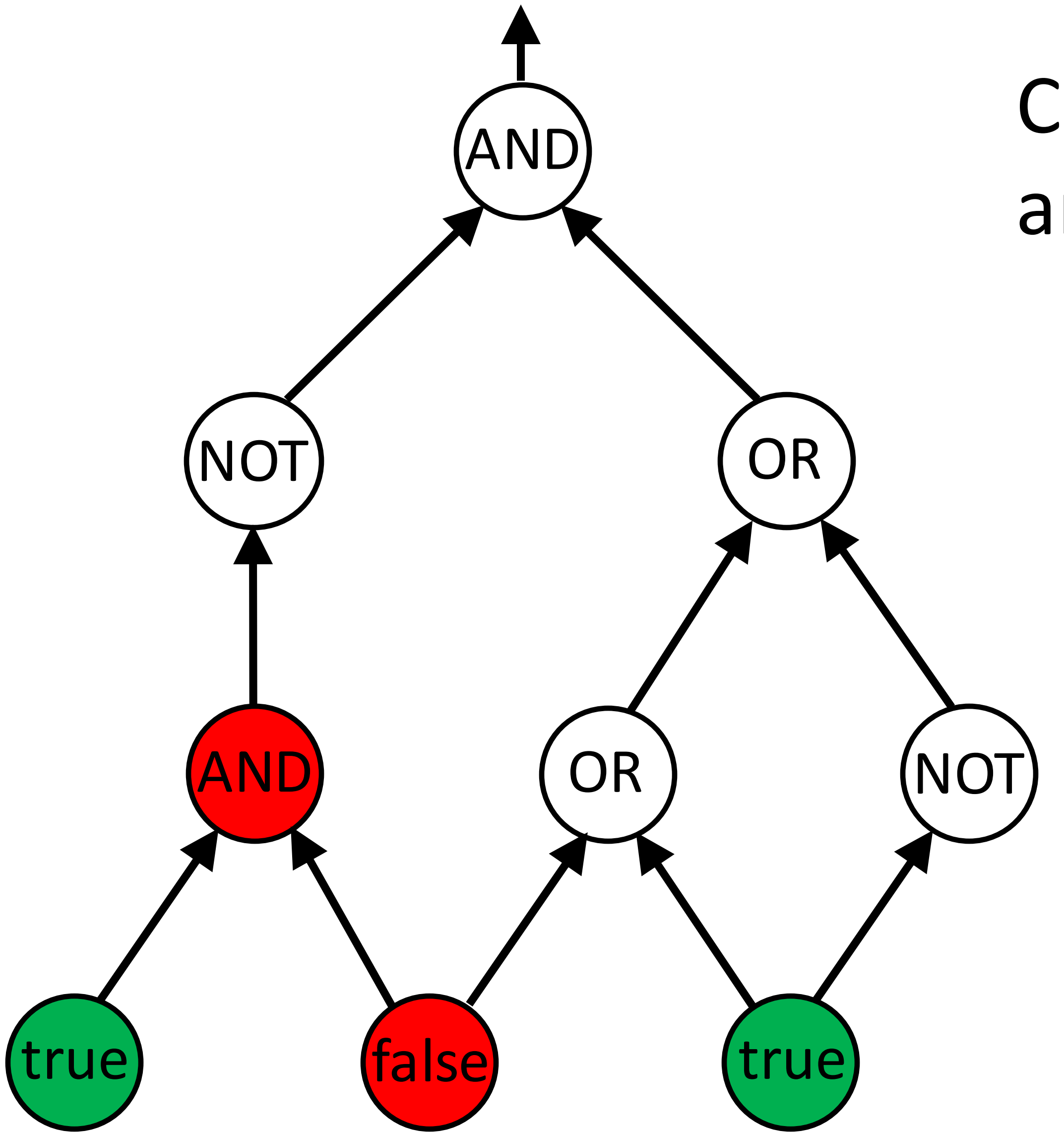

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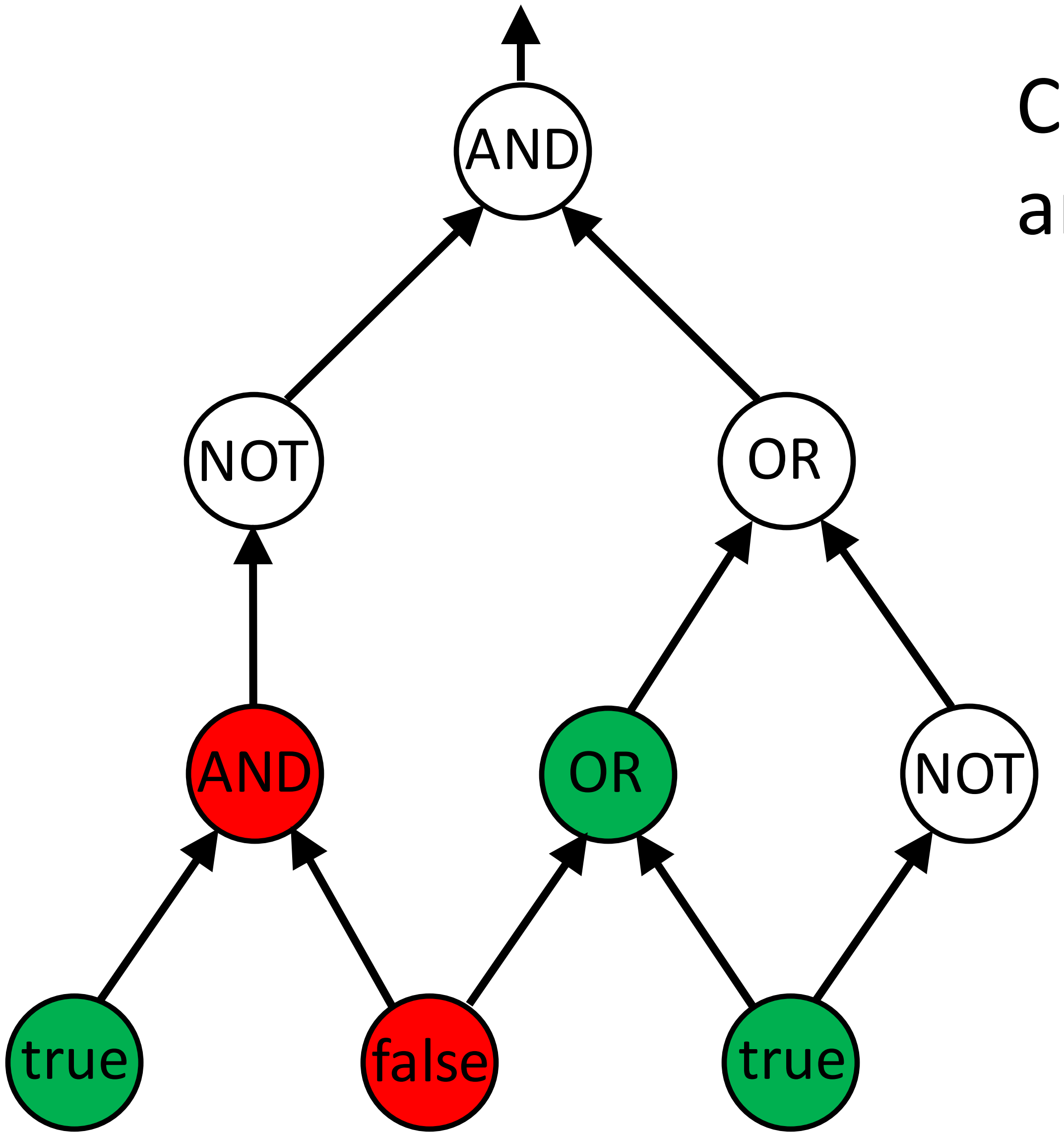
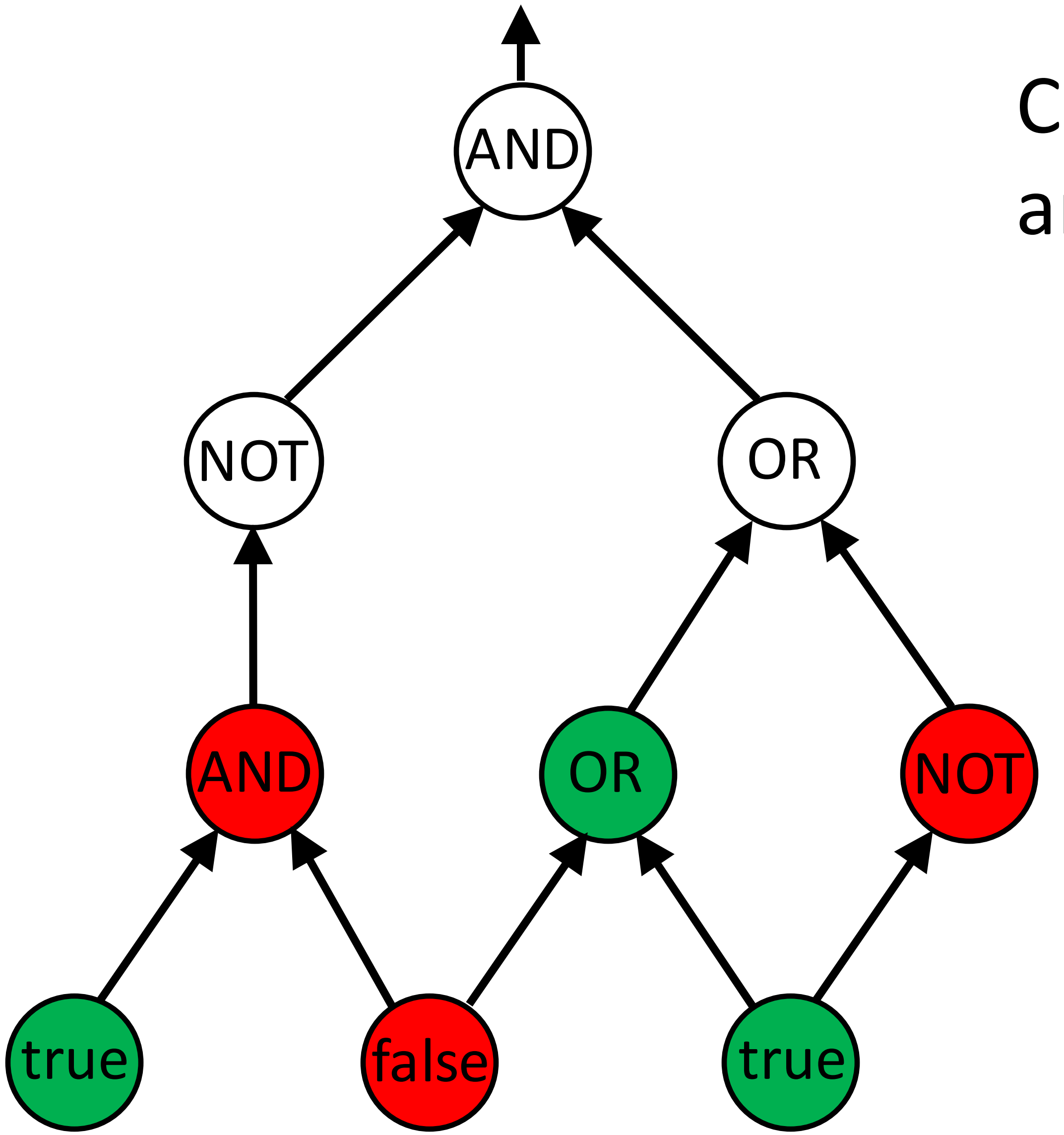


Circuit Value Problem: When the gates are evaluated, is the output true?

Given a circuit schematic, what is the answer?



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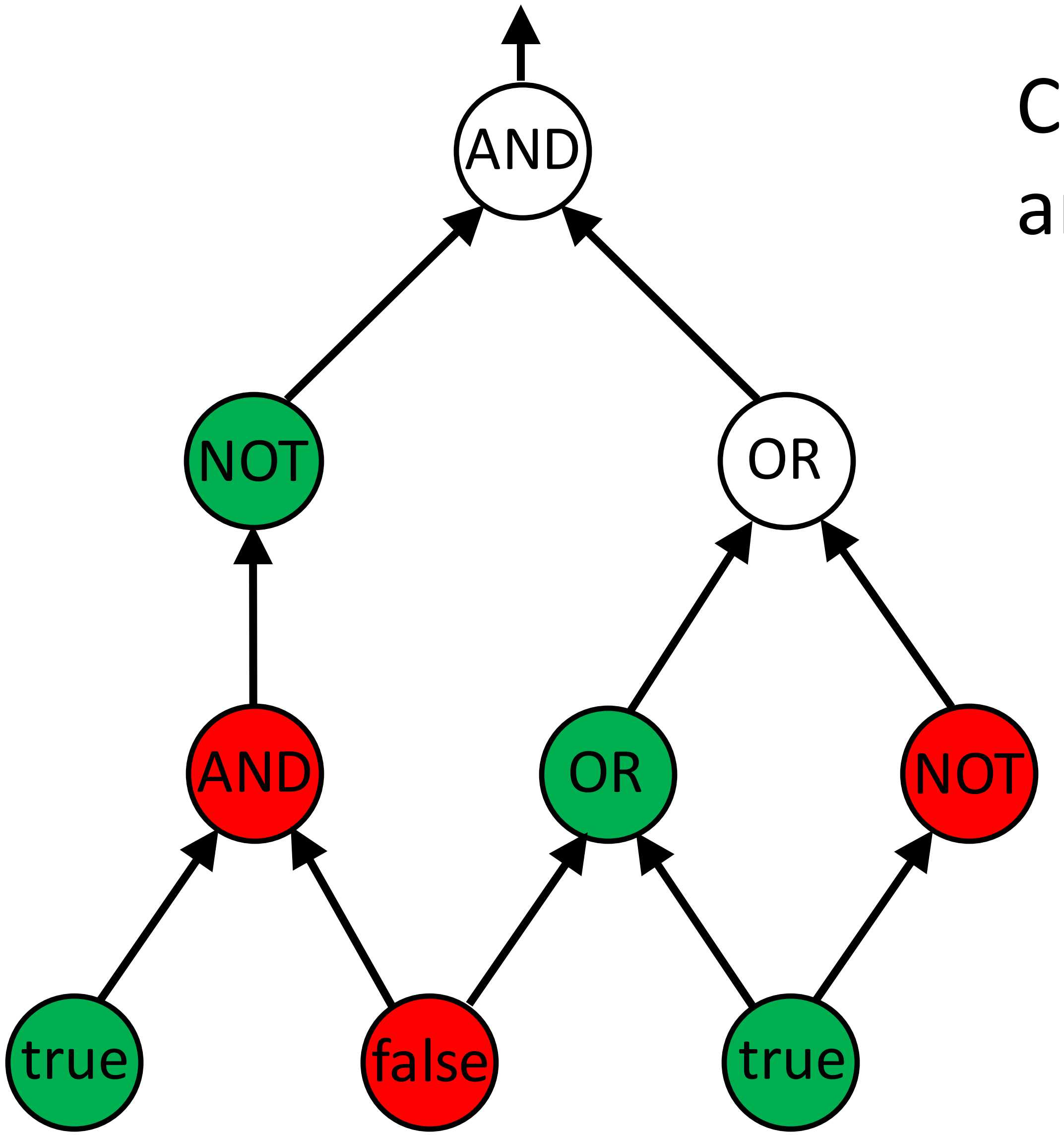
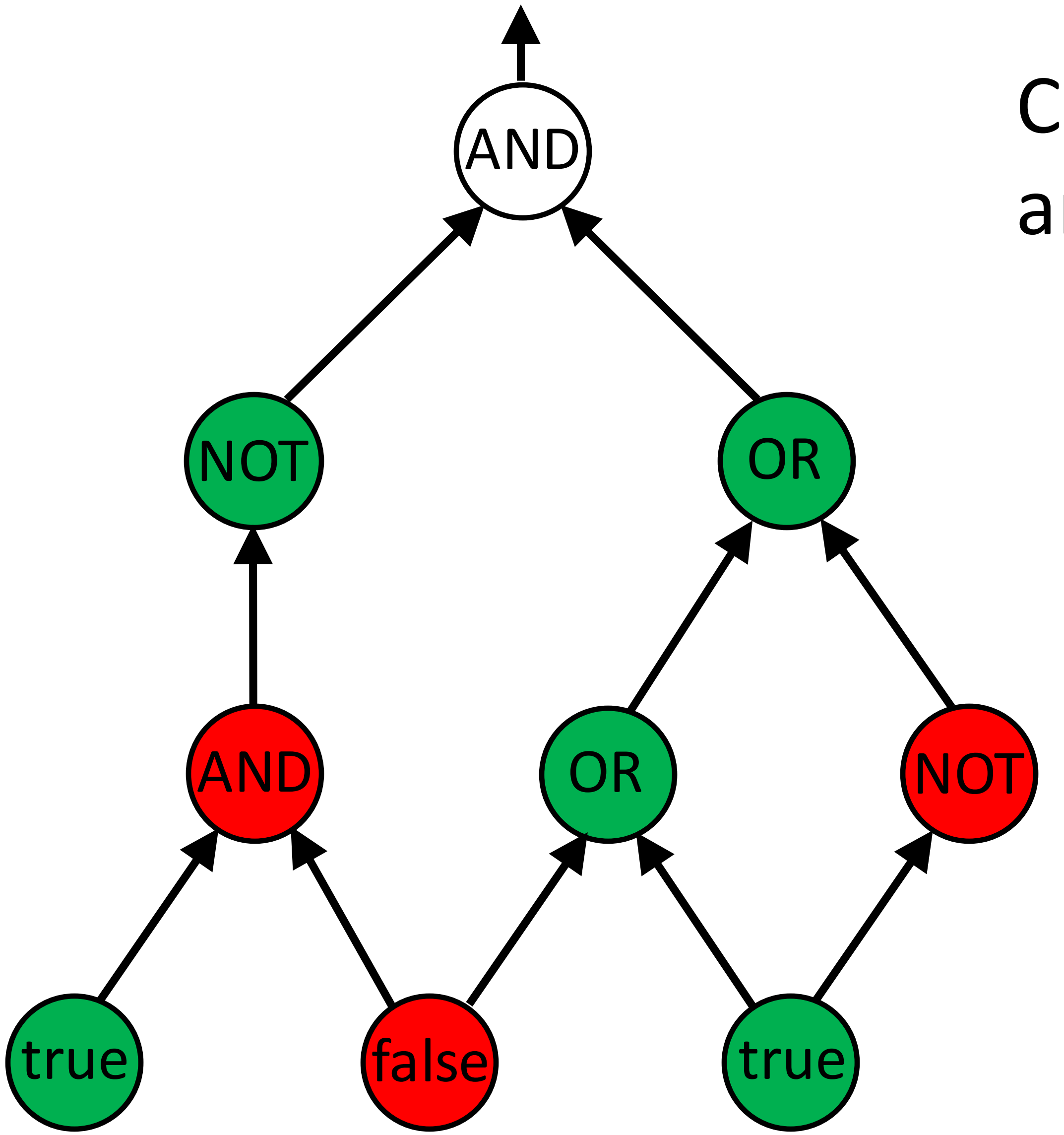
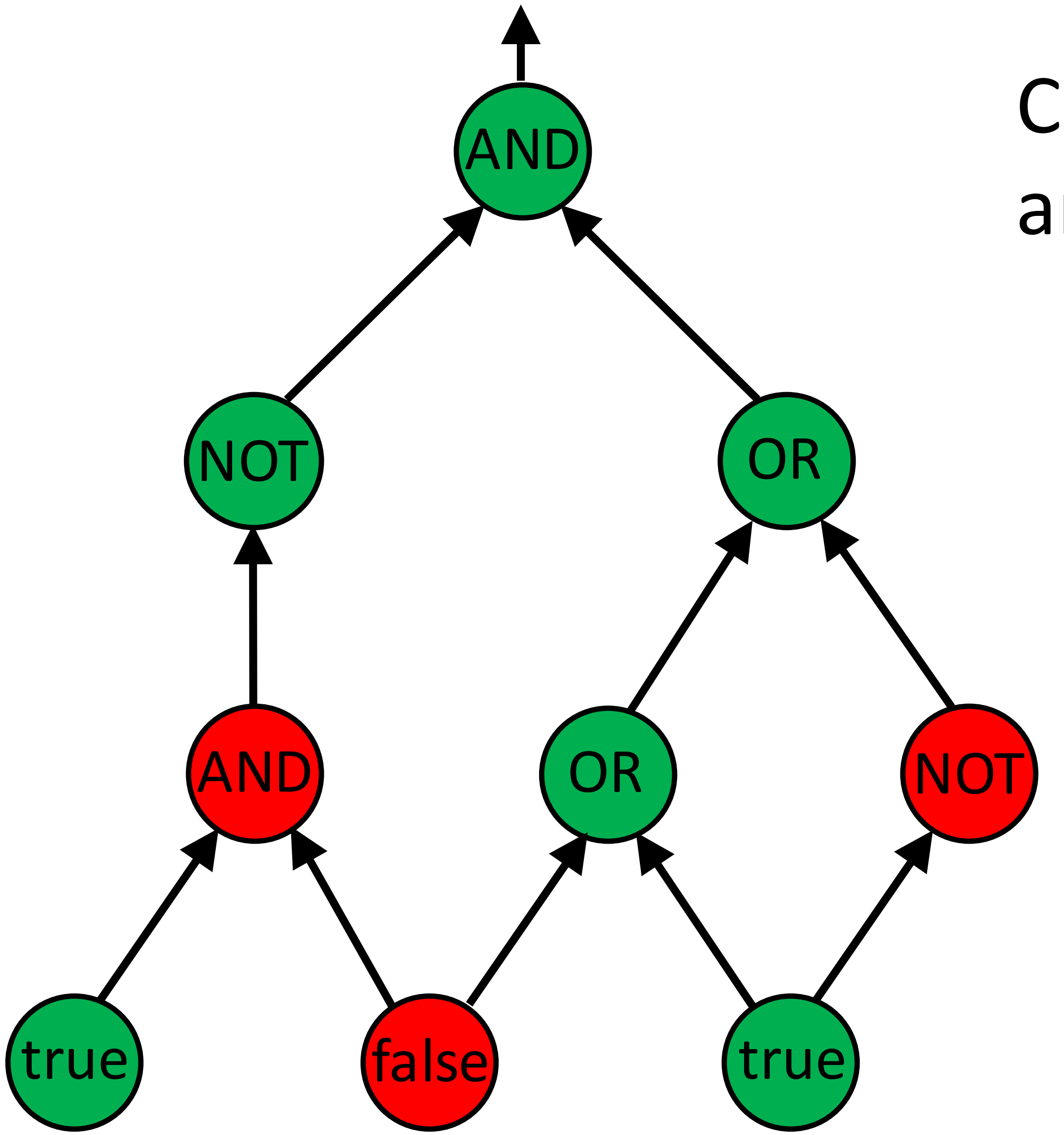


