Dynamic Programming
CSCI 432

Dynamic Programing Process

1. Identify optimal substructure:
   If we have an optimal solution of size X, what does that say about a sub-solution of X?
2. Recursively define value of optimal solution:
   $A_n =$ max or min of something related to A_{n-i}
3. Compute value of optimal solution.
   Fill out table for every optimal value $\leq A_n$
4. Construct optimal solution from computed information:
   Either backtrack or make a new table.

Edit Distance

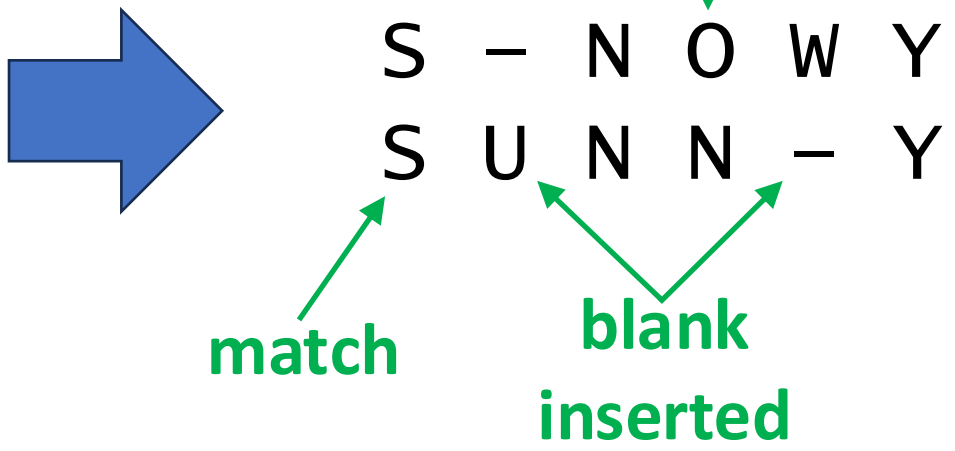
Given two strings, how many edits are needed to turn one string into another?

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SNOWY vs SUNNY
```

Edit Distance

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SNOWY vs SUNNY



Edit Distance

Given two strings, how many edits are needed to turn one string into another?

Input:

- Two strings
- Cost function
  (e.g., non-matches = +1)

S – N O W Y
S U N N – Y

Edit Distance

Given two strings, how many edits are needed to turn one string into another?

Input:

- Two strings
- Cost function
  (e.g., non-matches = +1)

S – N O W Y
S U N N – Y

cost = 3

Edit Distance

Given two strings, how many edits are needed to turn one string into another?

Input:

- Two strings
- Cost function
  (e.g., non-matches = +1)

```
- S N O W - Y
S U N - - N Y
```

cost = 5

Edit Distance

Given two strings, how many edits are needed to turn one string into another?

Input:

- Two strings
- Cost function
  (e.g., non-matches = +1)

Objective:

Find cheapest possible alignment.

```
- S N O W - Y
S U N - - N Y
```

cost = 5

Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Specifically, how must the optimal alignment end?

Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i y_j	
x_i $-$	
$-$ y_j	

Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i y_j	
x_i $-$	
$-$ y_j	

$$\begin{array}{cccccc} \ldots & x_{i-3} & x_{i-2} & x_{i-1} & - & x_i \\ \ldots & y_{j-3} & - & y_{j-2} & y_{j-1} & y_j \end{array}$$

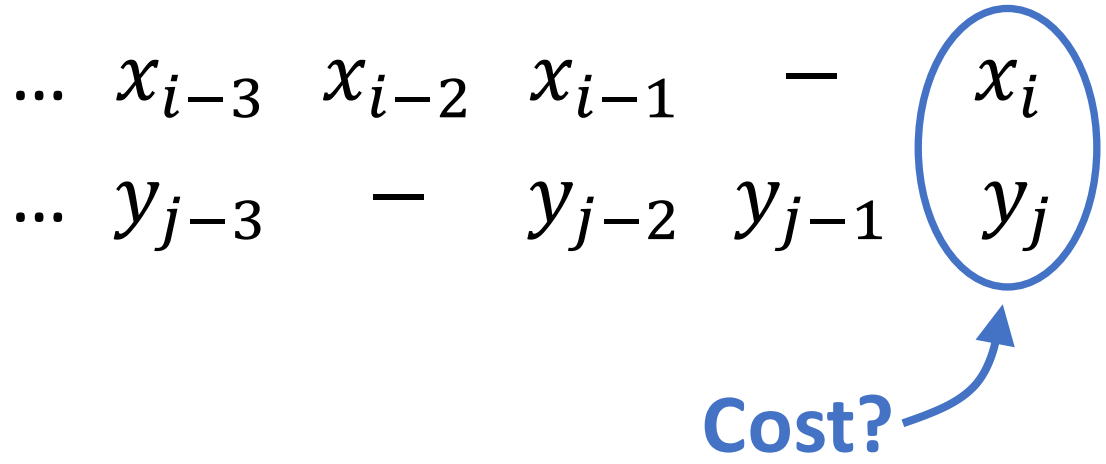
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Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i y_j	
x_i $-$	
$-$ y_j	



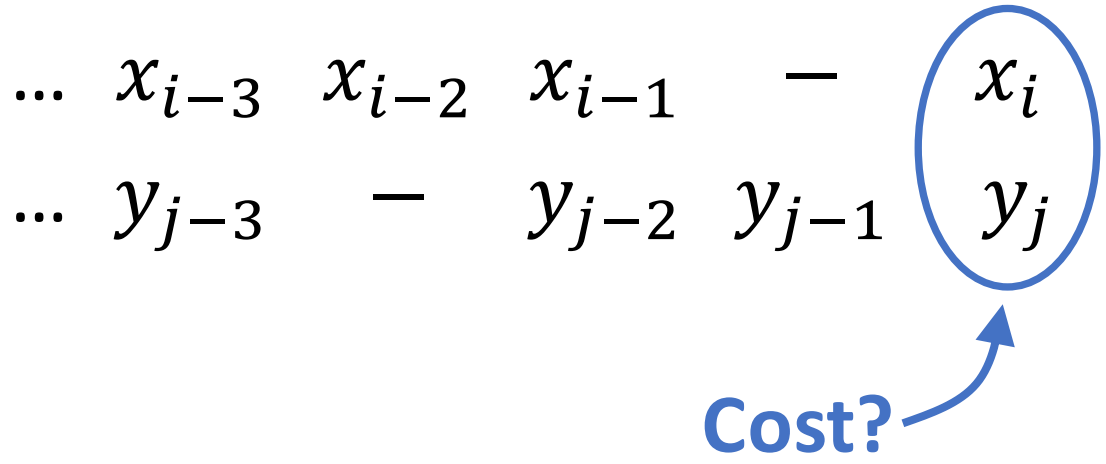
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We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i y_j	$\{0,1\}$
x_i $-$	
$-$ y_j	



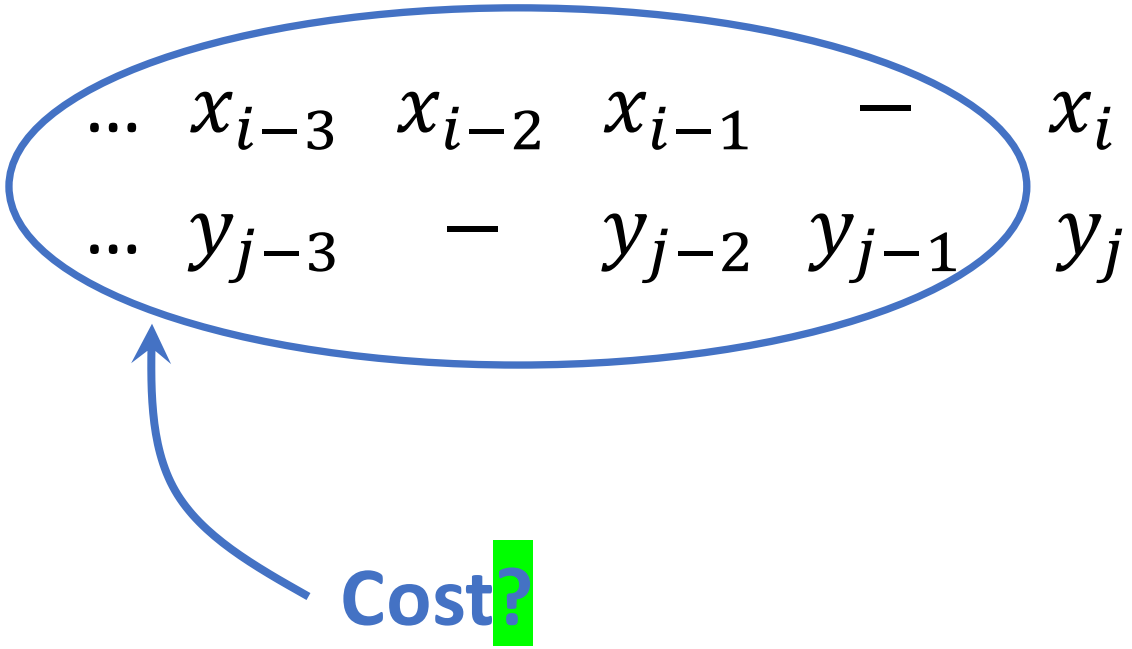
Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i / y_j	$\{0,1\}$ + ???
