Linear Programming
CSCI 532

Max Flow

Decision Variables:

- Real numbers = solvable in polynomial time (called LP).
- Integers = not (yet?) solvable in polynomial time (called integer linear program – ILP).

x_e = Amount of flow on edge e.

Objective: $\max \sum_{e \in \text{out}(s)} x_e$

Subject to:

$$\begin{cases} x_e \leq \text{capacity}_e, \forall e \in E \\ \sum_{e \in \text{in}(v)} x_e - \sum_{e \in \text{out}(v)} x_e = 0, \forall v \in V \setminus \{s, t\} \\ x_e \geq 0, \forall e \in E \end{cases}$$

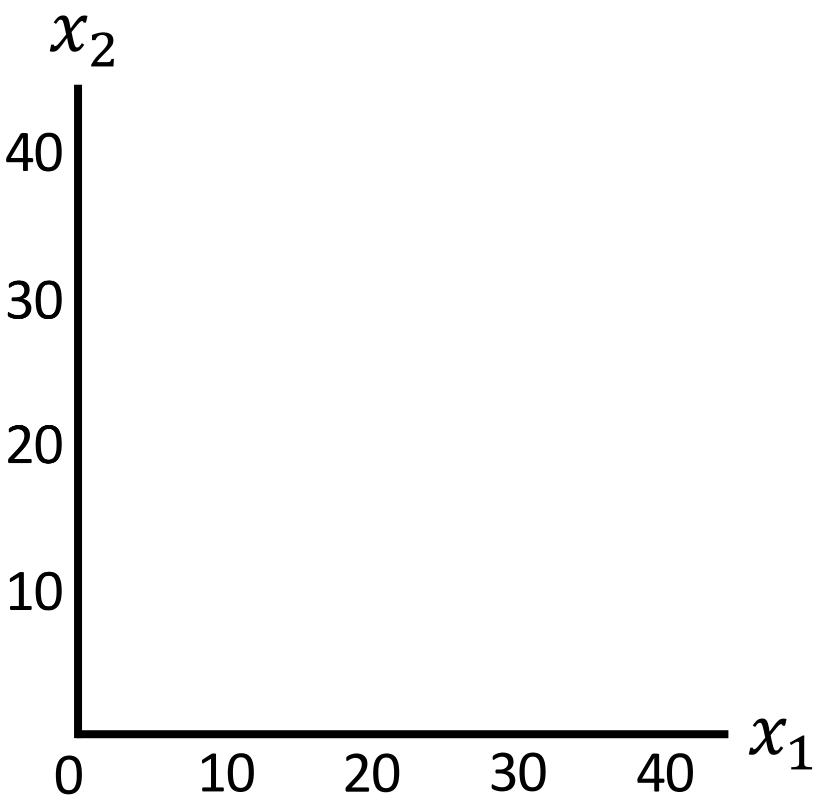
Objective:

- Can be minimization or maximization.
- Must be linear combinations of variables x_i (e.g. $a_1 x_1 + \cdots + a_n x_n$ for constants a_i, not $a_i x_1 x_2$).

Constraints:

- Can be $\leq$, $\geq$, $=$.
- Must be linear combinations of variables.

Example



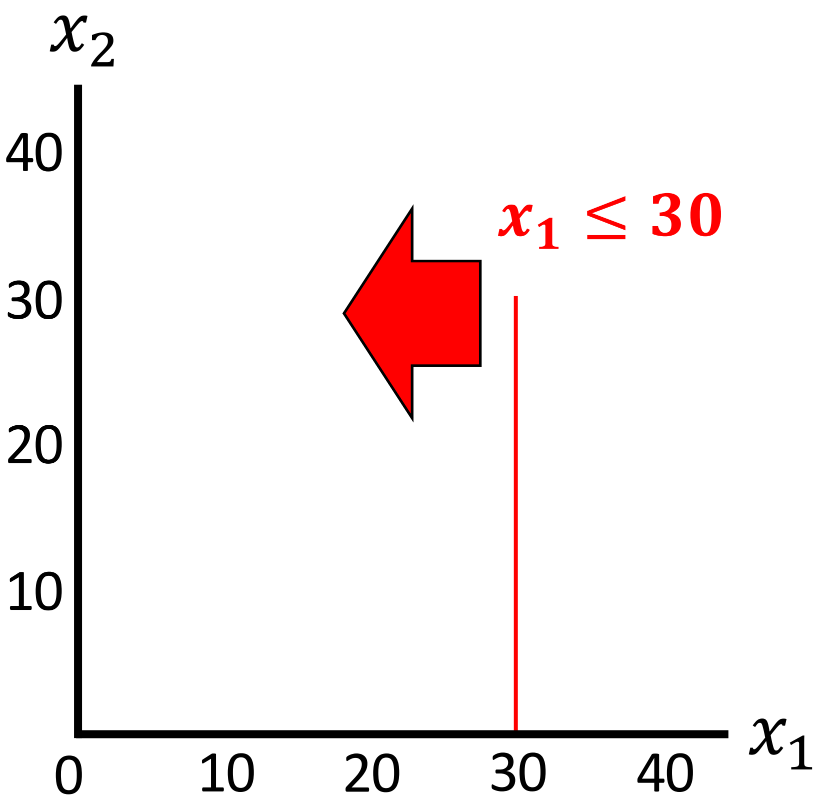
$x_1, x_2 \in \mathbb{R}$

Objective: $\max 100x_1 + 300x_2$

Subject to:

$$x_1 \leq 30$$
$$x_2 \leq 20$$
$$x_1 + x_2 \leq 40$$
$$x_1, x_2 \geq 0$$

Example

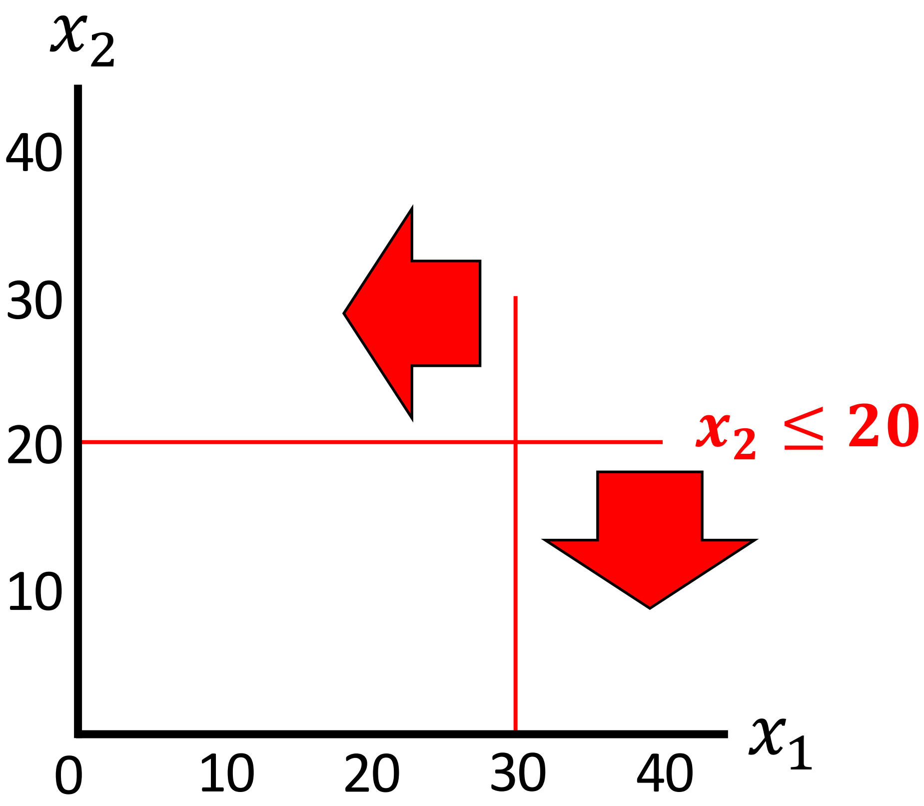


$x_1, x_2 \in \mathbb{R}$

Objective: $\max 100x_1 + 300x_2$

Subject to:
$x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$x_1, x_2 \geq 0$