Given a circuit schematic, what is the answer?



Circuit Value Problem: When the gates are evaluated, is the output true?

Given a circuit schematic, what is the answer?



Circuit Value Problem: When the gates are evaluated, is the output true?

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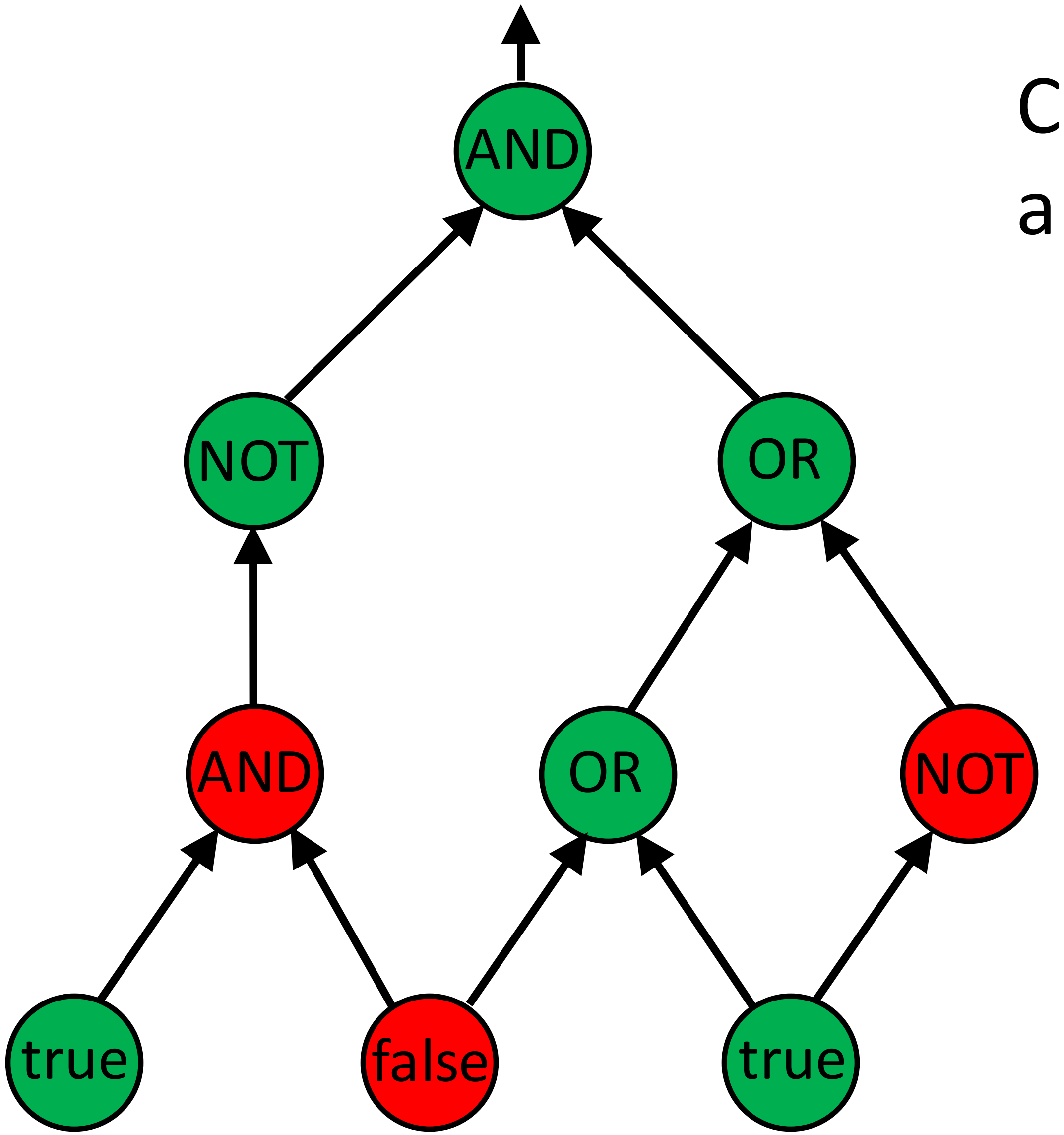
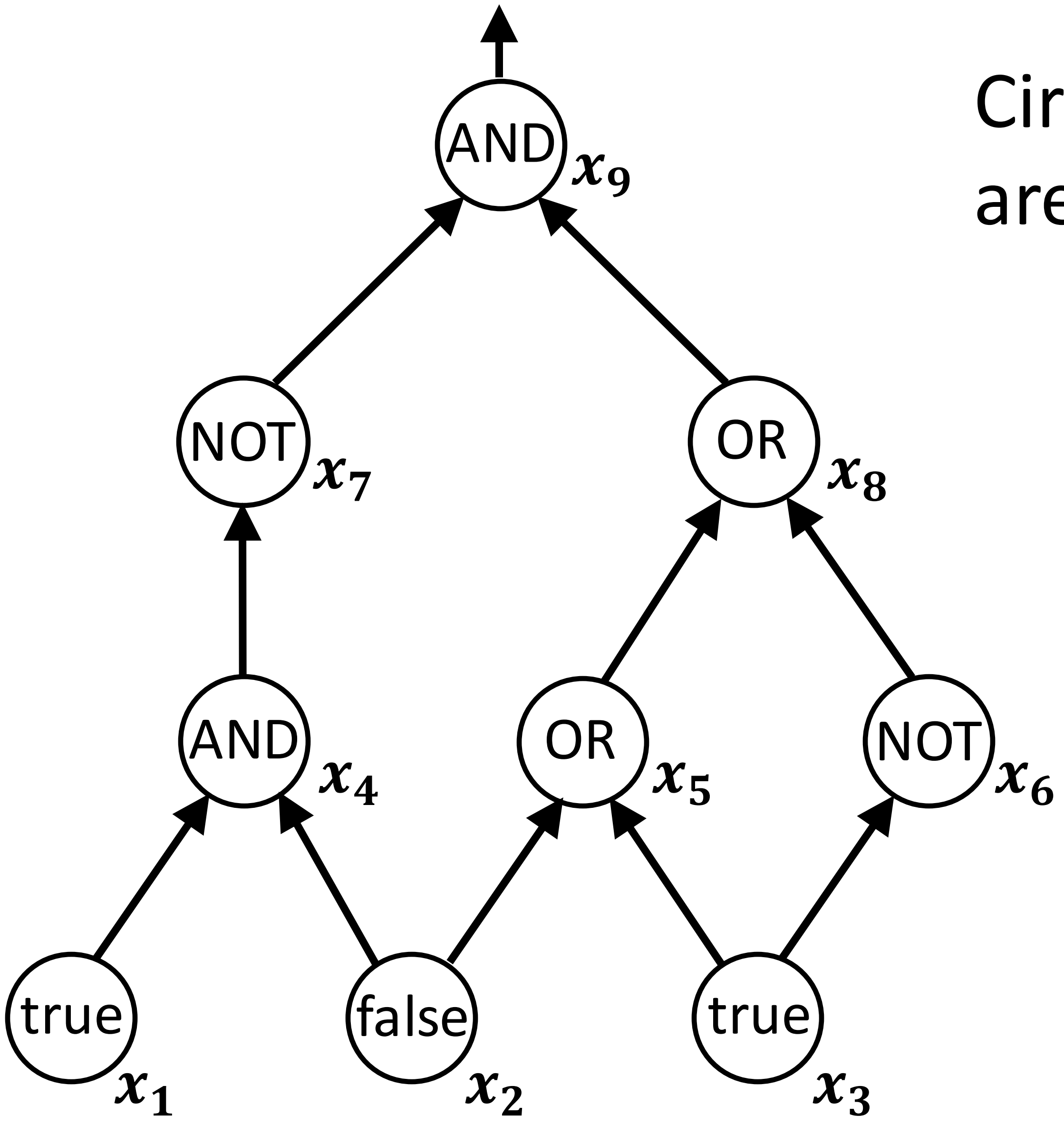


Circuit Value Problem: When the gates are evaluated, is the output true?

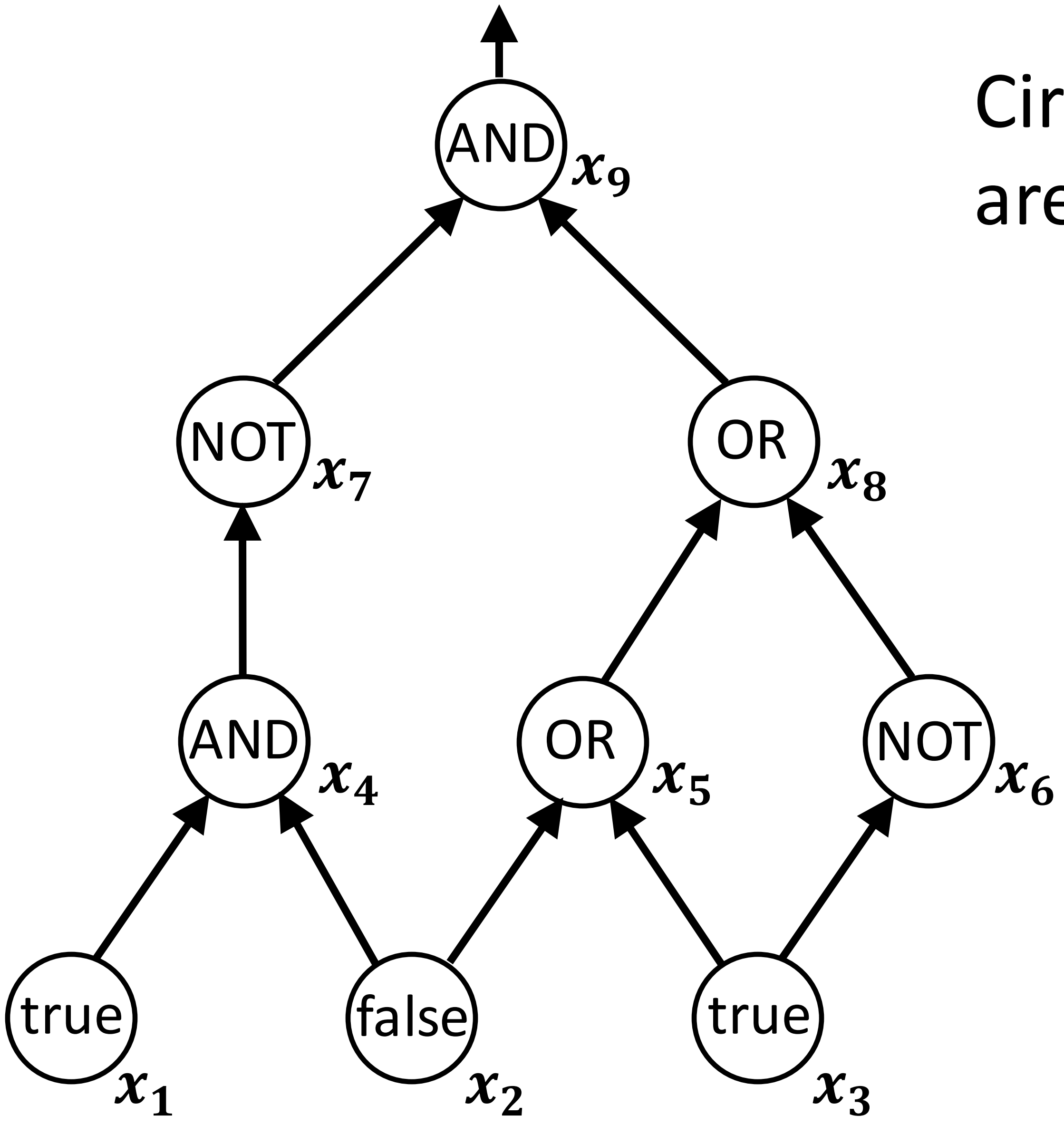
Given a circuit schematic, what is the answer?

Can we make a Linear Program that solves this?



Circuit Value Problem: When the gates are evaluated, is the output true?

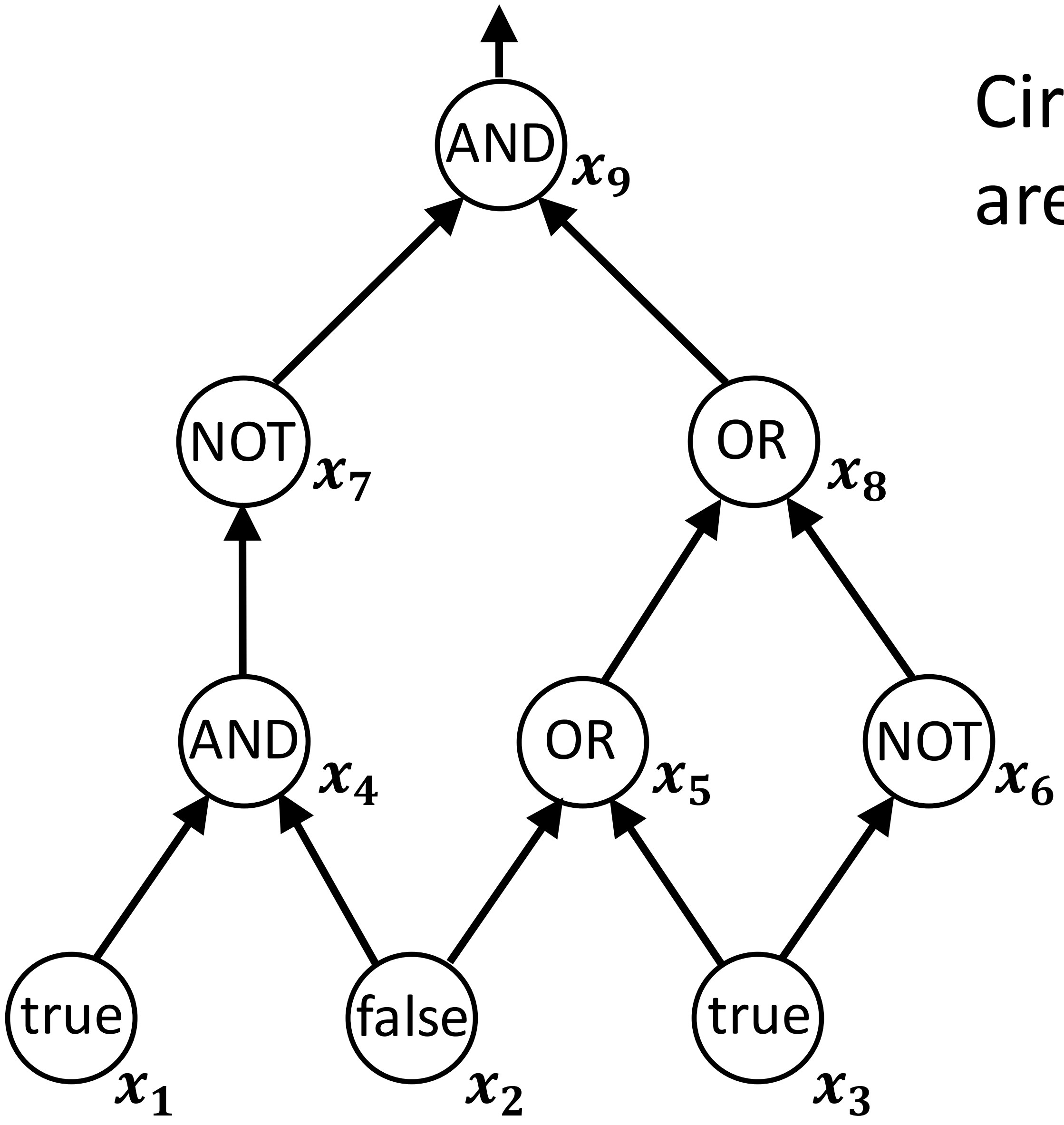
x_i = gate's evaluated value



Circuit Value Problem: When the gates are evaluated, is the output true?