x_i / $-$	
$-$ / y_j	



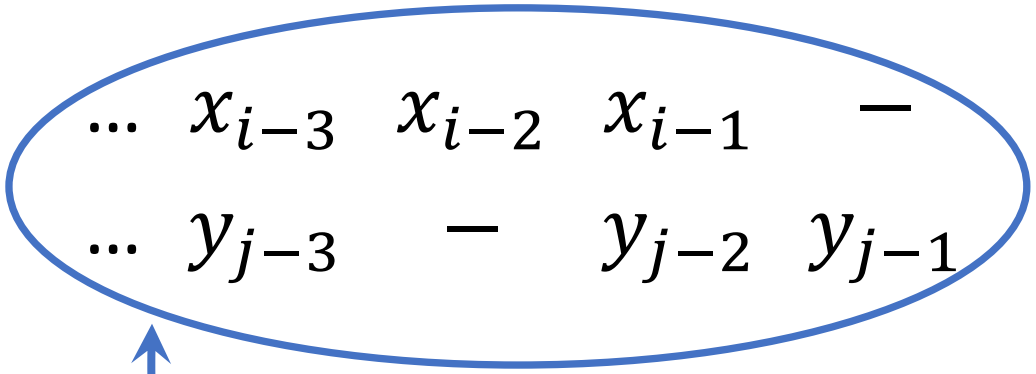
Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i / y_j	$\{0,1\}$ + ???
x_i / $-$	
$-$ / y_j	



$E(i-1, j-1)$: Alignment of $[x_1, \ldots, x_{i-1}]$ with $[y_1, \ldots, y_{j-1}]$

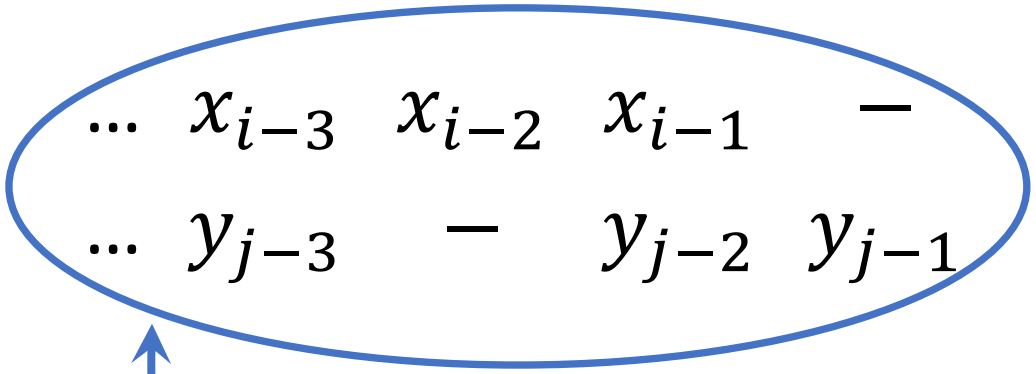
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We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i y_j	$\{0,1\} + E(i-1, j-1)$
x_i $-$	
$-$ y_j	



$E(i-1, j-1)$:
Alignment of $[x_1, \ldots, x_{i-1}]$ with $[y_1, \ldots, y_{j-1}]$

Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i y_j	$\{0,1\} + E(i-1, j-1)$
