Greedy Algorithms
CSCI 432

Greedy Algorithms:

Dynamic Programming:

- Characterize structure of optimal solution.
- Recursively define value of optimal solution.
- Compute value of optimal solution.
- Construct optimal solution from computed information.

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- Do not look ahead, plan, or revisit past decisions.
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Greedy algorithm for:

- Robbing a jewelry store?
- Eating at a fancy buffet?

Single Room Scheduling

Goal: Assign courses to a single classroom.

Single Room Scheduling

Input:

- $C = \{c_1, c_2, \ldots, c_n\}$ – set of courses that need rooms.
- $c_i = [s_i, f_i)$ – start and finish times for each course.

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Goal: Select a maximum sized subset of compatible courses.

I.e., Fill a single room up with the most possible courses.

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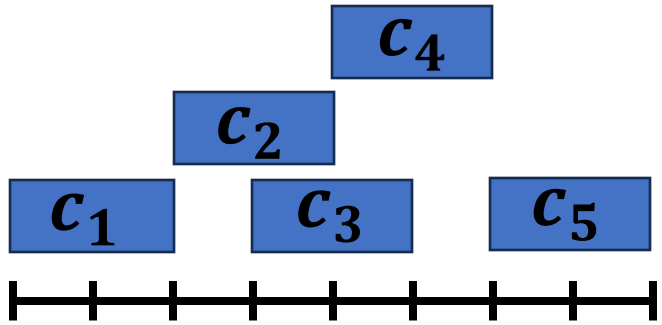

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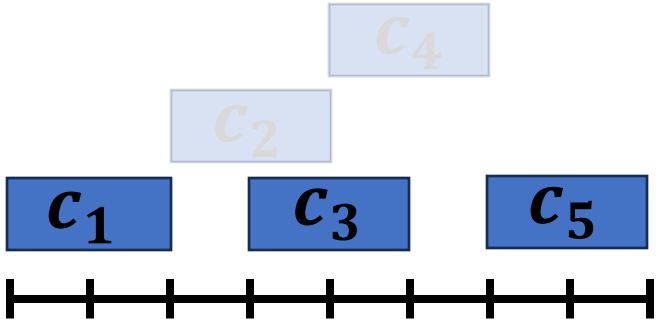

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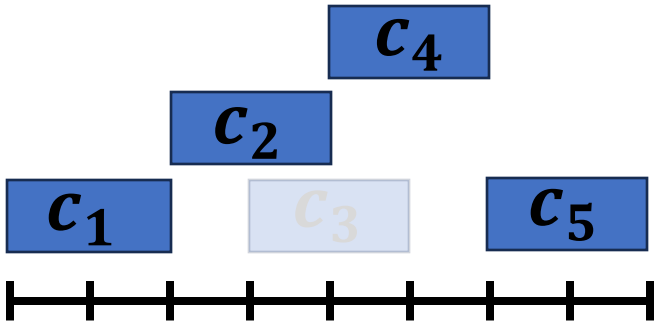

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Smallest conflict first.

In each iteration, pick the course that overlaps with the smallest number of other courses.

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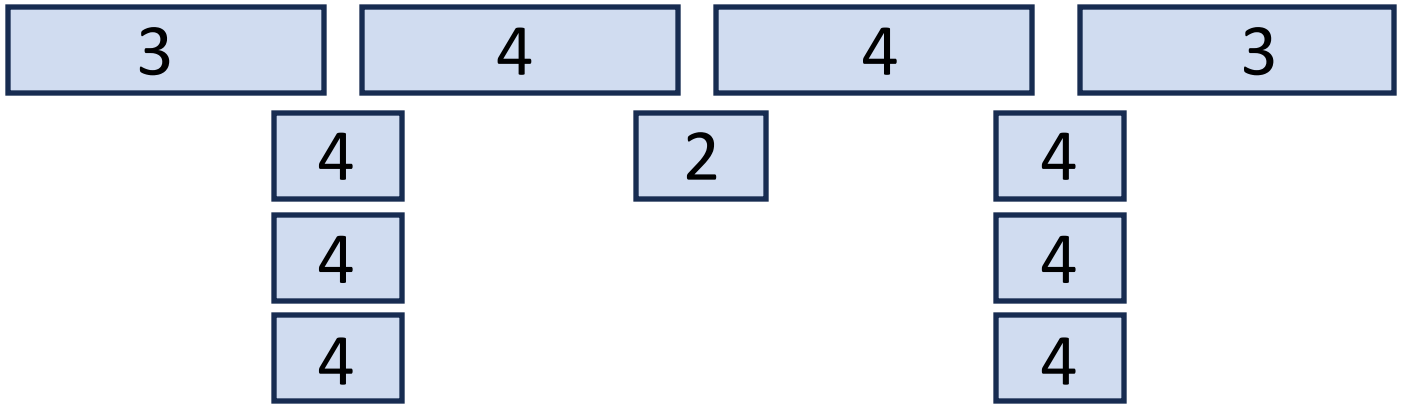

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Smallest duration first.