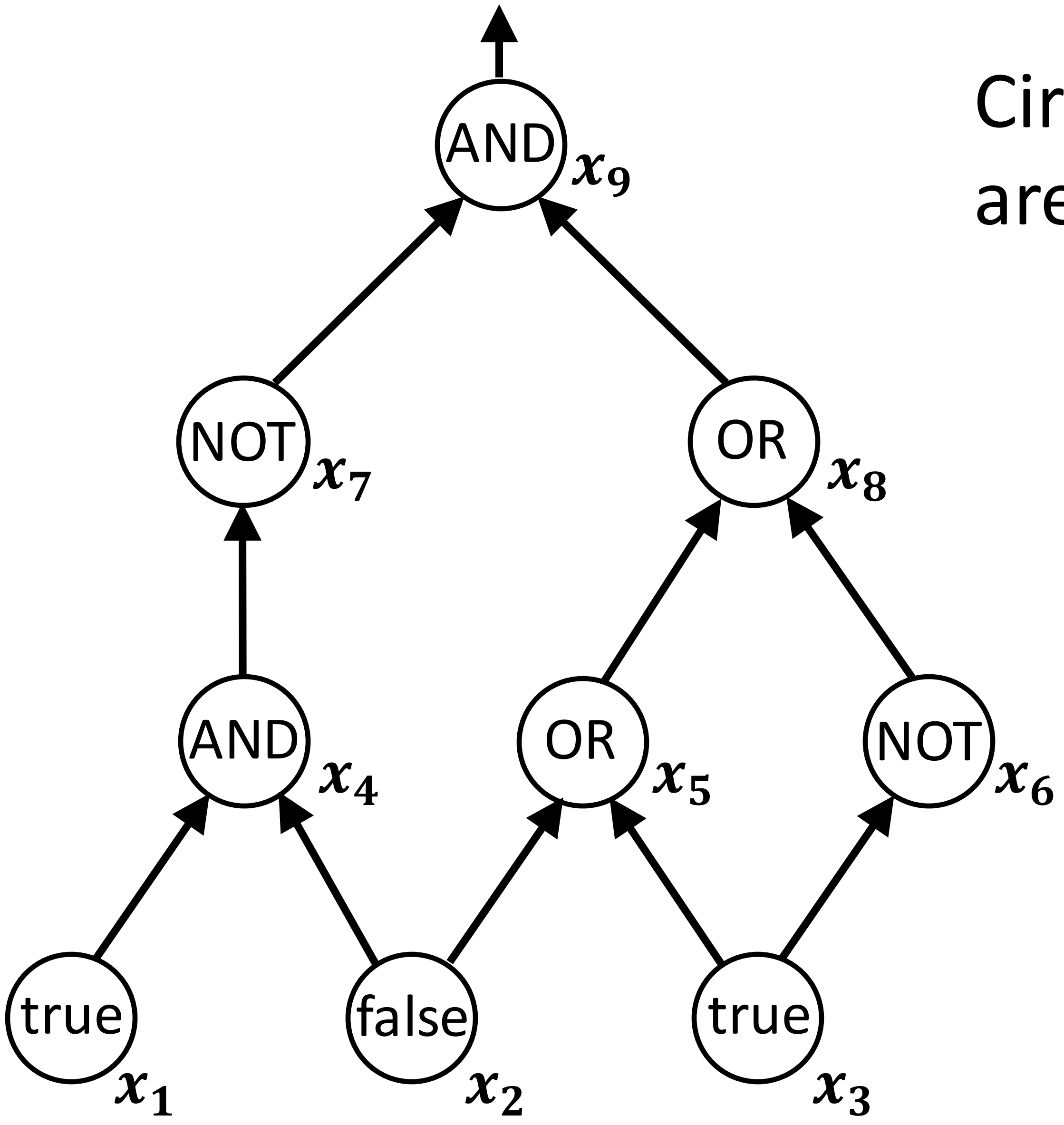
x_i = gate's evaluated value

What is x_9's value?



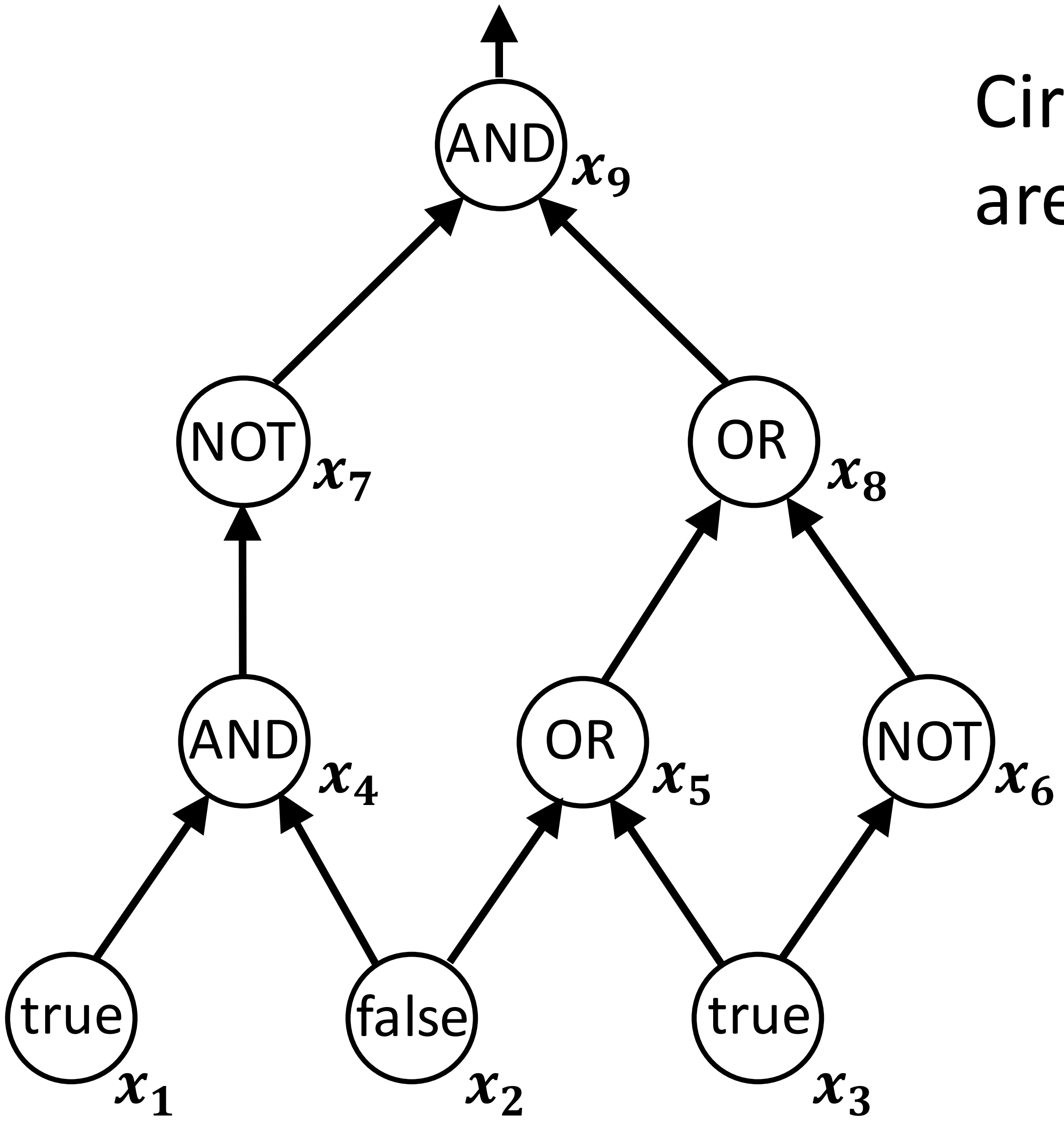
Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to:

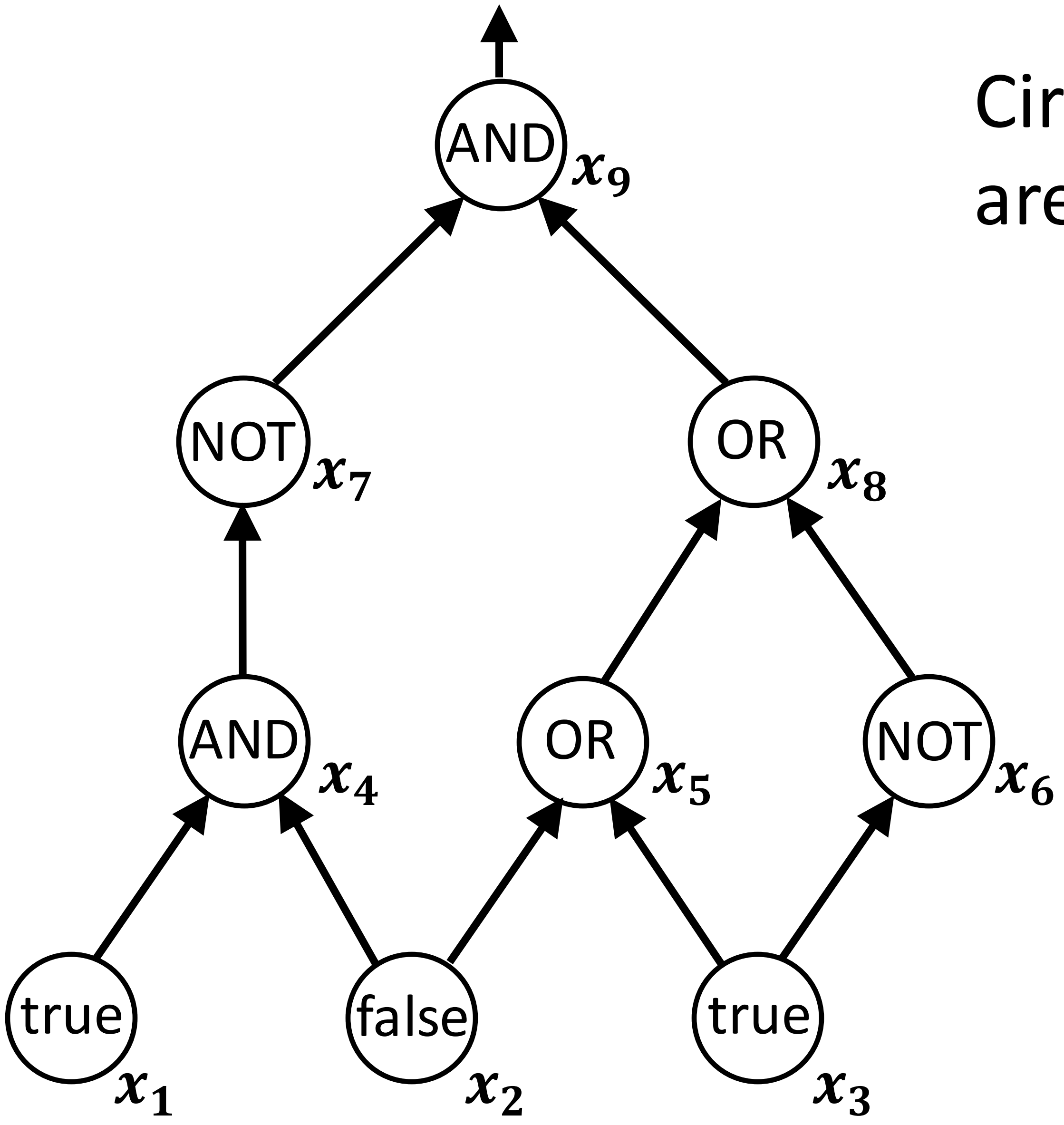


Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

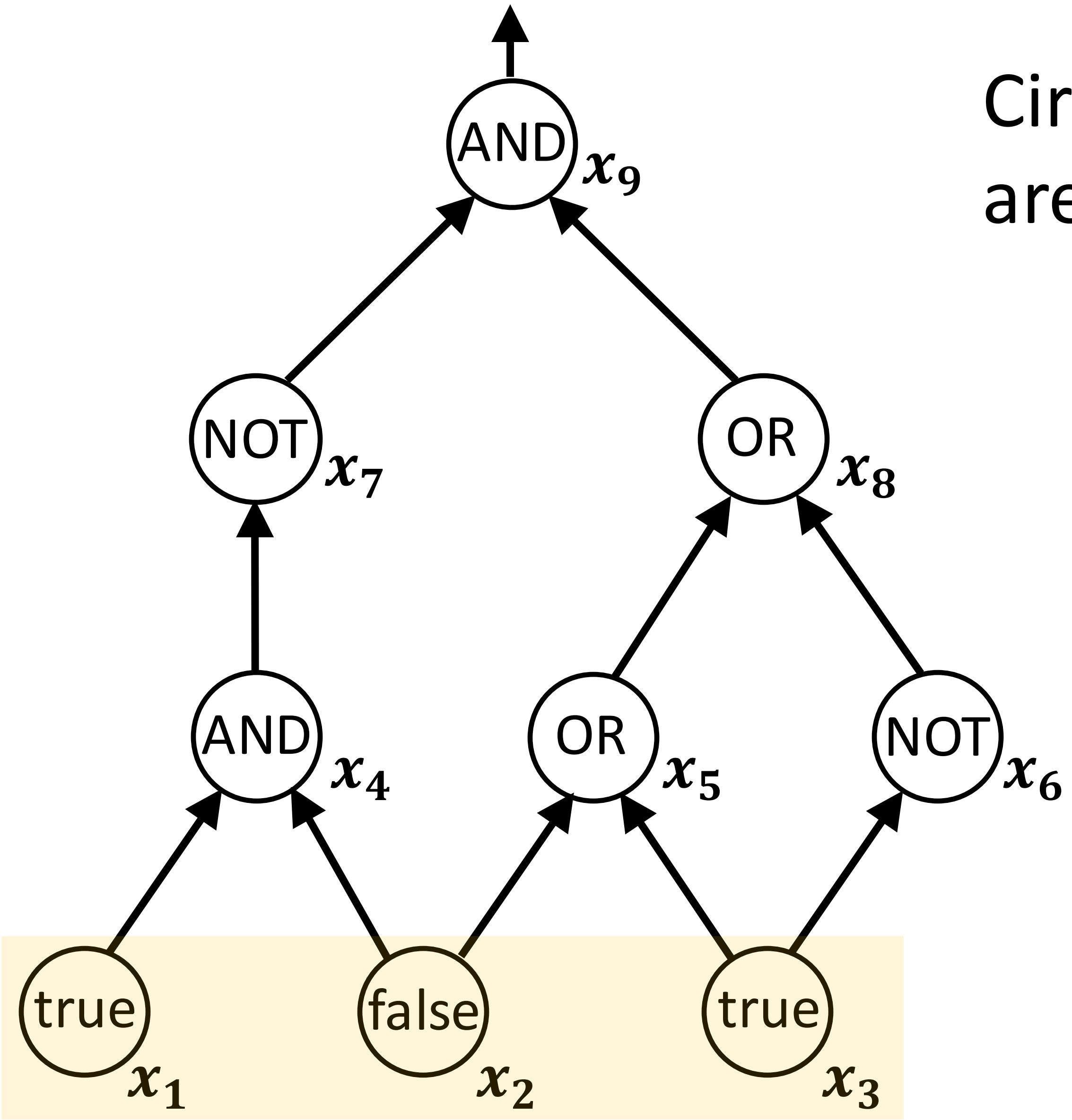
Subject to: $x_i \in \{0,1\}, \forall i$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in [0,1], \forall i$

We need $x_i \in \{0, 1\}$, but maybe we can make that happen without forcing it to be an ILP.



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

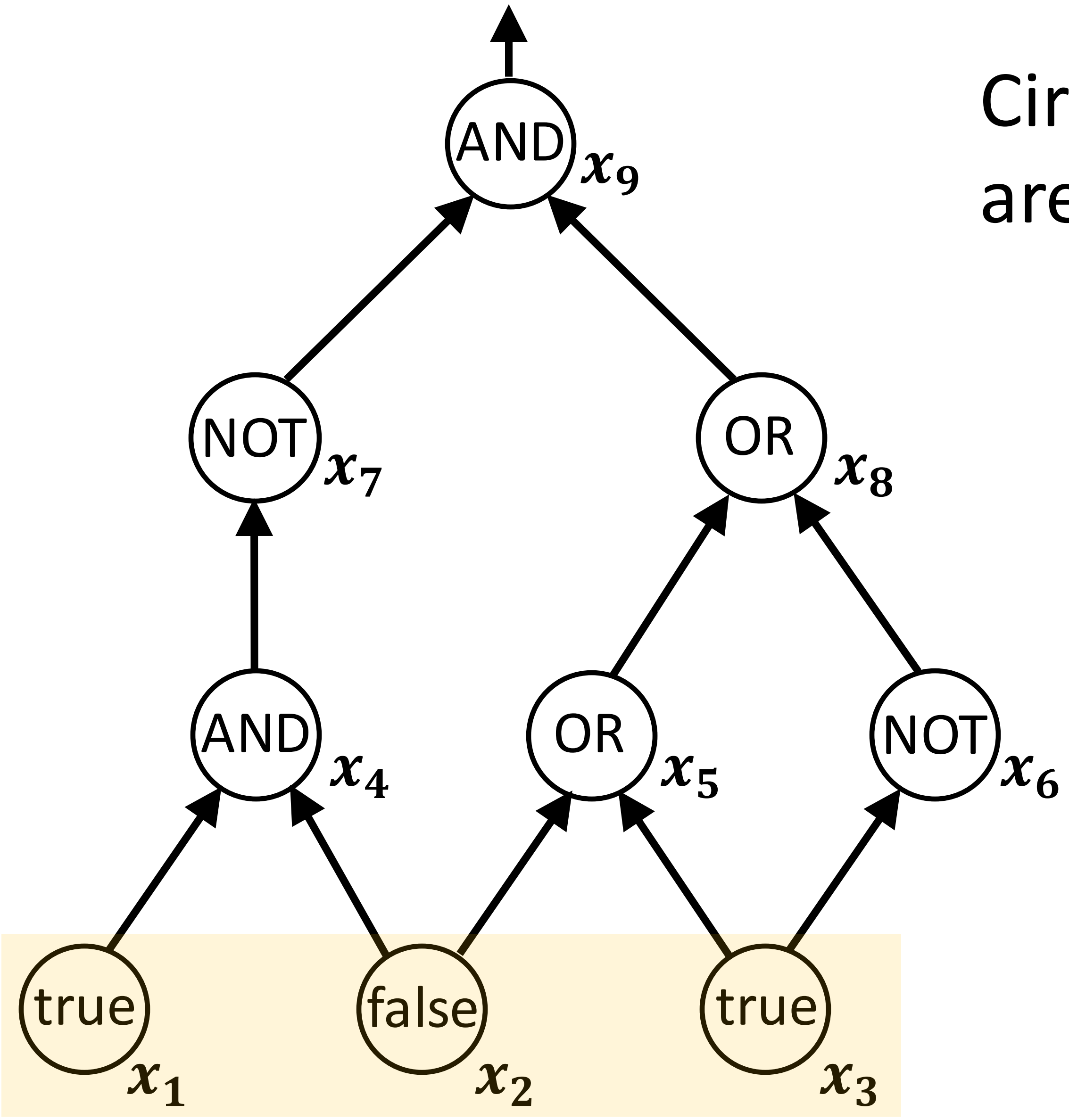
Objective: ??