x_i $-$	
$-$ y_j	

$$\begin{array}{cccccc} \ldots & x_{i-3} & x_{i-2} & x_{i-1} & - & x_i \\ \ldots & y_{j-2} & - & y_{j-1} & y_j & - \end{array}$$

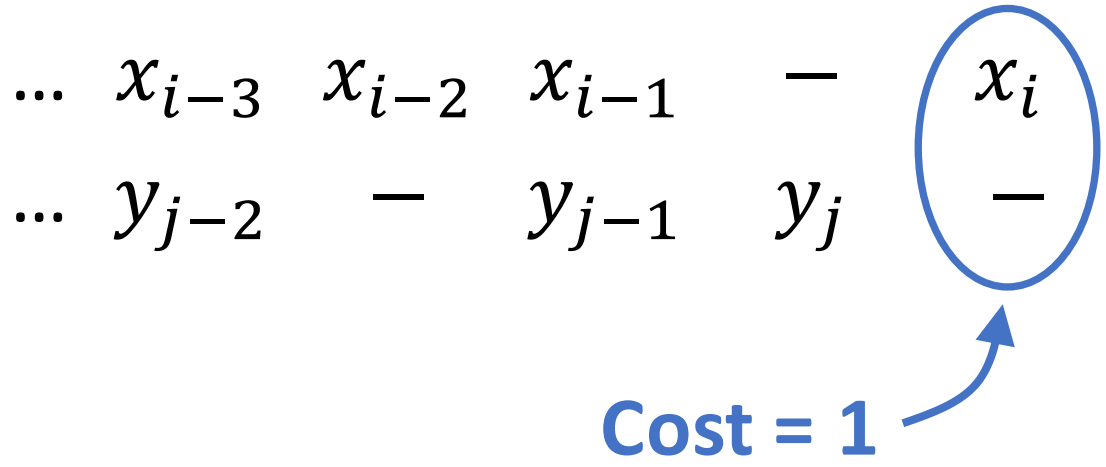
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$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i y_j	$\{0,1\} + E(i-1, j-1)$
x_i $-$	1
$-$ y_j	



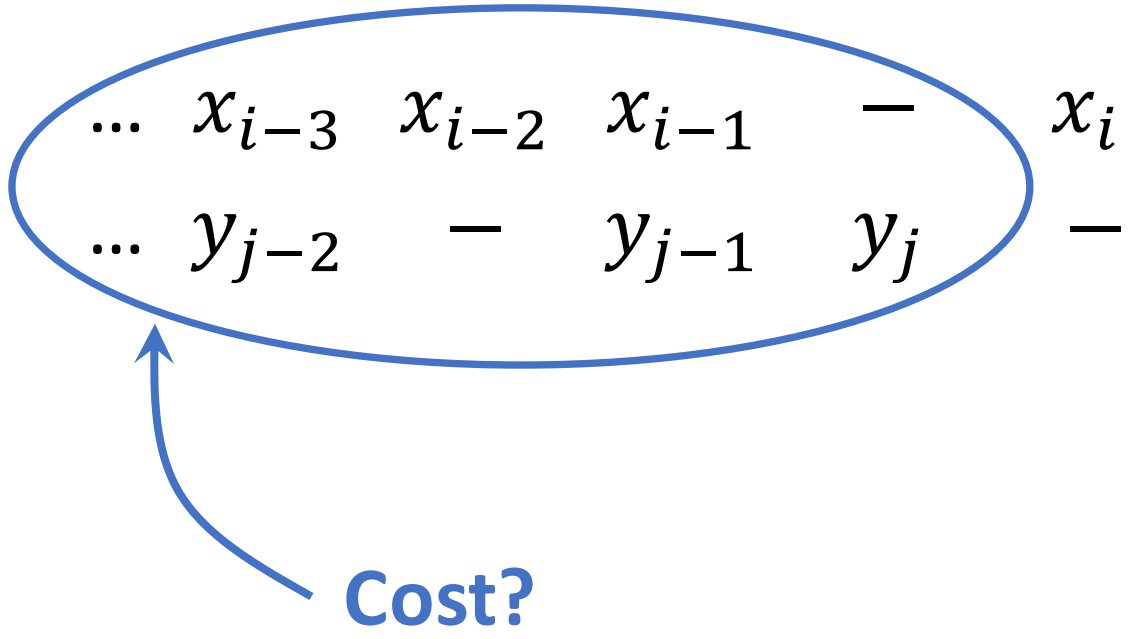
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We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i / y_j	$\{0,1\} + E(i-1, j-1)$
x_i / $-$	$1 +$???