In each iteration, pick the course that takes the least amount of time.

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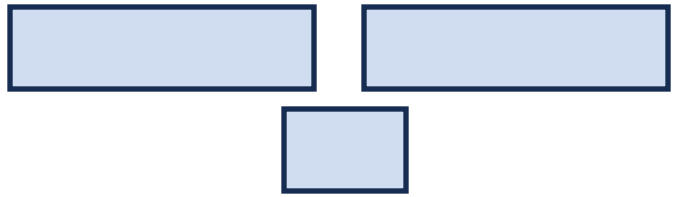

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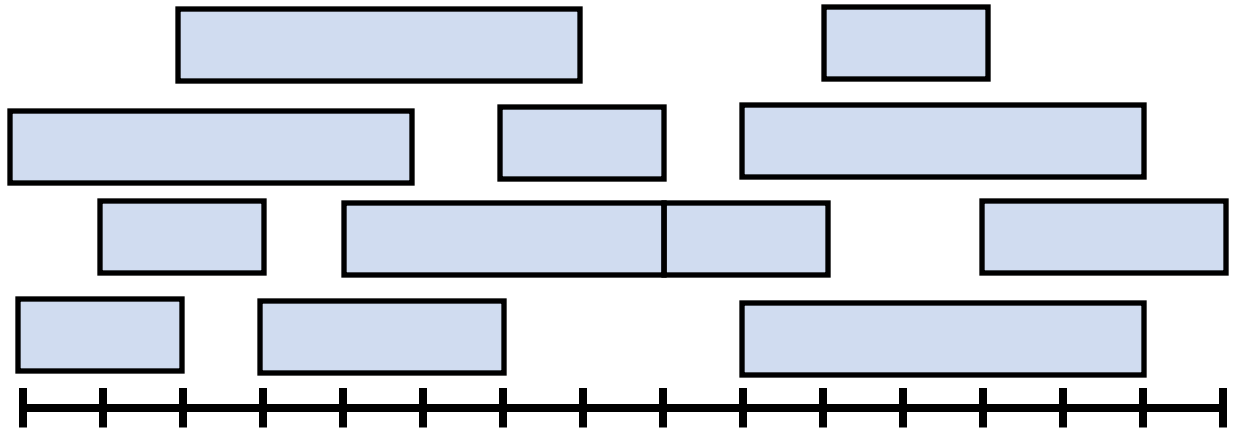

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In each iteration, pick the course that ends earliest and is compatible with existing schedule.

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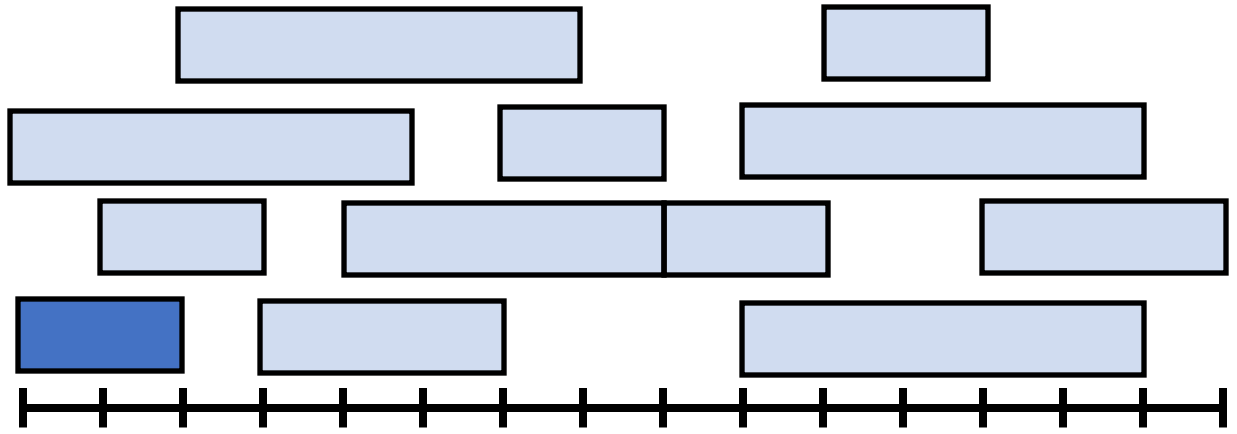