Subject to: $x_i \in [0,1], \forall i$

$x_1 = ?$

$x_2 = ?$

$x_3 = ?$

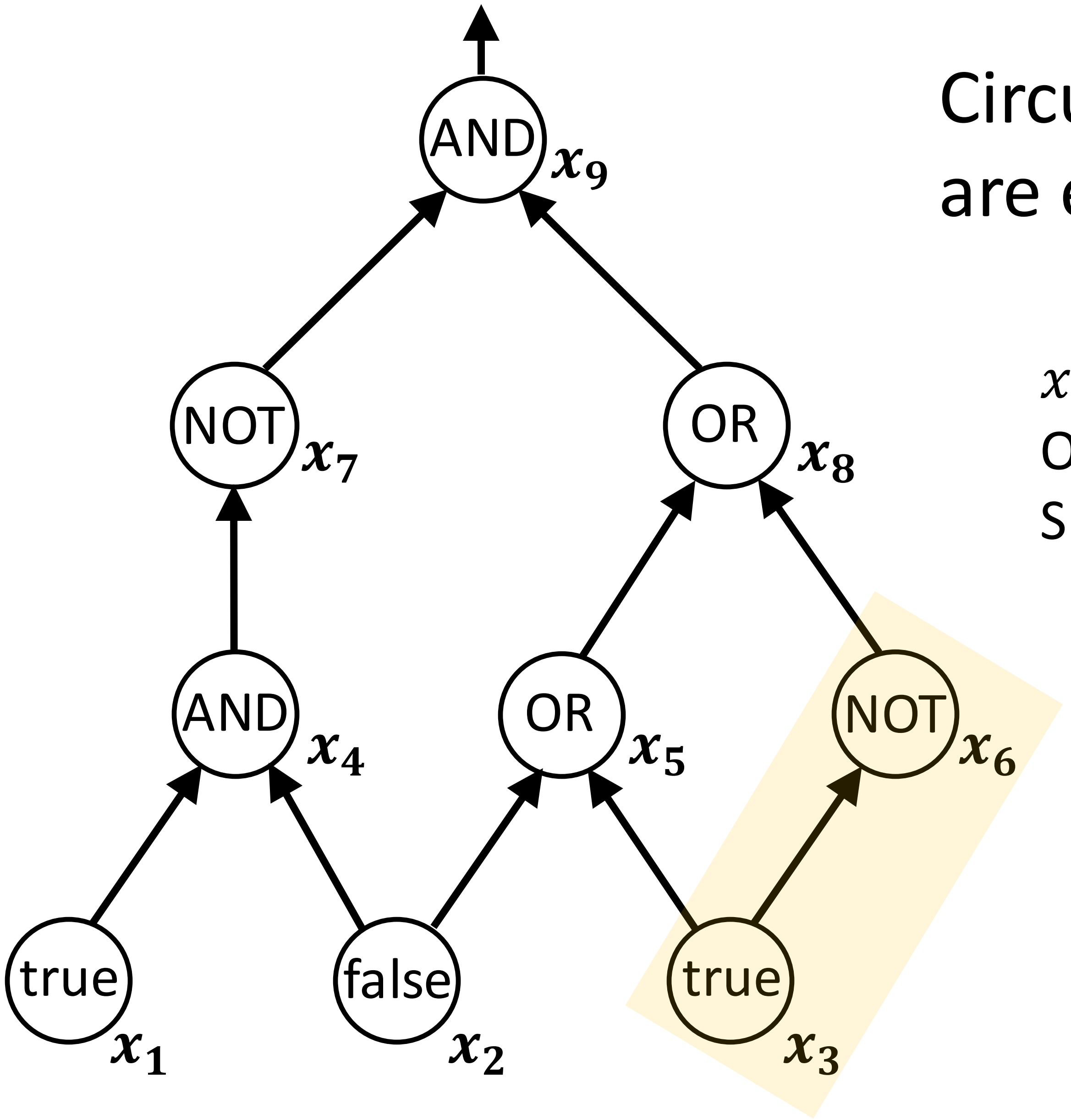


Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in [0,1], \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

In general, if gate i is initialized to true (or false), make $x_i = 1$ (or 0).



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

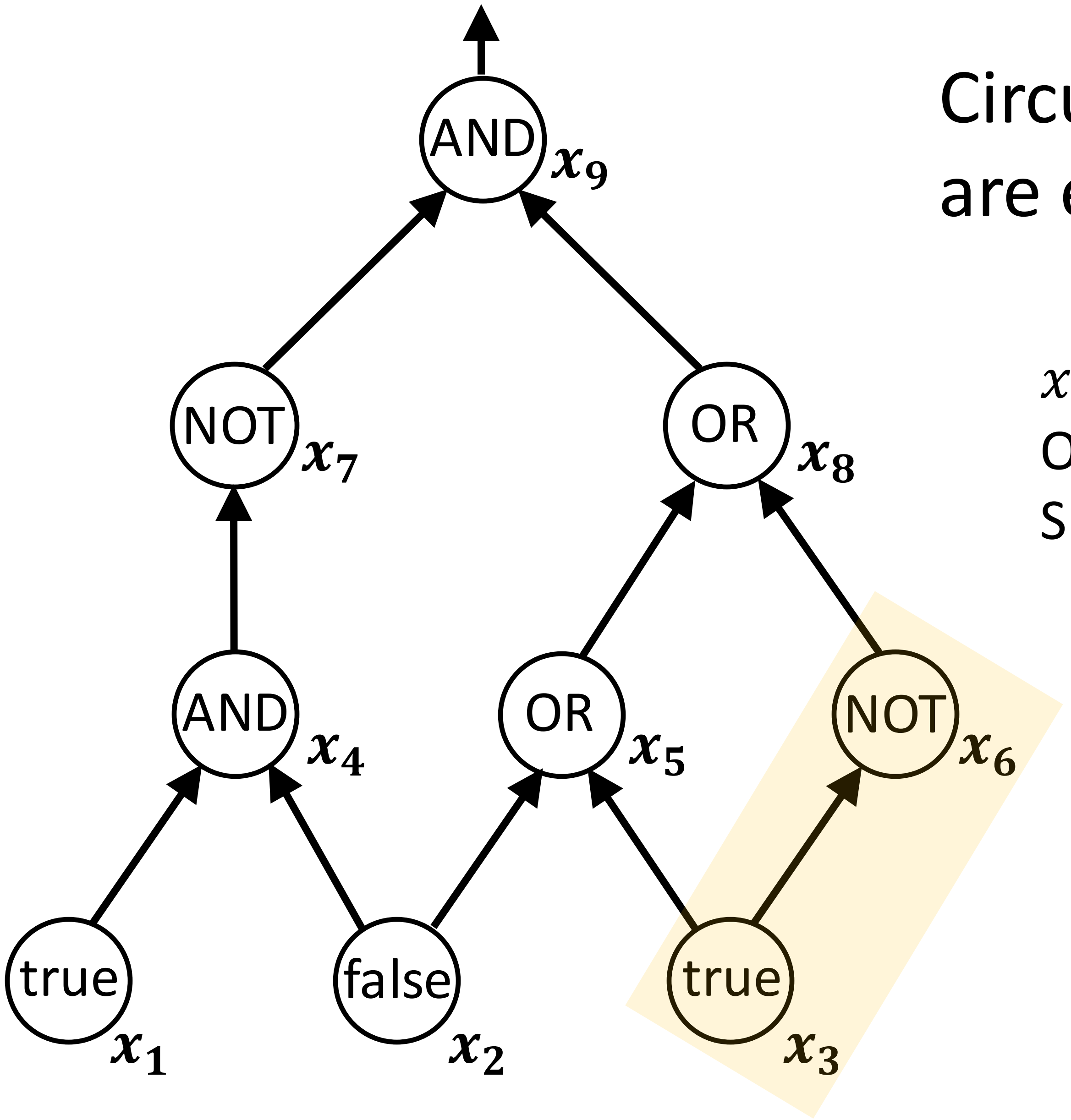
Subject to: $x_i \in [0,1], \forall i$

$x_1 = 1$

$x_2 = 0$

$x_3 = 1$

$x_6 = ??$



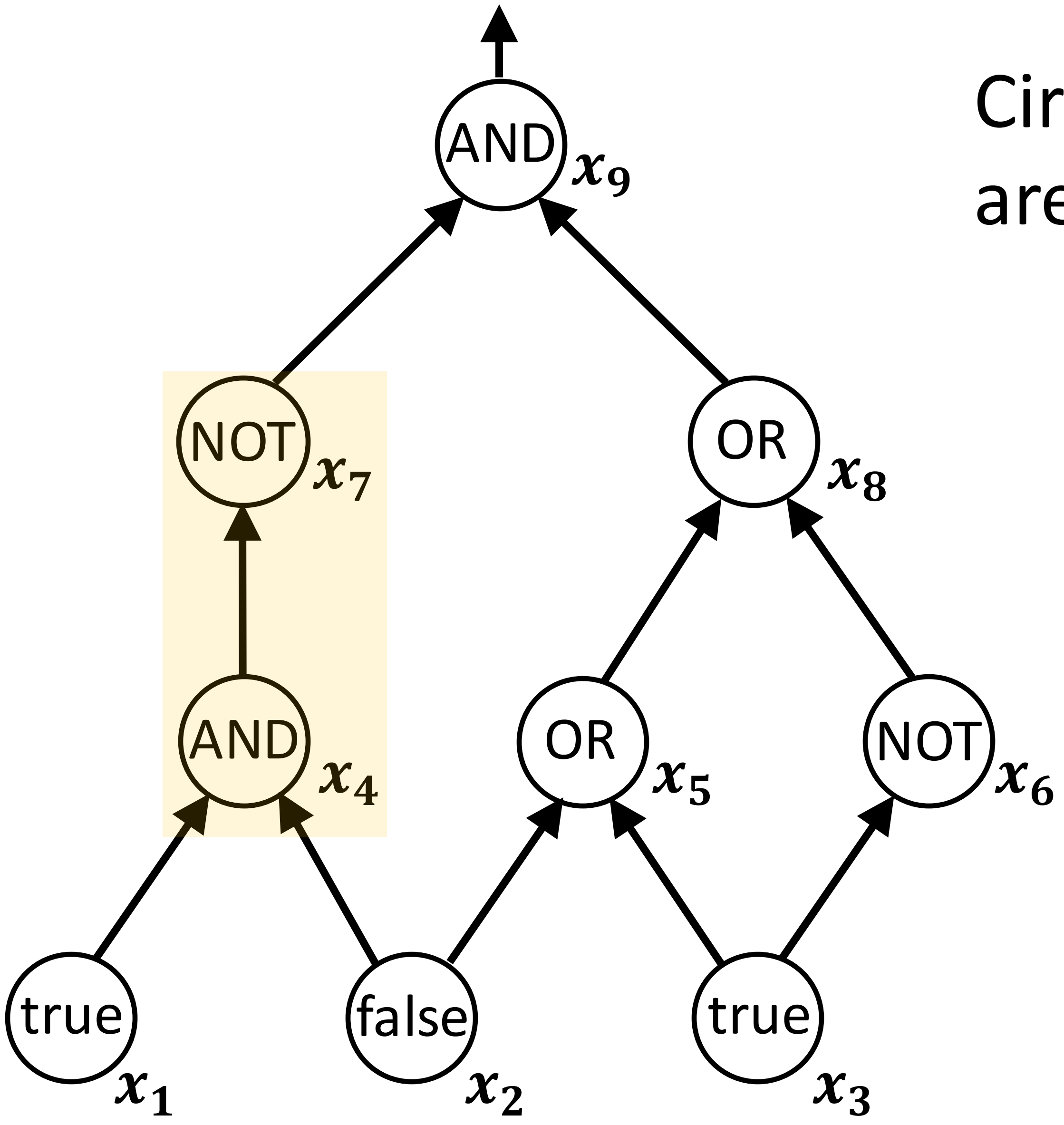
Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in [0,1], \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

$x_6 = 1 - x_3$

In general, if gate i is NOT gate j, make $x_i = 1 - x_j$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in [0,1], \forall i$

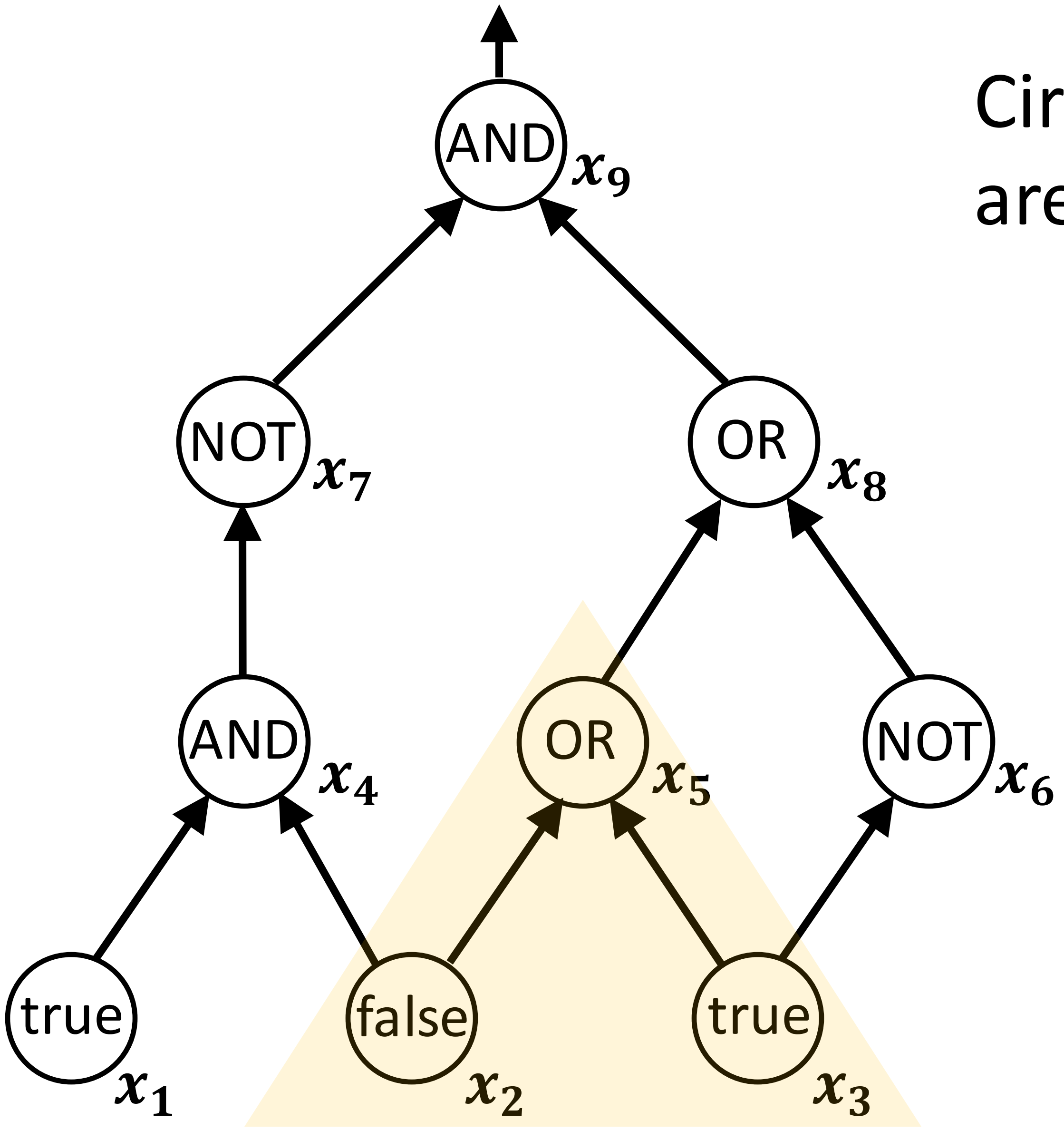
$x_1 = 1$

$x_2 = 0$

$x_3 = 1$

$x_6 = 1 - x_3$

$x_7 = 1 - x_4$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in [0,1], \forall i$

$x_1 = 1$

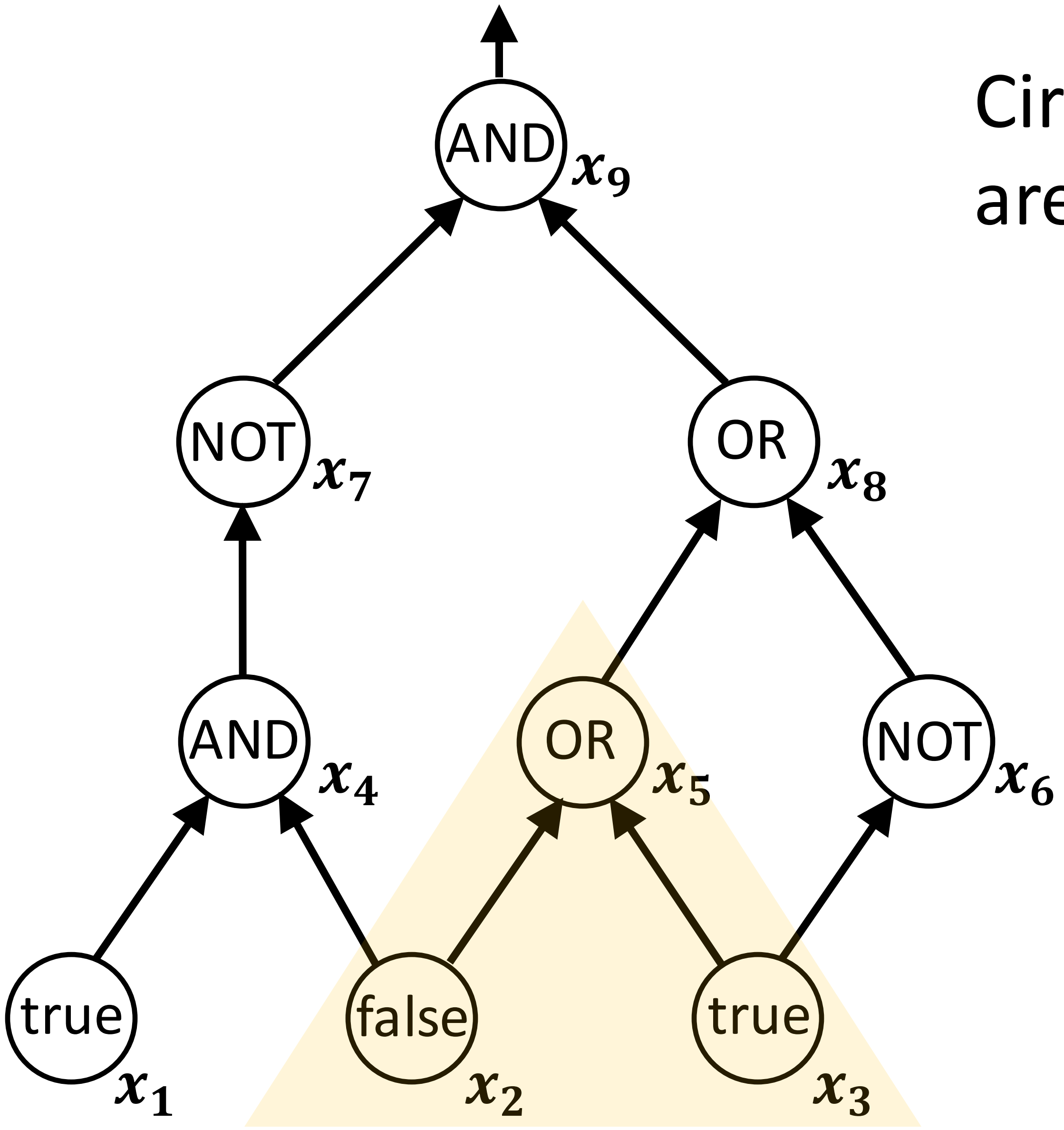
$x_2 = 0$

$x_3 = 1$

$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

??