Example



$x_1, x_2 \in \mathbb{R}$

Objective: $\max 100x_1 + 300x_2$

Subject to: $x_1 \leq 30$

$x_2 \leq 20$

$x_1 + x_2 \leq 40$

$x_1, x_2 \geq 0$

Example

$x_1, x_2 \in \mathbb{R}$

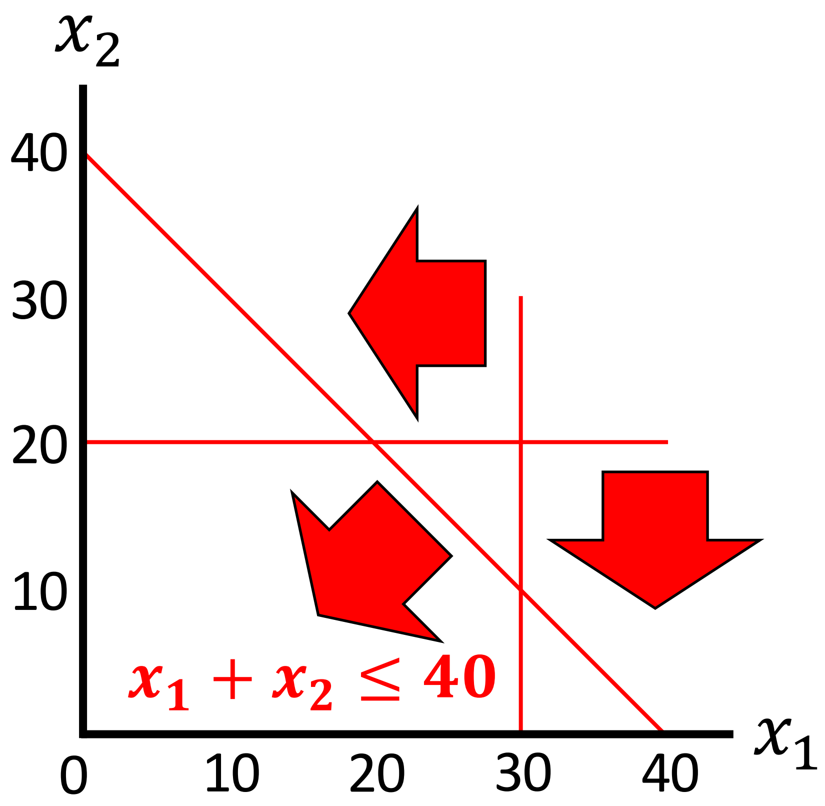
Objective: $\max 100x_1 + 300x_2$

Subject to: $x_1 \leq 30$

$x_2 \leq 20$

$x_1 + x_2 \leq 40$

$x_1, x_2 \geq 0$



Example

$x_1, x_2 \in \mathbb{R}$

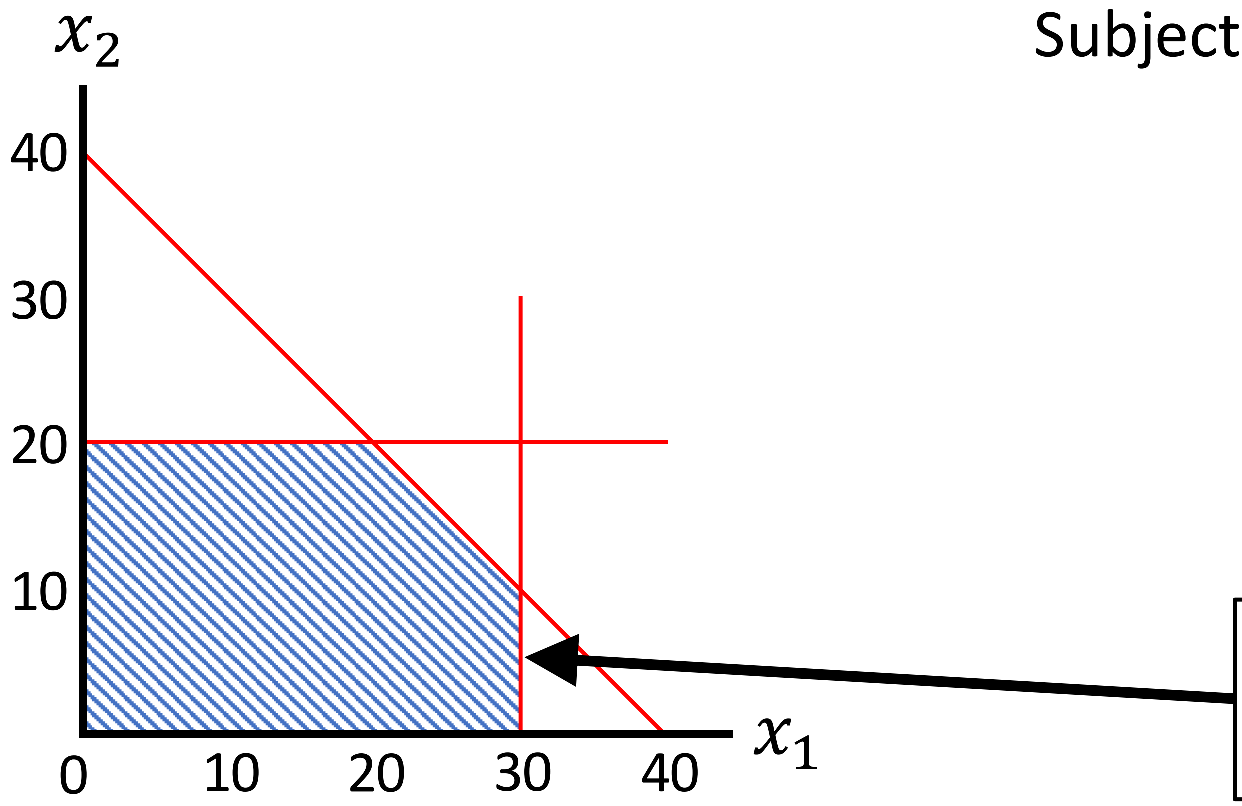
Objective: $\max 100x_1 + 300x_2$

Subject to: $x_1 \leq 30$

$x_2 \leq 20$

$x_1 + x_2 \leq 40$

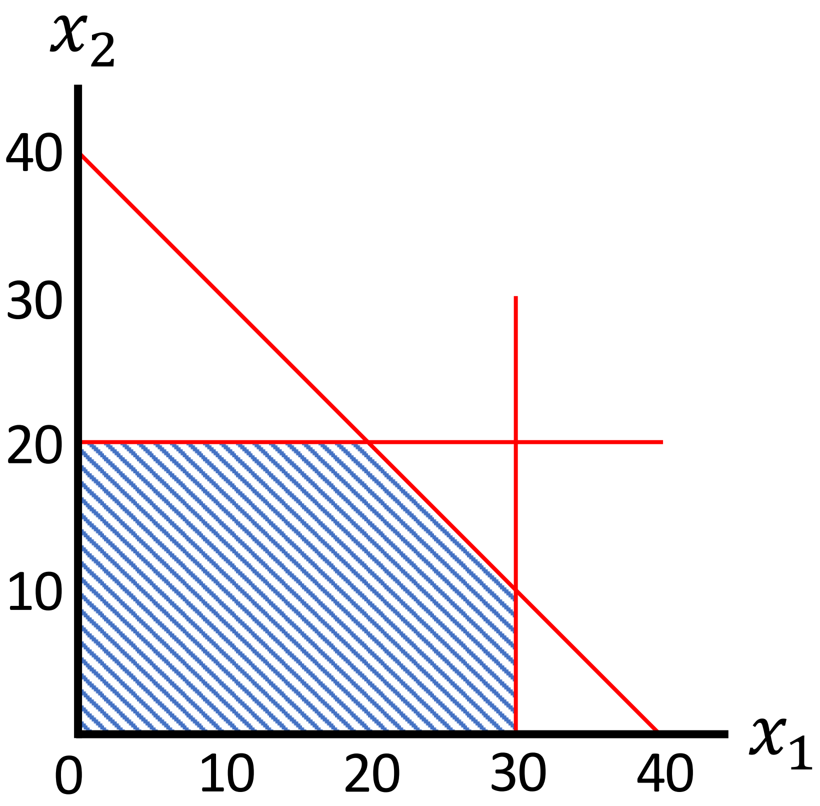
$x_1, x_2 \geq 0$



Feasible Region

(area where *all* valid solutions lie)

Example



$x_1, x_2 \in \mathbb{R}$

Objective: $\max 100x_1 + 300x_2$

Subject to:
$x_1 \leq 30$
$x_2 \leq 20$
$x_1 + x_2 \leq 40$
$x_1, x_2 \geq 0$

What is the optimal value?

Example

$x_1, x_2 \in \mathbb{R}$

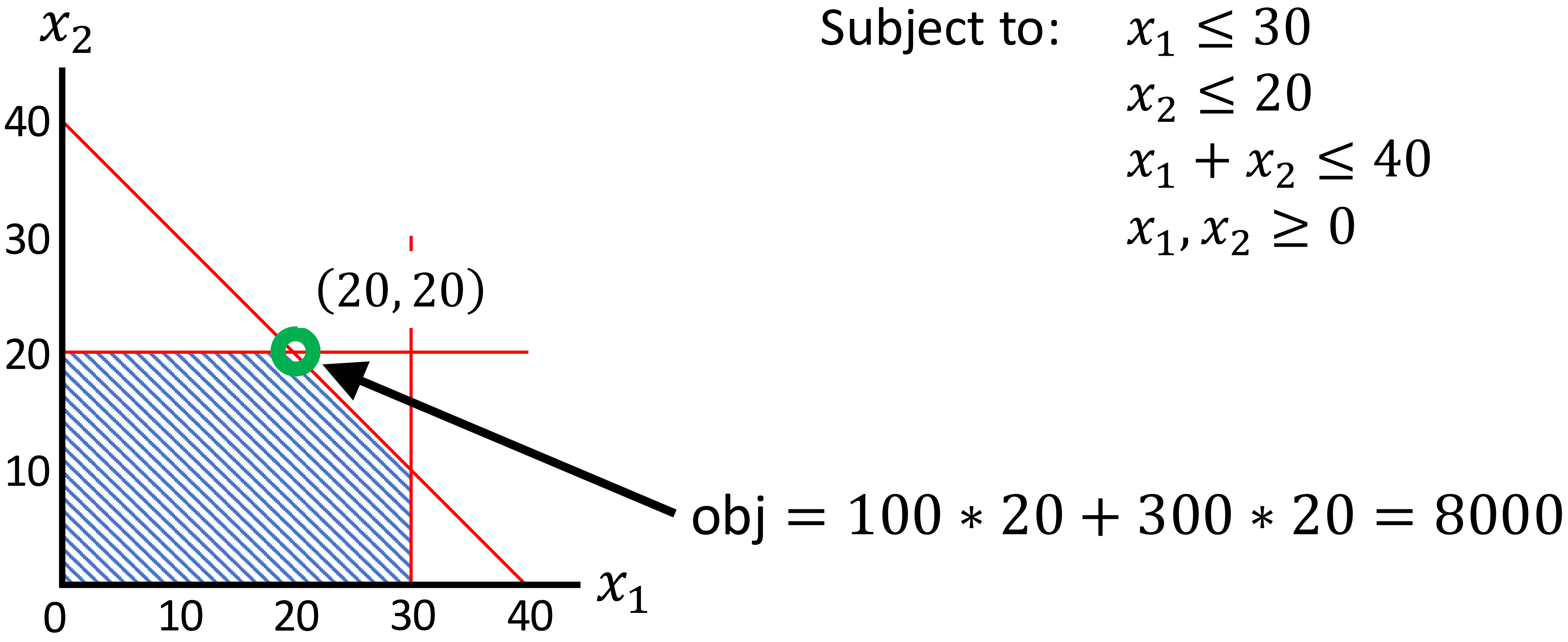
Objective: $\max 100x_1 + 300x_2$

Subject to: $x_1 \leq 30$

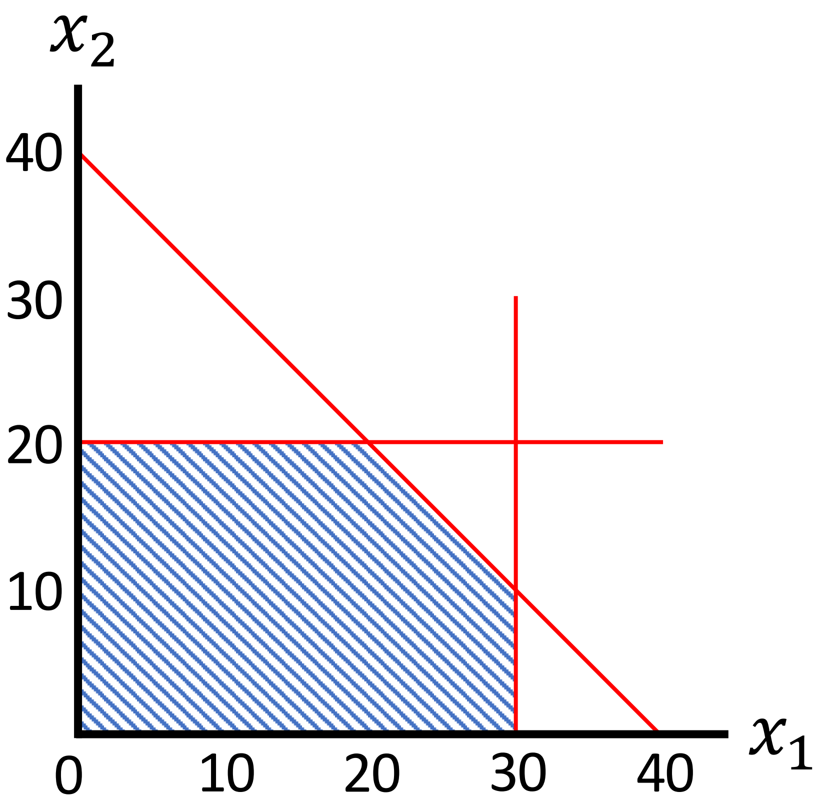
$x_2 \leq 20$

$x_1 + x_2 \leq 40$

$x_1, x_2 \geq 0$



Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to:

$$c_1(x_1, x_2)$$
$$c_2(x_1, x_2)$$
$$\vdots$$
$$c_n(x_1, x_2)$$

How can we efficiently find optimal solutions?

Identify two key properties of optimal solutions:

1. ?
2. ?

Optimal Value

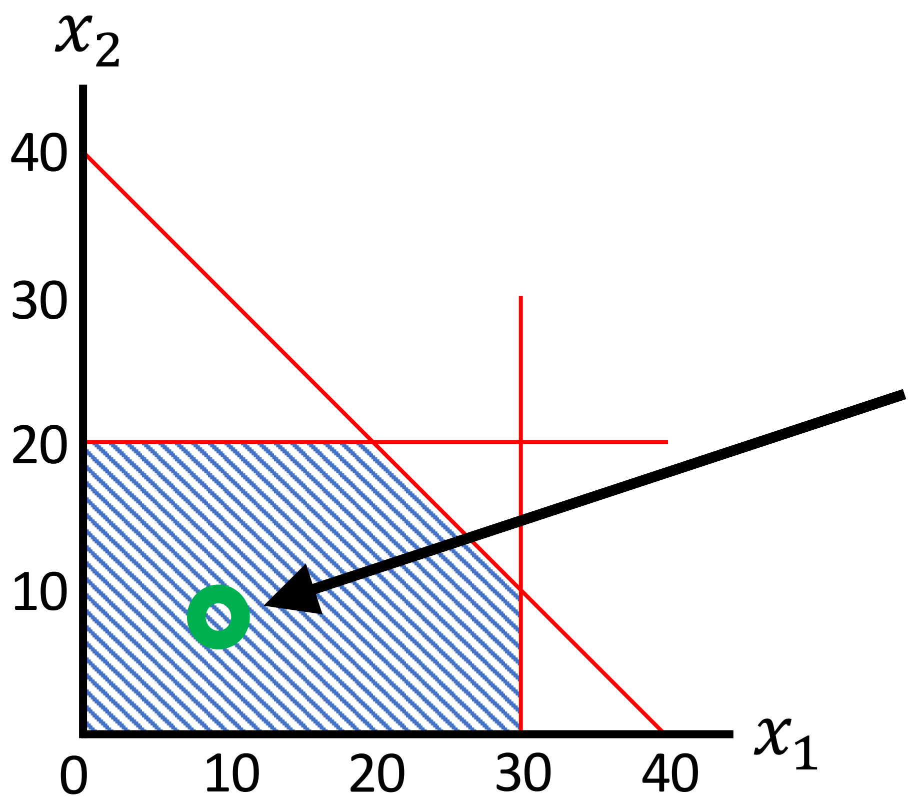
Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