$-$ / y_j	



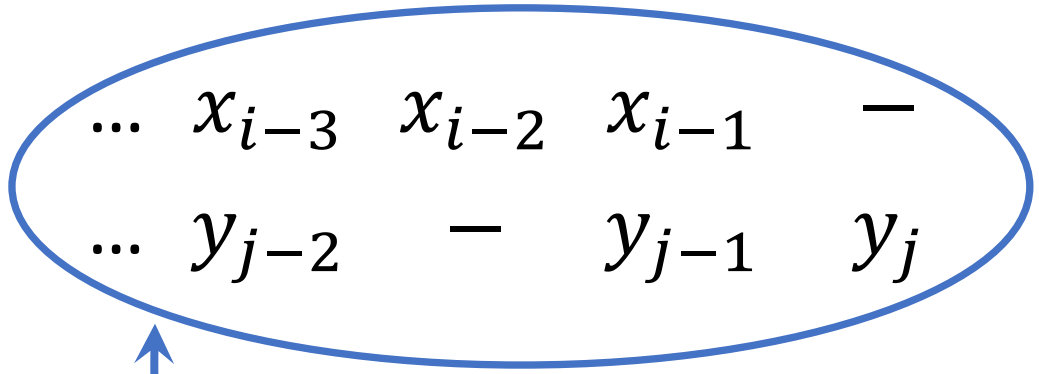
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We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
x_i y_j	$\{0,1\} + E(i-1, j-1)$
x_i $-$	$1 + E(i-1, j)$
$-$ y_j	



$E(i-1, j)$:
Alignment of $[x_1, \ldots, x_{i-1}]$ with $[y_1, \ldots, y_j]$

Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
$\begin{matrix} x_i \\ y_j \end{matrix}$	$\{0,1\} + E(i-1, j-1)$
$\begin{matrix} x_i \\ - \end{matrix}$	$1 + E(i-1, j)$
$\begin{matrix} - \\ y_j \end{matrix}$	$1 + E(i, j-1)$

$$\begin{matrix} \ldots & x_{i-2} & x_{i-1} & x_i & - & - \\ \ldots & y_{j-3} & - & y_{j-2} & y_{j-1} & y_j \end{matrix}$$

Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
$\begin{matrix} x_i \\ y_j \end{matrix}$	$\{0,1\} + E(i-1, j-1)$
$\begin{matrix} x_i \\ - \end{matrix}$	$1 + E(i-1, j)$
$\begin{matrix} - \\ y_j \end{matrix}$	$1 + E(i, j-1)$

$E(i, j) =$??

Edit Distance

We want to align two strings, $x = [x_1, \ldots, x_n]$ and $y = [y_1, \ldots, y_m]$.

$E(i, j)$ = optimal cost of aligning $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$.

Can we say anything about optimal alignment of $[x_1, \ldots, x_i]$ and $[y_1, \ldots, y_j]$?

Alignment	Cost
$\begin{matrix} x_i \\ y_j \end{matrix}$	$\{0,1\} + E(i-1, j-1)$
$\begin{matrix} x_i \\ - \end{matrix}$	$1 + E(i-1, j)$
$\begin{matrix} - \\ y_j \end{matrix}$	$1 + E(i, j-1)$

$$E(i,j) = \min \begin{cases} \text{diff}(i,j) + E(i-1, j-1) \\ 1 + E(i-1, j) \\ 1 + E(i, j-1) \end{cases}$$

$$\text{where diff}(i,j) = \begin{cases} 0, & x_i = y_j \\ 1, & x_i \neq y_j \end{cases}$$

Edit Distance

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & x[i] \neq y[j] \end{cases}$$

How should we find proceed?

Edit Distance

	j	0	1	2	3	4	5
i			S	U	N	N	Y
0		0					
1	S						
2	N						
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & x[i] \neq y[j] \end{cases}$$

How should we find proceed?

- Make 2d array that holds up to $E(n,m)$.