Circuit Value Problem: When the gates are evaluated, is the output true?

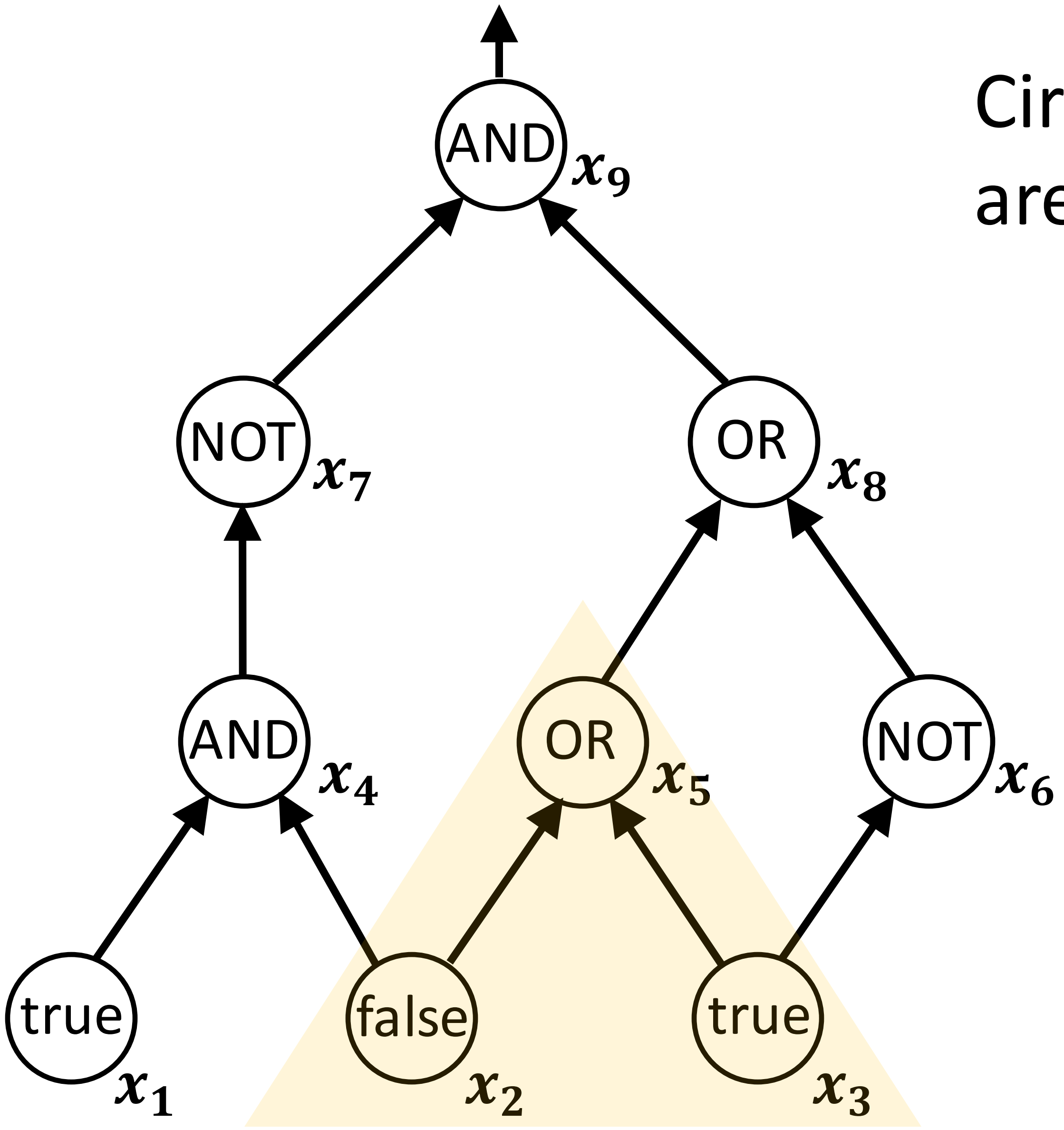
x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in [0,1], \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

x_j	x_k	$x_i = x_j$ OR x_k
0	0	0
1	0	1
0	1	1
1	1	1



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in [0,1], \forall i$

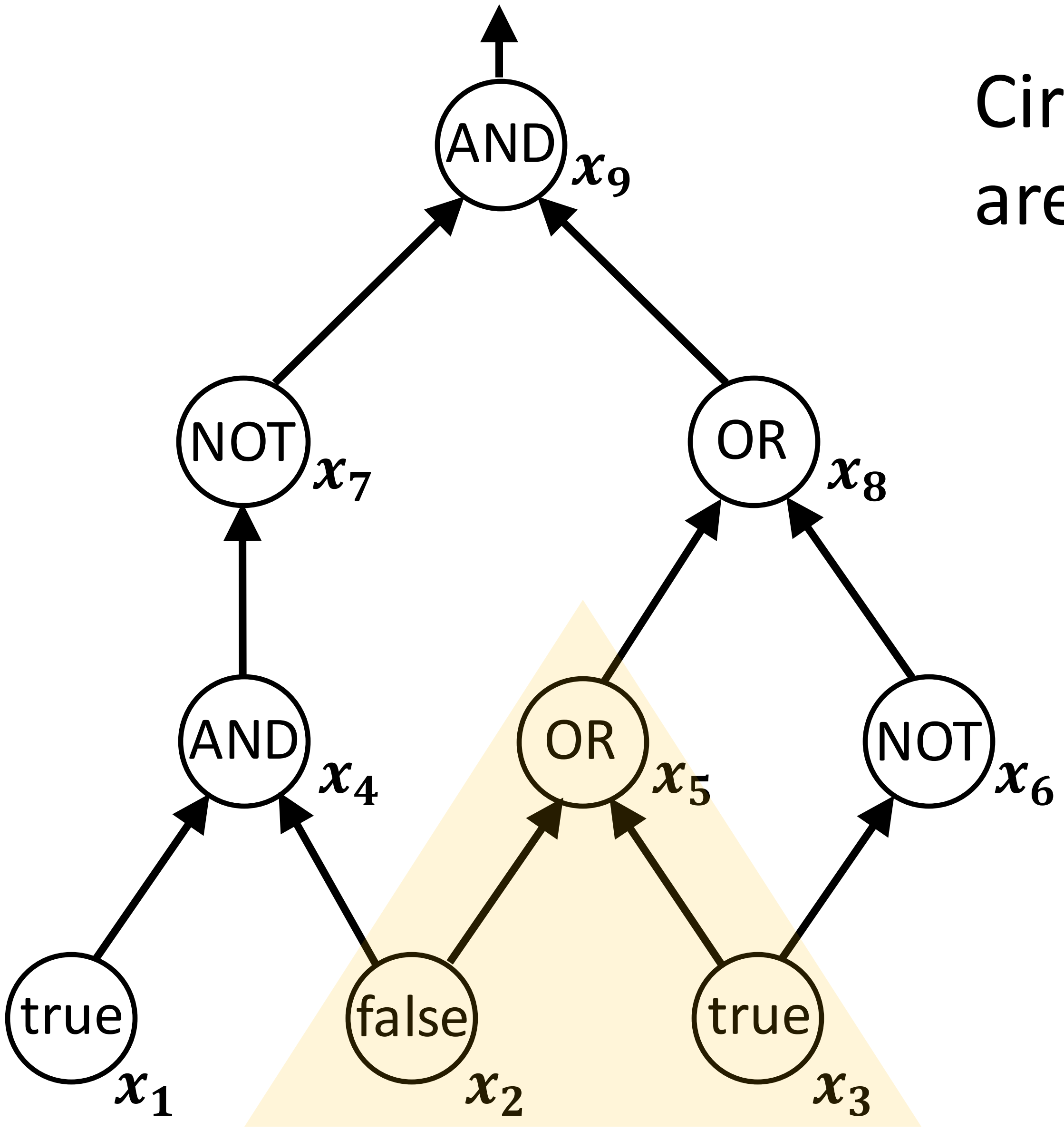
$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

$x_5 \geq x_2$

x_j	x_k	$x_i = x_j$ OR x_k
0	0	0
1	0	1
0	1	1
1	1	1

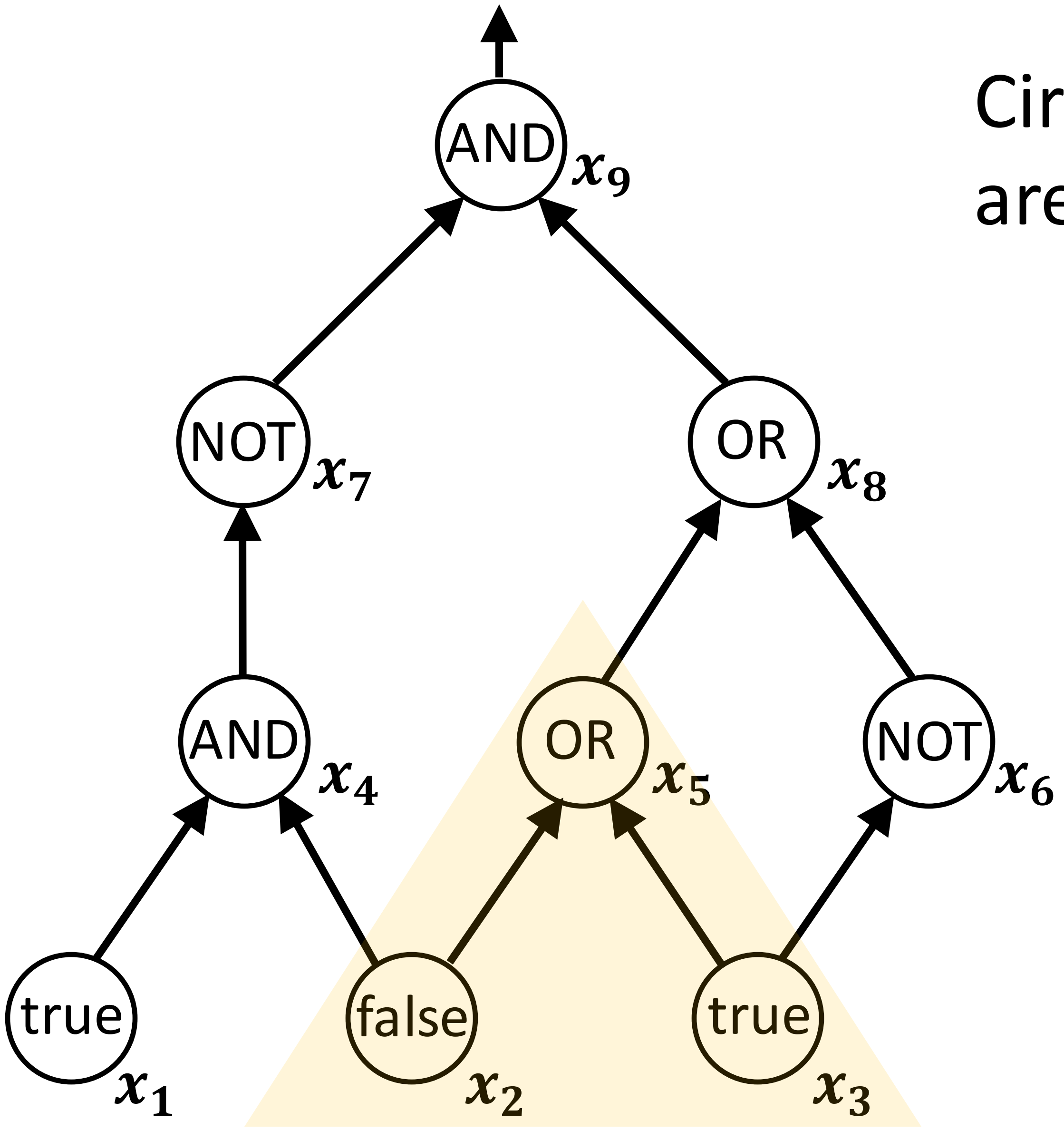


Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in [0,1], \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$
$x_6 = 1 - x_3$
$x_7 = 1 - x_4$
$x_5 \geq x_2$
$x_5 \geq x_3$

x_j	x_k	$x_i = x_j$ OR x_k
0	0	0
1	0	1
0	1	1
1	1	1



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

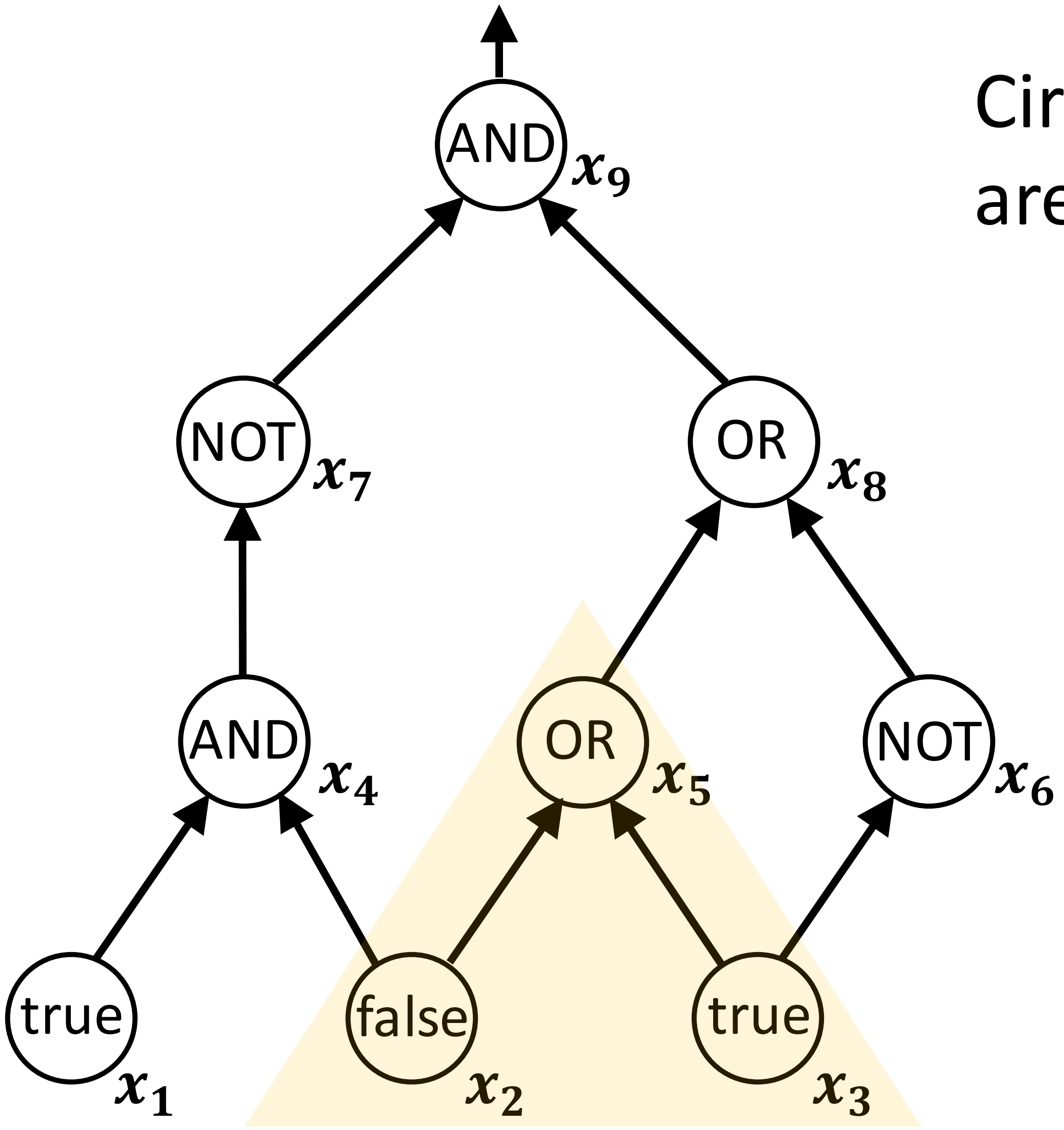
Objective: ??

Subject to: $x_i \in [0,1], \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_6 = 1 - x_3$$
$$x_7 = 1 - x_4$$
$$x_5 \geq x_2$$
$$x_5 \geq x_3$$

x_j	x_k	$x_i = x_j$ OR x_k
0	0	0
1	0	1
0	1	1
1	1	1

Is that enough?



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in [0,1], \forall i$

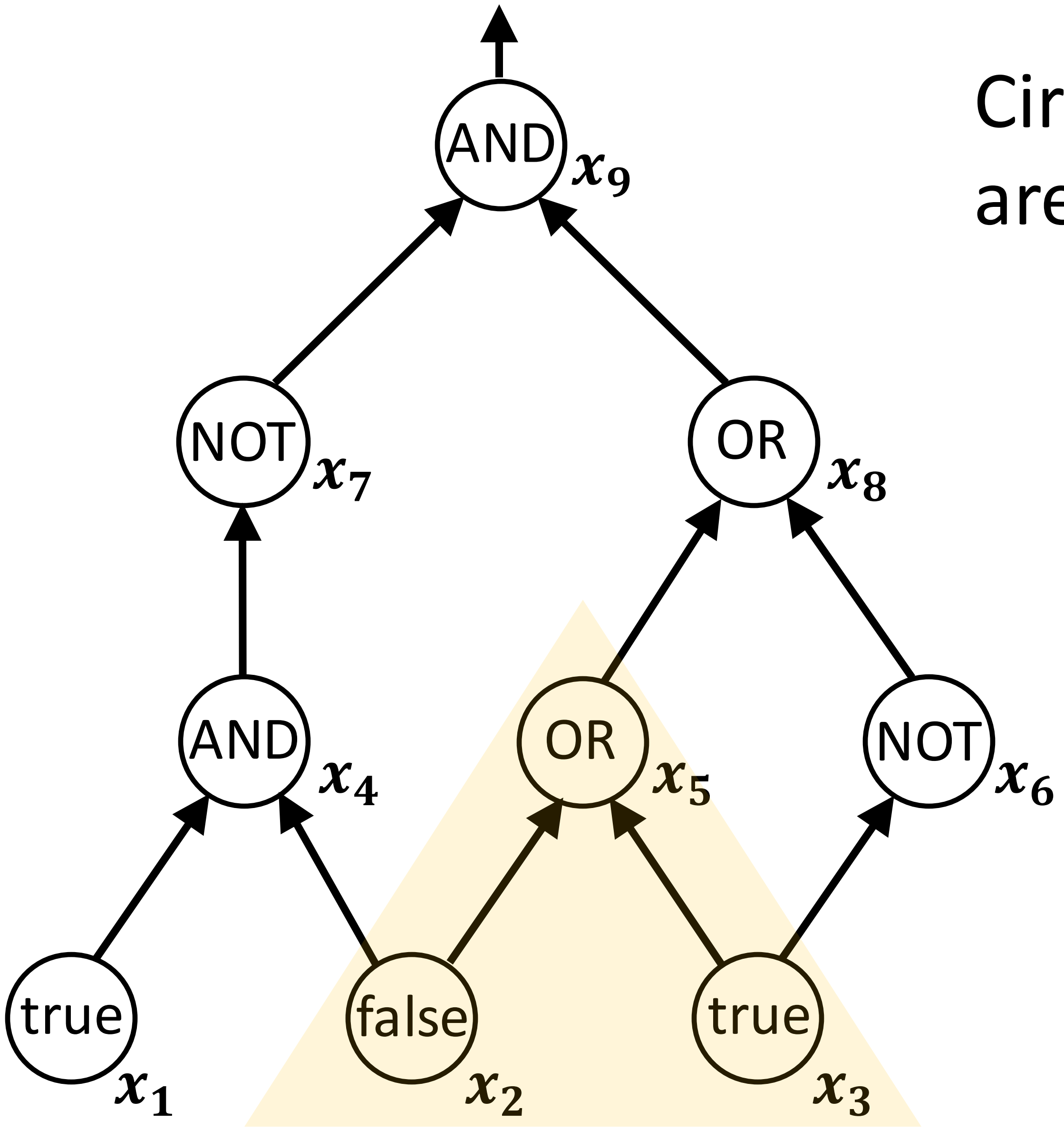
$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