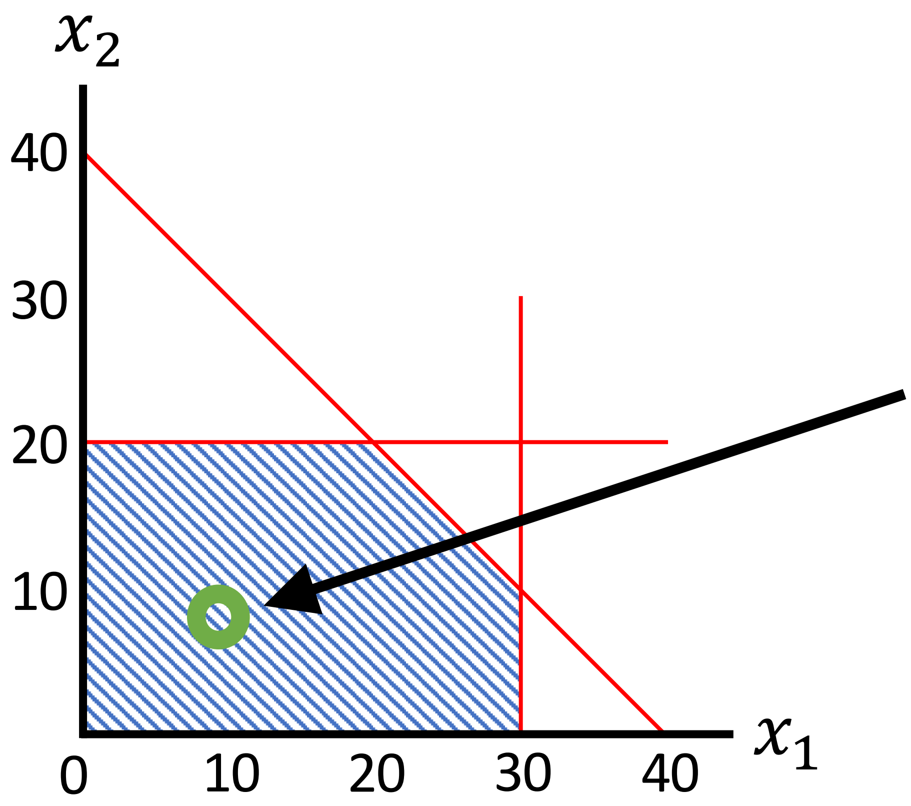
$\vdots$

$c_n(x_1, x_2)$



Could this ever be a maximum value of the objective function?

Optimal Value



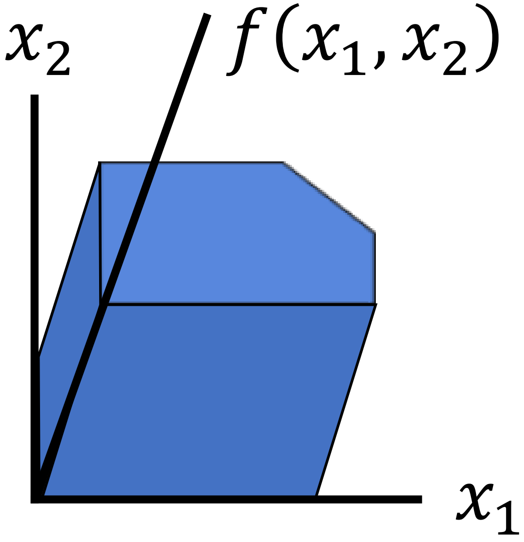
Objective: $\max f(x_1, x_2)$
Subject to: $c_1(x_1, x_2)$
$c_2(x_1, x_2)$
⋮
$c_n(x_1, x_2)$

Could this ever be a maximum value of the objective function?

Yes, if $f(x_1, x_2) = \text{constant}$.

Objective: $\max 5$
Subject to: $c_1(x_1, x_2)$
⋮

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

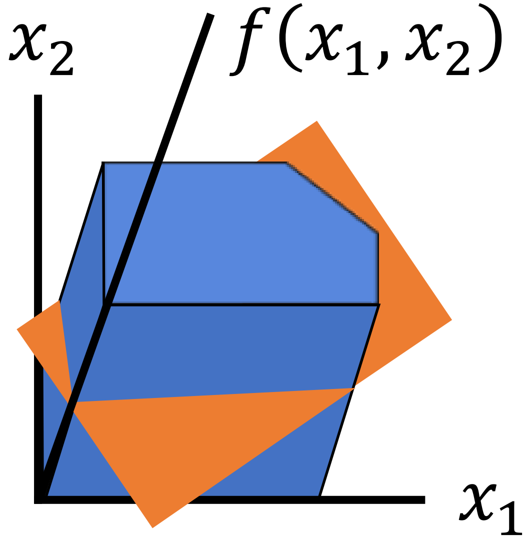
$\vdots$

$c_n(x_1, x_2)$

Could this ever be a maximum value of the objective function?

Yes, if $f(x_1, x_2) =$ constant.

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

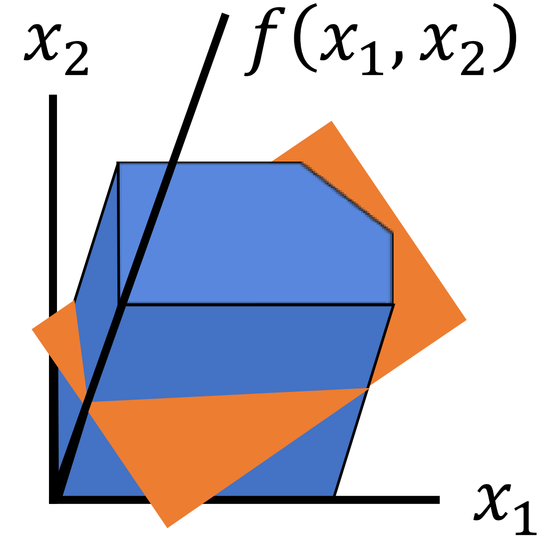
$c_n(x_1, x_2)$

Could this ever be a maximum value of the objective function?

Yes, if $f(x_1, x_2) = \text{constant}$.

$f(x_1, x_2)$ is a plane

Optimal Value



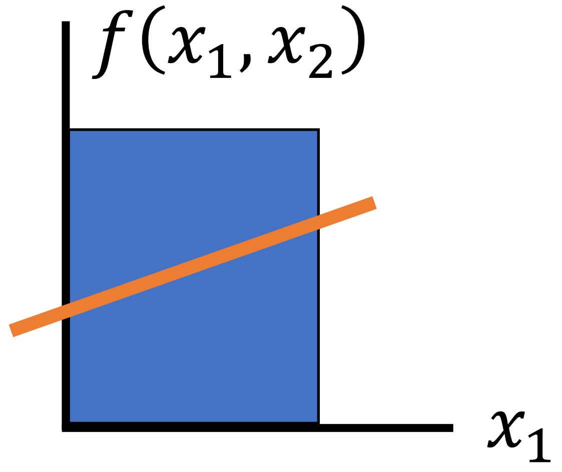
Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

$c_n(x_1, x_2)$

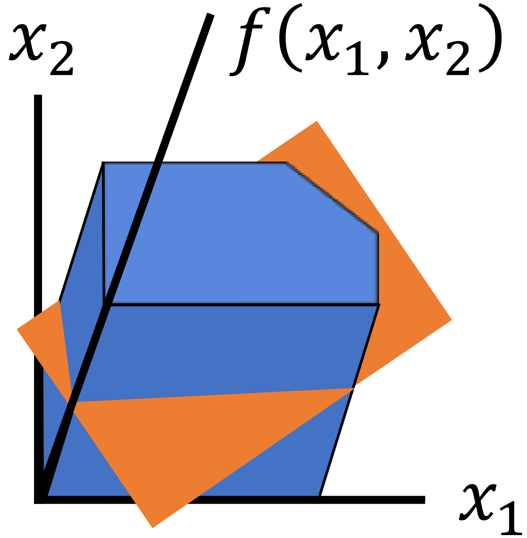


Could this ever be a maximum value of the objective function?

Yes, if $f(x_1, x_2) = \text{constant}$.

$f(x_1, x_2)$ is a plane $\Longrightarrow$ a max/min of $f(x_1, x_2)$ occurs on the boundary of the feasible region.

Optimal Value



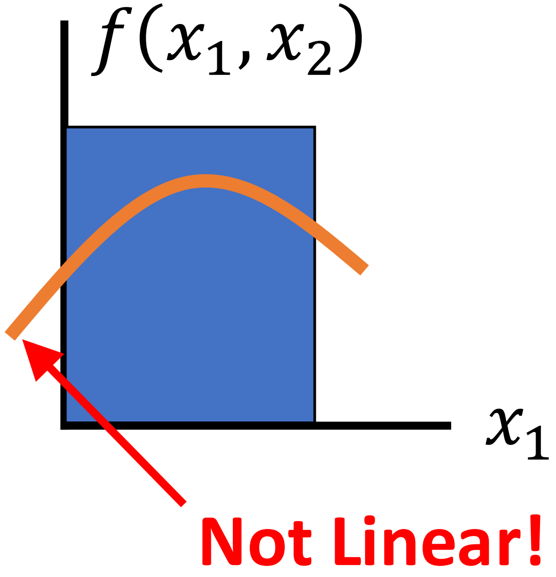
Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

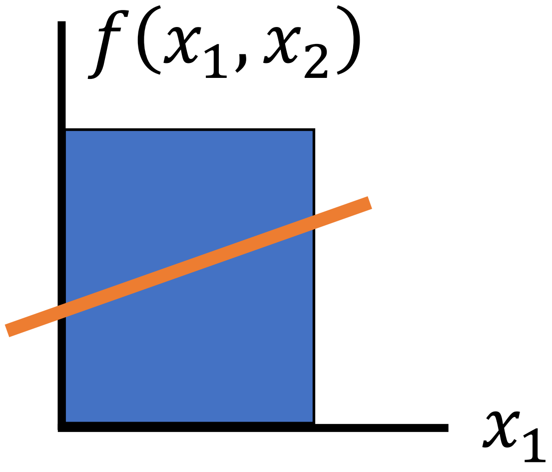
$c_2(x_1, x_2)$

$\vdots$

$c_n(x_1, x_2)$



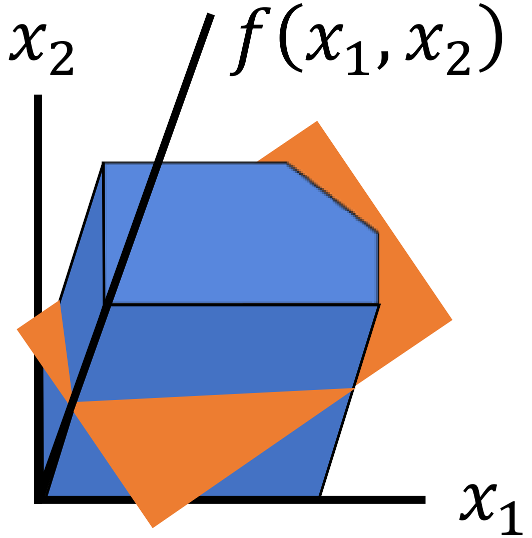
Could this ever be a maximum value of the objective function?



Yes, if $f(x_1, x_2) =$ constant.

$f(x_1, x_2)$ is a plane $\Longrightarrow$ a max/min of $f(x_1, x_2)$ occurs on the boundary of the feasible region.