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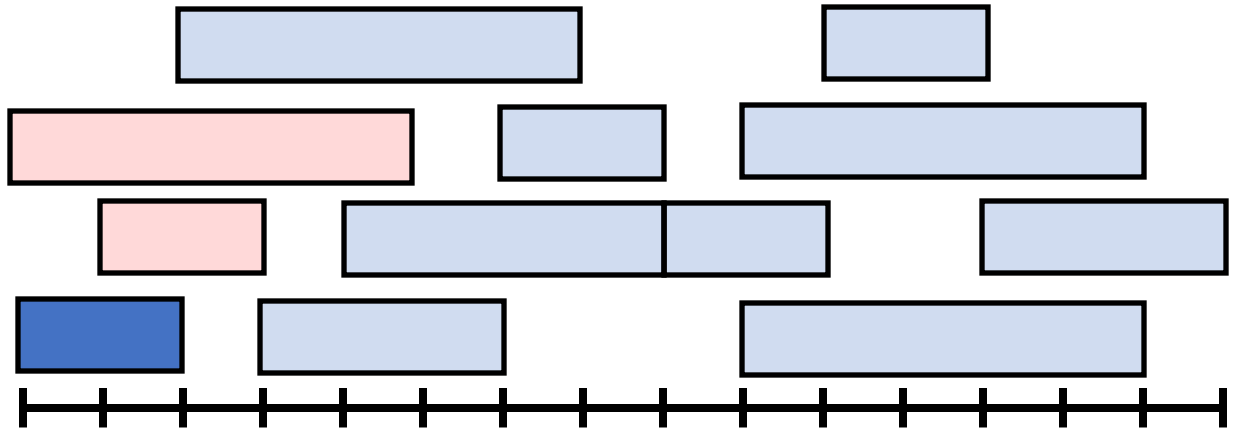

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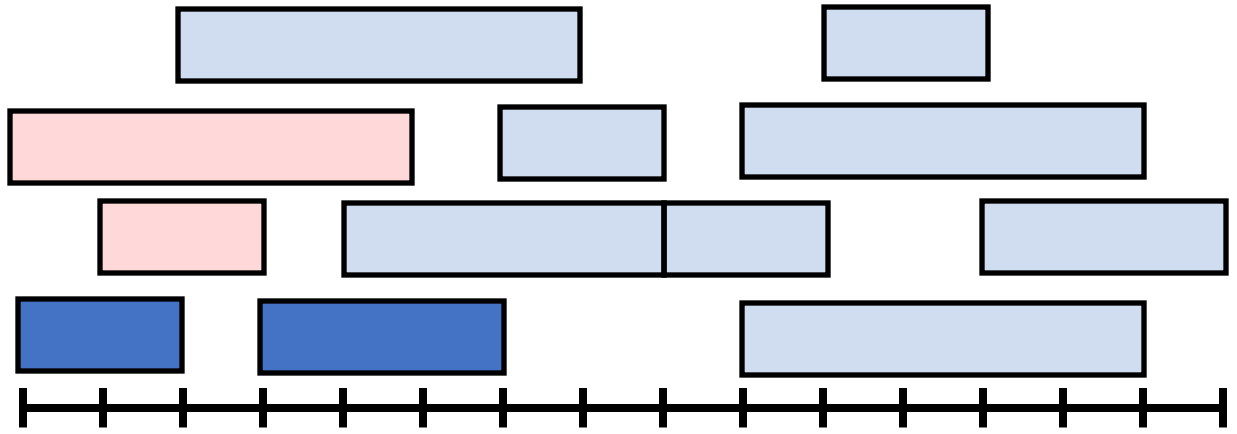