Edit Distance

i		j 0	1 S	2 U	3 N	4 N	5 Y
0		0					
1	S						
2	N						
3	O						
4	W						
5	Y						

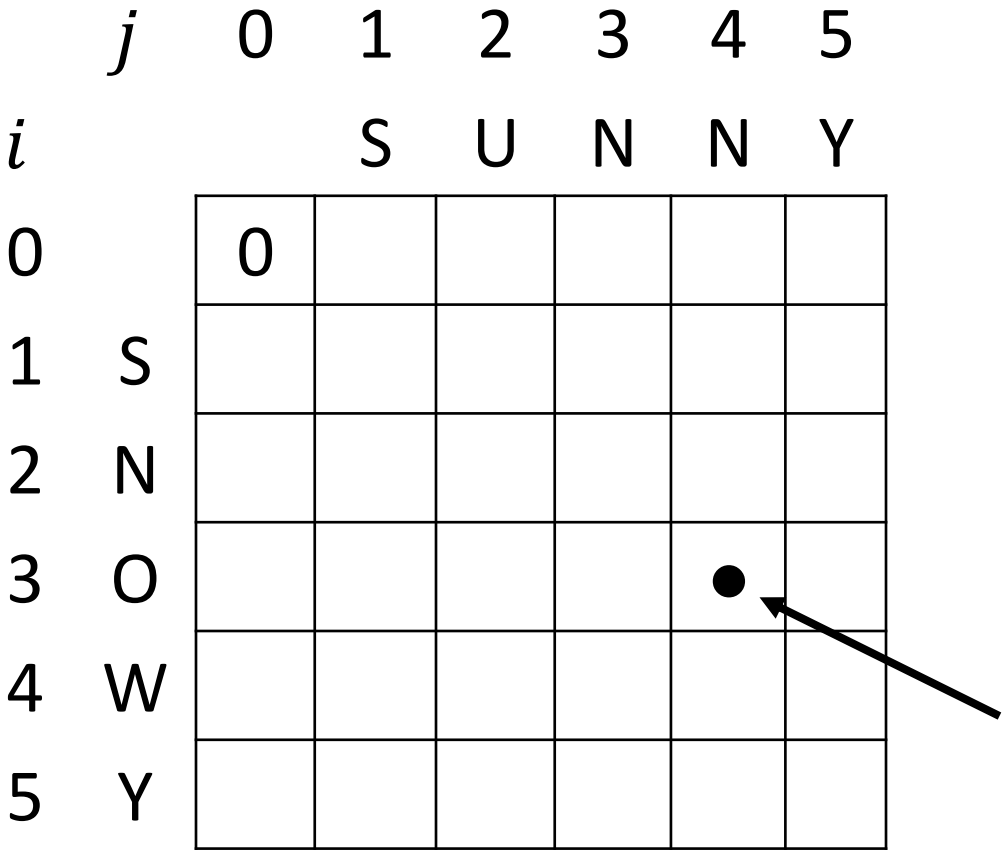
$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & x[i] \neq y[j] \end{cases}$$

How should we find proceed?

- Make 2d array that holds up to $E(n,m)$.
- Fill it out.
- Return $E(n,m)$.

Edit Distance

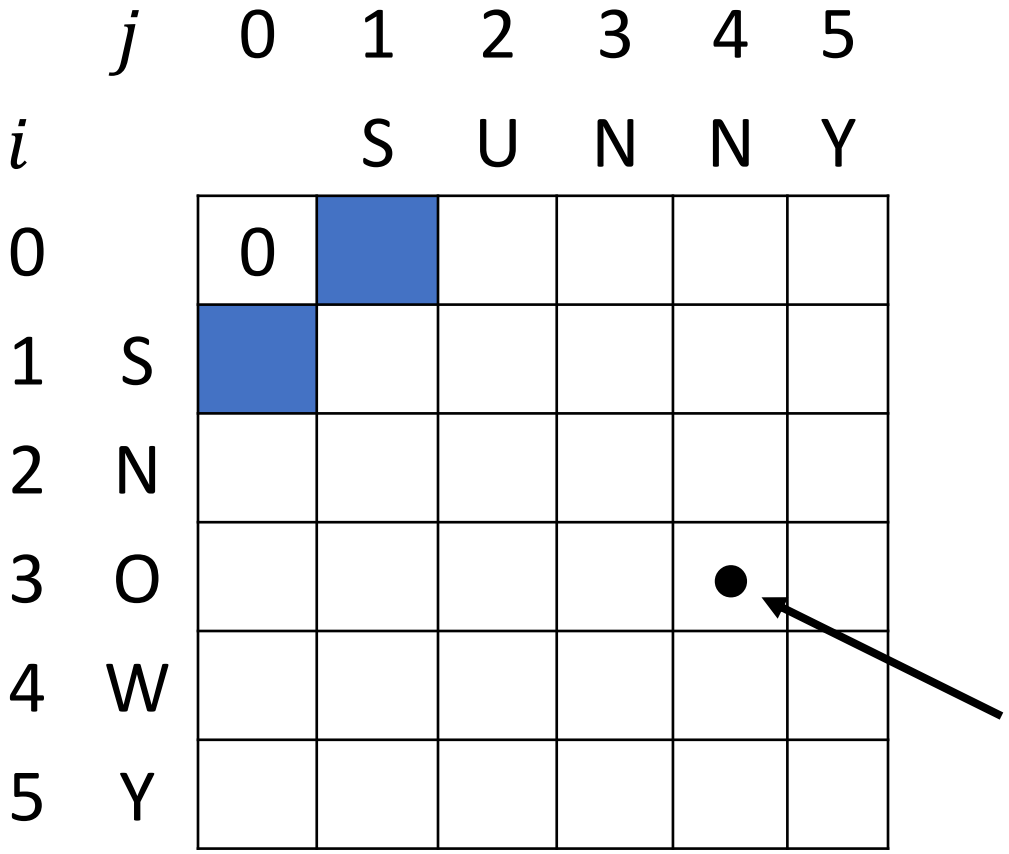


$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & x[i] \neq y[j] \end{cases}$$

Where can we start?