Optimal Value



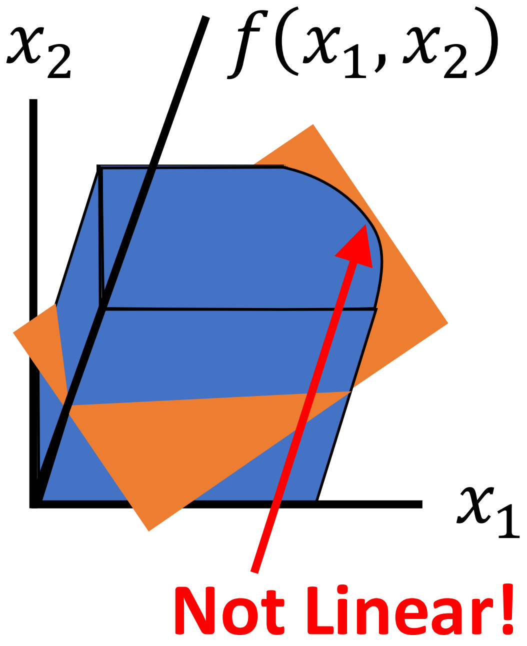
Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

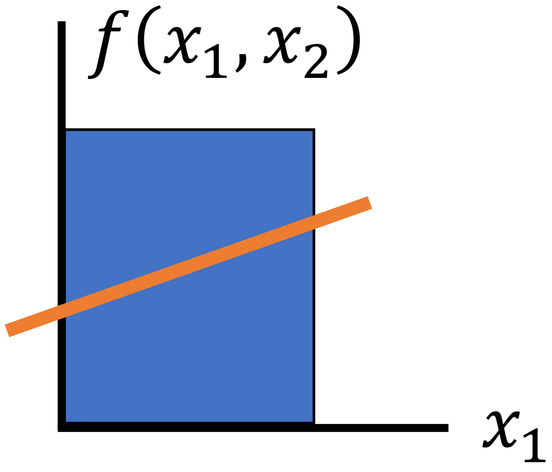
$c_2(x_1, x_2)$

$\vdots$

$c_n(x_1, x_2)$



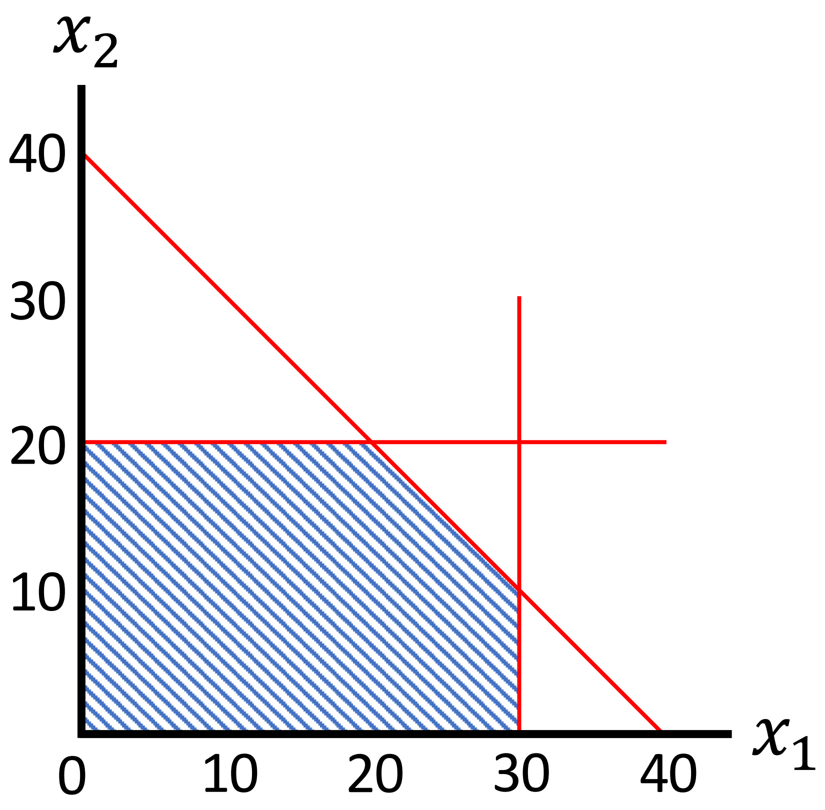
Could this ever be a maximum value of the objective function?



Yes, if $f(x_1, x_2) = \text{constant}$.

$f(x_1, x_2)$ is a plane $\Longrightarrow$ a max/min of $f(x_1, x_2)$ occurs on the boundary of the feasible region. Since feasible region has linear boundaries, max/min must occur at a vertex in the feasible region.

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

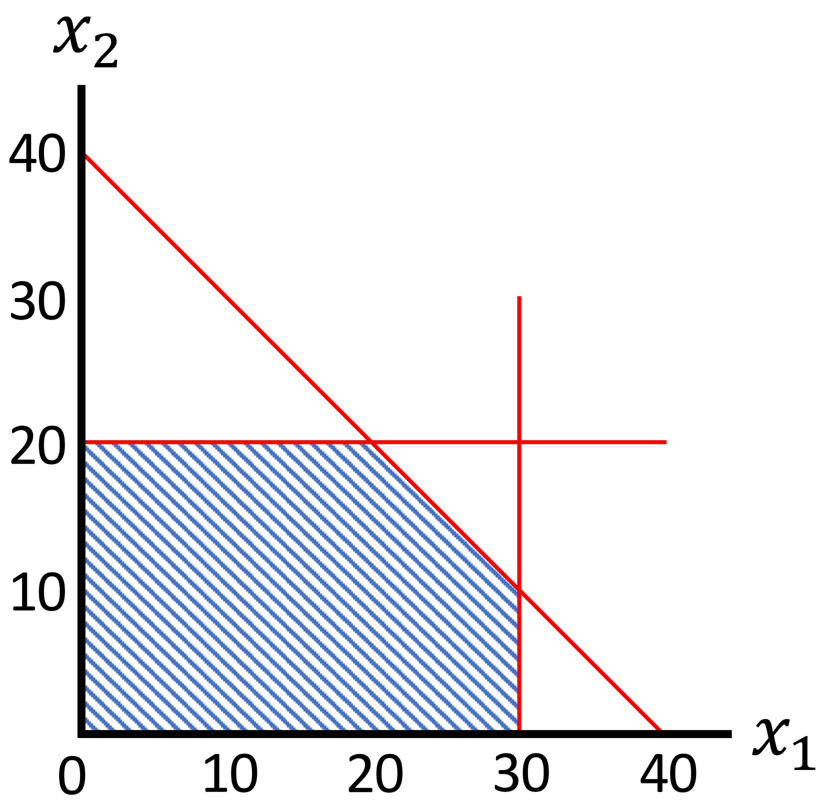
$c_n(x_1, x_2)$

How can we efficiently find optimal solutions?

Identify two key properties of optimal solutions:

1. Optimal value occurs at a vertex.
2. ?

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

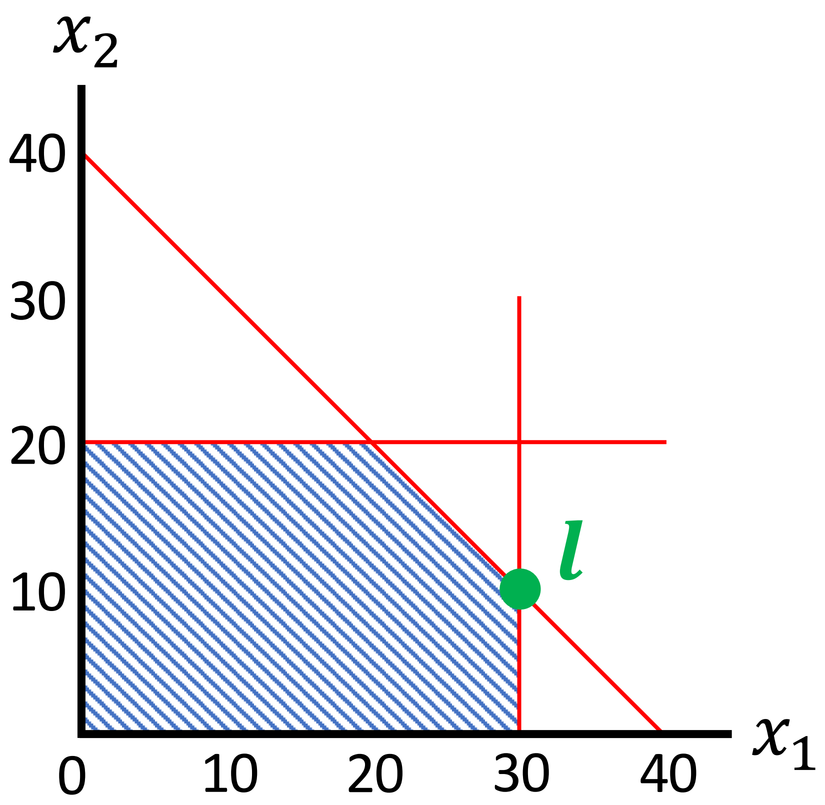
$c_2(x_1, x_2)$

$\vdots$

$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

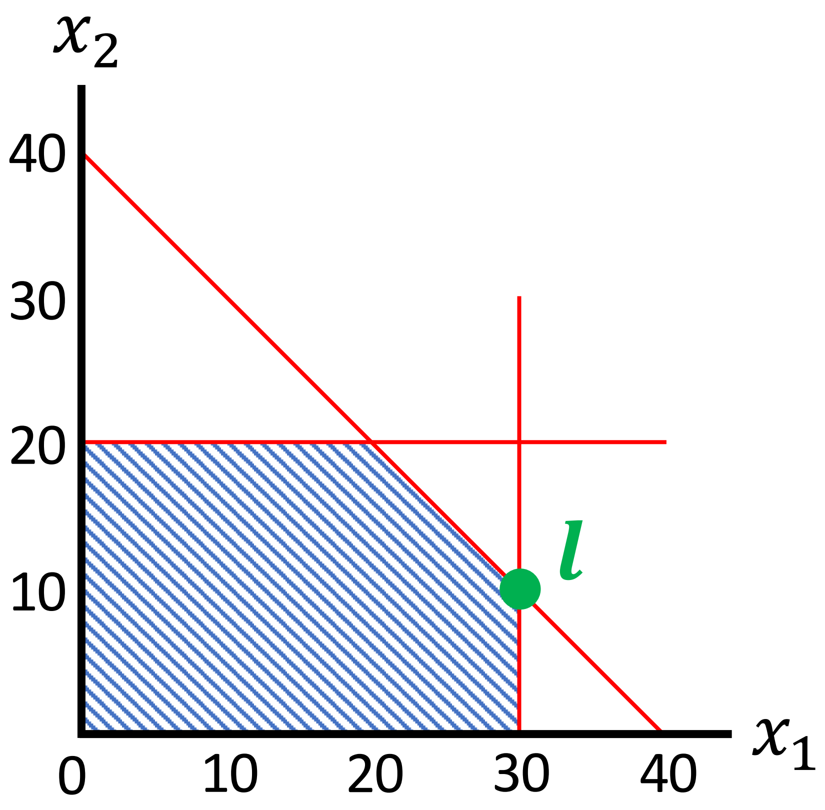
$\vdots$

$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$?

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

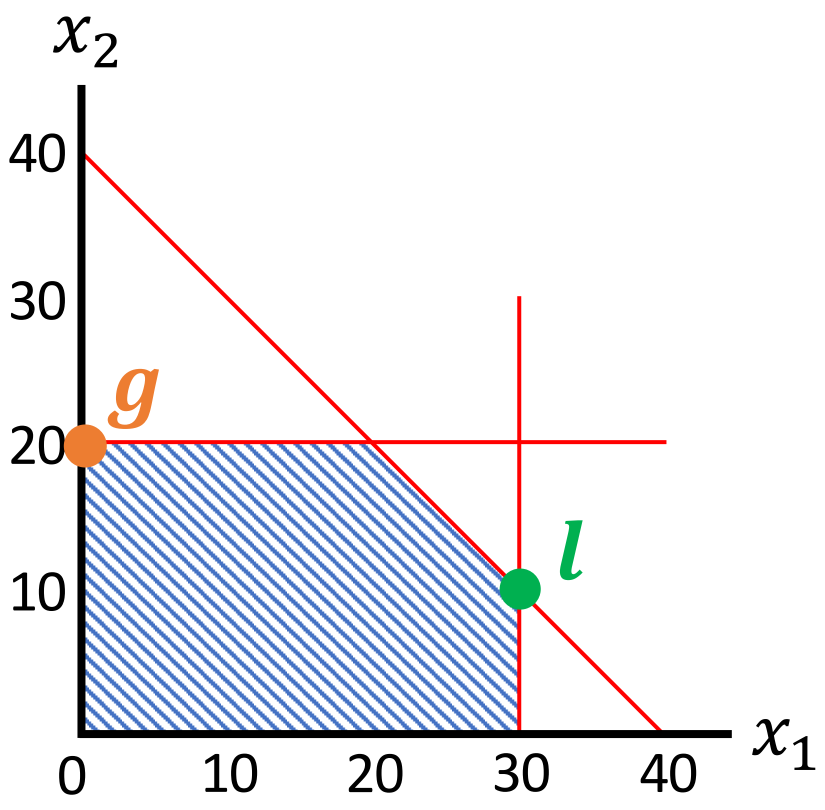
$\vdots$

$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.