$x_5 \geq x_2$
$x_5 \geq x_3$
$x_5 \leq x_2 + x_3$

x_j	x_k	$x_i = x_j$ OR x_k
0	0	0
1	0	1
0	1	1
1	1	1



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

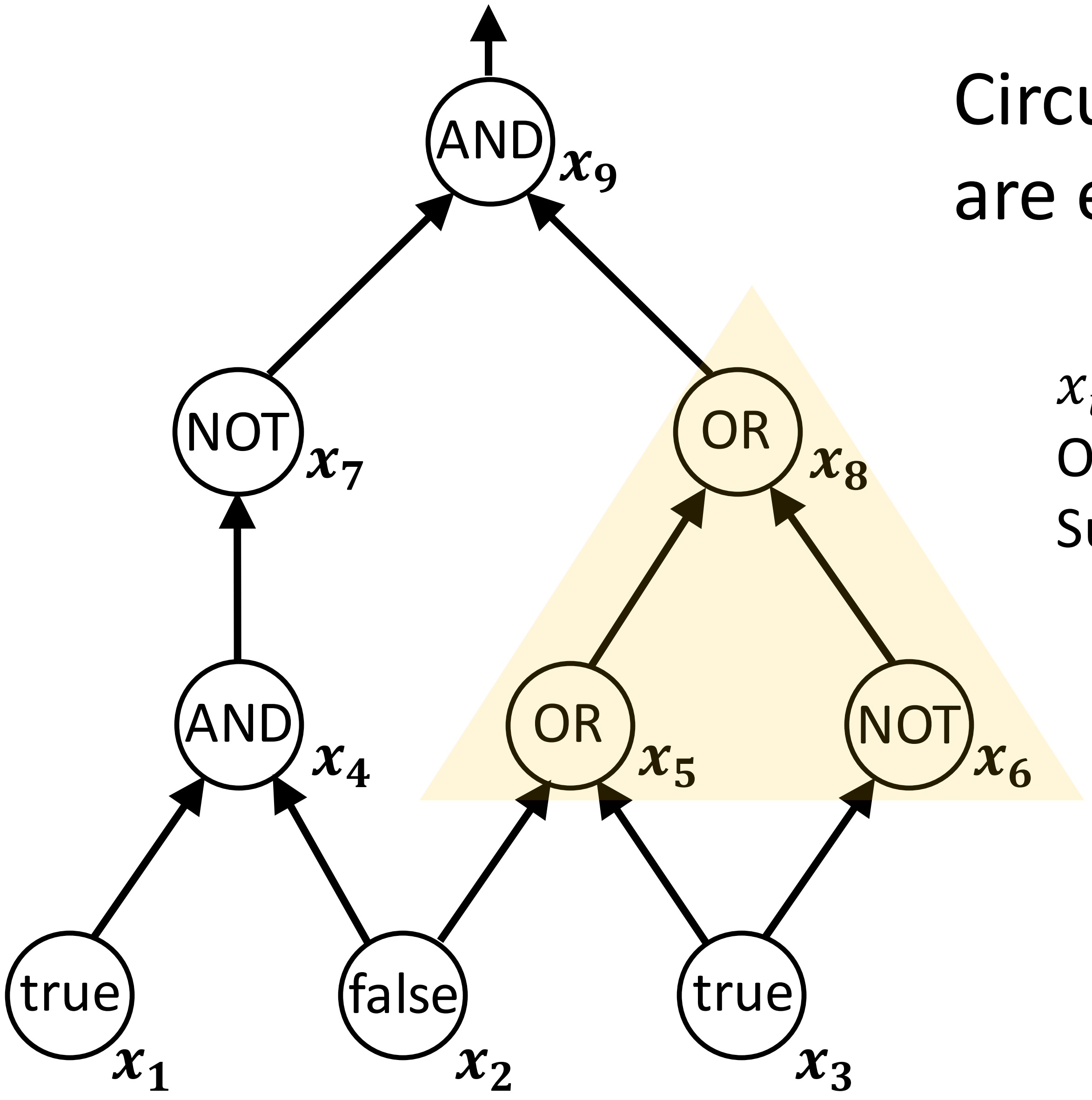
Objective: ??

Subject to: $x_i \in [0,1], \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$
$x_6 = 1 - x_3$
$x_7 = 1 - x_4$
$x_5 \geq x_2$
$x_5 \geq x_3$
$x_5 \leq x_2 + x_3$

In general, if gate i is OR gates j, k:

$$x_i \geq x_j$$
$$x_i \geq x_k$$
$$x_i \leq x_j + x_k$$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value
Objective: ??
Subject to: $x_i \in [0,1], \forall i$

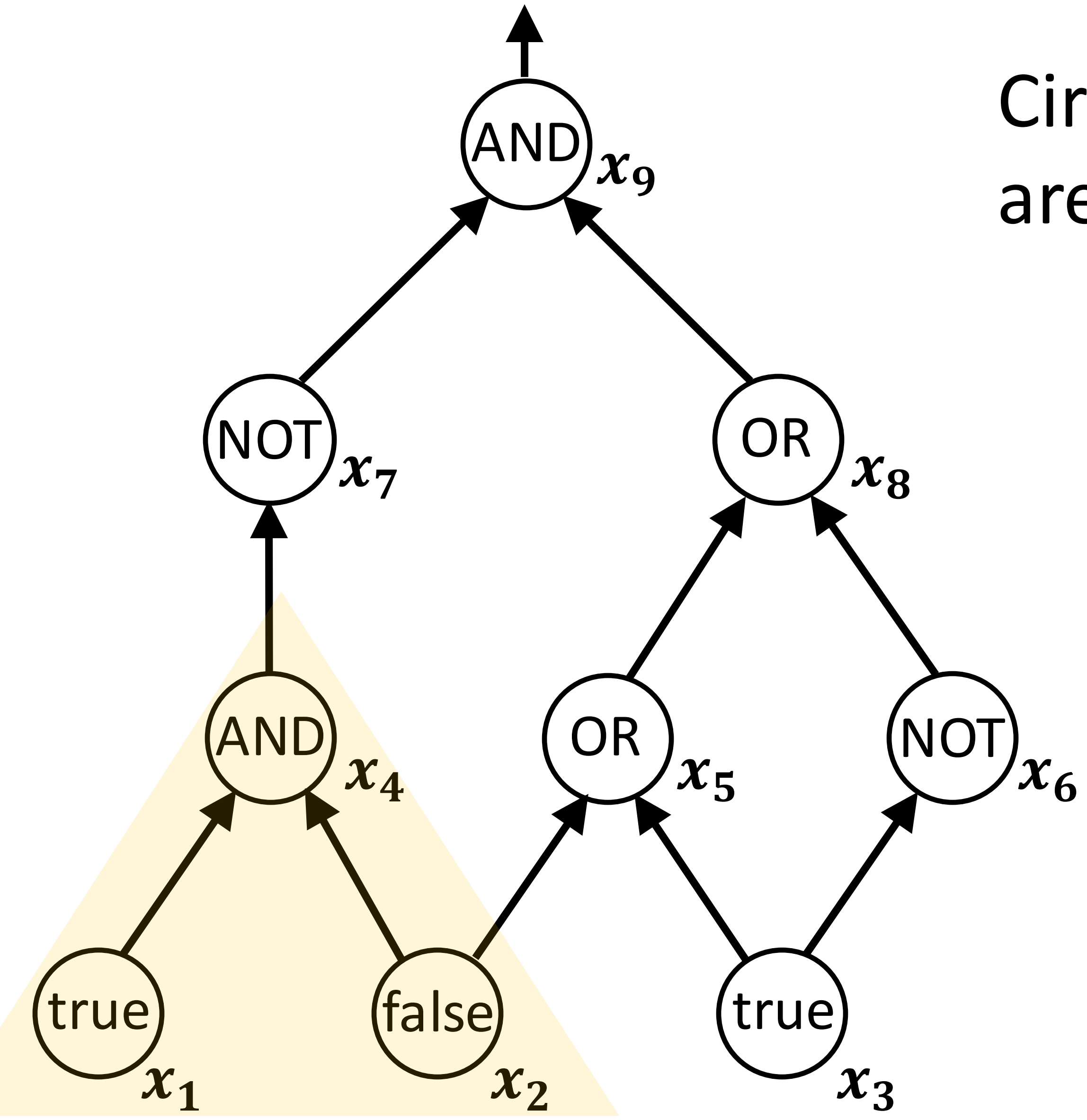
$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

$x_5 \geq x_2$
$x_5 \geq x_3$
$x_5 \leq x_2 + x_3$

$x_8 \geq x_5$
$x_8 \geq x_6$
$x_8 \leq x_5 + x_6$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in [0,1], \forall i$

$x_1 = 1$

$x_2 = 0$

$x_3 = 1$

$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

$x_5 \geq x_2$

$x_5 \geq x_3$

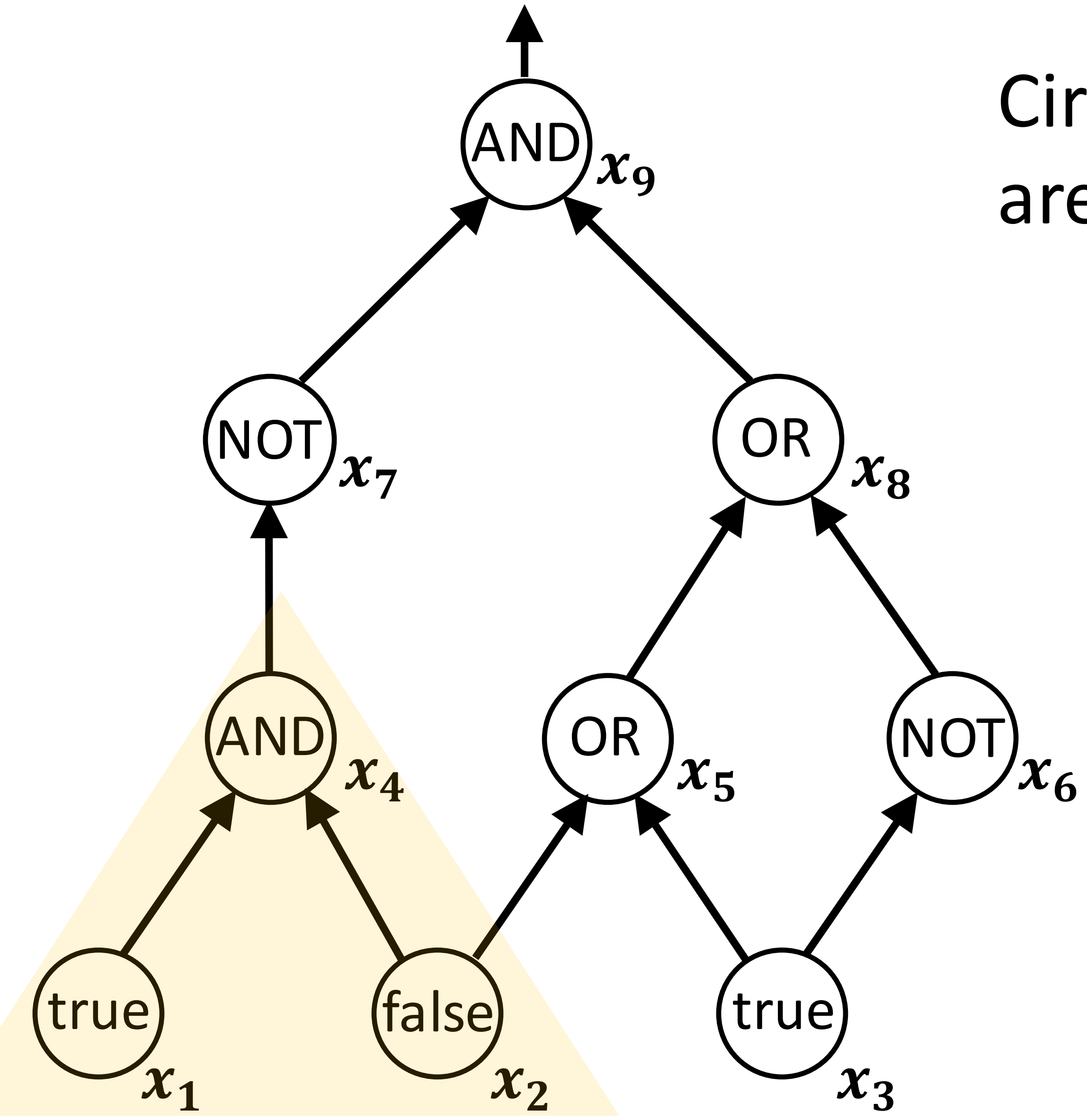
$x_5 \leq x_2 + x_3$

$x_8 \geq x_5$

$x_8 \geq x_6$

$x_8 \leq x_5 + x_6$

??



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in [0,1], \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

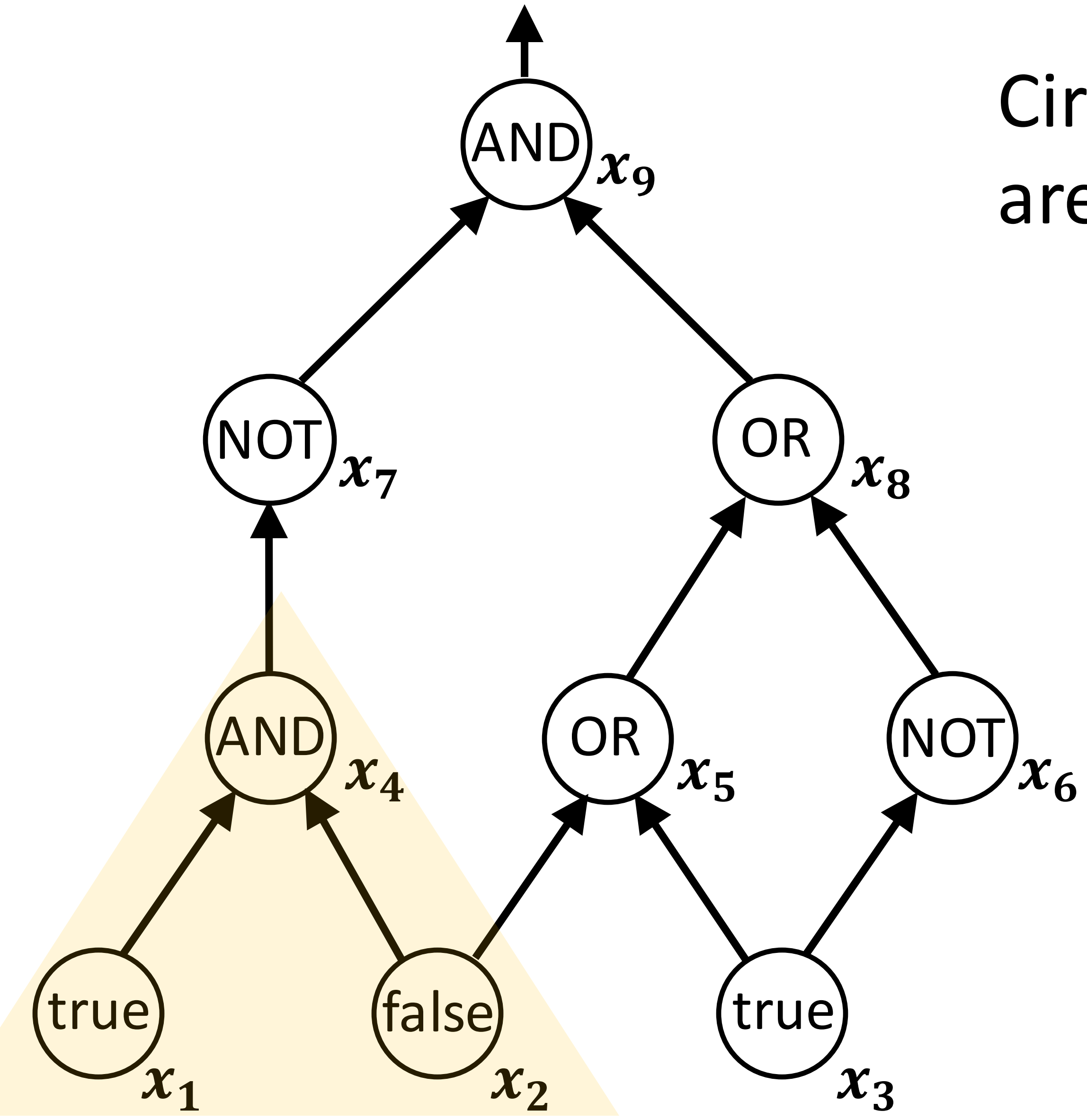
$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

$x_5 \geq x_2$
$x_5 \geq x_3$
$x_5 \leq x_2 + x_3$

$x_8 \geq x_5$
$x_8 \geq x_6$
$x_8 \leq x_5 + x_6$

$x_4 \leq x_1$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in [0,1], \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