Edit Distance



$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & x[i] \neq y[j] \end{cases}$$

Where can we start?

$E(0,1)$ or $E(1,0)$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0					
1	S						
2	N						
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(0,1) = \min \begin{cases} E(-1,1)+1 \\ E(0,0)+1 \\ E(-1,0)+1 \end{cases} = ?$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0					
1	S						
2	N						
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(0,1) = \min \begin{cases} \cancel{E(-1,1)+1} \\ E(0,0)+1 \\ \cancel{E(-1,0)+1} \end{cases} = ?$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1				
1	S						
2	N						
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(0,1) = \min \begin{cases} \sout{E(-1,1)+1} \\ E(0,0)+1 \\ \sout{E(-1,0)+1} \end{cases} = 1$$

Corresponds to aligning S from SUNNY with inserted – from SNOWY.

Edit Distance

j		0	1	2	3	4	5
i			S	U	N	N	Y
0		0	1				
1	S						
2	N						
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(1,1) = \min \begin{cases} E(0,1)+1 \\ E(1,0)+1 \\ E(0,0)+0 \end{cases} = ?$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1				
1	S						
2	N						
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

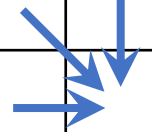
$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(1,1) = \min \begin{cases} E(0,1)+1 \\ \boldsymbol{E(1,0)+1} \\ E(0,0)+0 \end{cases} = ?$$

Not calculated yet!

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1				
1	S						
2	N						
3	O						
4	W						
5	Y						



$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

Need upper left-hand corner filled out before we can progress.

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2			
1	S						
2	N						
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(0,2) = \min \begin{cases} E(-1,2)+1 \\ E(0,1)+1 \\ E(-1,1)+1 \end{cases} = 2$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2			
1	S	1					
2	N						
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(1,0) = \min \begin{cases} E(0,0)+1 \\ E(1,-1)+1 \\ E(0,-1)+1 \end{cases} = 1$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2			
1	S	1	0				
2	N						
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(1,1) = \min \begin{cases} E(0,1)+1 \\ E(1,0)+1 \\ E(0,0)+0 \end{cases} = 0$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	?		
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	?		
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(2,3) = \min \begin{cases} E(1,3)+1 \\ E(2,2)+1 \\ E(1,2)+0 \end{cases}$$

Edit Distance

i		j 0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	?		
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(2,3) = \min \begin{cases} E(1,3)+1 = \mathbf{2+1} \\ E(2,2)+1 = \mathbf{1+1} \\ E(1,2)+0 = \mathbf{1+0} \end{cases}$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	?		
3	O						
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(2,3) = \min \begin{cases} E(1,3)+1 \\ E(2,2)+1 \\ E(1,2)+0 \end{cases} = 1$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	?		
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	?		
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(3,3) = \min \begin{cases} E(2,3)+1 = \mathbf{1+1} \\ E(3,2)+1 = \mathbf{2+1} \\ E(2,2)+1 = \mathbf{1+1} \end{cases}$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	?		