Let g be global max.

Optimal Value

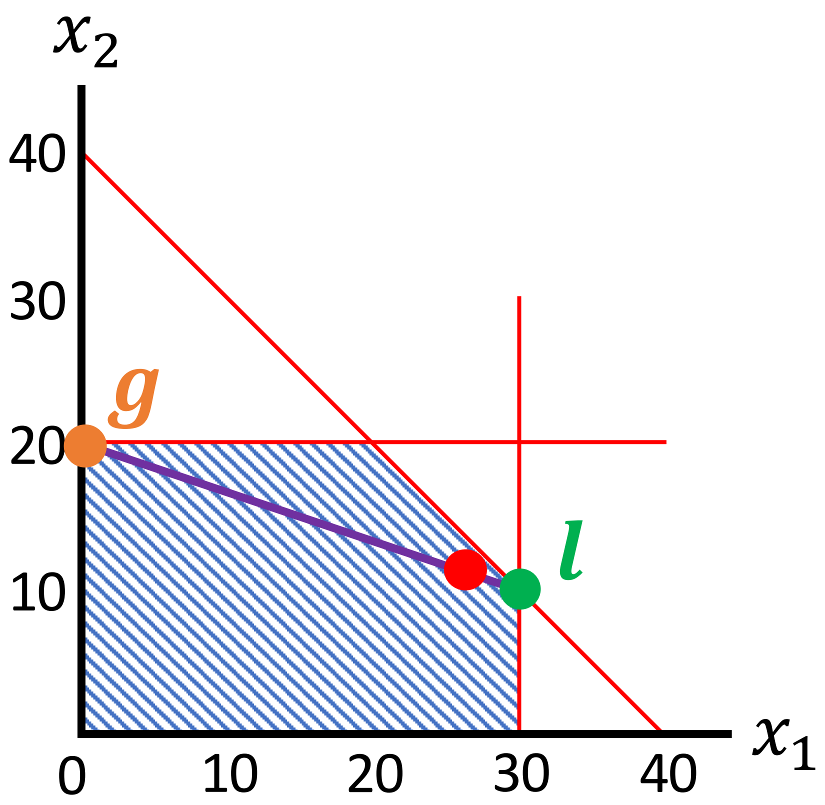
Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

$c_n(x_1, x_2)$

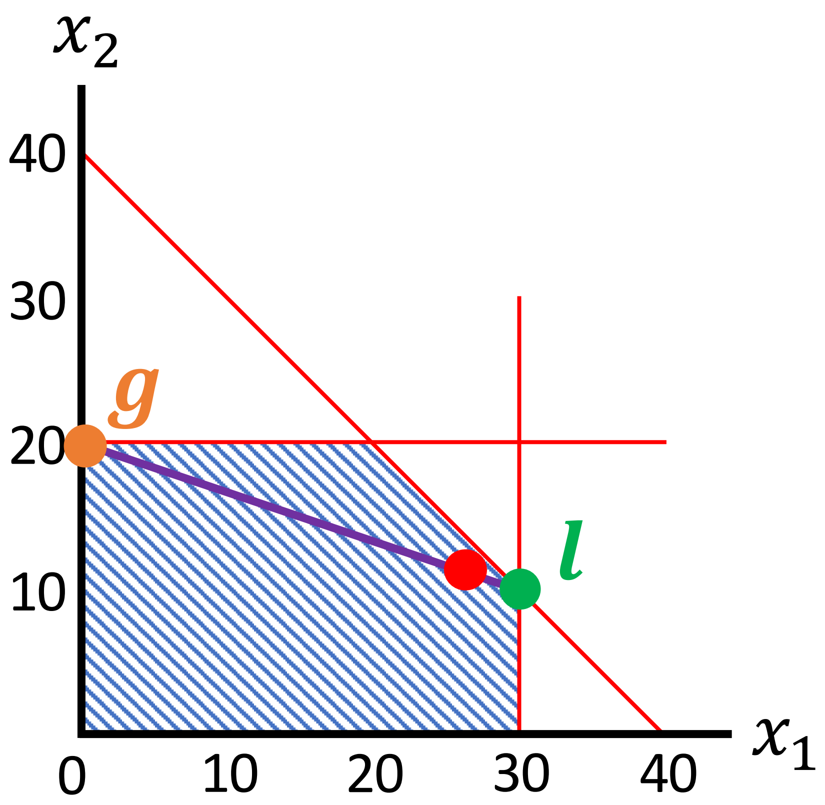


Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and …

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

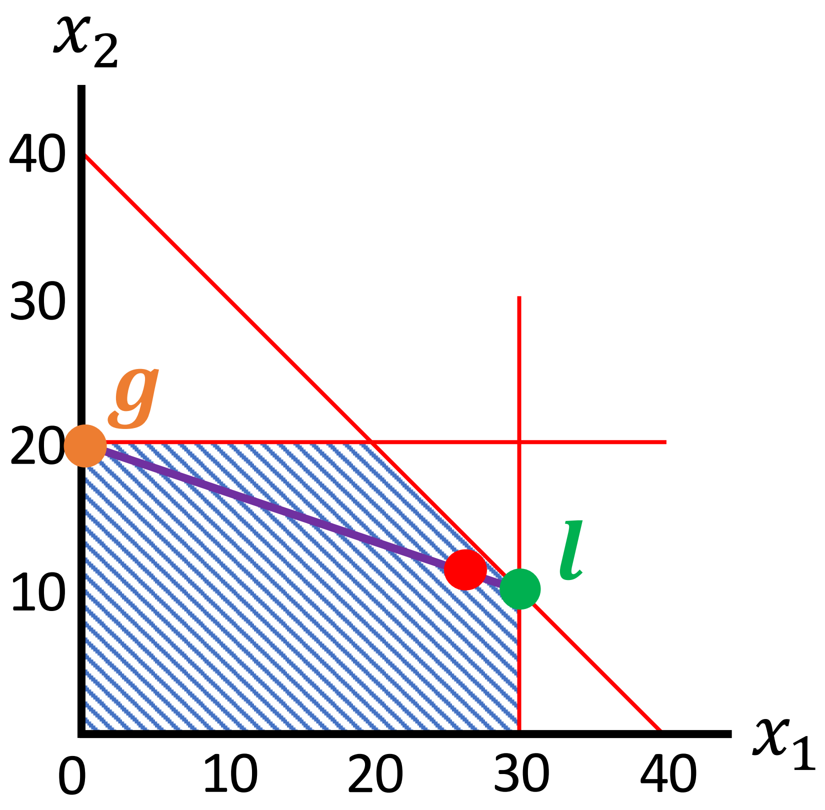
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (?).

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

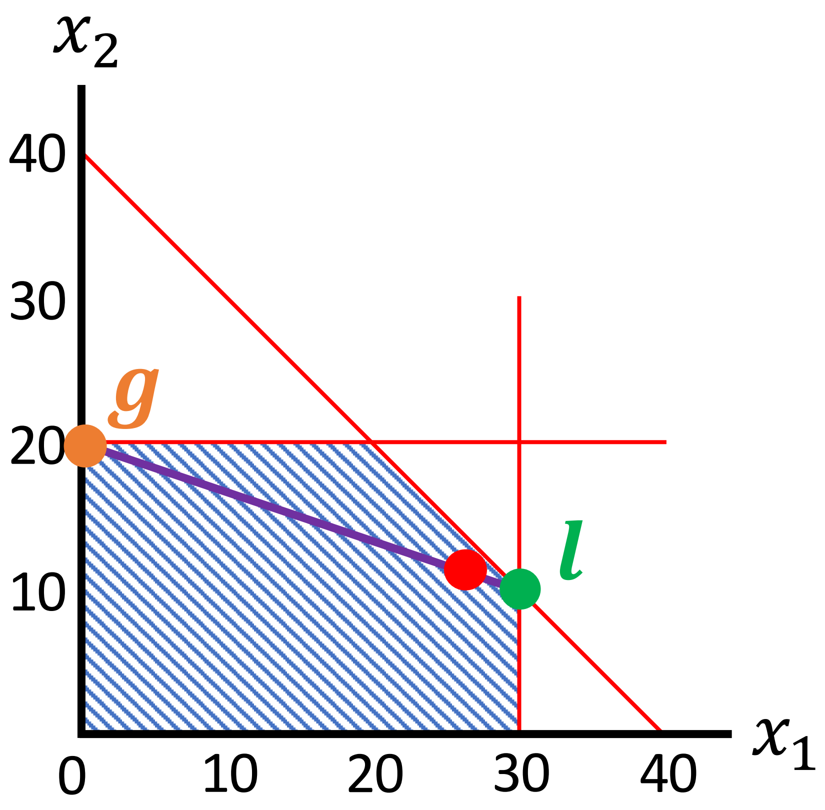
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region).

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

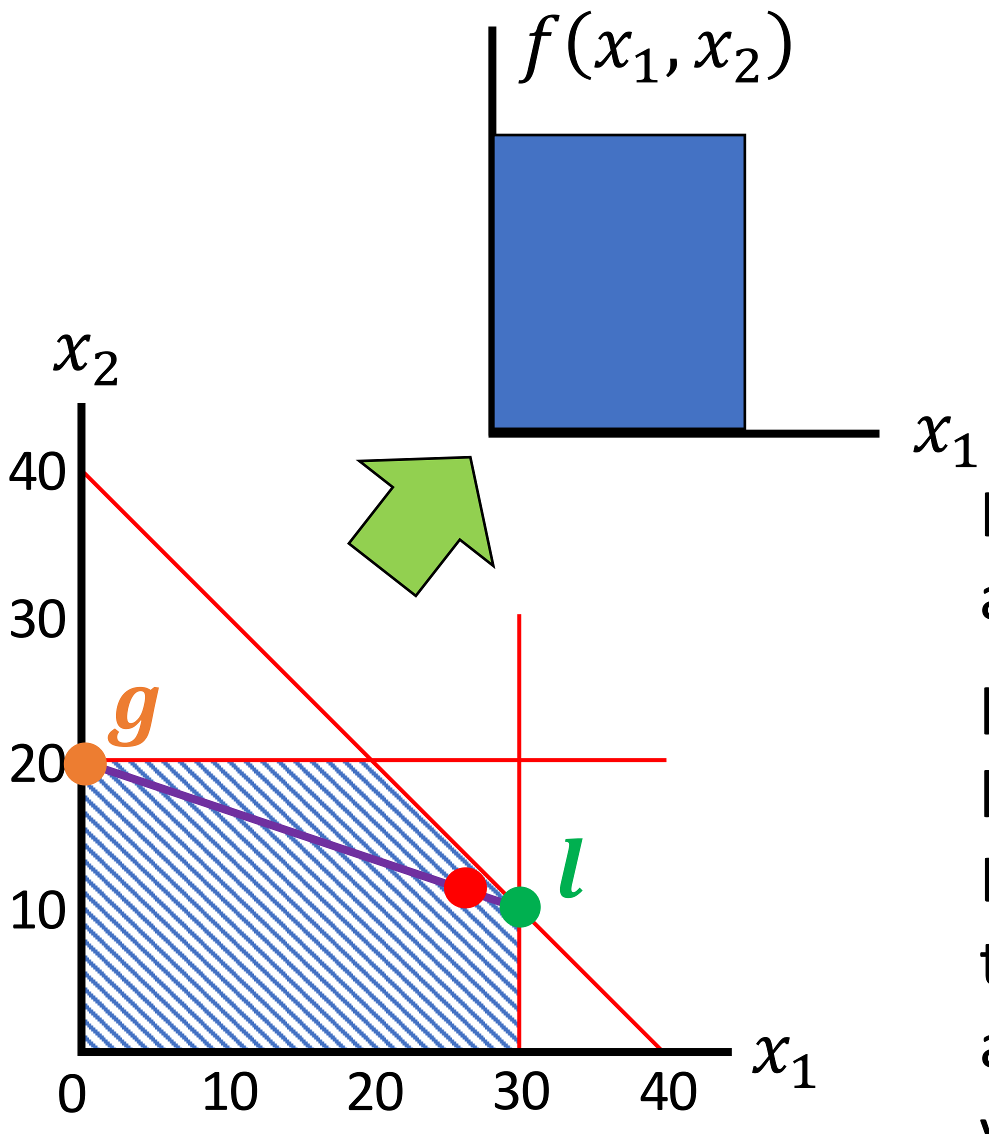
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region). Thus, objective values of the line is a line on objective hyperplane...