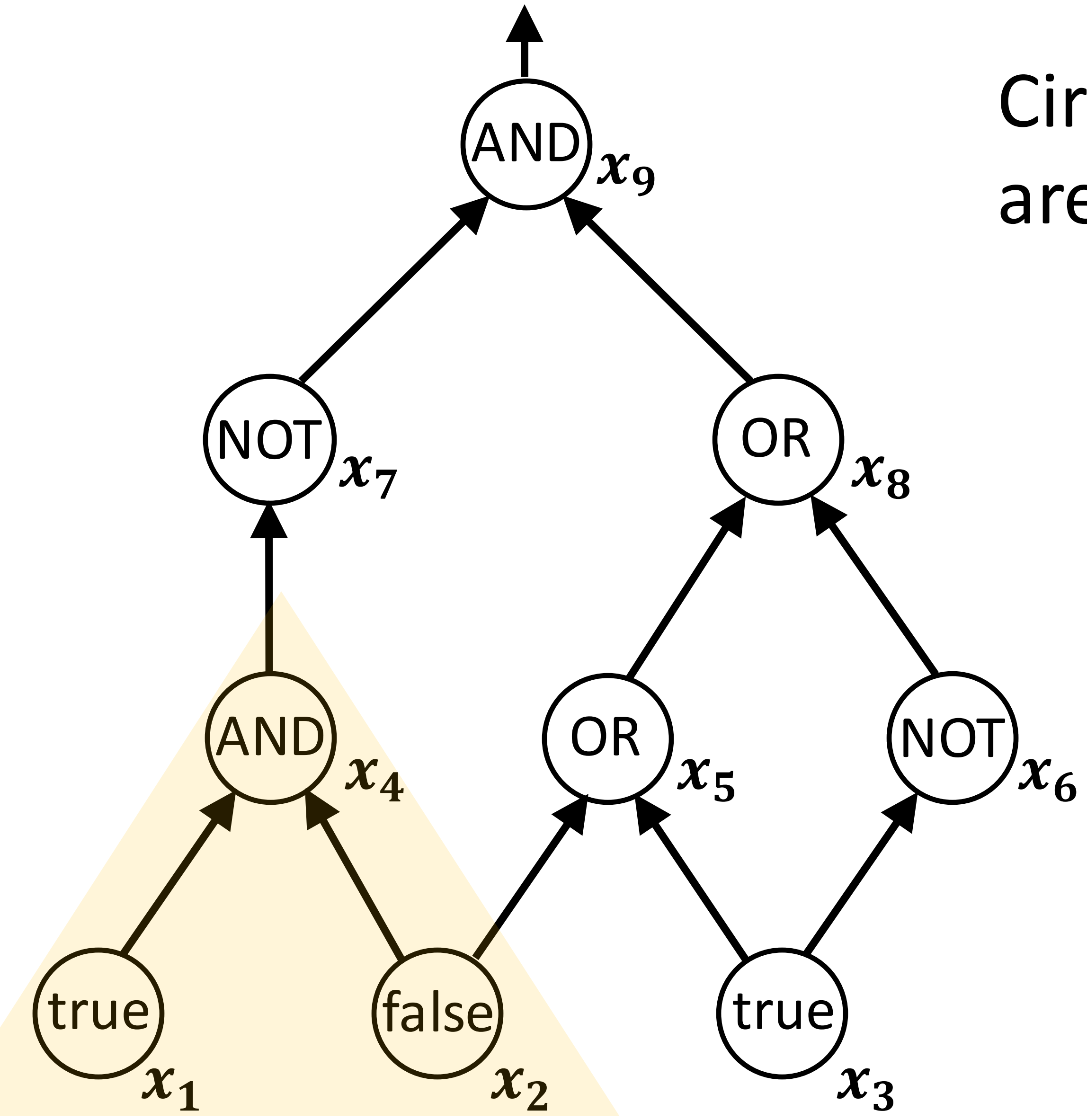
$x_6 = 1 - x_3$

$x_7 = 1 - x_4$

$x_5 \geq x_2$
$x_5 \geq x_3$
$x_5 \leq x_2 + x_3$

$x_8 \geq x_5$
$x_8 \geq x_6$
$x_8 \leq x_5 + x_6$

$x_4 \leq x_1$
$x_4 \leq x_2$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

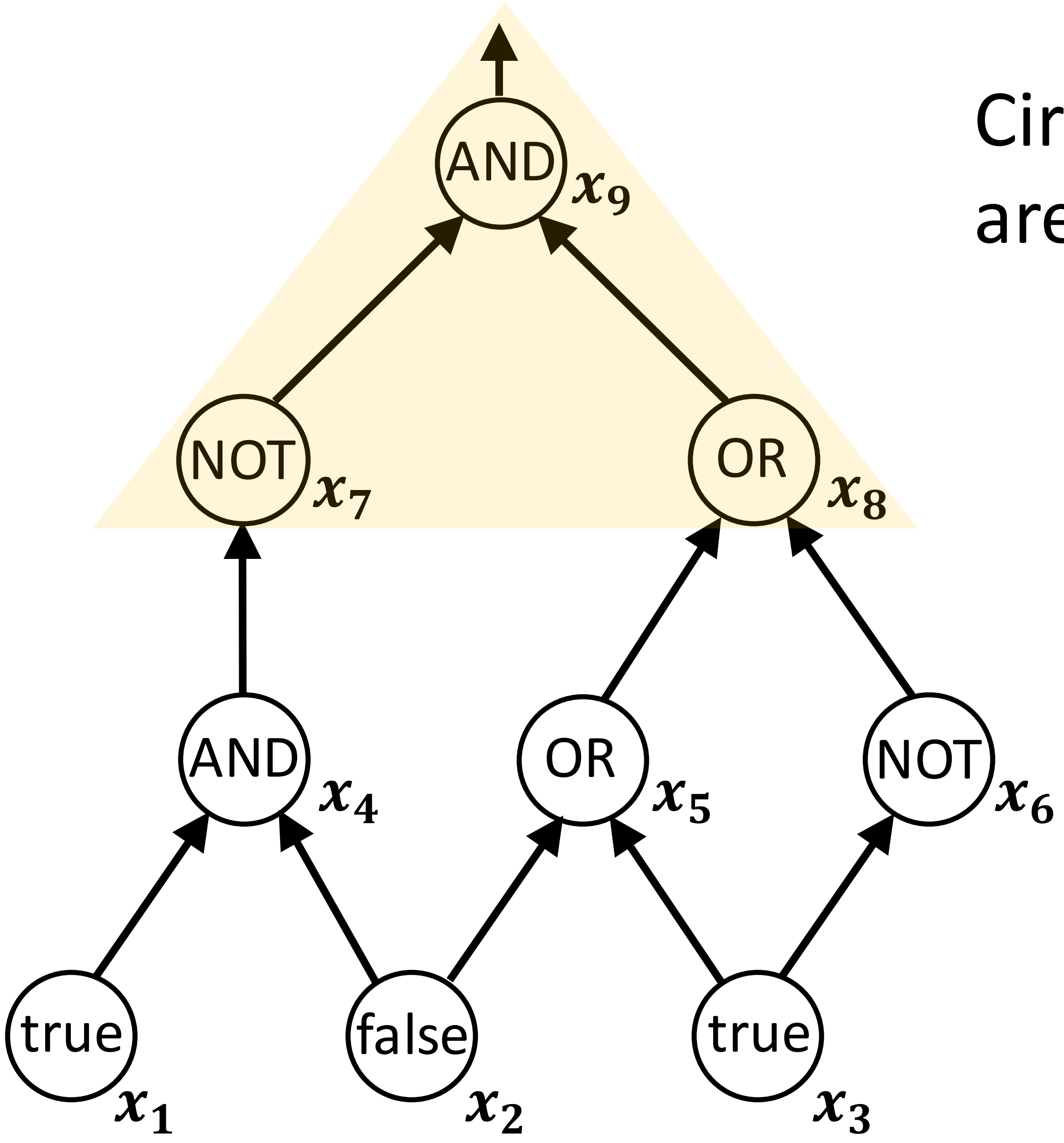
Subject to: $x_i \in [0,1], \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_6 = 1 - x_3$$
$$x_7 = 1 - x_4$$
$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$

$$x_8 \geq x_5$$
$$x_8 \geq x_6$$
$$x_8 \leq x_5 + x_6$$

$$x_4 \leq x_1$$
$$x_4 \leq x_2$$
$$x_4 \geq x_1 + x_2 - 1$$

Circuit Value Problem: When the gates are evaluated, is the output true?

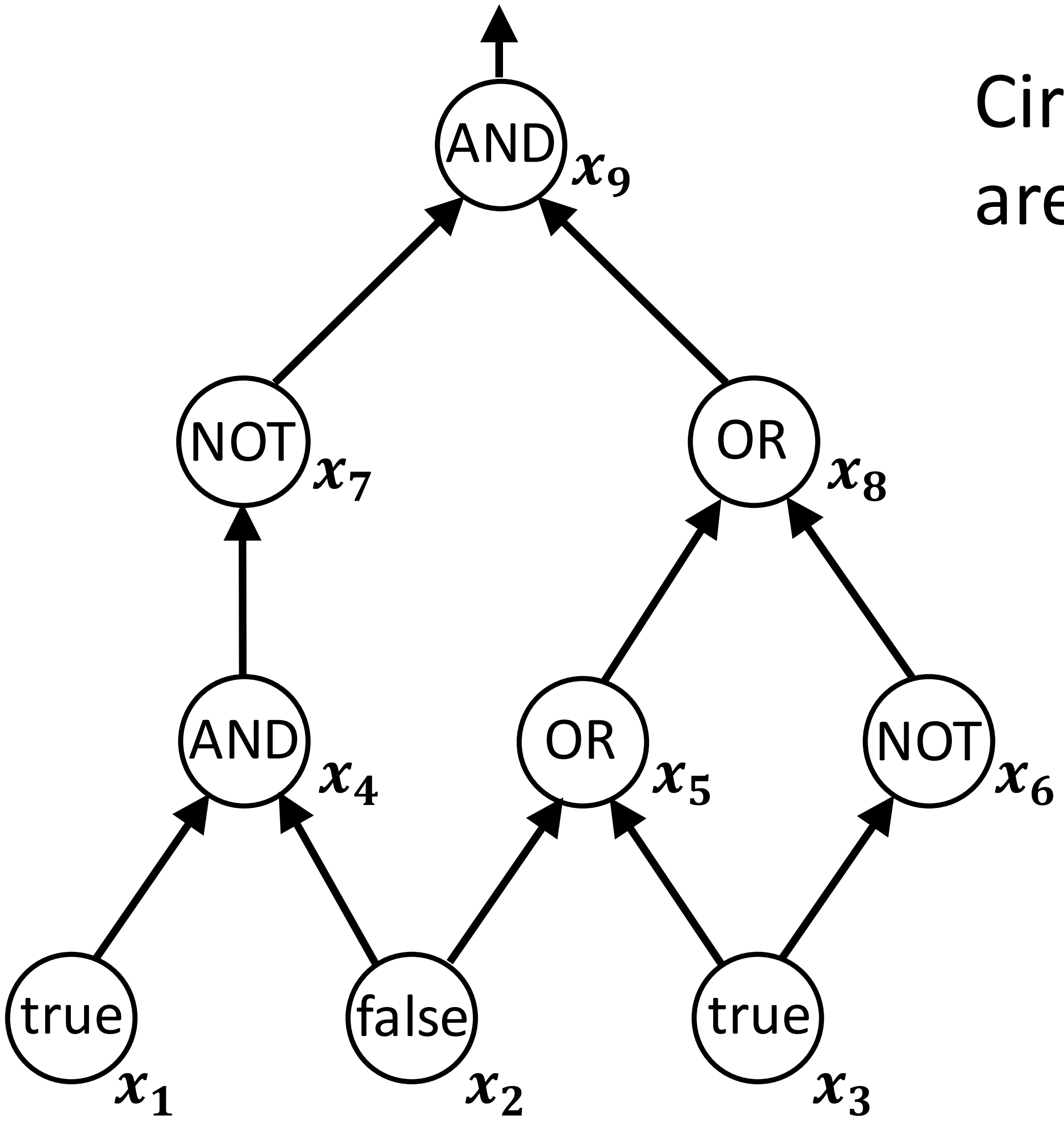


x_i = gate's evaluated value

Objective: ??

Subject to: $x_i \in [0,1], \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_6 = 1 - x_3$$
$$x_7 = 1 - x_4$$
$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$
$$x_8 \geq x_5$$
$$x_8 \geq x_6$$
$$x_8 \leq x_5 + x_6$$
$$x_4 \leq x_1$$
$$x_4 \leq x_2$$
$$x_4 \geq x_1 + x_2 - 1$$
$$x_9 \leq x_7$$
$$x_9 \leq x_8$$
$$x_9 \geq x_7 + x_8 - 1$$



Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: ??

Subject to:

$x_i \in [0,1], \forall i$

$x_1 = 1$
$x_2 = 0$
$x_3 = 1$

$x_6 = 1 - x_3$

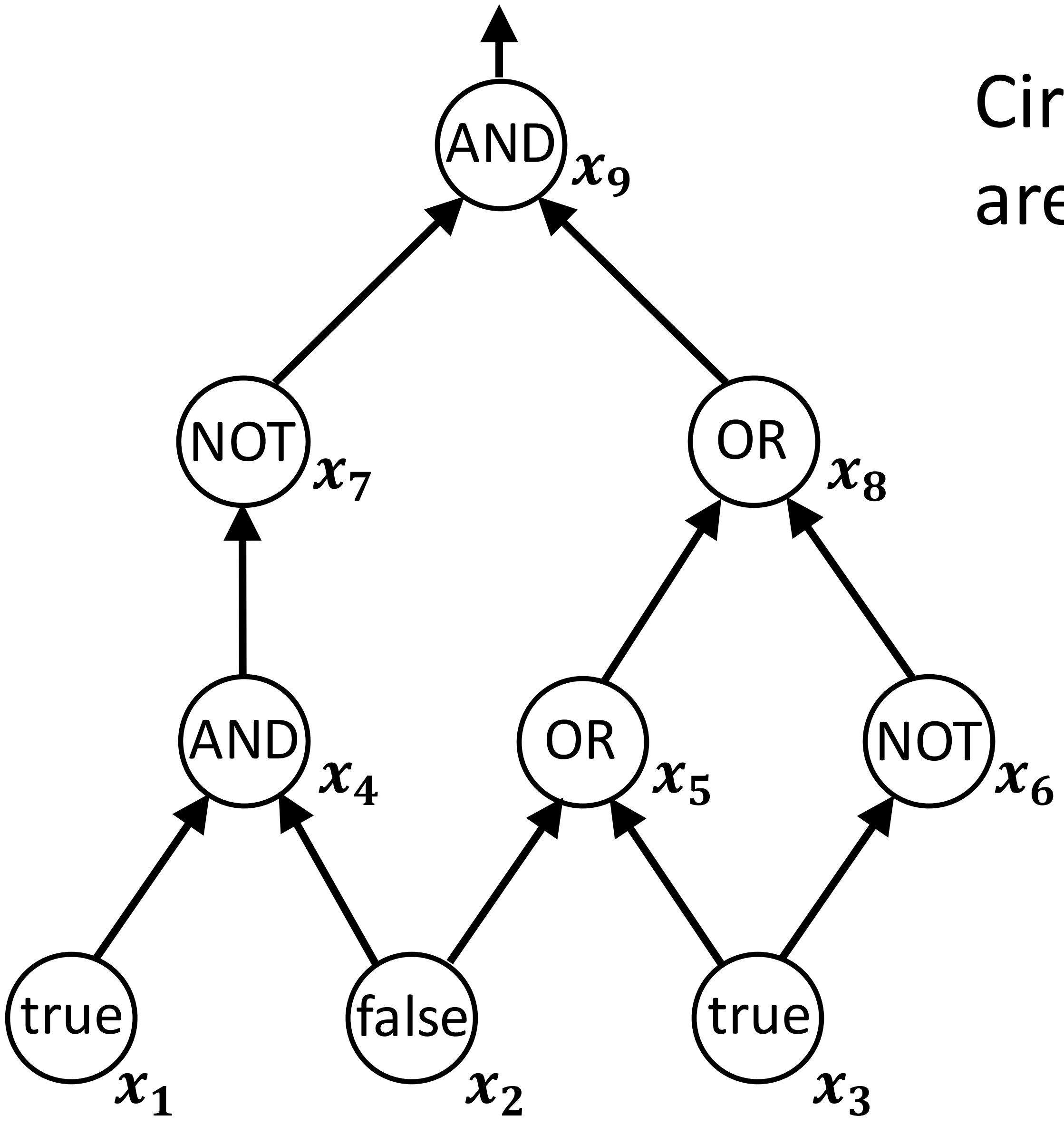
$x_7 = 1 - x_4$

$x_5 \geq x_2$
$x_5 \geq x_3$
$x_5 \leq x_2 + x_3$

$x_8 \geq x_5$
$x_8 \geq x_6$
$x_8 \leq x_5 + x_6$

$x_4 \leq x_1$
$x_4 \leq x_2$
$x_4 \geq x_1 + x_2 - 1$

$x_9 \leq x_7$
$x_9 \leq x_8$
$x_9 \geq x_7 + x_8 - 1$



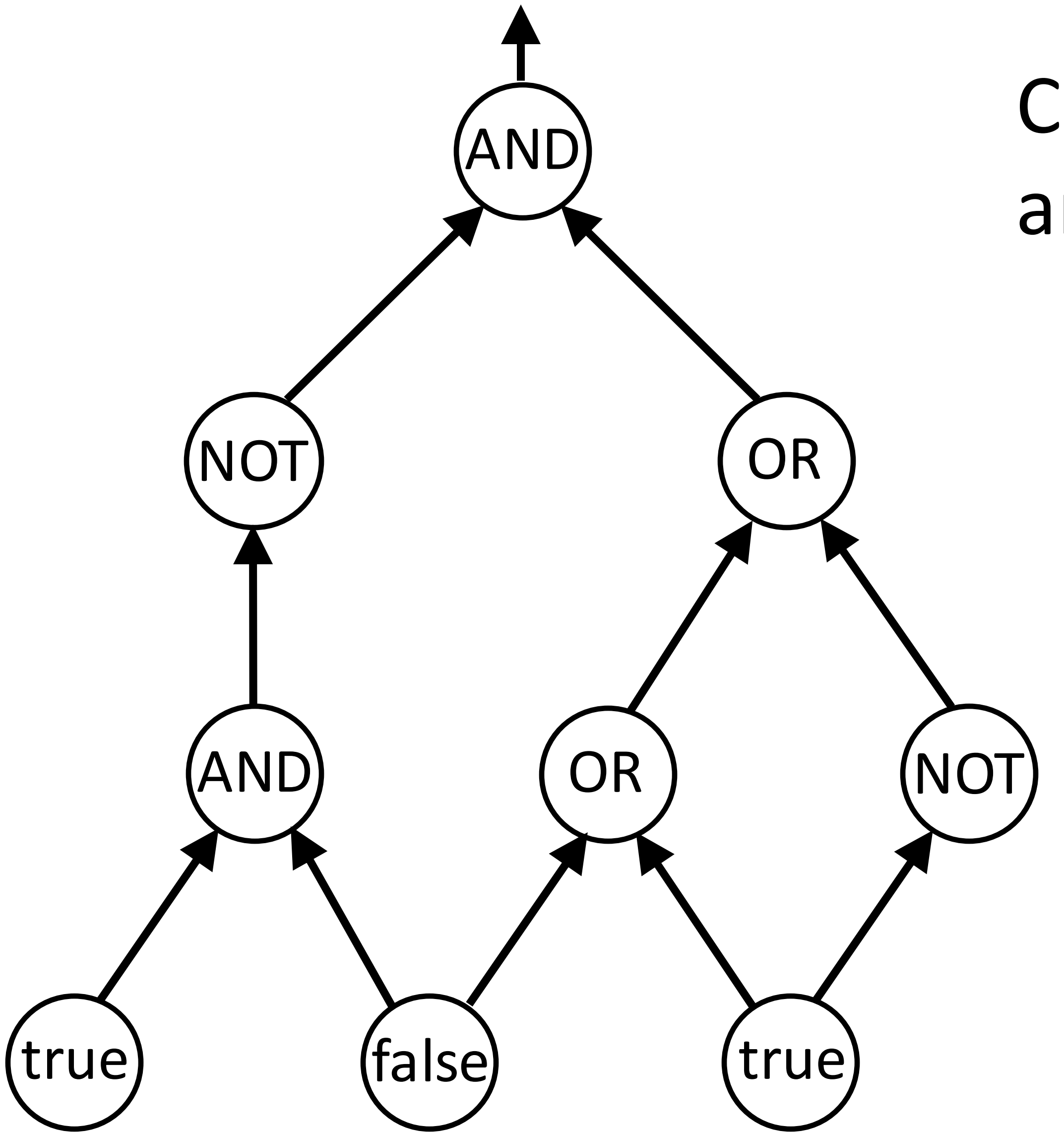
Circuit Value Problem: When the gates are evaluated, is the output true?

x_i = gate's evaluated value

Objective: $\min 0$

Subject to: $x_i \in [0,1], \forall i$

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_6 = 1 - x_3$$
$$x_7 = 1 - x_4$$
$$x_5 \geq x_2$$
$$x_5 \geq x_3$$
$$x_5 \leq x_2 + x_3$$
$$x_8 \geq x_5$$
$$x_8 \geq x_6$$
$$x_8 \leq x_5 + x_6$$
$$x_4 \leq x_1$$
$$x_4 \leq x_2$$
$$x_4 \geq x_1 + x_2 - 1$$
$$x_9 \leq x_7$$
$$x_9 \leq x_8$$
$$x_9 \geq x_7 + x_8 - 1$$



Circuit Value Problem: When the gates are evaluated, is the output true?