4	W						
5	Y						

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

$$E(3,3) = \min \begin{cases} E(2,3)+1 \\ E(3,2)+1 \\ E(2,2)+1 \end{cases} = 2$$

Edit Distance

	j	0	1	2	3	4	5
i			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

Running Time?

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

$$\text{diff}(i,j) = \begin{cases} 0, & x[i] = y[j] \\ 1, & \text{otherwise} \end{cases}$$

Running Time?

Fill out $n \times m$ table with constant operations: $O(nm)$

Edit Distance

	j	0	1	2	3	4	5
i			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	**3**

Edit distance = **3**.

How can we recreate the actual alignment?

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	**3**

Edit distance = **3**.

How can we recreate the actual alignment?

Backtracking.

Ask the question: “How did we get here?”

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	**3**

How did we get to $E(5,5)$?

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4 ←	**3**

How did we get to $E(5,5)$?
From $E(5,4)$?

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4 ←	**3**

How did we get to $E(5,5)$?

From $E(5,4)$? – No. Can never go down in cost.

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	**3**

How did we get to $E(5,5)$?

From $E(5,4)$? – No. Can never go down in cost.

From $E(4,5)$?

Edit Distance

i		j 0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	**3**

How did we get to $E(5,5)$?

From $E(5,4)$? – No. Can never go down in cost.

From $E(4,5)$? – No. Need +1 to move that direction.

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

Edit Distance

	j	0	1	2	3	4	5
i			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	**3**

How did we get to $E(5,5)$?

From $E(5,4)$? – No. Can never go down in cost.

From $E(4,5)$? – No. Need +1 to move that direction.

From $E(4,4)$?

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	**3**

How did we get to $E(5,5)$?

From $E(5,4)$? – No. Can never go down in cost.

From $E(4,5)$? – No. Need +1 to move that direction.

From $E(4,4)$? – Yes. Match Y's.

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

Edit Distance

	j	0	1	2	3	4	5
i			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

Continuing the process yields all of the optimal solutions.

Diagonal move indicates ?

Vertical move indicates ?

Horizontal move indicates ?

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

Continuing the process yields all of the optimal solutions.

Diagonal move indicates match.

Vertical move indicates ?

Horizontal move indicates ?

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

Edit Distance

i		j	0	1	2	3	4	5
				S	U	N	N	Y
0			0	1	2	3	4	5
1	S		1	0	1	2	3	4
2	N		2	1	1	1	2	3
3	O		3	2	2	2	2	3
4	W		4	3	3	3	3	3
5	Y		5	4	4	4	4	3

S – N O W Y

S U N N – Y

Continuing the process yields all of the optimal solutions.

Diagonal move indicates match.

Vertical move indicates space inserted in j.

Horizontal move indicates ?

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

Edit Distance

i		j 0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

S	–	N	O	W	Y
S	U	N	N	–	Y

Continuing the process yields all of the optimal solutions.

Diagonal move indicates match.

Vertical move indicates space inserted in j.

Horizontal move indicates space inserted in i.

$$E(i,j) = \min \begin{cases} E(i-1,j)+1 \\ E(i,j-1)+1 \\ E(i-1,j-1)+\text{diff}(i,j) \end{cases}$$

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

Diagonal move indicates match.

Vertical move indicates space inserted in j.

Horizontal move indicates space inserted in i.

S – N O W Y

S U N N – Y

Edit Distance

	j	0	1	2	3	4	5
i			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

Diagonal move indicates match.

Vertical move indicates space inserted in j.

Horizontal move indicates space inserted in i.

Alignment?

Edit Distance

	j	0	1	2	3	4	5
i			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

Diagonal move indicates match.

Vertical move indicates space inserted in j.

Horizontal move indicates space inserted in i.

S – N O W Y

S U N – N Y

Edit Distance

i	j	0	1	2	3	4	5
			S	U	N	N	Y
0		0	1	2	3	4	5
1	S	1	0	1	2	3	4
2	N	2	1	1	1	2	3
3	O	3	2	2	2	2	3
4	W	4	3	3	3	3	3
5	Y	5	4	4	4	4	3

Diagonal move indicates match.

Vertical move indicates space inserted in j.

Horizontal move indicates space inserted in i.

S N O W Y

S U N N Y