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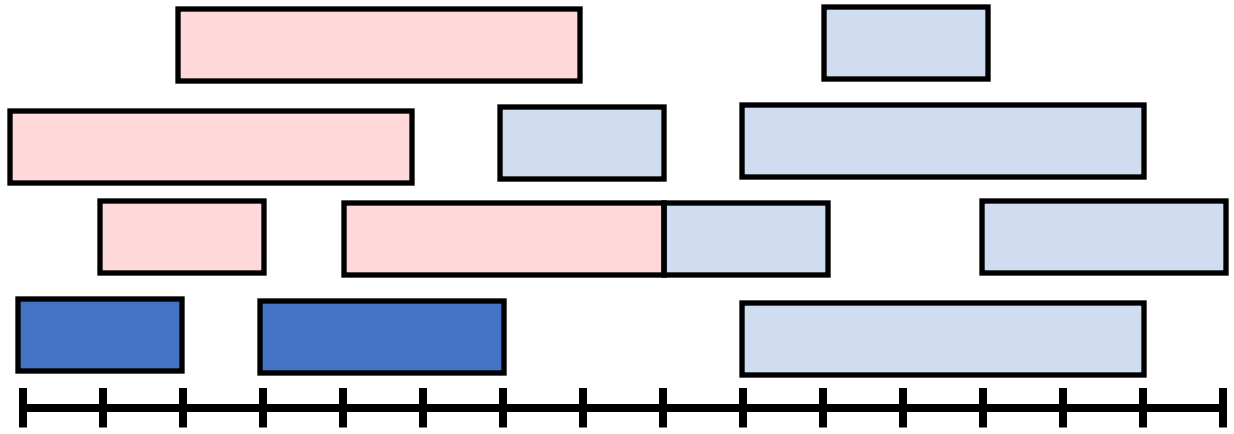

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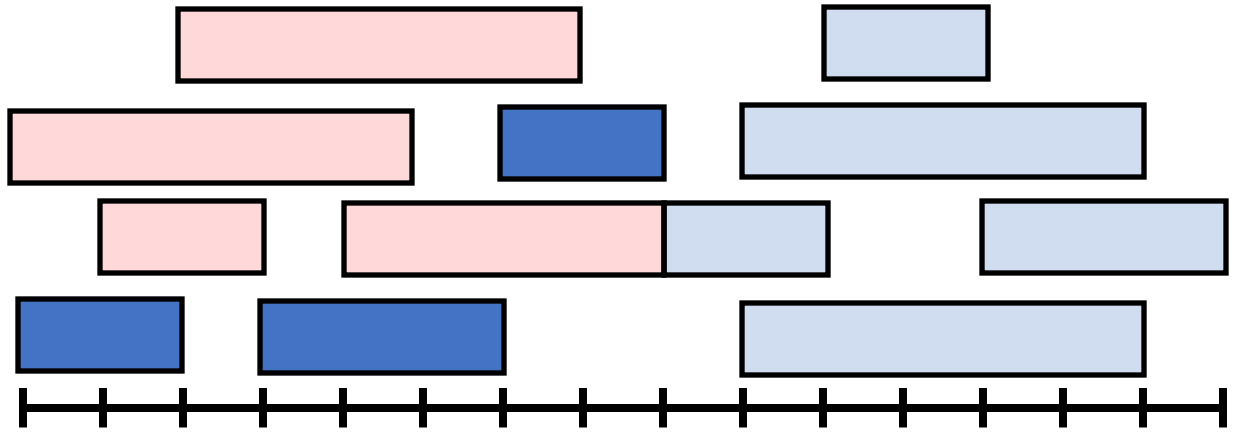

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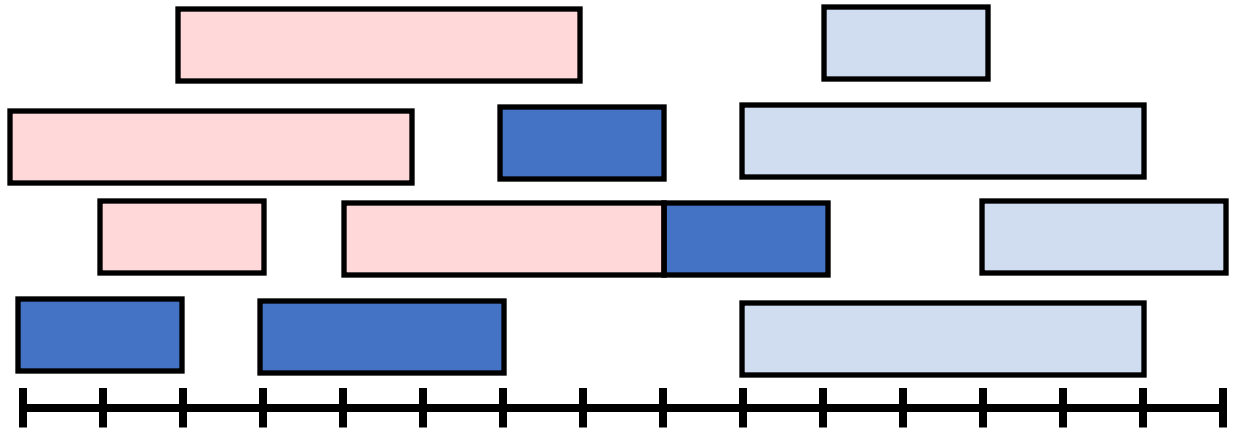