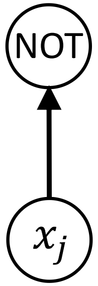
x_i = gate's evaluated value

Objective: $\min 0$

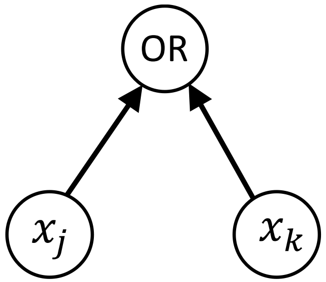
Subject to: $x_i \in [0,1]$

$x_i = 1$

$x_i = 0$



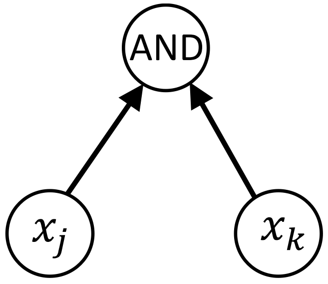
$x_i = 1 - x_j$



$x_i \geq x_j$

$x_i \geq x_k$

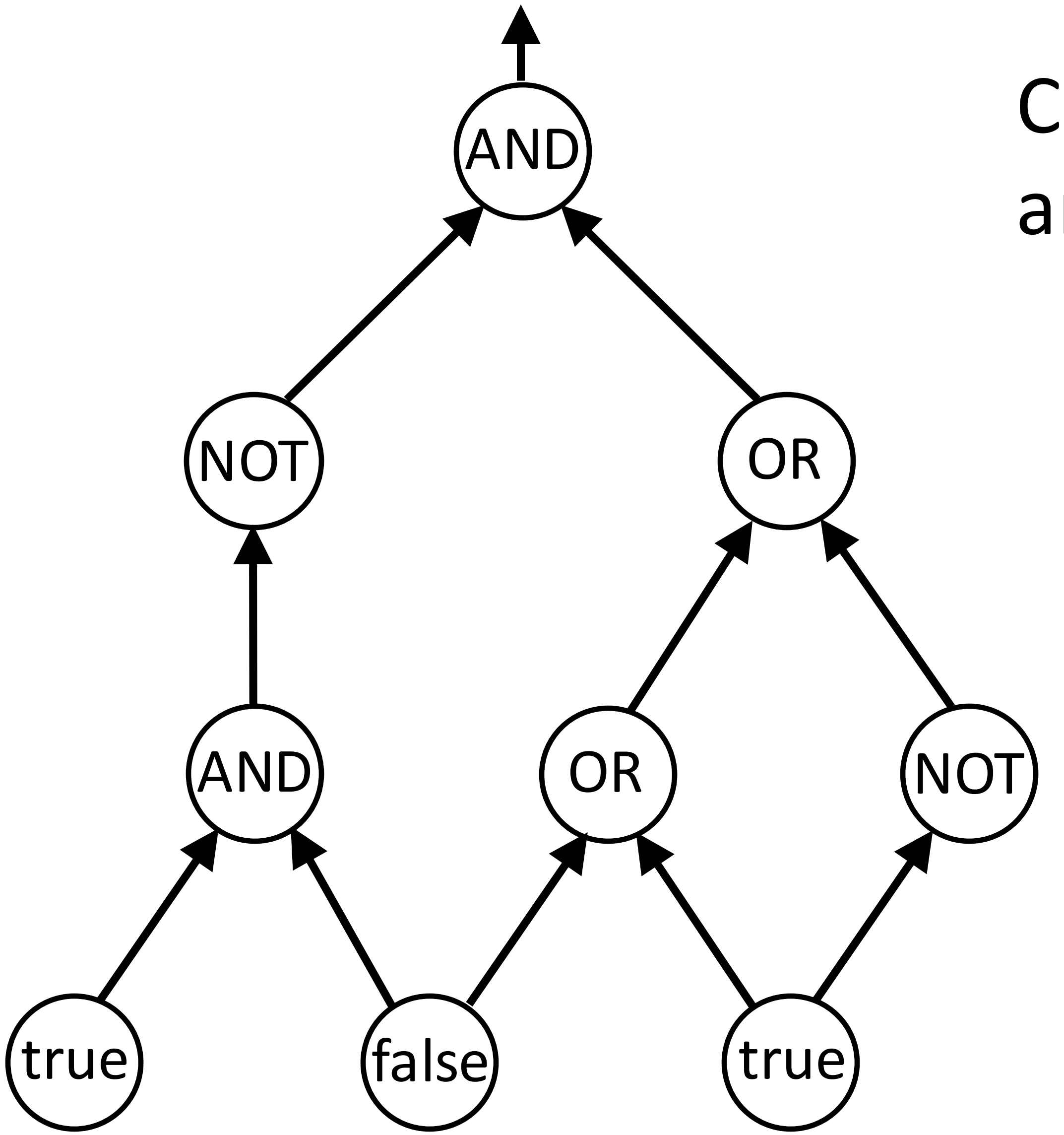
$x_i \leq x_j + x_k$



$x_i \leq x_j$

$x_i \leq x_k$

$x_i \geq x_j + x_k - 1$



Will answer be an integer?

Circuit Value Problem: When the gates are evaluated, is the output true?

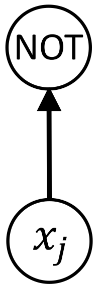
x_i = gate's evaluated value

Objective: $\min 0$

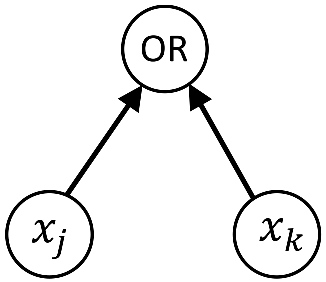
Subject to: $x_i \in [0,1]$

$x_i = 1$

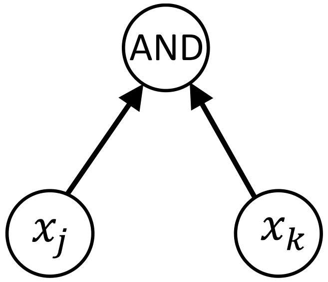
$x_i = 0$



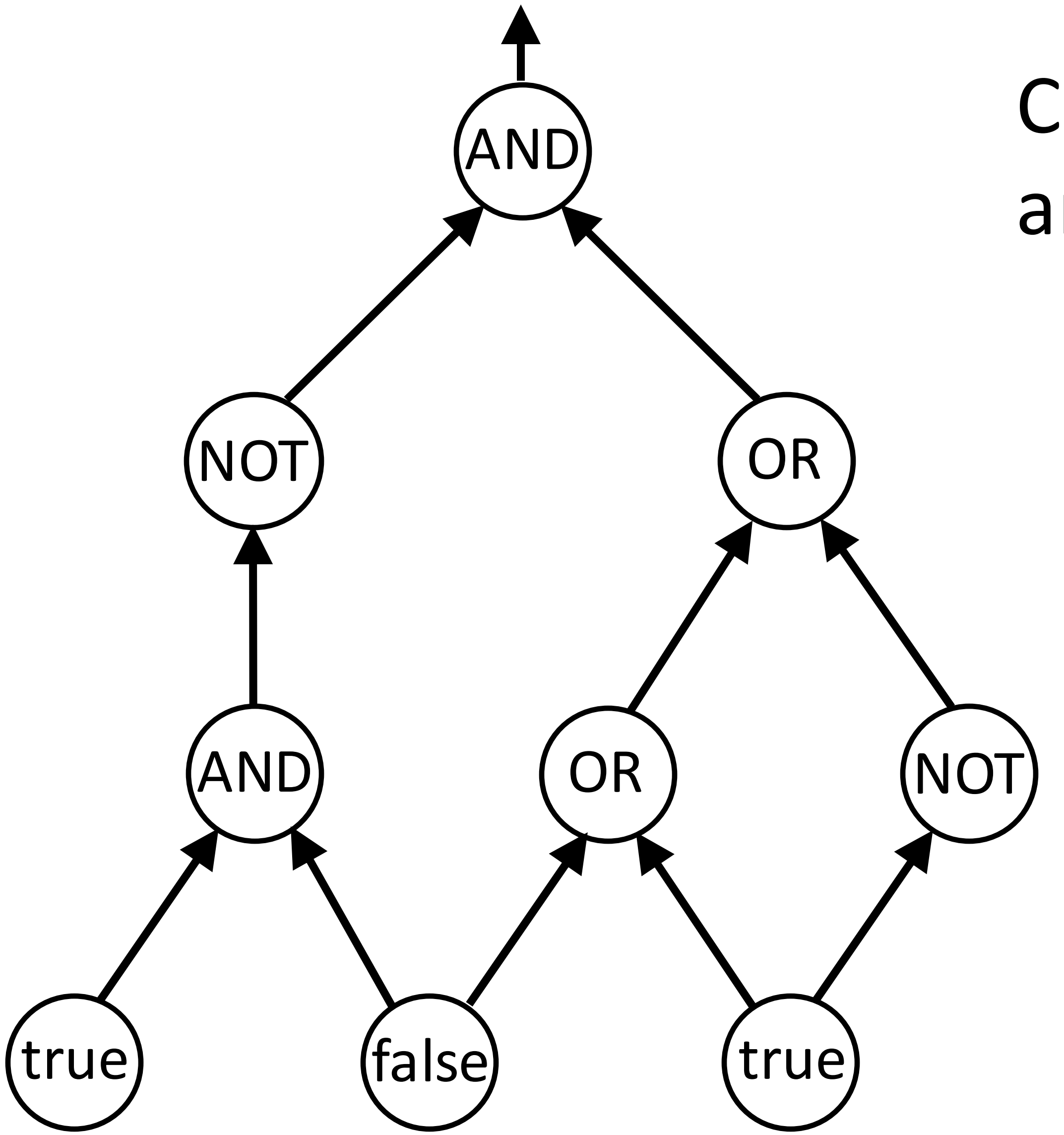
$x_i = 1 - x_j$



$x_i \geq x_j$
$x_i \geq x_k$
$x_i \leq x_j + x_k$



$x_i \leq x_j$
$x_i \leq x_k$
$x_i \geq x_j + x_k - 1$



Circuit Value Problem: When the gates are evaluated, is the output true?

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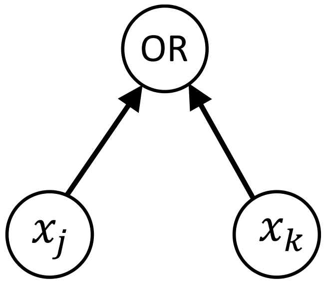

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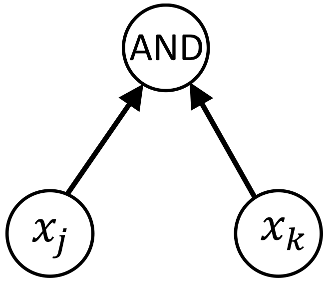
$x_i \geq x_k$

$x_i \leq x_j + x_k$



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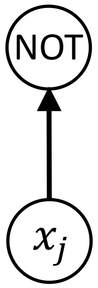
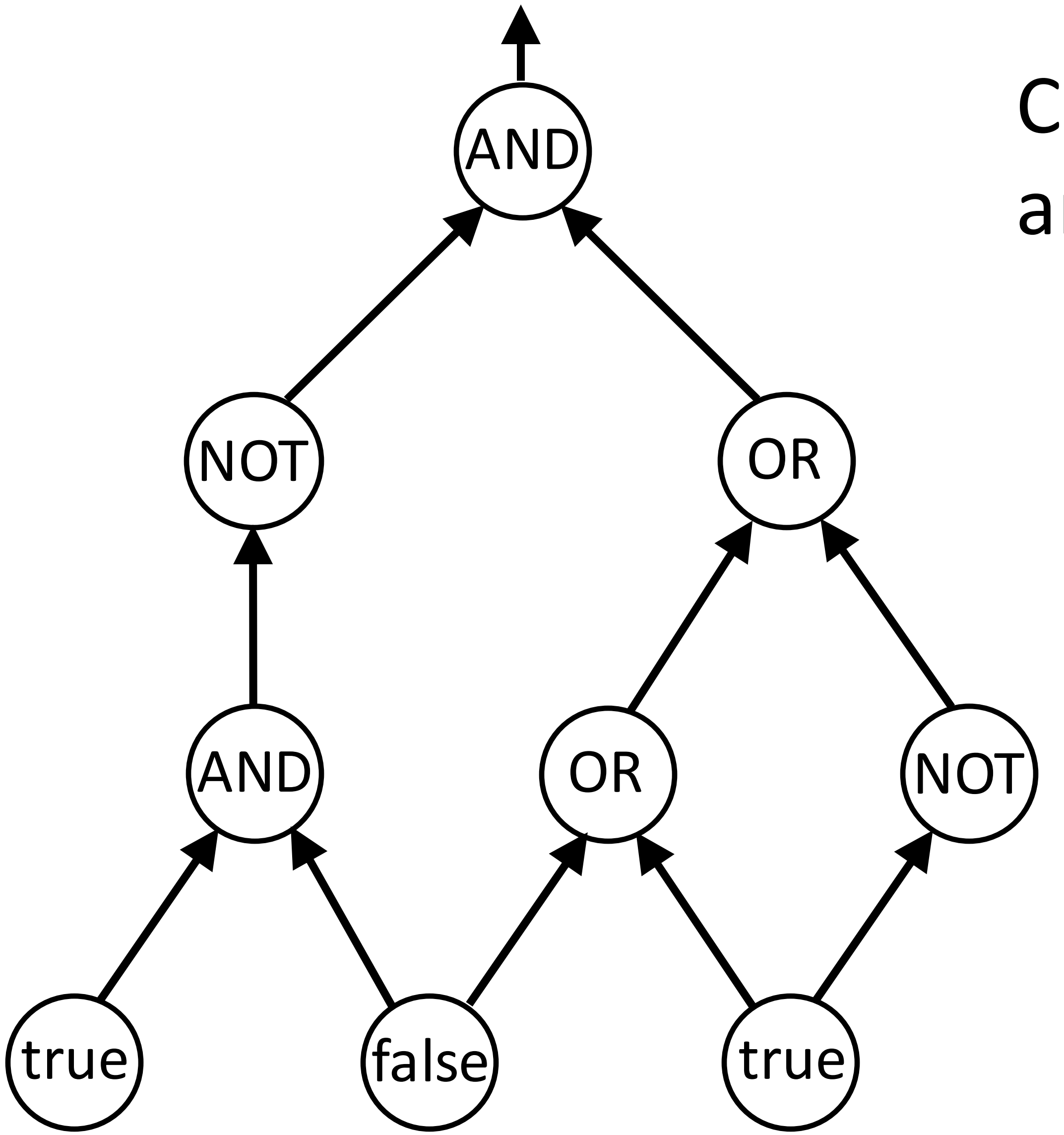


$x_i = 1 - x_j$

Will answer be an integer?

Yes! Input are integers and constraints keep them integers.



Anything that can run on a computer in poly time can be solved with an LP in poly time!

Circuit Value Problem: When the gates are evaluated, is the output true?

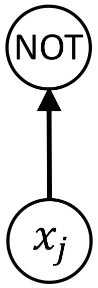
x_i = gate's evaluated value

Objective: $\min 0$

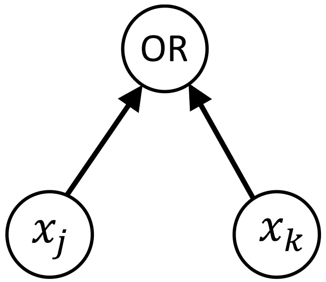
Subject to: $x_i \in [0,1]$

$x_i = 1$

$x_i = 0$



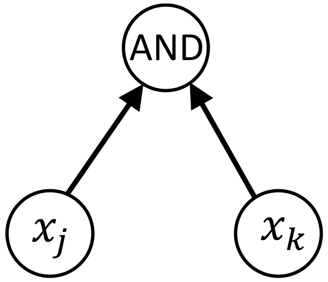
$x_i = 1 - x_j$



$x_i \geq x_j$

$x_i \geq x_k$

$x_i \leq x_j + x_k$



$x_i \leq x_j$

$x_i \leq x_k$

$x_i \geq x_j + x_k - 1$

Dual

Primal

Objective: max $c^T x$
Subject to: $A x \leq b$
$x \geq 0$

Dual

Objective: min $b^T y$
Subject to: $A^T y \geq c$
$y \geq 0$

Objective: $\max 100x_1 + 300x_2 + 150x_3$
Subject to:
$x_2 \leq 20$ A
$x_1 + x_2 + x_3 \leq 40$ B
$2x_1 + x_3 \leq 60$ C

Objective: $\min 20y_1 + 40y_2 + 60y_3$
Subject to:
$y_2 + 2y_3 \geq 100$
$y_1 + y_2 \geq 300$
$y_2 + y_3 \geq 150$
$y_1, y_2, y_3 \geq 0$

Dual

Primal

Objective: max $c^T x$

Subject to: $A x \leq b$

$x \geq 0$

Dual

Objective: min $b^T y$

Subject to: $A^T y \geq c$

$y \geq 0$

Theorem: The dual of a dual is the original primal.

Theorem: If $\bar{x}$ is any feasible solution to the primal and $\bar{y}$ is any feasible solution to the dual, then $c^T \bar{x} \leq b^T \bar{y}$.

Theorem: If $\bar{x}$ is the optimal solution to the primal and $\bar{y}$ is the optimal solution to the dual, then $c^T \bar{x} = b^T \bar{y}$.

Dual

Primal

Objective: max $c^T x$
Subject to: $A x \leq b$
$x \geq 0$

Maximum Flow

Dual

Objective: min $b^T y$
Subject to: $A^T y \geq c$
$y \geq 0$

???

Theorem: The dual of a dual is the original primal.

Theorem: If $\bar{x}$ is any feasible solution to the primal and $\bar{y}$ is any feasible solution to the dual, then $c^T \bar{x} \leq b^T \bar{y}$.

Theorem: If $\bar{x}$ is the optimal solution to the primal and $\bar{y}$ is the optimal solution to the dual, then $c^T \bar{x} = b^T \bar{y}$.

Dual

Primal

Objective: max $c^T x$

Subject to: $A x \leq b$

$x \geq 0$

Dual

Objective: min $b^T y$

Subject to: $A^T y \geq c$

$y \geq 0$

Maximum Flow

Minimum Cut

Theorem: The dual of a dual is the original primal.

Theorem: If $\bar{x}$ is any feasible solution to the primal and $\bar{y}$ is any feasible solution to the dual, then $c^T \bar{x} \leq b^T \bar{y}$.

Theorem: If $\bar{x}$ is the optimal solution to the primal and $\bar{y}$ is the optimal solution to the dual, then $c^T \bar{x} = b^T \bar{y}$.