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

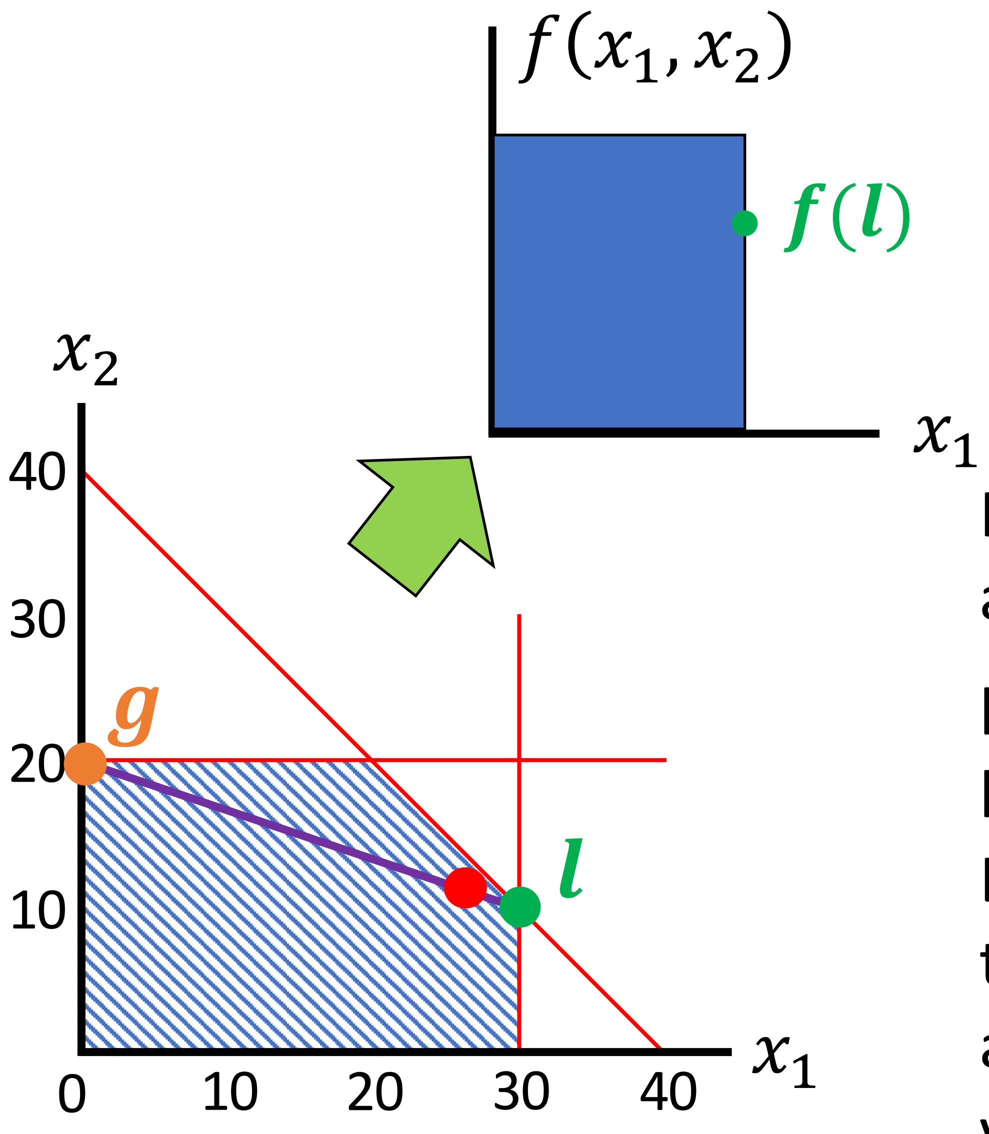
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region). Thus, objective values of the line is a line on objective hyperplane...

Optimal Value

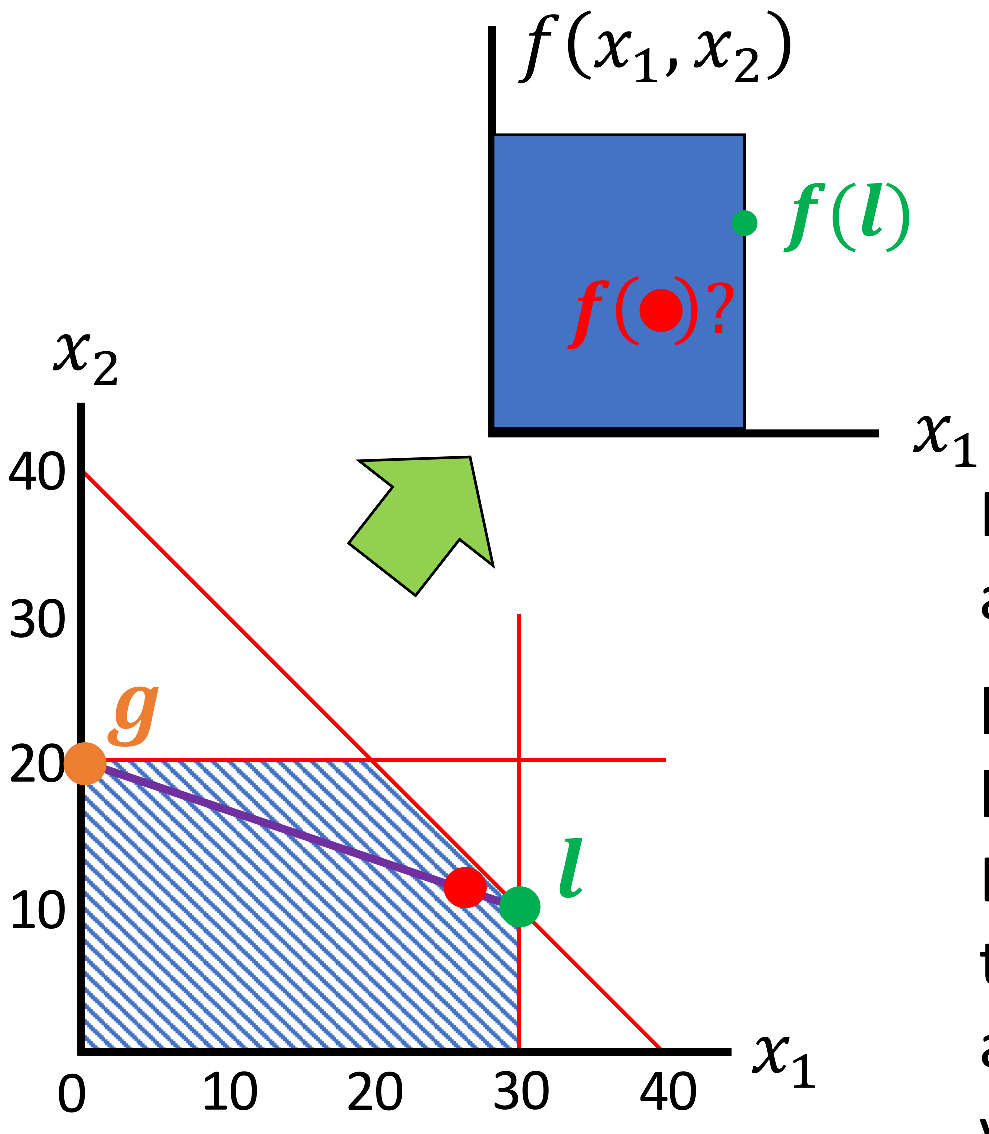


Objective:	$\max f(x_1, x_2)$
Subject to:	$c_1(x_1, x_2)$
	$c_2(x_1, x_2)$
	$\vdots$
	$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.
Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region). Thus, objective values of the line is a line on objective hyperplane...

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

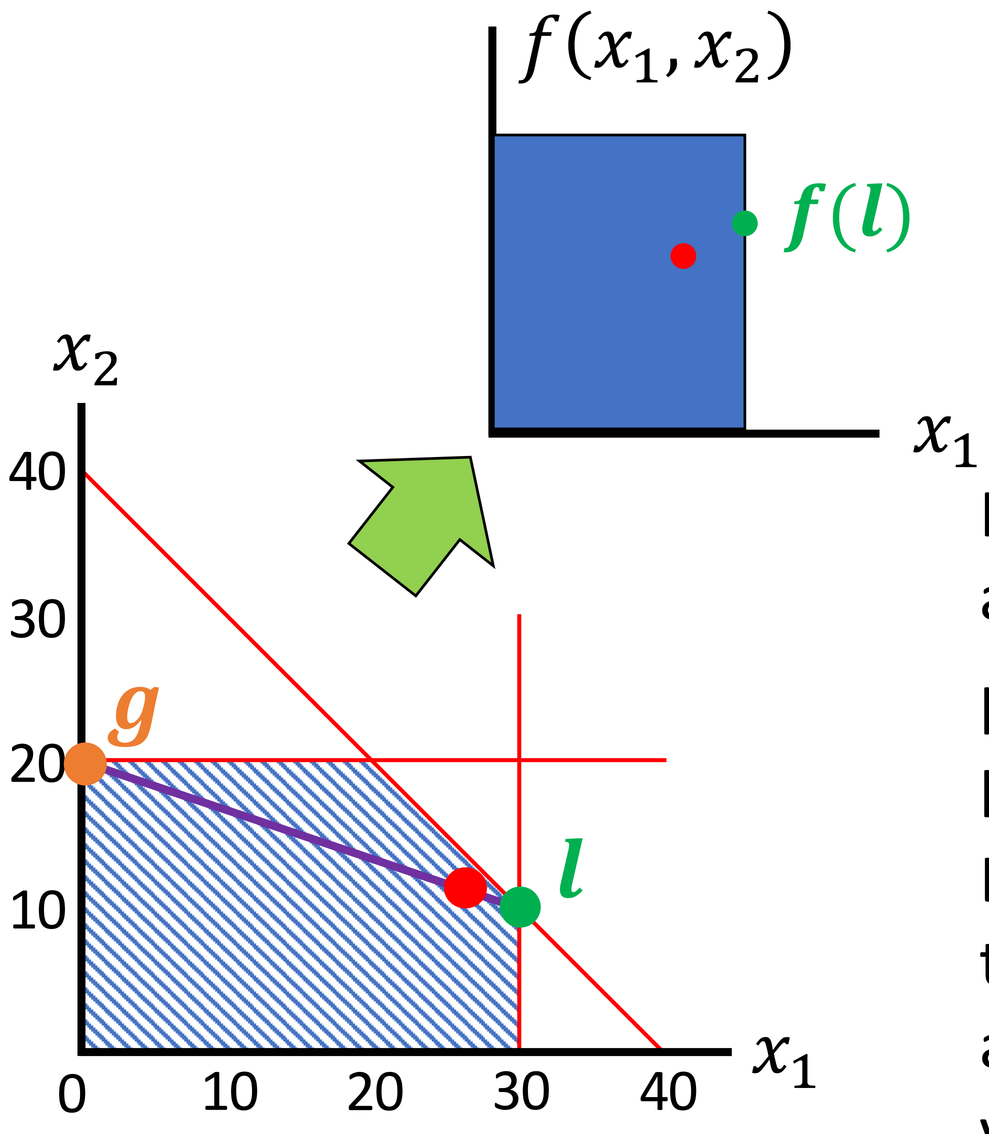
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max ⇒ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region). Thus, objective values of the line is a line on objective hyperplane...