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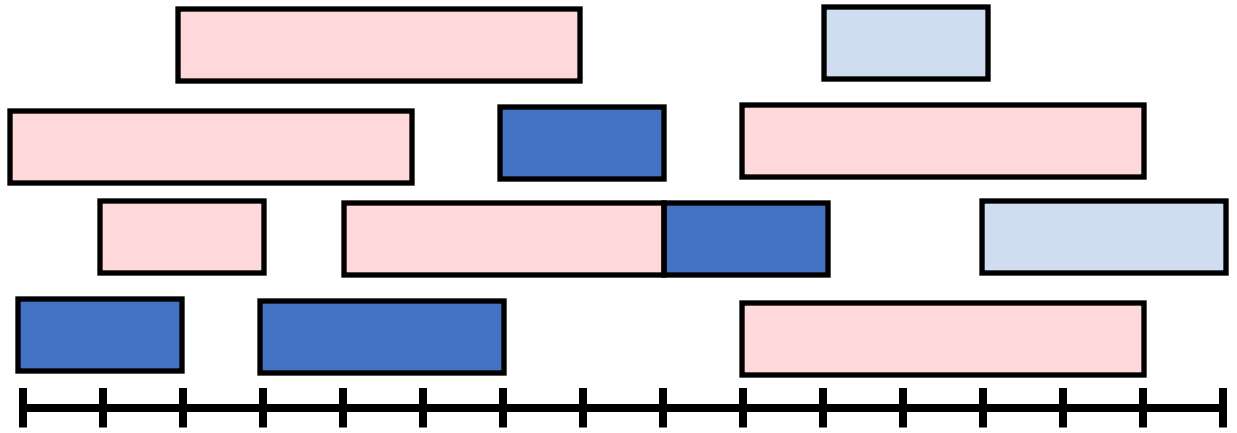

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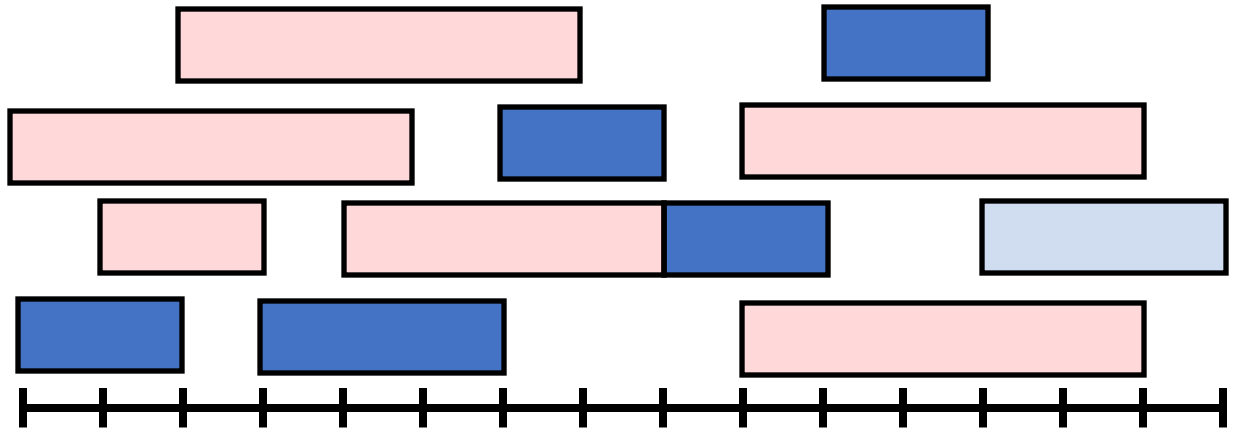

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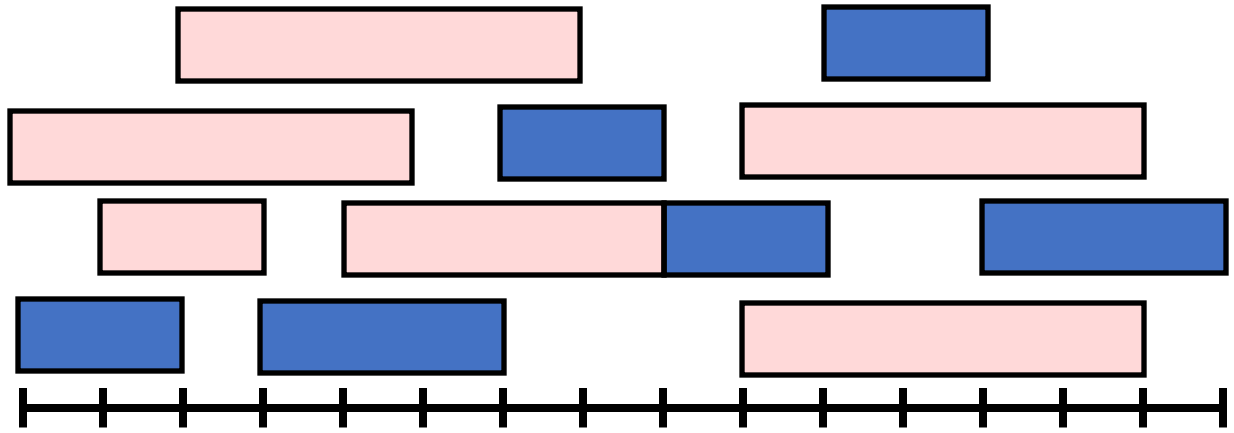

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Coding Plan?

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1. Sort by increasing finish times.
2. Select first course.
3. Iterate through list looking for first compatible course.
4. Repeat.

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```
room_scheduling(courses C)
   C.sort_by_finish()
   selected = {C[1]}
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   for i = 2 to C.length



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Test to see if `C[i]` is compatible with courses already selected?

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Running Time?

Validity?

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Running Time? $O(n \log n)$

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Validity? `selected` consists of compatible courses.

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Running Time? $O(n \log n)$

Validity? `selected` consists of compatible courses.

Performance? Is `selected` the largest possible subset?

Single Room Scheduling

Greedy decision: Select the next course with the earliest compatible finish time.

Proof of optimality:

Plan: Turn a hypothetical optimal solution into the algorithm's solution without changing the cost (i.e., number of courses) and without violating course compatibility.

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Proof of optimality: Let C be the set of courses, $S_{ALG} \subseteq C$ be the greedy algorithm's selection, and $S_{OPT} \subseteq C$ be an optimal selection, all sorted by increasing finish time.

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Suppose $S_{ALG}[i] = S_{OPT}[i]$, for all $i \leq k$ and $S_{ALG}[k] = c_i \neq c_j = S_{OPT}[k]$.

Suppose S_{ALG} and S_{OPT} schedule the same courses up until course k.

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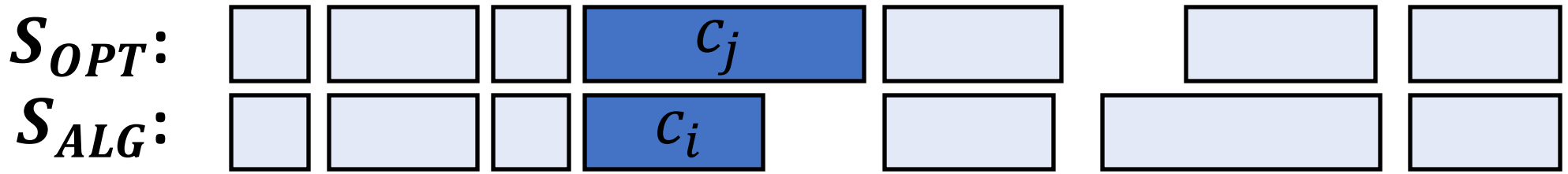

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Create the revised schedule $S'_{OPT} = S_{OPT} \setminus \{c_j\} \cup \{c_i\}$. (I.e., Swap $S_{ALG}[k]$ for $S_{OPT}[k]$)

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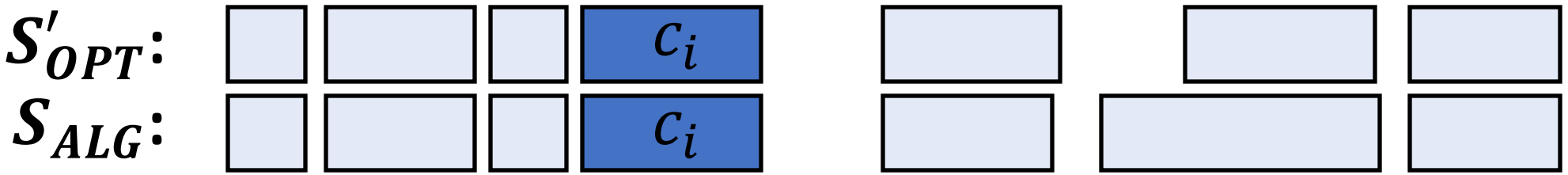