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

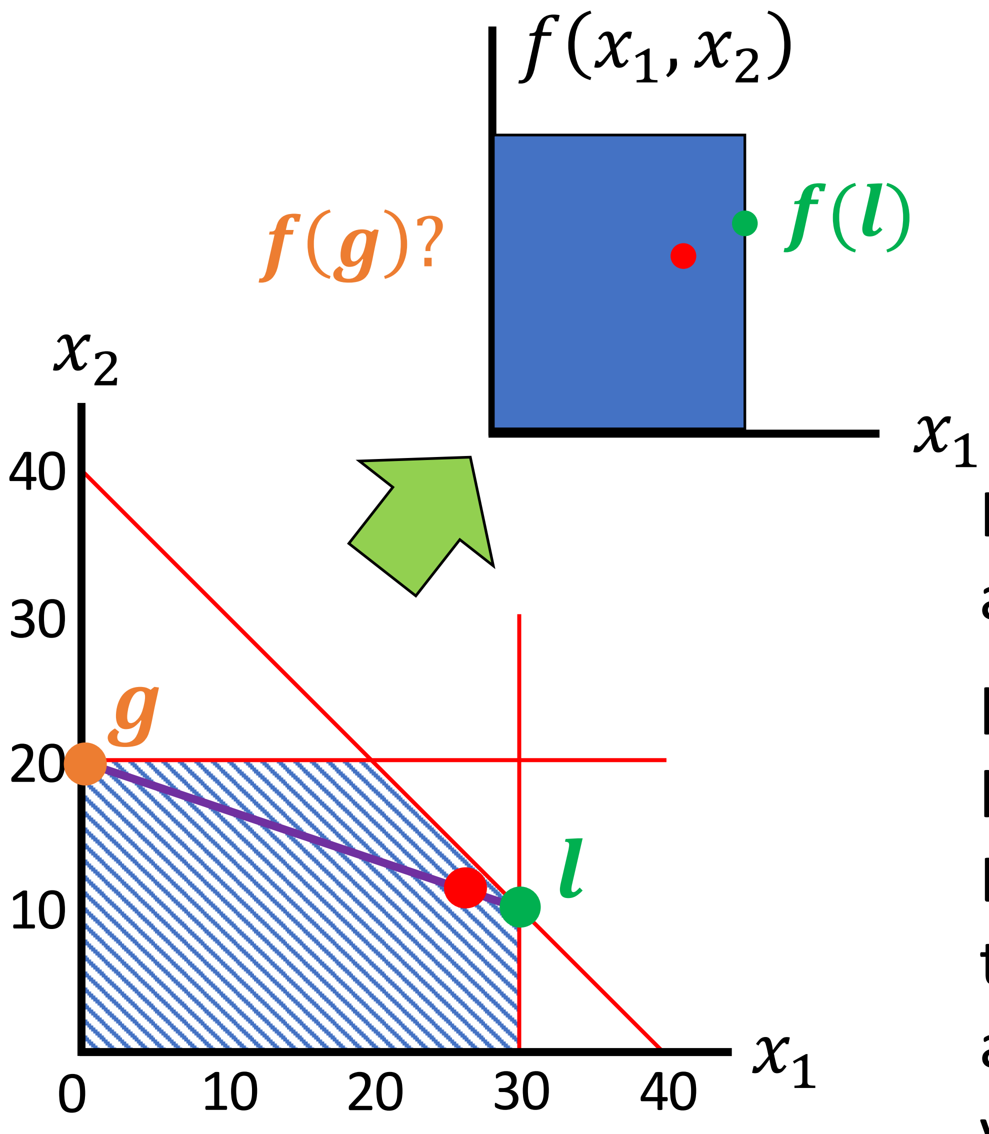
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region). Thus, objective values of the line is a line on objective hyperplane...

Optimal Value

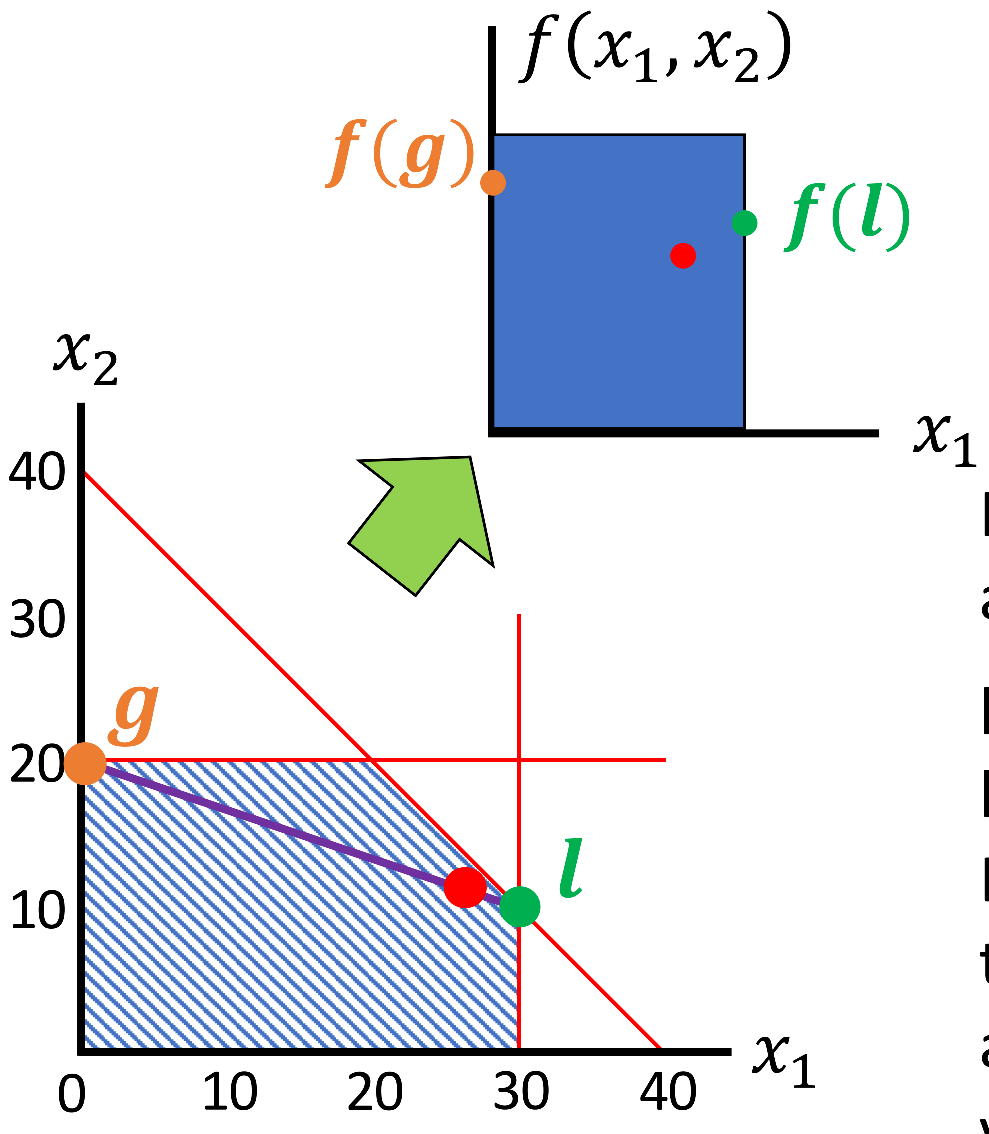


Objective: $\max f(x_1, x_2)$
Subject to: $c_1(x_1, x_2)$
$c_2(x_1, x_2)$
$\vdots$
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.
Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region). Thus, objective values of the line is a line on objective hyperplane...

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

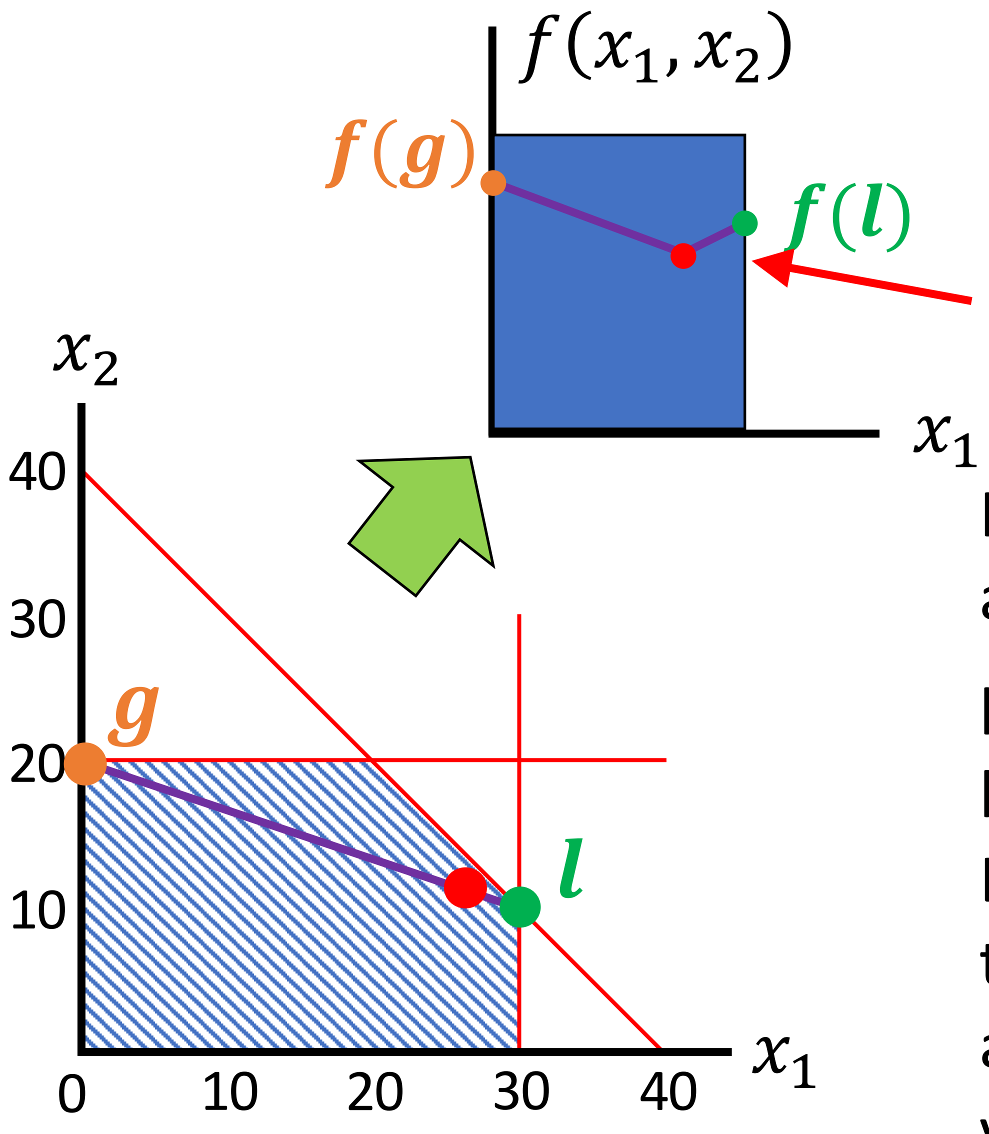
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region). Thus, objective values of the line is a line on objective hyperplane...

Optimal Value



Not Linear!

Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

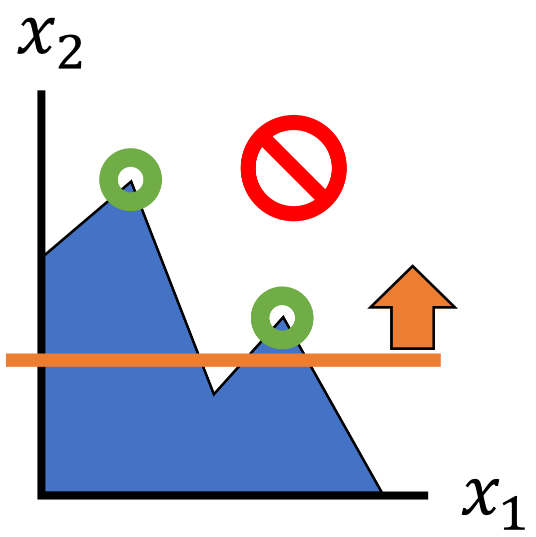
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min?

local max $\Rightarrow$ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region). Thus, objective values of the line is a line on objective hyperplane...

Optimal Value



The only way for local optimum ≠ global optimum ***and*** objective be linear is for feasible region to not be convex.

Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

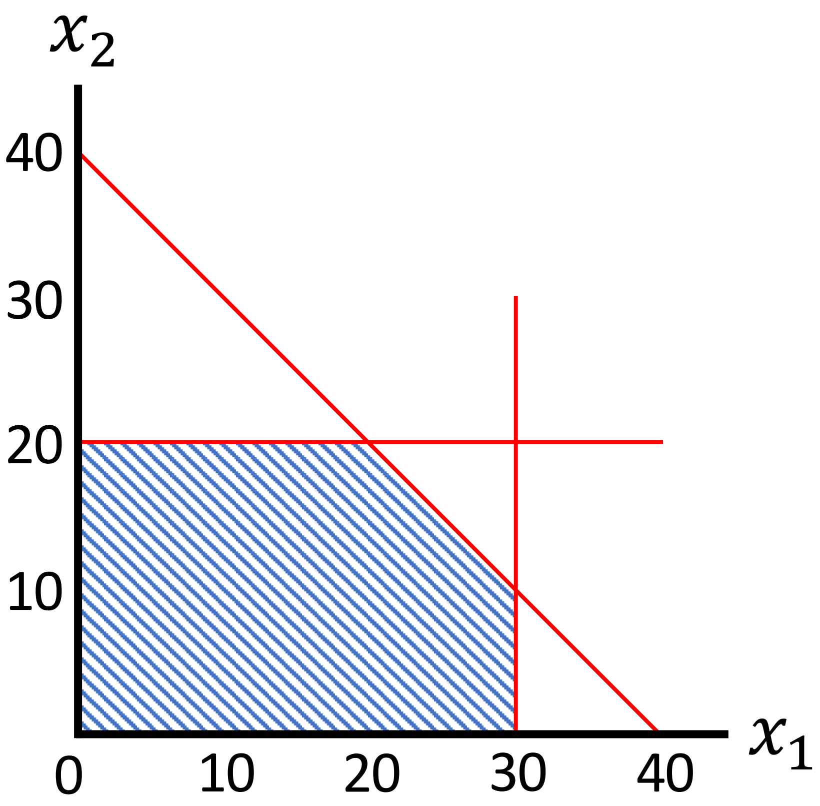
$c_n(x_1, x_2)$

Is there a relationship between a local max/min and a global max/min? local max/min = global max/min.

local max ⇒ all points in ε-neighborhood of l have lower objective values.

Let g be global max. Some point in ε-nbhd lies on the line between l and g and all points on that line are feasible (convex feasible region). Thus, objective values of the line is a line on objective hyperplane...

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

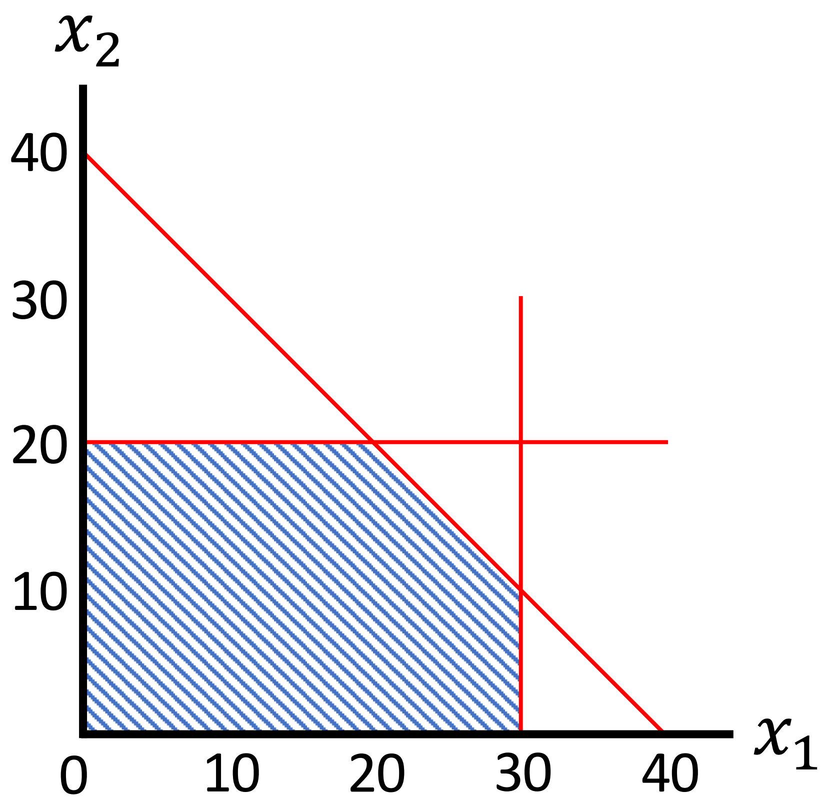
$c_n(x_1, x_2)$

How can we efficiently find optimal solutions?

Identify two key properties of optimal solutions:

1. Optimal value occurs at a vertex.
2. Local optimum is global optimum.

Optimal Value



Objective: $\max f(x_1, x_2)$

Subject to: $c_1(x_1, x_2)$

$c_2(x_1, x_2)$

$\vdots$

$c_n(x_1, x_2)$

Properties of optimal solutions:

1. Optimal value occurs at a vertex.
2. Local optimum is global optimum.

Algorithm to find optimal solution:

Test each vertex in order until no neighbors have larger (or smaller) value.

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	**Urban**	**Suburban**	**Rural**
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	**Urban**	**Suburban**	**Rural**
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

Step 1: Make variables.

“What are the decisions that need to be made?”

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	Urban	Suburban	Rural
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

Step 1: Make variables.

"What are the decisions that need to be made?"

x_1 = \$ spent on infrastructure.
x_2 = \$ spent on gun control.
x_3 = \$ spent on farm subsidies.
x_4 = \$ spent on gasoline tax.

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	Urban	Suburban	Rural
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

x_1 = \$ spent on infrastructure.
x_2 = \$ spent on gun control.
x_3 = \$ spent on farm subsidies.
x_4 = \$ spent on gasoline tax.

Step 2: Make objective.

"What are we trying to maximize or minimize?"

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	Urban	Suburban	Rural
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

Step 2: Make objective.

“What are we trying to maximize or minimize?”

x_1 = $ spent on infrastructure.
x_2 = $ spent on gun control.
x_3 = $ spent on farm subsidies.
x_4 = $ spent on gasoline tax.

Objective: $\min x_1 + x_2 + x_3 + x_4$

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	Urban	Suburban	Rural
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

x_1 = $ spent on infrastructure.
x_2 = $ spent on gun control.
x_3 = $ spent on farm subsidies.
x_4 = $ spent on gasoline tax.

Objective: $\min x_1 + x_2 + x_3 + x_4$

Step 3: Make constraints.

"What are the requirements for the solution to be valid?"

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	**Urban**	**Suburban**	**Rural**
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

x_1 = $ spent on infrastructure.
x_2 = $ spent on gun control.
x_3 = $ spent on farm subsidies.
x_4 = $ spent on gasoline tax.

Objective: $\min x_1 + x_2 + x_3 + x_4$
Subject to: $-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$

Step 3: Make constraints.

"What are the requirements for the solution to be valid?"

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	Urban	Suburban	Rural
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

Step 3: Make constraints.

"What are the requirements for the solution to be valid?"

x_1 = \$ spent on infrastructure.
x_2 = \$ spent on gun control.
x_3 = \$ spent on farm subsidies.
x_4 = \$ spent on gasoline tax.

Objective: $\min x_1 + x_2 + x_3 + x_4$
Subject to: $-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$
$5x_1 + 2x_2 \geq 100{,}000$
$3x_1 - 5x_2 + 10x_3 - 2x_4 \geq 25{,}000$

A district has an urban area (100,000 voters), suburban area (200,000 voters), and rural area (50,000 voters). A politician decided she needs at least half of the voters in each area to support her. Her campaign has four issues which are popular/unpopular with specific areas. The campaign has estimated the number of voters gained or lost based on each $1 spent advertising an issue. The campaign aims to minimize advertising expenses.

Issue	Urban	Suburban	Rural
Infrastructure	-2	+5	+3
Gun Control	+8	+2	-5
Farm Subsidies	+0	+0	+10
Gasoline Tax	+10	+0	-2

Step 3: Make constraints.

"What are the requirements for the solution to be valid?"

x_1 = \$ spent on infrastructure.
x_2 = \$ spent on gun control.
x_3 = \$ spent on farm subsidies.
x_4 = \$ spent on gasoline tax.

Objective: $\min x_1 + x_2 + x_3 + x_4$

Subject to:

$$-2x_1 + 8x_2 + 10x_4 \geq 50{,}000$$
$$5x_1 + 2x_2 \geq 100{,}000$$
$$3x_1 - 5x_2 + 10x_3 - 2x_4 \geq 25{,}000$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	**# Required**
7	276
9	100
12	250

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste in feet (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	**# Required**
7	276
9	100
12	250

Linear Program?

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	# Required
7	276
9	100
12	250

x_1 = {7’, 7’} cut (6’ waste).
x_2 = {9’, 7’} cut (4’ waste).
x_3 = {9’, 9’} cut (2’ waste).
x_4 = {12’, 7’} cut (1’ waste).

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	**# Required**
7	276
9	100
12	250

x_1 = {7’, 7’} cut (6’ waste).
x_2 = {9’, 7’} cut (4’ waste).
x_3 = {9’, 9’} cut (2’ waste).
x_4 = {12’, 7’} cut (1’ waste).

Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	**# Required**
7	276
9	100
12	250

x_1 = {7’, 7’} cut (6’ waste).
x_2 = {9’, 7’} cut (4’ waste).
x_3 = {9’, 9’} cut (2’ waste).
x_4 = {12’, 7’} cut (1’ waste).

Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$
Subject to: $2x_1 + x_2 + x_4 \geq 276$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	**# Required**
7	276
9	100
12	250

x_1 = {7’, 7’} cut (6’ waste).
x_2 = {9’, 7’} cut (4’ waste).
x_3 = {9’, 9’} cut (2’ waste).
x_4 = {12’, 7’} cut (1’ waste).

Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$
Subject to: $2x_1 + x_2 + x_4 \geq 276$
$x_2 + 2x_3 \geq 100$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	**# Required**
7	276
9	100
12	250

x_1 = {7’, 7’} cut (6’ waste).
x_2 = {9’, 7’} cut (4’ waste).
x_3 = {9’, 9’} cut (2’ waste).
x_4 = {12’, 7’} cut (1’ waste).

Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$

Subject to:

$$2x_1 + x_2 + x_4 \geq 276$$
$$x_2 + 2x_3 \geq 100$$
$$x_4 \geq 250$$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	**# Required**
7	276
9	100
12	250

x_1 = {7', 7'} cut (6' waste).
x_2 = {9', 7'} cut (4' waste).
x_3 = {9', 9'} cut (2' waste).
x_4 = {12', 7'} cut (1' waste).

Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$

Subject to:

$$2x_1 + x_2 + x_4 \geq 276$$
$$x_2 + 2x_3 \geq 100$$
$$x_4 \geq 250$$
$$x_1, x_2, x_3 \geq 0$$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	**# Required**
7	276
9	100
12	250

Does solution need to be integer?

x_1 = {7', 7'} cut (6' waste).
x_2 = {9', 7'} cut (4' waste).
x_3 = {9', 9'} cut (2' waste).
x_4 = {12', 7'} cut (1' waste).

Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$

Subject to:

$2x_1 + x_2 + x_4 \geq 276$

$x_2 + 2x_3 \geq 100$

$x_4 \geq 250$

$x_1, x_2, x_3 \geq 0$

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	# Required
7	276
9	100
12	250

x_1 = {7', 7'} cut (6' waste).
x_2 = {9', 7'} cut (4' waste).
x_3 = {9', 9'} cut (2' waste).
x_4 = {12', 7'} cut (1' waste).

Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$

Subject to:

$$2x_1 + x_2 + x_4 \geq 276$$
$$x_2 + 2x_3 \geq 100$$
$$x_4 \geq 250$$
$$x_1, x_2, x_3 \geq 0$$

Does solution need to be integer?

Is solution guaranteed to be integer?

Suppose a lumber mill produces 20-foot boards. They want to cut these boards in such a way as to provide the finished pieces in the table. They wish to minimize the waste (cut pieces that are not used). Extra boards cut to one of the desired lengths are fine.

Length (feet)	**# Required**
7	276
9	100
12	250

x_1 = {7', 7'} cut (6' waste).
x_2 = {9', 7'} cut (4' waste).
x_3 = {9', 9'} cut (2' waste).
x_4 = {12', 7'} cut (1' waste).

Objective: $\min 6x_1 + 4x_2 + 2x_3 + x_4$

Subject to:

$$2x_1 + x_2 + x_4 \geq 276$$
$$x_2 + 2x_3 \geq 100$$
$$x_4 \geq 250$$
$$x_1, x_2, x_3 \geq 0$$

Does solution need to be integer?

Is solution guaranteed to be integer?

If feasible region is defined by integer vertices, the solution will be integer.