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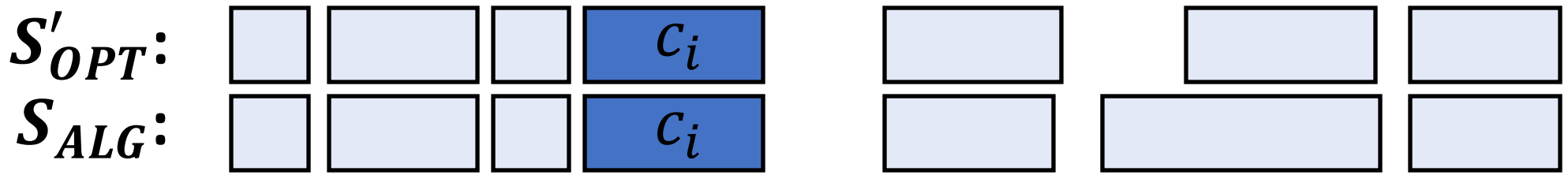

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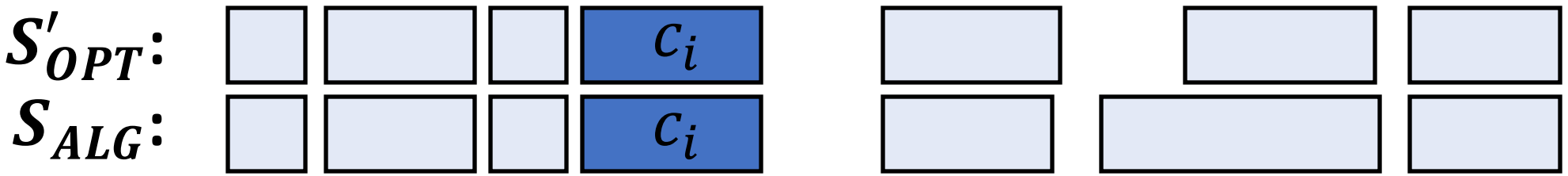
Will S'_{OPT} be valid?
Need to check and see if c_j
messed up any compatibilities.



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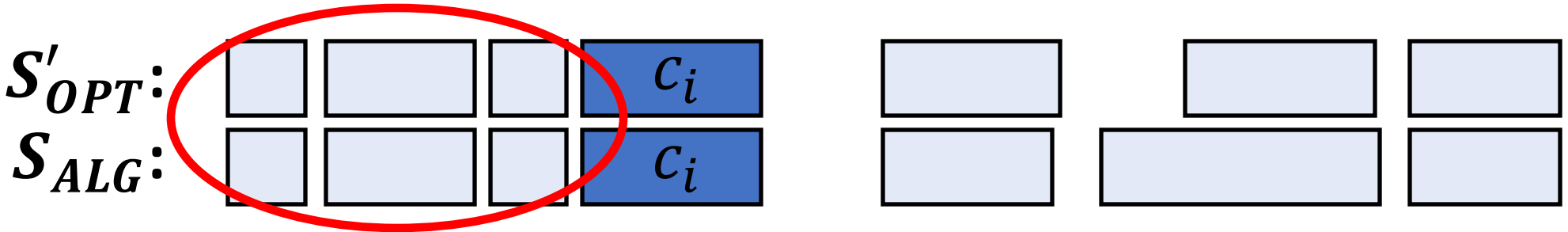
Create the revised schedule $S'_{OPT} = S_{OPT} \setminus \{c_j\} \cup \{c_i\}$. (I.e., Swap $S_{ALG}[k]$ for $S_{OPT}[k]$)

c_i is compatible with previous courses in S'_{OPT} since $S_{ALG}[i] = S_{OPT}[i] = S'_{OPT}[i]$, for all $i \leq k$

c_i is compatible with S_{ALG}, and S_{OPT} and S'_{OPT} share the same courses before c_i.



Single Room Scheduling

Greedy decision: Select the next course with the earliest compatible finish time.

Proof of optimality: Let C be the set of courses, $S_{ALG} \subseteq C$ be the greedy algorithm's selection, and $S_{OPT} \subseteq C$ be an optimal selection, all sorted by increasing finish time.

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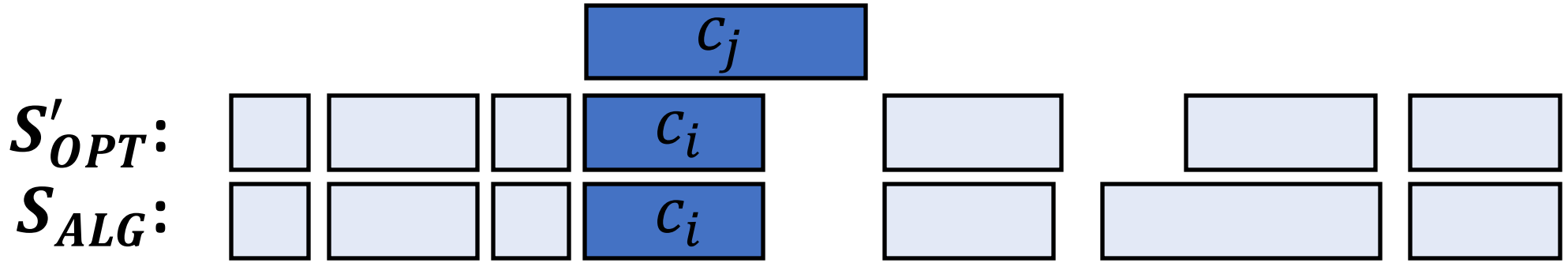
c_i is compatible with subsequent courses in S'_{OPT} since $f_i \leq f_j$. Otherwise, the greedy algorithm would have selected c_j instead of c_i.



Single Room Scheduling

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Proof of optimality: Let C be the set of courses, $S_{ALG} \subseteq C$ be the greedy algorithm's selection, and $S_{OPT} \subseteq C$ be an optimal selection, all sorted by increasing finish time.

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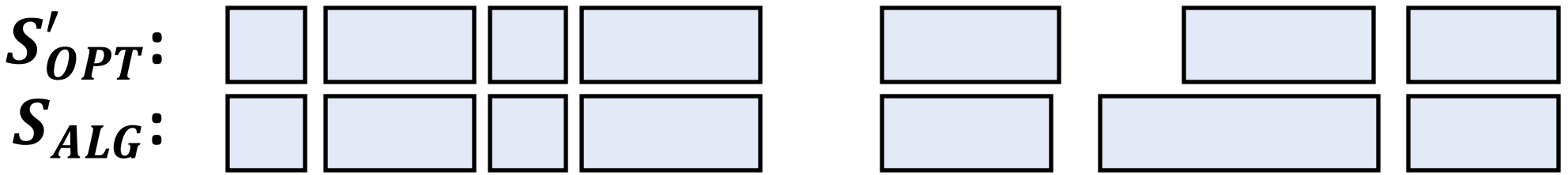
So S'_{OPT} is a valid schedule with the same number of courses as S_{OPT}, so S'_{OPT} is also optimal.



Single Room Scheduling

Greedy decision: Select the next course with the earliest compatible finish time.

Proof of optimality: Let C be the set of courses, $S_{ALG} \subseteq C$ be the greedy algorithm's selection, and $S_{OPT} \subseteq C$ be an optimal selection, all sorted by increasing finish time.

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So S'_{OPT} is a valid schedule with the same number of courses as S_{OPT}, so S'_{OPT} is also optimal.

We can then proceed inductively and show that each course in S_{OPT} can be replaced by the corresponding course in S_{ALG} without violating compatibility.

Single Room Scheduling

Greedy decision: Select the next course with the earliest compatible finish time.

Proof of optimality: Let C be the set of courses, $S_{ALG} \subseteq C$ be the greedy algorithm's selection, and $S_{OPT} \subseteq C$ be an optimal selection, all sorted by increasing finish time.

Suppose $S_{ALG}[i] = S_{OPT}[i]$, for all $i \leq k$ and $S_{ALG}[k] = c_i \neq c_j = S_{OPT}[k]$.

Create the revised schedule $S'_{OPT} = S_{OPT} \setminus \{c_j\} \cup \{c_i\}$. (I.e., Swap $S_{ALG}[k]$ for $S_{OPT}[k]$)

c_i is compatible with previous courses in S'_{OPT} since $S_{ALG}[i] = S_{OPT}[i] = S'_{OPT}[i]$, for all $i \leq k$

c_i is compatible with subsequent courses in S'_{OPT} since $f_i \leq f_j$. Otherwise, the greedy algorithm would have selected c_j instead of c_i.

So S'_{OPT} is a valid schedule with the same number of courses as S_{OPT}, so S'_{OPT} is also optimal.

We can then proceed inductively and show that each course in S_{OPT} can be replaced by the corresponding course in S_{ALG} without violating compatibility. Since replacing every course in S_{OPT} with the courses in S_{ALG} keeps the solution optimal, S_{ALG} must be optimal. (i.e., we translated S_{OPT} into S_{ALG} at no extra cost).